

Ebert, Udo; Welsch, Heinz

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The social evaluation of income distribution: An assessment based on happiness surveys

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**The Social Evaluation of Income Distribution:
An Assessment Based on Happiness Surveys**

Udo Ebert and Heinz Welsch

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Udo Ebert and Heinz Welsch

**The Social Evaluation of Income Distribution:
An Assessment Based on Happiness Surveys**

Address: Department of Economics, University of Oldenburg,
D-26111 Oldenburg, Germany
Tel.: (+49) (0)441-798-4113 (U.E.) and 798-4112 (H.W.)
Fax: (+49) (0)441-798-4101
e-mail: ebert@uni-oldenburg.de
welsch@uni-oldenburg.de

Abstract

The paper investigates how the income distribution affects the subjective well-being of society. Starting from the concept of a social welfare function defined on income distributions it assesses the preference for equality in European countries. It examines how mean income and the distribution of income in a country (measured by an appropriate inequality measure) determine the subjective well-being of its inhabitants which is identified with social welfare. This issue is addressed under alternative assumptions concerning the adjustment of needs due to differences in household size. The results derived are relevant for the design and monitoring of redistribution policies.

JEL Codes: D63, D31, I31

Keywords: Income distribution, social welfare, preference for equality, happiness data, subjective well-being

1. Introduction

Policies of income redistribution can be thought as justified if and to the degree in which society has a preference for income equality. A recently explored method of assessing the preference for equality is by investigating how the distribution of income prevailing in a society affects the subjective well-being of the members of society. Evidence that income inequality may impair subjective well-being has been presented by Morawetz et al. (1977) with respect to two Israeli communities and, more recently, by Alesina et al. (2004) with respect to the U.S. and several European countries. The latter paper suggests that Americans and Europeans are different in terms of their respective equality preference. This assessment is based on using a single inequality measure, the Gini coefficient.

Concentrating on one measure and only on inequality may be an approach which is rather specific for an evaluation of income distributions. Obviously the concept of income inequality is a vague one and, in addition to inequality, the average income is also a necessary determinant of subjective well-being. Therefore the notion of inequality and equality preference has to be defined precisely and different possibilities have to be considered. Moreover, and even more fundamentally, the concept of an income distribution is also not self-evident: Income distribution data are usually given in terms of household incomes, and households may differ in size. This gives rise to different needs of individuals living in different types of households. Accommodating differences in needs can be achieved in several ways which leads to different income distributions for individuals.

The purpose of the present paper is to investigate the linkage between the distribution of income and subjective well-being in a more comprehensive and systematical way. At a qualitative level, we will examine how various aspects of the distribution of income (especially levels and ranks of incomes) affect subjective well-being. At a quantitative level we will investigate the respective degree of equality preference. Both of these issues will be addressed under alternative assumptions concerning the adjustment for needs. We identify average subjective well-being with social welfare. Therefore the way how the distribution of income affects subjective well-being will be taken to reflect the social evaluation of the distribution of income. Given the validity of this assumption, the results that emerge may be important for the design and monitoring of redistribution policies.

The theoretical starting point for addressing our research issues is a social welfare ordering over income distributions and its representation by a social welfare function. We observe that the latter can be written (in reduced form) as a function of mean income and a measure of

income inequality (trade-off function). It is the trade-off function which we seek to identify empirically. In doing so, we will use as the dependent variable average subjective well-being in 10 European countries, 1978-1997. This variable will be our measure of social welfare. The main explanatory variables will be the respective mean incomes, and various measures of income inequality. Since several *families* of inequality measures capture different aspects of inequality, and *members* of the respective families correspond to different degrees of equality preference, estimating empirical trade-off functions will allow us to draw inference on the social evaluation of income distributions. By using equivalence scales and applying two different weighting schemes to account for household size, we are able to examine how this inference varies with the procedure adopted for incorporating differences in needs.

In implementing this research strategy, we are faced with the problem that the number of conceivable inequality measures is virtually infinite. We therefore confine ourselves to the two families of measures which have received the bulk of attention in research on inequality: the Atkinson and the Gini families. Both belong to the Atkinson-Kolm-Sen (AKS) type of inequality measures, that is, they can be derived from welfare orderings over income distributions which incorporate some sort of equality preference (or inequality aversion). However, the two families differ in that they take into account different aspects of inequality: While the Atkinson measures merely focus on the levels of income and neglect their ranking, the Gini measures also take the ranking of incomes into account. In the light of our research questions it is thus useful to examine whether the two families and special members thereof – which reflect different degrees of equality preference – differ with respect to their ‘relevance’ for subjective well-being.

In order to adjust the income distribution for different needs each household is assigned its equivalent income (the living standard of a representative household member) and a weight representing its size or needs. The equivalent income is based on equivalence scales. In the literature the weight is usually set equal to the number of household members. Some recent papers (e.g. Ebert 1997, Ebert and Moyes 2003) suggest to use the number of equivalent adults instead since then the measures employed reflect the redistribution of household income between households of different size in the way one would expect. We use both kinds of weights and try to infer from the data which of the two approaches are appropriate. Anyway, in our paper we always consider the needs-adjusted income distribution and examine its average and (in)equality.

The procedure of the paper is as follows. In a first step we discuss the AKS approach of deriving relative (in)equality measures from homothetic welfare functions over income distributions, and introduce the trade-off function. We also introduce the important concept of self-consistency: A given inequality measure is self-consistent for a given trade-off function if it coincides with the corresponding AKS measure. Self-consistency assures that the equality preference inherent in a given inequality measure is identical with the equality preference of the associated welfare ordering. Moreover, we describe the Atkinson and Gini families by characterizing the underlying welfare ordering. Finally, we discuss ways how to construct measures of needs-adjusted income when household size differs.

In a second step we estimate trade-off functions that are specified in such a way that self-consistency can be checked. The inequality measures used in these estimations are members of the Atkinson-Gini family and the welfare measure is average self-reported subjective well-being¹ in 10 European countries. Our estimates allow us to evaluate the inequality measures considered with respect to self-consistency as well as with respect to statistical significance in their role as determinants of social welfare. By identifying which members of the Atkinson-Gini family may be considered self-consistent, we are able to draw inference on the degree of equality preference. The significance levels of the self-consistent Atkinson and Gini measures (as well as mixtures thereof) allow us to draw some conclusions with respect to which aspects of inequality matter for social well-being and which concepts of needs-adjusted income are relevant.

In our empirical assessment we use two sources of data. The first is the Eurobarometer survey series which contains sample data of self-rated subjective well-being (referred to as happiness data). The second is the Luxembourg Income Study which allows to compute mean equivalent incomes as well as any desired inequality measure on the basis of comparable micro-data of income distributions.

A key finding from our investigation is that there are marked differences among the inequality measures considered, in their role as determinants of social welfare. This is surprising since these measures are highly correlated to each other. Indices from the Gini family are generally found more significant than indices from the Atkinson family. This suggests that the *ranking* of incomes plays an important role in people's evaluations of inequality, over and above the *level* of income. The indices which can be considered self-consistent differ from those conventionally used in empirical assessments of inequality. This suggests that the conven-

¹ See section 2 in Rivallion and Lokshin (2002) for a brief survey of various approaches using subjective data in welfare measurement.

tional measures may be inadequate with respect to the implied degree of equality preference. In addition, we find that the evaluation of inequality varies considerably with the way of adjustment, that is, with the weighting applied to correct for household size.

Previous literature on the subjective assessment of inequality is sparse: As mentioned above, one strand of literature has examined whether and how inequality affects subjective well-being, using a particular, arbitrarily chosen inequality measure (the Gini coefficient; see Alesina et al. 2004). Ravallion and Lokshin (2002) use data on self-rated poverty to test the appropriateness of the ways in which poverty lines are usually constructed (and to assess which variables (in addition to income) may contribute to subjective poverty perceptions). Their focus, however, is the individuals' personal evaluation of their welfare, not the social evaluation of the income distribution. Other studies have focused on the properties of the inequality measures directly: Amiel and Cowell have investigated people's value judgements on inequality measurement in a series of papers (see Amiel and Cowell 1999 for a summary). They employed questionnaires in which they asked various groups of people in different countries to compare or to rank income distributions which are consistent with or represent properties often postulated in inequality measurement. It turned out that some properties belonging to the core of theoretical analysis are not supported by a majority of respondents. There are two major differences between Amiel and Cowell's investigations and the analysis performed in this paper. First, Amiel and Cowell's methodology is completely individualistic whereas in our approach a representative individual is considered and aggregate measures of the level and distribution of income are used. Second, they examine whether certain axioms of inequality measurement are approved or not, while we are interested in the way income distributions are evaluated by a typical individual.

