Muduli, Silu; Dash, Shridhar Kumar

Preprint

Financial Trust in Social Economic Network and Credit Risk

Suggested Citation: Muduli, Silu; Dash, Shridhar Kumar (2019) : Financial Trust in Social Economic Network and Credit Risk, ZBW – Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:
http://hdl.handle.net/10419/190918

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Financial Trust in Social Economic Network and Credit Risk *

Silu Muduli † Shridhar Kumar Dash‡

Abstract

The paper models lender’s decision on project riskiness, trust from borrower’s socio-economic network, and social cost of default for the borrower. The paper suggests a methodology to estimate aggregate level of trustworthiness of borrower in socio-economic network and uses the same to link the social cost of default to credit default of the borrower. The proposed model finds that a relatively safer project executed by a borrower with lower social cost of default is likely to be a willful defaulter. Similarly, relatively safer project executed by a borrower with high social cost of default is likely to pay-back the loan.

Keywords: Social Economic Network, Trust, Credit Risk.

JEL Classification: D85, G21, L14.

*The views expressed in the paper are those of the author(s) and not necessarily those of the institution to which they belong.

†Manager, Department of Economic and Policy Research, Reserve Bank of India, India. Email: silu-muduli@rbi.org.in

‡Professor, Xavier Institute of Management, Bhubaneswar, India. Email: shridhar@ximb.ac.in
1 Introduction

It is difficult for lenders to assess credit risk of borrowers without having access to hard information. Non-availability of hard information leads to credit rationing problem (Stiglitz and Weiss, 1981). Our model uses the trustworthiness and social cost of default of the borrower. Trustworthiness is estimated from the borrower’s socio-economic network through ordinal ranking of trust levels by different individuals in borrower’s network. Social cost of default are the cost borrower incurs when she defaults. The cost may be reduction in future social trust, difficulty in accessing credits in future, social stigma, etc. Though social costs are not observable, it becomes a friction in availing credit for the borrower in future (Guiso et al., 2004).

2 The Model

There is an undirected network $G$ of $m$ individuals with nodes set $N$ and edge set $E$.

$$G = (N = \{1, 2, 3, ..., m\}, E)$$

We say $(i, j) \in E$ if there is or was any financial relationship between $i$ and $j$. The borrower $b \in N$ needs one unit of capital for a project that generate return $R$ with probability $q$ (observable by borrower only), else zero. $b$ can borrow from lender $l \in N \setminus \{b\}$ at interest rate $r$ such that $r < R$. The lender $l$ forms a belief about borrower $b$ on the basis of trustworthiness of borrower $b$ in the network. Lender $l$ can acquire trustworthiness of borrower $b$ at a cost $C(n)$, where $n = |N_b \cap N_l|$ ($|.|$ represent cardinality of the set) , $N_b$ is neighbourhood of borrower, $N_l$ is neighbourhood of lender. $d$ (where $d < r$) is cost of fund. Total cost of lender $l$ is $C(n) + 1 + d$ where,

$$\frac{dC}{dn} \geq 0 \quad , \quad 0 \leq C(n) + (1 + d) < (1 + r) \quad , \quad C(0) = 0$$

2.1 Estimating Aggregate Trustworthiness in Network

As trust is not measurable, an individual can assign rank to different individuals in her neighbourhood based on trust, mistrust, and no-trust. We define rank function $R_i$ for individual $i$ to be

$$R_i : N_i \rightarrow \mathbb{N}$$
Where \( N \) is set of all natural numbers and \( N_i \) is neighbourhood of node \( i \). In case the individual \( i \) has trust on individuals \( j \) and \( k \), we assign \( R_i(j) \geq R_i(k) \) if \( j \) is at least as trustworthy as \( k \). By normalizing,
\[
T(i, j) = \frac{R_i(j) - \min(R_i)}{\max(R_i) - \min(R_i)}
\]

\( T(i, j) \) represents normalized trust (henceforth trust) of \( i \) on \( j \) and \( T(j, i) \) represents trust of \( j \) on \( i \). In case the individual \( i \) has mistrust on individuals \( j \) and \( k \), \( R_i(j) \geq R_i(k) \) if \( i \) mistrust \( j \) higher than \( k \). By normalizing,
\[
T(i, j) = \frac{\min(R_i) - R_i(j)}{\max(R_i) - \min(R_i)}
\]

In case the individual \( j \) has no-trust for the individual \( j \), it assigns
\[
T(i, j) = 0
\]