The paper is organized as follows. In section 2 we set up our theoretical framework by discussing social welfare orderings and the AKS class of inequality measures and by characterizing two of its sub-classes, the Atkinson and Gini families. In section 3 we discuss our empirical data and approach, and in section 4 we present our results. Section 5 concludes.

2. Theoretical framework

In this section we introduce the basic concepts and present the framework used for the empirical investigation. Therefore we will discuss the relationship between welfare, mean income, and (in)equality and characterize some classes of welfare functions. The literature on these topics is huge. We confine ourselves to the essentials and provide some references for

further reading.² In subsections 2.1-2.3 single individuals are examined, starting with the case of a fixed population and then addressing the problem of comparing populations with different size. Subsection 2.4 is devoted to an investigation of a heterogeneous population, i.e. individuals form households which may differ in size.

We first consider a population of fixed size $n \geq 3$. It is assumed that the individuals are identical except for income. They are numbered arbitrarily by a number i ($1 \leq i \leq n$). Individual i has income $X_i > 0$. Therefore the set of feasible income distributions $X = (X_1, \dots, X_n)$ is given by \mathbb{R}_{++}^n . Let $X_{\downarrow} = (X_{[1]}, \dots, X_{[n]})$ be the ordered vector X , i.e. X_{\downarrow} is a permutation of X and we have $X_{[i]} \geq X_{[i+1]}$ for $i = 1, \dots, n-1$. The vector consisting of n ones is abbreviated by $\mathbf{1}$, and $\mu(X)$ denotes the average income.

2.1 From welfare to (in)equality

We want to compare vectors of income $X, Y \in \mathbb{R}_{++}^n$ in terms of social welfare. Therefore we introduce a social welfare ordering \succsim_W defined on \mathbb{R}_{++}^n . Let \sim_W denote its symmetric and \succ_W its asymmetric part. Any function $W : \mathbb{R}_{++}^n \rightarrow \mathbb{R}$ representing the welfare ordering is called a welfare function ($X \succsim_W Y \Leftrightarrow W(X) \geq W(Y)$). It is an ordinal concept. We will assume that the ordering \succsim_W is

- continuous [i.e. a welfare function reacts to a small change of income with a small change],
- increasing along rays [i.e. welfare increases if all incomes are increased in the same proportion],
- anonymous [i.e. welfare does not depend on the way individuals are numbered],
- equity-preferring [i.e. a progressive transfer (that is a transfer which redistributes a small amount of income from a richer to a poorer individual without changing the ranking of incomes) increases welfare], and
- homothetic [i.e. the ordering of two distributions in terms of welfare is preserved if all incomes are changed in the same proportion].

² We offer a detailed exposition of the results reported in this section to the interested reader (see Appendix A).

A particular representation of \succsim_w (particular welfare function) is the equally-distributed-equivalent income (EDEI) $\xi(X)$ defined by $\xi(X)\mathbf{1} \sim_w X$. It denotes the level of income which – if enjoyed by each individual – yields the same level of welfare as X and is cardinally significant. Given the above properties of \succsim_w the EDEI is continuous, increasing along rays, anonymous, equity-preferring, and linearly homogeneous ($\xi(\lambda X) = \lambda \xi(X)$).

The concept of welfare takes into account efficiency *and* the distribution (inequality) of incomes.

At first we consider inequality: Every welfare ordering allows us to define an inequality (AKS) measure by

$$I_{AKS}(X) := \frac{\mu(X) - \xi(X)}{\mu(X)}. \quad (1)$$

This definition goes back to Kolm (1969), Atkinson (1970), and Sen (1973). It represents the welfare loss due to inequality and is equal to the share of total income which is not necessary to attain the level of welfare implied by X . It is also cardinally significant. We will use the corresponding equality measure $E_{AKS}(X) := 1 - I_{AKS}(X) = \xi(X)/\mu(X)$ with $0 \leq E_{AKS}(X) \leq 1$. The equality measure is

- continuous, anonymous, equity-preferring, and
- a relative measure [i.e. equal proportional changes of all incomes leave the measure unchanged ($E(\lambda X) = E(X)$ for $\lambda > 0$, $X \in \mathbb{R}_{++}^n$)].

For the rest of the paper we will in general employ an equality measure E (instead of the corresponding inequality measure $I = 1 - E$).

Taking into account efficiency and distribution jointly it can be noted that in the case of homothetic welfare orderings corresponding welfare functions, especially the EDEI, can be decomposed in a way which captures these two aspects:

$$\xi(X) = \mu(X) \left(\frac{\xi(X)}{\mu(X)} \right) =: V(\mu(X), E_{AKS}(X)). \quad (2)$$

$V(\mu, E)$ denotes the trade-off function which expresses welfare by means of average income and the level of equality. The function V is increasing in both arguments and reflects the trade-off between average income μ and equality E . Ebert (1987) demonstrates that a welfare

ordering implies a unique equality ordering and trade-off ordering. For simplicity we use here only *functions* representing the trade-off ordering.

2.2 From (in)equality to welfare

We now consider a given equality measure $E: \mathbb{R}_{++}^n \rightarrow \mathbb{R}_+$ and trade-off function $V(\mu, E)$. Then a corresponding welfare function (ordering \succsim_w) can be derived, i.e. the above proceeding can be reversed: Using V and E , a representation of \succsim_w is defined by the welfare function $V(\mu(X), E(X))$. If E is continuous, anonymous, equity-preferring and a relative measure and if V is increasing in both arguments the resulting welfare ordering possesses the properties discussed in section 2.1.

But in general the equality measure $E(X)$ will not be the AKS measure of \succsim_w . Therefore we introduce the concept of self-consistency: we call an equality measure E self-consistent for $V(\mu, E)$ if it coincides with the corresponding AKS measure. Self-consistency ensures that the equality preference implicit in a given equality measure is consistent with the equality preference of the associated welfare ordering.

It is easy to see that a given E with $E(\mathbf{1})=1$ is self-consistent for $V(\mu, E)$ if and only if $V(\mu, E) = f(\mu E)$ where f is strictly increasing. The proof is immediate: The EDEI is defined by $V(\xi(X), E(\mathbf{1})) = V(\mu(X), E(X))$. Then

$$V\left(\mu(X) \frac{\xi(X)}{\mu(X)}, E(\mathbf{1})\right) = V(\mu(X) E_{AKS}(X), E(\mathbf{1})) =: f(\mu(X) E_{AKS}(X)).$$

Conversely

$$\begin{aligned} V(\mu, E) = f(\mu E) &\Rightarrow f(\mu(X) E(X)) = g(\mu(X) E_{AKS}(X)) \\ &\Rightarrow f(\mu(X)) = g\left(\mu(X) \frac{E_{AKS}(X)}{E(X)}\right) \end{aligned}$$

This implies that $E_{AKS} = E$ since $E_{AKS}/E = \text{constant}$ and $E_{AKS}(\mathbf{1}) = E(\mathbf{1}) = 1$.

Self-consistency is one of the main criteria which we will employ in the empirical evaluation of equality measures.