\( T(j, i) \) is not a symmetric function, and \( T(i, j) \in [-1, 1] \). In case the individual assigns same rank (say \( R \)) to all individuals in a subset \( N \subseteq N_i \), then each individuals are assigned \( \frac{R}{|N|} \). Using this formulation, we can estimate the aggregate trustworthiness by using eigenvalue centrality weighted method. Let \( A \) be the adjacency matrix of the sub graph
\[
G' = (N', E')
\]

Where, \( N' = (N_b \cap N_l) \cup \{l, b\} \) and \( E' \) is set of nodes that contains pairs from \( N' \). We propose that:

**Proposition 1.** The constructed subgraph \( G' \) is a strongly connected graph\(^1\).

*Proof.* Let \( i \) and \( j \) be two arbitrary nodes in \( G' \). Since \( i, j \in N' \) and \( N' \) is a undirected graph, then there exist an edges \((i, b)\) and \((b, j)\) in \( E' \). This implies there exist a walk \( \{i, b, j\} \). Therefore, every node has a walk in \( G' \). Hence, \( G' \) is a strongly connected graph. \(\blacksquare\)

Since graph \( G' \) is strongly connected, the adjacency matrix is an irreducible matrix\(^2\). According to Perron - Frobenius theorem (Perron, 1907; Frobenius, 1912), for a non-negative real irreducible square matrix \( A \) there exist an maximum eigenvalue \( \lambda_{\text{max}} \), and the eigenvector \( \mathbf{v} \) corresponding to \( \lambda_{\text{max}} \) contains all positive real number. Given that there are \( n + 2 \) nodes (including \( a, b \), let

\(^1\)A graph is called strongly connected if for any two nodes there exist a walk.

\(^2\)A matrix \( M \) is said to be irreducible matrix if there does not exist a permutation matrix \( P \) such that \( PAP^T \) is in lower block triangular form. Where \( P^T \) is transpose of the matrix \( P \).
\( v = (v_1, v_2, ..., v_n, v_l, v_b) \) be the eigenvector. With this parameters we can estimate the aggregate trust of the borrower as

\[
\mathcal{T} = \left[ \sum_{i \in N_l \cap N_b} w_i \times T(l, i) \times T(i, b) \times \mathbb{I}(l, i, b) \right] + w_l T(l, b)
\]

Where

\[
\mathbb{I}(l, i, b) = \begin{cases} 
1 & : \text{if } \max\{T(l, i), T(i, b)\} > 0 \\
0 & : \text{if } \max\{T(l, i), T(i, b)\} \leq 0
\end{cases}, \quad w_i = \frac{v_i}{\sum_{i=1}^{n} v_i + v_i}
\]

The term \( T(l, i) \times T(i, b) \) represents how the lender \( l \) trusts the neighbour and how much neighbour trusts the borrower \( b \). Logically, if lender \( l \) mistrusts a neighbour and the neighbour trust the borrower \( b \), then in overall the lender \( l \) mistrusts the borrower \( b \). However, when \( l \) mistrust \( i \) and \( i \) mistrusts \( b \) then \( T(l, i) \times T(i, b) > 0 \) i.e. \( l \) trusts \( b \). To eliminate this misleading social behaviour we have defined \( \mathbb{I}(l, i, b) \) to be zero. This aggregate trust helps the formation of belief about the repayment of the borrower \( b \).

### 2.2 Lender

Lender \( l \) is risk neutral and operates in a competitive credit market. Taking aggregate trust \( \mathcal{T} \) into account, the lender sets a belief function \( F(\mathcal{T}) \) defined by

\[
F(\mathcal{T}) = \text{Prob}[\text{Borrower with trust level } \mathcal{T} \text{ will repay the loan}]
\]

with following properties.

\begin{enumerate}
  \item \( \frac{dF(T)}{dT} \geq 0 \)
  \item \( \lim_{\mathcal{T} \to -1} F(\mathcal{T}) = 0 \)
  \item \( \lim_{\mathcal{T} \to 1} F(\mathcal{T}) = 1 \)
\end{enumerate}

Taking the above belief the lender’s expected profit is given by

\[
E(\pi_l) = F(\mathcal{T})(1 + r) - C(n) - (1 + d)
\]
The lender $l$ extends the credit if $E(\pi_l) \geq 0$. That is

$$r \geq \frac{C(n) + (1 + d)}{F(T)} - 1 = \bar{r} \quad (say)$$

The interest rate $r$ decreases if the borrower has very high trust in the network and increases with the increase in cost of getting the information.