2.3 Atkinson-Gini welfare functions

Below we will examine a general class of welfare functions comprising the Atkinson and Gini family as special subclasses (see Ebert 1988 and also Araar and Duclos 2003). The underlying welfare orderings can be represented by a welfare function

$$\xi_{\delta,\varepsilon}^n(X) = \begin{cases} \left(\sum_{i=1}^n w_i^n(\delta) X_{[i]}^{1-\varepsilon} \right)^{1/(1-\varepsilon)} & \text{for } \varepsilon \geq 0, \quad \varepsilon \neq 1 \\ \prod_{i=1}^n X_{[i]}^{w_i^n(\delta)} & \text{for } \varepsilon = 1 \end{cases} \quad (3)$$

$$\text{where } w_i^n(\delta) = (i/n)^\delta - ((i-1)/n)^\delta \text{ and } \delta \geq 1, \quad \varepsilon \geq 0, \quad \delta + \varepsilon > 1. \quad (4)$$

We obtain the Atkinson family for $\delta = 1$ and $\varepsilon > 0$, the Gini family³ for $\delta > 1$ and $\varepsilon = 0$ (see also Kakwani 1980 and Donaldson and Weymark 1980) and hybrid functions for $\delta > 1$ and $\varepsilon > 0$. The (conventional) Gini coefficient is implied by $\xi_{\delta,\varepsilon}^n(X)$ for $\delta = 2$ and $\varepsilon = 0$.

Equality preference (or inequality aversion) is increasing with δ and ε . For the Atkinson family only the level of income is relevant. For the Gini family and for mixed welfare functions both the levels and rankings of incomes are taken into account. The welfare functions with $\delta = 2$ provide an interesting reference case: The difference between two neighbored weights is always the same. If $1 < \delta < 2$ the differences are decreasing, if $\delta > 2$ they are increasing in the rank i . Thus, compared to the case $\delta = 2$, higher incomes are relatively favored if $1 < \delta < 2$ and lower incomes for $\delta > 2$: An increase in δ increases the weights attached to the lower end and decreases the weight put on the higher end of the income distribution in the social welfare function and the equality measure (!).⁴

Now we address the problem of comparing populations of different size. Therefore we assume that n is variable and want to present a characterization of (3). The orderings can be characterized by two sets of properties.

(A) Firstly, suppose that for every n a welfare ordering \succsim_W^n is given which is

- continuous, increasing along rays, anonymous, equity-preferring and homothetic

and satisfies

³ Donaldson and Weymark called them S-Ginis (S = single-parameter generalization of the Gini indices).

⁴ This aspect is also discussed by Yitzhaki (1983). He considers the corresponding inequality measures (Gini family) in a continuous framework and proves that they can be interpreted as a weighted integration of the area below the Lorenz curve.

- a general form of independence for rank-ordered vectors [i.e. if for two rank-ordered income vectors which are equivalent with respect to welfare and which have the same i -th components the i -th positions are changed simultaneously (without modifying the ranking), then the new vectors are still equivalent].

The latter property implies the separability of the welfare function in the rank-ordered incomes.

Then these properties already yield a particular form of the corresponding EDEI

$$\xi^n(X) = \begin{cases} \left(\sum_{i=1}^n w_i^n X_{[i]}^{1-\varepsilon_n} \right)^{1/(1-\varepsilon_n)} & \text{for } \varepsilon_n \geq 0, \quad \varepsilon_n \neq 1 \\ \prod_{i=1}^n X_{[i]}^{w_i^n} & \text{for } \varepsilon_n = 1 \end{cases}$$

where either $(\varepsilon_n = 0 \text{ and } w_i^n < w_{i+1}^n)$ or $(\varepsilon_n > 0 \text{ and } w_i^n \leq w_{i+1}^n)$.

(B) Secondly, suppose that the welfare orderings $\{\succsim_W^n\}_{n \geq 3}$ satisfying the above properties (A) are related by

- the population principle [which requires that the equivalence of two income vectors of size n depends only on the distribution of incomes (in the statistical sense). Then $X \sim_W^n Y$ implies that the m -fold replicated distributions are also equivalent: $X^{(m)} \sim_W^{nm} Y^{(m)}$], and
- a principle of restricted aggregation [which postulates that social welfare is not changed if the k highest incomes X_{\downarrow}^k ($k \geq 3$) in an income vector X of size $n > k$ are simultaneously replaced by the corresponding equally distributed income $\xi^k(X_{\downarrow}^k)$ determined by \succsim_W^k (substitution principle)].

Then these properties (A) and (B) imply that $\varepsilon_n = \varepsilon$ for all $n \geq 3$ and that the weights possess the form shown in (4).

In summary, if the welfare orderings $\{\succsim_W^n\}_{n \geq 3}$ satisfy both sets of properties (A) and (B), there are unique parameters δ and ε such that these orderings are represented by $\xi_{\delta, \varepsilon}^n(X)$, and conversely, every sequence of orderings represented by $\xi_{\delta, \varepsilon}^n(X)$ satisfies these properties.

Given the entire class we are now able to characterize two *subclasses*, the Atkinson and Gini family, by adding, respectively, one more property.

The Atkinson family satisfies a further principle of restricted aggregation: For any $k < n$ the k *lowest* incomes can be replaced by the corresponding equally distributed income. Then a general aggregation property is implied and it is impossible to take into account the ranking of incomes. Thus $\delta = 1$ (the weights w_i^n are identical).

On the other hand the Gini family satisfies the above properties and is in addition translatable; i.e. the welfare ordering of two income distributions is preserved⁵ if all incomes are changed by the same (absolute) amount. This property can be fulfilled only by linear functions and therefore $\varepsilon = 0$.

2.4 Different household types

Now we still assume that all individuals are identical, but that they may form households of different size. Let h be the number of individuals belonging to a household ($1 \leq h \leq H$) and N be the number of households. Then a household i can be described by its income and its size: (X_i, h_i) with $1 \leq h_i \leq H$. Thus X_i is no longer an individual's, but household i 's income. Therefore the size of households and the differing needs have to be taken into account when evaluating the income distribution. This is achieved by employing equivalence scales $m(h)$ and type-specific weights $\omega(h)$ with $m(1) = \omega(1) = 1$. The equivalence scale $m(h)$ represents the needs of a household with h individuals. It allows us to define the equivalent income $X^* = X/m(h)$ which represents the income a one-person household needs in order to be as well off as each person living in a household with h individuals and household income X . Equivalent income is an indicator of living standard. Therefore the equivalence scale can be interpreted as the number of equivalent adults (representing a household's needs). Two versions of type-specific weights are considered in the literature: Either the weight is set equal to the number of household members ($\omega(h) = h$) or it is identified with the number of equivalent adults ($\omega(h) = m(h)$). Independent of this choice we have to use the needs-adjusted income distribution described by $(X^*, \omega) = (X_1^*, \omega_1, \dots, X_N^*, \omega_N)$.

Then the form of the EDEI is changed to

⁵ Nevertheless the level of social welfare is changed.

$$\xi_{\delta,\varepsilon}(X^*, \omega) = \begin{cases} \left(\sum_{i=1}^N w_i^{*n}(\delta) X_{[i]}^{*(1-\varepsilon)} \right)^{1/(1-\varepsilon)} & \text{for } \varepsilon \neq 1, \quad \varepsilon \geq 0 \\ \prod_{i=1}^N X_{[i]}^{*w_i^{*n}(\delta)} & \text{for } \varepsilon = 1 \end{cases} \quad (5)$$

$$\text{where } w_i^{*n}(\delta) = \left(\sum_{j=1}^i \omega(h_j) / \sum_{j=1}^N \omega(h_j) \right)^\delta - \left(\sum_{j=1}^{i-1} \omega(h_j) / \sum_{j=1}^N \omega(h_j) \right)^\delta \quad (6)$$

and $X_{[i]}^* \geq X_{[i+1]}^*$ for $i = 1, \dots, N-1$.

A characterization of a more general class of welfare orderings with different household types is presented in Ebert (2004).

For a derivation of (in)equality measures we employ the corresponding weighted (!) average of equivalent income defined by

$$\mu(X^*, \omega) := \sum_{i=1}^N \frac{\omega(h_i)}{\sum \omega(h_j)} X_i^*. \quad (7)$$

The inequality and equality measures we will employ below are then given⁶ by

$$I_{\delta,\varepsilon}(X^*, \omega) = 1 - \xi_{\delta,\varepsilon}(X^*, \omega) / \mu(X^*, \omega) \text{ for } \delta \geq 1, \quad \varepsilon \geq 0, \quad \delta + \varepsilon > 1$$

and $E_{\delta,\varepsilon}(X^*, \omega) = 1 - I_{\delta,\varepsilon}(X^*, \omega)$, respectively.