### 2.3 Borrower

The risk neutral borrower $b$’s expected profit will be

$$\pi_b = q(1 + R) - (1 + r)$$

Where $q \in [0, 1]$ is the probability of success of the project known privately by the borrower who may be willful defaulter. Success of the project will depend upon several factors like effort of the borrower, external shocks, business environment, regulations etc. A defaulting borrower will lose the social capital - the social cost of default. $S(T)$ is the social cost of default, which is privately observed by the borrower, and it is an increasing function of aggregate trust $T$. We assume $T$ is positively related to trustworthiness of borrower and social cost of default, and lie in functional space $\mathcal{F}^+[-1, 1]$ (where $\mathcal{F}^+[-1, 1]$ is set of all increasing functions with domain $[-1, 1]$). Payoff of the borrower $b$ in case of default will be,

$$\pi_{bd} = q(1 + R) - S(T)$$

The borrower $b$ will repay the credit only if

$$\pi_b \geq \pi_{bd} \implies r \leq S(T) - 1 = \bar{r}$$

For borrowers there is a threshold interest rate $\bar{r}$ depending on social cost of default, above which it is optimal for her to default.
3 Game Setup

In this game, there are two players - lender $l$ and borrower $b$. The lender’s strategy space is to set interest rate in $[0, \infty)$. Borrower $b$ observes her type which is decided by nature - social cost of default ($S$) and riskiness of the project ($q$). Borrower $b$’s strategy space is given by - Not Accept (NA), Accept and Not Default (A,ND), Accept and Default (A, D). The game follows the following sequence:

1. The lender $l$ estimate the aggregate trust $\mathcal{T}$, and forms a belief about borrower $b$. Depending on the belief and cost of getting the trustworthiness information offers an interest rate $r$.

2. Borrower $b$ observes $r$ and realizes $q$, $S$ privately.

3. Decides to accept the offer or not to accept the offer.

4. If she does not accept, game ends and both get payoff zero.

5. If she accepts the offer, then she decides whether to default or not to default. Payoff is discussed in the subsection 2.2 and 2.3.

The extensive form of game has been shown in Figure 1. Using backward induction, the equilibrium strategy for the lender $l$ is to set interest rate based on belief $F(\mathcal{T}, n, d)$. For borrower, type space is given by $\mathcal{F}^+[-1, 1] \times [0, 1]$ consisting of an ordered pair of a social cost of default function and the project riskiness. Hence equilibrium strategy for the borrower $s$ is a function defined by

$$s : \mathcal{F}^+[-1, 1] \times [0, 1] \rightarrow \{(NA), (A, D), (A, ND)\}$$

3.1 Equilibrium Analysis

Equilibrium analysis is presented through two propositions - Proposition 2 and Proposition 4.

**Proposition 2 (Lender’s Equilibrium).** In a competitive credit market, it is weakly dominant strategy for the lender $l$ to set $r = \underline{r}$.

**Proof.** Suppose the lender $l$ sets $r > \underline{r}$. Borrower may have a project with probability of success $q$ such that

$$\frac{1 + \underline{r}}{1 + R} < q < \frac{1 + r}{1 + R}$$


Figure 1: Extensive Form of the Game. In the payoff tuple, left side represents lender’s payoff and right side represents borrower’s payoff.

Lender $l$ will lose this borrower. Since she is operating in a competitive credit market, lender could have set an interest rate $\bar{r} + \epsilon$ for sufficiently small $\epsilon > 0$ such that $q(1 + R) - (1 + \bar{r} + \epsilon) > 0$ and retain the borrower. Again $\bar{r} + \epsilon > \bar{r}$. Arguing similarly, it is weakly dominant strategy for lender to charge $r = \bar{r}$.

According to this proposition lender $l$ fixes interest rate at $r = \frac{C(n) + (1 + d)}{F(T)} - 1$. The interest rate $r$ will decrease with increase in trustworthiness of the borrower, and decrease in cost acquiring the trustworthiness information. Depending on $r$ the borrower decides to accept or not to accept the offer, and depending on type space of borrower decides to default or not to default (in case accepts the offer of the lender). Therefore Equilibrium strategy of the borrower $b$ depends on: social cost of default and the riskiness of the project, which is discussed proposition 4.

Proposition 3. In equilibrium, if $\lim_{T \to 1} S(T) > C(n) + 1 + d$ and borrower accepts the offer, then there exist a threshold trust level $T < 1$, above which it is optimal for borrower to repay-back the credit.