It turns out that the choice of the weights $\omega(h)$ is relevant for the properties of the corresponding welfare functions and equality measures (see e.g. Ebert 1997). If the weights equal the number of household members the measures are individualistic (each individual is taken into account explicitly), but they violate the principle of between-type transfers. This principle requires that a progressive transfer from a richer household to a poorer one increases welfare and equality where ‘rich’ and ‘poor’ is measured by the equivalent income. If the weights are equal to the number of equivalent adults this principle is always satisfied.

⁶ The corresponding Gini coefficient $I_{2,0}(X^*, \omega)$ for $\omega(h) = m(h)$ has also been characterized by Pyatt (1985).

3. Empirical Framework

Our objective is to investigate how the income distribution affects the well-being of society. In theoretical terms, we want to determine a social welfare ordering \succsim_W by estimating a social welfare function $W = W(X^*, \omega)$ representing \succsim_W where (X^*, ω) denotes the needs-adjusted income distribution. To this end we will use the decomposability of $W(X^*, \omega)$ discussed in section 2 and will estimate the welfare function by means of the associated trade-off function $V(\mu(X^*, \omega), E(X^*, \omega))$. But, of course, further choices and assumptions have to be made to carry out this task. We can distinguish five steps.

At first appropriate data have to be chosen (subsection 3.1). Second, starting with data on the distribution of household income and household size we construct the distribution of needs-adjusted (equivalent) income (X^*, ω) . It forms the basis for the computation of mean equivalent income $\mu(X^*, \omega)$ and equality $E(X^*, \omega)$ where the equality measure E is chosen from a given family of measures (subsection 3.2). In a third step we formulate our empirical model by specifying an empirical trade-off function (subsection 3.3). Next, this model is translated into a form suitable for estimation and an appropriate method of estimation is chosen (subsection 3.4).

Finally, the steps 2-4 can be performed for any choice of an equality measure E from the given family. Our aim is to identify a social welfare function which represents the underlying welfare ordering. The criteria used are self-consistency and statistical significance (subsection 3.5).

We will now describe these steps in more detail.

3.1 Basic Data

The required variables are derived from data on subjective well-being and income distribution in 10 European countries, 1978-1997. Our choice of countries and years is based on data availability. For each country, between 2 and 5 observations within this time frame are available, totaling 37 observations.⁷

⁷ The countries are Belgium, Denmark, France, (West) Germany, Ireland, Italy, Luxembourg, the Netherlands, Spain, and the U.K. With respect to Germany we deliberately confine ourselves to years prior to the unification to avoid the risk of structural break. Within the overall time frame, 1982 and 1993 have no observations.

The dependent variable is mean subjective-wellbeing H (happiness). H is interpreted as the well-being of a representative individual (of the respective country) and is taken as a proxy for social welfare W (see subsection 3.3).⁸

Our data on subjective well-being comes from the *World Database of Happiness* (Veenhoven 2002). The variable of interest is average self-reported life satisfaction by country and year. The data are based on the Eurobarometer Survey Series⁹ in which the following type of question is asked: “On the whole, are you very satisfied, fairly satisfied, not very satisfied or not at all satisfied with the life you lead.” The qualitative responses are rated as follows: “very satisfied” = 4, “fairly satisfied” = 3, “not very satisfied” = 2, “not at all satisfied” = 1.¹⁰

Happiness surveys are successfully used in a growing literature in economics. We will now discuss some of the arguments made in the previous literature in favor of using happiness data in economics (see Frey and Stutzer 2002).¹¹

A basic requirement is that the original qualitative response data be at least ordinal in character and satisfy conventional quality standards. Whether happiness measures meet these conditions has been widely assessed in decades of validation research. In these studies measures of happiness are generally found to have a high scientific standard in terms of internal consistency, reliability and validity, and a high degree of stability over time (Diener et al. 1999). Different happiness measures correlate well with each other and, according to factor analyses, represent a single unitary construct. Happiness responses are correlated with physical reactions that can be thought of as describing true, internal happiness: People reporting to be happy tend to smile more and show lower levels of stress responses (heart rate, blood pressure). They are more frequently described by others as being happy and they are less likely to commit suicide. As concerns the comparability across nations, no indication has been found that cultural or linguistic bias may prevent a comparison of happiness across nations (Veenhoven 1993).

⁸ The study of societal happiness functions is quite common in the literature (see Di Tella, McCulloch and Oswald 2001, to mention just one example).

⁹ The Eurobarometer public opinion surveys are conducted on behalf of the European Commission, DG Press and Communication. Each consists of approximately 1000 face-to-face interviews per Member State of persons aged 15 and over.

¹⁰ Although observations are available with three or five response categories, the four-item data from the Eurobarometer is the only complete data set available for European countries within our time frame.

¹¹ With respect to the conceptual foundations of the happiness approach, Kahnemann, Wakker and Sarin (1997) provide an axiomatic defense of what they call experienced utility (happiness). Ng (1997) discusses the measurability of happiness.

The explanatory variables are mean equivalent income and a corresponding measure of equality. They are based on the distribution of needs-adjusted income (X^*, ω) . The distribution (X^*, ω) is derived from the *Luxembourg Income Study* (LIS), see LIS (2004). This database contains micro-data on distributions of household income and household size in selected countries and years, taken from national surveys and harmonized by LIS. The basic income concept we use is disposable household income as defined in the LIS data set. In principle all households surveyed are included, but the bottom and the top of the income distributions are truncated (see below).

Conclusions on equality rely heavily on the underlying quality and comparability of income data. With respect to general quality, the LIS data satisfy at least the following minimum standards: they are based on household surveys, the population covered is representative of the entire country (or can be adjusted appropriately), and the measure of household income is comprehensive including income from self-employment, property income, and transfers (net of direct taxes and social security contributions). The LIS database allows a comparison of cross-national income distributions in a unified household income database environment created explicitly for this purpose and has been successfully used for comparisons of income distributions in OECD countries (see Buhmann et al. 1988, OECD 1995). Any remaining differences in the bases used to compute our equality measures are at least partially controlled for by including country dummies in our trade-off function regressions.

It may be added that the LIS database is the only appropriate data source for our purposes since it contains micro-data and thus permits to compute any desired equality measure. ‘Secondary’ data sets on income inequality that are frequently used (such as Deininger and Squire 1996) contain only pre-fabricated observations on Gini coefficients and do not allow to compute other inequality measures.¹²

3.2 Construction of Income Distributions and Choice of Measures

We can construct the income distribution (X^*, ω) which is based on equivalent incomes X_i^* and weights ω_i , for $i = 1, \dots, n$. Firstly, equivalence scale values $m(h)$ have to be chosen ($h = 1, \dots, H$). We use the square root of the number of household members (see OECD 1995, p. 21). Secondly, the weights have to be determined. Here two alternative weighting schemes

¹² Inequality databases are discussed by Atkinson and Brandolini (2001).

are considered: Either the number of household members or the number of equivalent adults (given by the corresponding equivalence scale value) are chosen. Furthermore, according to the LIS standard all households with income less than 1 percent of equivalent mean income or with income higher than 10 times the median of household income are ignored (top and bottom coding). Corresponding to the two types of weight applied to account for household size we then obtain the needs-adjusted income distribution (X^*, ω) which allows us to compute the mean equivalent income $\mu(X^*, \omega)$ and any equality measure. The mean equivalent incomes derived from the LIS data are nominal figures in national currency. For inclusion in the empirical trade-off function we converted them to a common currency (\$US at 1990 price levels) using purchasing power parities from OECD (1998, 2003).

Next the admissible equality measures E have to be described. We suppose that $E_{\delta, \varepsilon}$ belongs to the family $\mathcal{E} = \{E_{\delta, \varepsilon} \mid \delta \geq 1, \varepsilon \geq 0, \delta + \varepsilon > 1\}$. Thus the parameters (δ, ε) determine the equality measure precisely, given the functional form of measures belonging to \mathcal{E} (the Atkinson-Gini family). As discussed in subsection 2.3, higher values of δ or ε , respectively, reflect higher equality preference. For given (δ, ε) the measure $E_{\delta, \varepsilon}(X^*, \omega)$ is then calculated.¹³

3.3 Empirical Model

Since we want to derive social welfare implied by (X^*, ω) we choose a trade-off function $U(\mu, E)$ which is to satisfy

$$W(X^*, \omega) = U(\mu(X^*, \omega), E_{\delta, \varepsilon}(X^*, \omega)).$$

Our assumption that social welfare W is proxied by the subjective well-being of a representative individual H is captured by postulating that there exists a continuous and strictly increasing function F such that $H = F(W)$. Then $H(X^*, \omega)$ turns out to be also a representation of the welfare ordering.

Observing that $H = F(w)$ we obtain

$$H(X^*, \omega) = F(W(X^*, \omega)) = F(U(X^*, \omega), E_{\delta, \varepsilon}(X^*, \omega)).$$

¹³ Appendix B contains the programs for calculating the equality measures and the mean equivalent incomes.

In order to make the problem manageable we specify particular functional forms for U and F by assuming

$$U(\mu, E) = c\mu^a E^b \text{ and } F(W) = dW^e$$

where $c > 0$ and $d > 0$. The problem is then well defined since $U \geq 0$ and $F \geq 0$. Using this specification we get

$$\begin{aligned} H &= d \left(c\mu(X^*, \omega) E_{\delta, \varepsilon}(X^*, \omega)^b \right)^e = (d c^e) \mu(X^*, \omega)^{ae} E_{\delta, \varepsilon}(X^*, \omega)^{be} \\ &=: \gamma \mu(X^*, \omega)^\alpha E_{\delta, \varepsilon}^\beta =: V(\mu(X^*, \omega), E_{\delta, \varepsilon}(X^*, \omega)) \end{aligned}$$

The equation defines the empirical trade-off function V which is to be estimated.

3.4 Empirical Strategy

In terms of empirical variables, the trade-off function derived in the preceding subsection can be written as follows:

$$H_{it} = \gamma_i \mu_{it}^\alpha E_{it}^\beta.$$

H_{it} denotes the mean subjective well-being by country i and year t . μ_{it} is average equivalent income and E_{it} is a measure of equality of the adjusted distribution in country i and year t . While the parameters α and β are assumed to be identical across countries, the trade-off function is admitted to differ among countries with respect to the scaling parameter.

Our empirical approach is to estimate the logarithmic version of the trade-off function introduced above:

$$\log(H_{it}) = \tilde{\gamma}_i + \alpha \log(\mu_{it}) + \beta \log(E_{it}) + \varepsilon_{it}, \quad (8)$$

where, in addition to the previously introduced notation, $\tilde{\gamma}_i := \log(\gamma_i)$ and ε_{it} denotes an error term. The country dummies $\tilde{\gamma}_i$ control for possible cross-country differences in the bases used to compute mean equivalent income and equality, as well as time-invariant omitted-variable bias.¹⁴ Especially, the country dummies accommodate country-specific differences in the distribution of the individuals' personal characteristics not captured by the data.

¹⁴ Note that our aim is not to capture all determinants of social welfare, but only to identify the influence of the income distribution in an unbiased way.

Standard methods of estimating eq. (8) are fixed-effects or random-effects estimators. The random effect estimates are more efficient than the fixed effect estimates, but may be inconsistent.¹⁵ Especially, if there are missing observations so that the number of observations varies among cross-sections (countries), as is the case in our data, the random-effects method is consistent only when the number of missing observations is asymptotically negligible. In our data, with 10 countries and 18 relevant years¹⁶, there are 180 possible observations, of which 143 are missing. Random-effects is not an appropriate method in these circumstances. Instead, the fixed-effects method is used throughout. To account for cross-section heteroskedasticity, generalized least squares (GLS) is used.

3.5 Assessment of Results

We are able to estimate $\alpha(\delta, \varepsilon)$, $\beta(\delta, \varepsilon)$, and $\gamma(\delta, \varepsilon)$ for every admissible (δ, ε) . In order to discriminate between different welfare functions we impose the requirement that $E_{\delta, \varepsilon}$ is self-consistent for V , i.e. we look for (δ, ε) such that $E_{\delta, \varepsilon}$ is the AKS measure of $V\left(\mu\left(X^*, \omega\right), E_{\delta, \varepsilon}\left(X^*, \omega\right)\right)$. Simple computation demonstrates that $E_{\delta, \varepsilon}$ is self-consistent for V if and only if $\alpha(\delta, \varepsilon) = \beta(\delta, \varepsilon)$, a condition which can be checked easily. Since by construction $\alpha = ae$ and $\beta = be$ we recognize that – given the functional form of F – the parameter e (as well as d) has no impact on the identification of the self-consistent measure.

The second criterion that we employ is the marginal significance level (p -value).

4. Empirical Results

Now we present and discuss the results. We are interested in finding self-consistent equality measures and then evaluating them on the basis of statistical significance. In a further step we are able to derive the implications.

¹⁵ The major difference between these two techniques is the information utilized to calculate the coefficients. The fixed effects estimates are calculated from differences within each country across time; the random effects estimates are more efficient, since they incorporate information across individual countries as well as across periods. The random effects method is consistent only if the country-specific effects are uncorrelated with the other explanatory variables.

¹⁶ Recall from footnote 6 that there are no observations for 1982 and 1993.

4.1 Number of Household Members Used for Weighting

We estimated eq. (8) with δ and ε varied at steps of 0.1 over an appropriate range. The results are presented in Table 1. The top panel refers to the Gini measures, the middle panel to the Atkinson measures, and the bottom panel to selected ‘hybrid’ measures.

To begin with, it can be stated that the mean equivalent income is highly significant in all cases examined and the estimated coefficient on mean income (α) shows little variation with respect to which equality measure is included.

Concerning the Gini equality measures we find that, as δ increases, the coefficient on equality (β) declines in a continuous fashion. Given this continuity, self-consistency can be expected to occur at δ between 1.6 and 1.7. Moreover, the Gini measures are always at least weakly significant, and they are significant at levels of about 3 percent for those values of δ where self-consistency is obtained (p -values).

With respect to the Atkinson equality measures we find that, as ε increases, the coefficient on equality declines in a continuous way, and self-consistency is expected to occur at ε between 0.6 and 0.7. However the precision of the estimated β coefficients is low (cf. the t -statistics); in fact the coefficients are never significant at any usual level (cf. the p -values). Considering the probability that $\alpha = \beta$ (F -test), the low precision of the estimated β ’s entails that self-consistency is more likely to occur at $\varepsilon = 0.5$ ($\text{prob}(\alpha = \beta) = 0.866$) than at $\varepsilon = 0.7$ ($\text{prob}(\alpha = \beta) = 0.716$).

The bottom panel of Table 1 reports the results for selected (δ, ε) -configurations, namely those that nest the self-consistent equality measures. These configurations can be identified because the estimates of β change continuously as δ is varied (at any given ε) and as ε is varied (at any given δ). It can be seen that for self-consistency to occur, low (high) values of the δ parameter require relatively high (low) levels of the ε parameter. Thus, the two parameters of equality preference tend to be ‘substitutes’ for each other. However, the coefficient of the equality measure (β) is (weakly) significant only when δ exceeds a certain threshold value (about 1.2).

If we compare the self-consistent equality measures in terms of marginal significance (p -value) we see that marginal significance improves (p falls) as ε is reduced and δ is increased. The pure Atkinson measures in the self-consistent range ($\varepsilon \in [0.6, 0.7]$) are insignificant at any usual level. The lowest p -values among the self-consistent measures are

obtained in the case of the pure Gini measures. These are significant at a level of about 3 percent.

4.2 Number of Equivalent Adults Used for Weighting

The results for this case are presented in Table 2, which has the same structure as Table 1.

Again, the mean equivalent income is always highly significant and the estimated α coefficients vary little with respect to which equality measure is included. However, the coefficients are somewhat larger than in the previous case.

As concerns the Gini equality measures, there is again a continuous decline of the estimated β coefficient as δ increases. Self-consistency now occurs at δ between 2.1 and 2.2. The Gini measures are always highly significant, and the probability of self-consistency ($\text{prob}(\alpha = \beta)$) is greater than 0.9 at both $\delta = 2.1$ and $\delta = 2.2$.