Proof. In equilibrium lender sets $r = \bar{r} = \frac{C(n) + (1 + d)}{F(T)} - 1$ and $\lim_{T \to 1} \bar{r} = C(n) + 1 + d - 1$. Since $\bar{r} = S(T) - 1$ increases with $T$ and according to the condition as $T \to 1$, $S(T) - 1 > C(n) + 1 + d - 1$ i.e. $\bar{r} > \bar{r}$, therefore there exist a threshold trust level $T < 1$ above which the borrower is more likely to pay-back the credit.

Proposition 4 (Borrower’s Equilibrium). For the borrower $b$, the equilibrium strategy $s^*$ is
Given by
\[
\begin{aligned}
\mathbf{s}^* = \left\{ 
\begin{array}{ll}
(NA) & : \text{if } S \geq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \\
(NA) & : \text{if } S \leq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \cdot \frac{S(T)}{(1+R)} \geq q \\
(A, ND) & : \text{if } S \geq \frac{C(n)+(1+d)}{F(T)}, \ q \geq \frac{C(n)+(1+d)}{F(T)(1+R)} \\
(A, D) & : \text{if } S < \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \cdot \frac{S(T)}{(1+R)} \leq q \\
(A, D) & : \text{if } S \leq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)}
\end{array}
\right.
\end{aligned}
\]

Proof. Consider the following five cases.

Case - I: \( S \geq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \)

In this case
\[
\pi_b = q(1+R) - (1+r) = q(1+R) - \frac{C(n) + (1+d)}{F(T)} < 0
\]

Therefore it is not profitable to accept the project. Hence optimal strategy is to reject the offer.

Case - II: \( S \leq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \cdot \frac{S(T)}{(1+R)} \geq q \)

Same argument as case - I.

Case - III: \( S \geq \frac{C(n)+(1+d)}{F(T)}, \ q \geq \frac{C(n)+(1+d)}{F(T)(1+R)} \)

In this case
\[
\pi_b = q(1+R) - (1+r) = q(1+R) - \frac{C(n) + (1+d)}{F(T)} \geq 0
\]

and
\[
\pi_d - \pi_{bd} = S - \frac{C(n) + (1+d)}{F(T)} \geq 0
\]

Therefore the optimal strategy is to accept the project and not to default.

Case - IV: \( S < \frac{C(n)+(1+d)}{F(T)}, \ q \geq \frac{C(n)+(1+d)}{F(T)(1+R)} \)

In this case
\[
\pi_b = q(1+R) - (1+r) = q(1+R) - \frac{C(n) + (1+d)}{F(T)} \geq 0
\]

and
\[
\pi_d - \pi_{bd} = S - \frac{C(n) + (1+d)}{F(T)} < 0
\]

Therefore the optimal strategy is to accept the project and default.

Case - V: \( S \leq \frac{C(n)+(1+d)}{F(T)}, \ q < \frac{C(n)+(1+d)}{F(T)(1+R)} \cdot \frac{S(T)}{(1+R)} \leq q \). In this case
\[
\pi_b = q(1+R) - (1+r) = q(1+R) - \frac{C(n) + (1+d)}{F(T)} < 0
\]
\[
\pi_{bd} = q(1 + R) - S(T) \geq 0
\]

and
\[
\pi_d - \pi_{bd} = S - \frac{C(n) + (1 + d)}{F(T)} < 0
\]

If borrower decides to repay the loan then it ends up having negative payoff. But if it accepts and does not pay then it has non-negative payoff. Therefore, the optimal strategy of the borrower is to accept and default.

The borrower b’s equilibrium strategy linked to social cost of default, and quality of projects undertaken. In case the social cost of default is low, the borrower is likely to default, even if the project quality is good, these borrowers are willful defaulter. Similarly, if the borrower’s social cost of default is high, then she will not borrow for the bad projects or make every effort to pay-back the loan. The lender can use this information to make better credit decision by taking into cost of getting the information, which is shown in Figure 2. The region \((A, ND)\) is the best region for the lender to operate.

![Figure 2: The above figure shows the regions of borrower’s equilibrium](image)

4 Conclusion

Our model provides insights about the borrower’s equilibrium strategy, which a lender can use for better credit decisions. By incorporating the creditworthiness information from borrower’s network,
the lender can reduce problem of information asymmetry and reduce the cost of credit. Besides, it enables the lender to identify willful defaulters. Similarly, the borrower can get insight for lender’s equilibrium strategy to be in the network of individuals of good creditworthiness. This will enable them to get credit at a lower interest rate. Therefore, our model makes a small effort in reducing credit rationing problem for good quality borrowers.

References