With respect to the Atkinson equality measures we again find that, as ε increases, the coefficient on equality declines in a continuous way. Self-consistency now occurs at ε between 0.7 and 0.8. The precision of the estimated β coefficients (cf. the t -statistics) is in general higher than before, and the coefficients are at least weakly significant as long as ε does not exceed 0.8 (cf. the p -values).

The results for the hybrid measures are qualitatively similar to those described in 4.1, that is, the two parameters of equality preference tend to be substitutes for each other. In contrast to the previous case, however, the β coefficient is now at least weakly significant for all (δ, ε) -configurations.

Similar as in the preceding subsection, the worst marginal significance levels among the self-consistent equality measures are obtained in the case of the pure Atkinson measures. This time, however, they imply weak significance. Starting from the Atkinson measures, marginal significance again improves as ε is reduced and δ is increased. The lowest p -values among the self-consistent measures are obtained in the case of the pure Gini measures. These are significant at a level of about one half percent.

4.3 Comparison of weighting schemes

As discussed in subsections 2.4 and 3.2 equivalent incomes are weighted either by the number of household members or by the number of equivalent adults. A general assessment of our

results entails that, to begin with, the mean equivalent income is always ‘relevant’ (statistically significant) for social welfare¹⁷, independent of which of the two weighting schemes is applied and which of the considered equality measures is included in the estimation. In contrast to this, the significance of equality depends on the specific measure of equality. Equality in the sense of the Gini measures tends to be significant under both weightings, whereas equality in the sense of the Atkinson measures is significant only if the number of equivalent adults is used for weighting. Gini measures are in general significant at stricter levels than Atkinson measures (if the latter are significant at all).

Concentrating now on self-consistent measures we observe that they are more significant and more equality-preferring when the number of equivalent adults is used for weighting. Thus the conventionally used Atkinson ($\delta = 1$ and $\varepsilon = 0.5$) and Gini ($\delta = 2.0$ and $\varepsilon = 0$) indices do not appear to be self-consistent in this case. More specifically, the intrinsic equality preference inherent in these indices seems to be too small.

Thus the data supports weighting households by equivalence scales since the corresponding equality measures and welfare functions are preferred to the conventional ones. Therefore we can conclude that also the underlying principle is preferred. Choosing between individualism and the between-type transfer principle the representative individual is more interested in the topic of redistribution and decides for the transfer principle. The welfare function is egalitarian in this sense since a progressive transfer between households of different types improves welfare.

That there are so marked differences among the equality measures considered is quite surprising, all the more so since they are highly correlated. For instance, any of the Atkinson indices based on the number of household members shows a correlation of more than 0.99 with its counterpart based on the number of equivalent adults, and the same is true for the Gini indices. Likewise, the frequently used Atkinson ($\delta = 1$ and $\varepsilon = 0.5$) and the Gini ($\delta = 2.0$ and $\varepsilon = 0$) indices display a correlation of more than 0.98 with each other under both types of weighting. Our results suggest that – in spite of their high correlation – the notions of inequality incorporated in these indices affect well-being substantially different.

¹⁷ This is consistent with the majority of findings in recent research on cross-national subjective well-being. See Hagerty and Veenhoven (2003) for an assessment of the literature and additional new evidence.

5. Conclusions

The purpose of the paper was to examine how income distributions are evaluated by a representative individual. The approach was to estimate welfare functions over income distributions (trade-off functions) which decompose welfare into mean equivalent income and a measure of equality. The estimations were performed using a variety of equality measures that are members of the Atkinson-Gini family of measures. Since different members of this family represent different implicit notions of equality preference, the estimation results allowed us to derive inference about people's value judgements concerning income distribution. In addition, we considered two different weighting schemes usually applied to account for differences in household size.

Our results imply that social well-being is affected by income inequality measured not only in terms of the levels of income, but also in terms of their ranking. This is consistent with earlier findings that *relative income* – relative to some reference group – is important for self-related welfare, rather than absolute income (for recent evidence see, e.g., Easterlin 1995, Clark and Oswald 1996, and McBride 2001). Moreover the focus of people's concern is on the lower rather than the upper part of the income distribution. These results refer to people in Western Europe. Since it has been found in earlier literature that Europeans seem to have different general attitudes towards inequality than, say, Americans, it is an open question whether the above findings apply to members of non-European societies.

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Table 1: Estimation Results with Number of Household Members Used for Weighting

Gini δ ε		1.1 0.0	1.2 0.0	1.3 0.0	1.4 0.0	1.5 0.0	1.6 0.0	1.7 0.0	1.8 0.0	1.9 0.0	2.0 0.0
α	coefficient	0.124	0.125	0.125	0.125	0.126	0.127	0.126	0.126	0.127	0.127
	<i>t</i> -statistic	6.880	7.129	7.386	7.640	7.867	8.048	8.253	8.425	8.540	8.686
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.463	0.263	0.199	0.164	0.144	0.131	0.119	0.112	0.109	0.101
	<i>t</i> -statistic	1.903	1.996	2.098	2.160	2.220	2.270	2.299	2.329	2.356	2.370
	<i>p</i> -value	0.068	0.057	0.046	0.040	0.035	0.032	0.030	0.028	0.026	0.026
α/β		0.267	0.475	0.632	0.766	0.875	0.964	1.059	1.131	1.174	1.253
prob($\alpha=\beta$)		0.145	0.246	0.372	0.548	0.735	0.919	0.865	0.695	0.601	0.437
Adj. R2		0.908	0.908	0.908	0.908	0.908	0.908	0.908	0.908	0.908	0.908

Atkinson δ ε		1.0 0.1	1.0 0.2	1.0 0.3	1.0 0.4	1.0 0.5	1.0 0.6	1.0 0.7	1.0 0.8	1.0 0.9	1.0 1.0
α	coefficient	0.120	0.120	0.120	0.119	0.115	0.116	0.114	0.112	0.110	0.109
	<i>t</i> -statistic	7.452	7.577	7.633	7.692	7.610	7.829	7.876	7.937	8.040	8.139
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.853	0.444	0.288	0.210	0.130	0.126	0.091	0.061	0.037	0.020
	<i>t</i> -statistic	1.647	1.690	1.630	1.566	1.262	1.417	1.210	0.946	0.652	0.416
	<i>p</i> -value	0.112	0.103	0.115	0.129	0.218	0.168	0.237	0.353	0.520	0.680
α/β		0.140	0.271	0.415	0.569	0.882	0.921	1.250	1.832	3.014	5.397
prob($\alpha=\beta$)		0.154	0.201	0.308	0.457	0.866	0.896	0.716	0.342	0.111	0.026
Adj. R2		0.907	0.907	0.907	0.907	0.906	0.906	0.906	0.906	0.906	0.906

Hybrid δ ε		1.1 0.5	1.1 0.6	1.2 0.4	1.2 0.5	1.3 0.3	1.3 0.4	1.4 0.2	1.4 0.3	1.5 0.1	1.5 0.2	1.6 0.0	1.6 0.1
α	coefficient	0.120	0.119	0.123	0.121	0.124	0.123	0.124	0.124	0.126	0.126	0.127	0.126
	<i>t</i> -statistic	7.843	7.904	7.902	7.980	7.937	8.058	8.003	8.134	8.027	8.153	8.048	8.209
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.129	0.104	0.134	0.114	0.134	0.118	0.130	0.117	0.133	0.121	0.131	0.122
	<i>t</i> -statistic	1.638	1.502	1.851	1.752	1.998	1.922	2.093	2.030	2.201	2.153	2.270	2.245
	<i>p</i> -value	0.114	0.145	0.076	0.091	0.056	0.066	0.046	0.053	0.037	0.041	0.032	0.034
α/β		0.929	1.143	0.915	1.071	0.924	1.047	0.953	1.061	0.948	1.037	0.964	1.035
prob($\alpha=\beta$)		0.890	0.794	0.851	0.878	0.854	0.910	0.904	0.877	0.887	0.921	0.919	0.921
Adj. R2		0.907	0.907	0.907	0.907	0.907	0.907	0.908	0.907	0.908	0.908	0.908	0.908

Note: Fixed-effects estimation of $\log(H_{it}) = \tilde{\gamma}_i + \alpha \log(\mu_{it}) + \beta \log(E_{it}) + \varepsilon_{it}$. Method: GLS. Number of observations: 37.

Table 2: Estimation Results with Number of Equivalent Adults Used for Weighting

Gini δ ε		1.1 0.0	1.2 0.0	1.3 0.0	1.4 0.0	1.5 0.0	1.6 0.0	1.7 0.0	1.8 0.0	1.9 0.0	2.0 0.0	2.1 0.0	2.2 0.0	2.3 0.0
α	coefficient	0.133	0.134	0.135	0.135	0.136	0.136	0.137	0.137	0.137	0.137	0.138	0.138	0.138
	<i>t</i> -statistic	6.755	7.000	7.220	7.445	7.669	7.860	8.052	8.255	8.433	8.590	8.724	8.867	8.976
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.660	0.376	0.283	0.235	0.206	0.186	0.172	0.161	0.153	0.146	0.139	0.135	0.130
	<i>t</i> -statistic	2.435	2.556	2.663	2.745	2.822	2.872	2.915	2.972	3.009	3.037	3.058	3.089	3.107
	<i>p</i> -value	0.022	0.017	0.013	0.011	0.009	0.008	0.007	0.006	0.006	0.005	0.005	0.005	0.005
α/β		0.201	0.356	0.476	0.577	0.660	0.733	0.796	0.850	0.897	0.943	0.986	1.022	1.058
prob($\alpha=\beta$)		0.044	0.070	0.108	0.165	0.273	0.354	0.446	0.560	0.682	0.817	0.954	0.925	0.804
Adj. R2		0.909	0.909	0.909	0.909	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910

Atkinson δ ε		1.0 0.1	1.0 0.2	1.0 0.3	1.0 0.4	1.0 0.5	1.0 0.6	1.0 0.7	1.0 0.8	1.0 0.9	1.0 1.0
α	coefficient	0.127	0.126	0.126	0.125	0.124	0.122	0.120	0.117	0.115	0.113
	<i>t</i> -statistic	7.663	7.787	7.891	8.036	8.178	8.316	8.497	8.659	8.894	9.224
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	1.291	0.641	0.424	0.308	0.236	0.183	0.141	0.106	0.076	0.049
	<i>t</i> -statistic	2.400	2.391	2.361	2.312	2.237	2.120	1.965	1.736	1.458	1.117
	<i>p</i> -value	0.024	0.024	0.026	0.029	0.034	0.044	0.060	0.094	0.157	0.274
α/β		0.098	0.197	0.297	0.405	0.523	0.666	0.848	1.106	1.517	2.284
prob($\alpha=\beta$)		0.032	0.049	0.079	0.133	0.229	0.410	0.721	0.821	0.355	0.083
Adj. R2		0.908	0.908	0.908	0.908	0.908	0.907	0.907	0.907	0.907	0.906

Hybrid δ ε		1.1 0.7	1.1 0.8	1.2 0.6	1.2 0.7	1.3 0.5	1.3 0.6	1.4 0.5	1.4 0.6	1.5 0.4	1.5 0.5	1.6 0.3	1.6 0.4
α	coefficient	0.122	0.119	0.125	0.123	0.129	0.126	0.129	0.127	0.131	0.130	0.133	0.132
	<i>t</i> -statistic	8.475	8.647	8.392	8.555	8.358	8.509	8.491	8.622	8.480	8.624	8.468	8.625
	<i>p</i> -value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.125	0.097	0.137	0.122	0.146	0.124	0.133	0.115	1.149	0.124	0.144	0.131
	<i>t</i> -statistic	2.100	1.873	2.354	2.185	2.543	2.419	2.598	2.464	2.721	2.638	2.826	2.761
	<i>p</i> -value	0.046	0.072	0.026	0.038	0.017	0.023	0.015	0.021	0.012	0.014	0.009	0.010
α/β		0.975	1.228	0.916	1.091	0.883	1.016	0.969	1.102	0.942	1.043	0.924	1.008
prob($\alpha=\beta$)		0.947	0.587	0.802	0.797	0.702	0.961	0.916	0.736	0.835	0.879	0.776	0.975
Adj. R2		0.907	0.907	0.908	0.908	0.908	0.908	0.908	0.908	0.909	0.909	0.909	0.909

Hybrid δ ε		1.7 0.3	1.7 0.4	1.8 0.2	1.8 0.3	1.9 0.1	1.9 0.2	2.0 0.1	2.0 0.2	2.1 0.0	2.1 0.1
α	coefficient	0.134	0.132	0.135	0.134	0.136	0.135	0.137	0.135	0.138	0.137
	t-statistic	8.624	8.754	8.610	8.755	8.612	8.748	8.746	8.879	8.724	8.870
	p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β	coefficient	0.136	0.123	0.140	0.129	0.143	0.133	0.137	0.128	0.139	0.132
	t-statistic	2.858	2.783	2.932	2.885	2.992	2.954	3.015	2.987	3.058	3.044
	p-value	0.008	0.010	0.007	0.008	0.006	0.007	0.006	0.006	0.005	0.005
α/β		0.983	1.070	0.965	1.037	0.953	1.015	0.998	1.058	0.986	1.039
prob($\alpha=\beta$)		0.948	0.790	0.890	0.885	0.850	0.952	0.993	0.812	0.954	0.870
Adj. R2		0.909	0.909	0.909	0.909	0.910	0.909	0.910	0.909	0.910	0.910

Note: Fixed-effects estimation of $\log(H_{it}) = \tilde{\gamma}_i + \alpha \log(\mu_{it}) + \beta \log(E_{it}) + \varepsilon_{it}$. Method: GLS. Number of observations: 37.

Appendix A: Ethical inequality measures – A general discussion¹⁸

A.1 Separable welfare orderings

We consider a population of size $n \geq 3$. At first the number n of individuals is fixed. It is assumed that the individuals are numbered arbitrarily by a number i ($1 \leq i \leq n$). A vector $X = (X_1, \dots, X_n)$ denotes the individuals' real incomes. The set of feasible incomes is given by \mathbb{R}_{++}^n , i.e. each individual possesses a strictly positive income. Let $X_{\downarrow} = (X_{[1]}, \dots, X_{[n]})$ be the ordered vector X , i.e. X_{\downarrow} is a permutation of X and we have $X_{[i]} \geq X_{[i+1]}$ for $i = 1, \dots, n-1$.

We want to compare vectors of income $X, Y \in \mathbb{R}_{++}^n$ in terms of social welfare. Therefore we introduce a social welfare ordering \succsim_W defined on \mathbb{R}_{++}^n . Let \sim_W denote its symmetric and \succ_W its asymmetric part. We will impose

Property CONT (Continuity)

\succsim is continuous.

Property MON (Monotonicity)

$\lambda X \succ_W X$ for $\lambda > 1$.

These properties imply

Proposition 1 (Debreu 1954)

Assume CONT and MON.

\succsim_W can be represented by a continuous social welfare function $W : \mathbb{R}_{++}^n \rightarrow \mathbb{R}$, i.e.

$$\text{for all } X, Y \in \mathbb{R}_{++}^n : X \succsim_W Y \Leftrightarrow W(X) \geq W(Y).$$

Continuity implies that the social welfare function reacts to a small change in income itself with a small change. Monotonicity guarantees that social welfare increases if all incomes increase in the same proportion. Both properties jointly imply the existence of a social welfare function representing the ordering \succsim_W . An indicator W of \succsim_W is only unique up to a strictly

¹⁸ The presentation is based on Ebert (1988).

increasing transformation of W . Thus the framework considered is entirely ordinal. But a particular ordinal representation of the ordering \succsim_W will play an important part below: the equally distributed equivalent income.

Definition 1

The function $\xi: \mathbb{R}_{++}^n \rightarrow \mathbb{R}$ denotes the equally-distributed-equivalent income (EDEI), if it satisfies

$$\xi(X)\mathbf{1} \sim_W X \text{ for all } X \in \mathbb{R}_{++}^n$$

where $\mathbf{1}$ is the vector $(1, \dots, 1)$ containing the number 1 n -times.

It is clear by definition that the EDEI $\xi(X)$ is a cardinal concept: it denotes that level of income which if enjoyed by each individual yields the same level of social welfare as X . The concept is due to Kolm (1969), Atkinson (1970), and Sen (1973).

We propose two more basic properties:

Property SYM (Symmetry or Anonymity)

For all $X \in \mathbb{R}_{++}^n : X \sim_W X^\pi$ where $X^\pi \in \mathbb{R}_{++}^n$ is any vector of incomes which can be obtained by a permutation of the components X_i of X .

Symmetry requires that social welfare does not depend on the way the individuals are numbered.

Property IND (Independence)

Take any $X, Y \in \mathbb{R}_{++}^n$ such that $X \sim_W Y$ and assume that there is $i \in \{1, \dots, n\}$ with $X_i = Y_i$. Then $X_{-i}\kappa \sim_W Y_{-i}\kappa$ for all $\kappa > 0$ where $X_{-i}\kappa$ is the income vector which is identical with X in all components $j \neq i$, and in which the i -th component is equal to $\kappa \in \mathbb{R}_{++}$.

Independence postulates that certain changes do not alter the equivalence of two vectors X and Y . If they are equivalent and if their i -th component (income) is the same, this income may be modified in the same way in X and Y without destroying the equivalence. The properties introduced up to now will be abbreviated by WELF (Welfare) since they represent the basic properties for social welfare orderings.

Definition 2

The property WELF comprises CONT, MON, SYM and IND.

These properties have strong implications:

Proposition 3

The ordering \succsim_W satisfies WELF if and only if \succsim_W can be represented by a social welfare function $W(X) = \sum_{i=1}^n g(X_i)$ where the function $g: \mathbb{R}_{++} \rightarrow \mathbb{R}$ is continuous and strictly increasing.

Thus the property WELF implies the complete strict separability of W . Furthermore W is symmetric in X_i , for $i = 1, \dots, n$.

A.2 Basic properties: Atkinson family

In the next step we discuss the homogeneity of \succsim_W .

Property REL (Relative homogeneity)

For all $X, Y \in \mathbb{R}_{++}^n : (X \sim_W Y \Rightarrow \lambda X \sim_W \lambda Y \text{ for all } \lambda > 0)$.

Property ABS (Absolute homogeneity)

For all $X, Y \in \mathbb{R}_{++}^n : (X \sim_W Y \Rightarrow X + \lambda \mathbf{1} \sim_W Y + \lambda \mathbf{1} \text{ for all } \lambda > 0)$.

In case of REL the equivalence of X and Y is preserved for a simultaneous multiplication of all incomes X_i, Y_i by the same scalar. Then the income shares remain constant. We obtain $\xi(\lambda X) = \lambda \xi(X)$; i.e. ξ is homothetic. ABS requires that the simultaneous addition of the same amount $\lambda > 0$ to all incomes does not affect the equivalence of X and Y . Then the absolute income differences are not changed and we get $\xi(X + \lambda \mathbf{1}) = \xi(X) + \lambda$, i.e. ξ is translatable.

The implications of REL are given by

Proposition 4

The ordering \succsim_W satisfies WELF and REL if and only if there is $\varepsilon \in \mathbb{R}^1$ such that \succsim_W can be represented by

$$W^\varepsilon(X) = \begin{cases} \left(\frac{1}{n} \sum_{i=1}^n X_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} & \text{for } \varepsilon \neq 1 \\ \prod_{i=1}^n X_i^{1/n} & \text{for } \varepsilon = 1. \end{cases}$$

It turns out that WELF and REL yield orderings which possess a particular form of welfare function. The social welfare functions are linearly homogeneous functions. They are called Atkinson welfare functions. The implications of ABS will be examined below.

Next we turn to the topic of redistribution.

Definition 3

X is obtained from Y by a progressive transfer if there are $i, j \in \{1, \dots, n\}$ and $\mathcal{G} > 0$ such that $X_h = Y_h$ for $h \neq i, h \neq j$, $X_i = Y_i + \mathcal{G}$, $X_j = Y_j - \mathcal{G}$, and $X_i \leq X_j$.

A progressive transfer redistributes a (small) amount of income from a richer individual to a poorer one without changing their ranking of incomes. It is evaluated by

Property PT (Progressive Transfers)

If X is derived from Y by a progressive transfer we have $X \succ_W Y$.

Thus the principle of progressive transfers PT requires that this kind of redistribution improves social welfare. The social welfare function is equity-preferring. Its implications are described by

Proposition 5

The ordering \succsim_W satisfies WELF, REL, and PT if and only if \succsim_W can be represented by $W^\varepsilon(X)$ for $\varepsilon > 0$.

Now we will discuss different population sizes m . It is obvious that the properties discussed do not suffice to make orderings \succsim_W^n and \succsim_W^m for $n \neq m$ consistent. We suppose that for

every $n \geq 3$ an ordering \succsim_W^n is given satisfying the properties WELF, REL, and PT. Then the family $\{\succsim_W^n\}_{n \geq 3}$ can be characterized by a sequence of social welfare functions

$$W_n(X) = g_n^{-1} \left(\frac{1}{n} \sum g_n(X_i) \right) \quad (A1)$$

for $X \in \mathbb{R}_{++}^n$ where

$$g_n(t) = \begin{cases} \frac{1}{1-\varepsilon_n} t^{1-\varepsilon_n} & \text{for } \varepsilon > 0 \\ \log t & \text{for } \varepsilon_n = 1. \end{cases} \quad (A2)$$

A priori the parameters ε_n need not be equal for different n .

We introduce Dalton's principle of population

Property PP (Principle of Population)

For all $n \geq 3, m \geq 2$ and $X, Y \in \mathbb{R}_{++}^n$ such that $X \succsim_W^n Y$ we have $X^{(m)} \succsim_W^{n \cdot m} Y^{(m)}$ where $X^{(m)}$ and $Y^{(m)}$ are the m -fold replications of X and Y , respectively.

If two vectors of income X and Y are equivalent for a population of size n the equivalence also holds for the replicated distributions in the m -fold replicated population. Then the equivalence depends only on the distribution of incomes (in the statistical sense) or on the distribution functions. Then we obtain

Proposition 6

The orderings $\{\succsim_W^n\}_{n \geq 3}$ satisfy the properties WELF, REL, PT, and PP if and only if there is exactly one parameter $\varepsilon > 0$ such that the ordering \succsim_W^n can be represented by (A1) and (A2) for any $n \geq 3$.

This proposition characterizes the class of Atkinson social welfare functions for every n .

A.3 Basic properties: Gini family

In a second step we generalize the framework by weakening the property IND. We propose

Property IND \downarrow (Independence for rank-ordered vectors)

Take any $X, Y \in \mathbb{R}_{++}^n$ such that $X_{\downarrow} \sim_W Y_{\downarrow}$ and assume that there is $i \in \{1, \dots, n\}$ with $X_{[i]} = Y_{[i]}$.

Then $X_{\downarrow-i} \kappa \sim_W Y_{\downarrow-i} \kappa$ for all $\kappa > 0$.

It considers ordered vectors of income being equivalent with respect to social welfare. If their i -th components coincide the income vectors can be changed simultaneously in the i -th position without modifying the ranking. Then the new vectors are still equivalent; i.e. the equivalence does not depend on the level $X_{[i]} = Y_{[i]}$. It should be stressed that this property does not imply that $X_{\downarrow} \sim_W X_{\downarrow-i} \alpha$. This equivalence will in general not hold.

Abbreviating the properties CONT, MON, SYM, and IND \downarrow by WELF \downarrow we obtain

Proposition 3*

The ordering \succsim_W satisfies WELF \downarrow if and only if \succsim_W can be represented by a social welfare function

$$W(X) = g_0^{-1} \left(\sum_{i=1}^n g_i(X_{[i]}) \right)$$

where the functions $g_i : \mathbb{R}_{++} \rightarrow \mathbb{R}$ for $i = 1, \dots, n$ and $g_0 : \mathbb{R} \rightarrow \mathbb{R}$ are continuous and strictly increasing.

In this case the ranking of incomes has to be taken into account. Individuals i 's contribution to social welfare depends on the position of X_i in X_{\downarrow} .

Similarly we get

Proposition 4*

The ordering \succsim_W satisfies WELF \downarrow and REL if and only if there are $\varepsilon \in \mathbb{R}^1$ and

$w_i > 0, i = 1, \dots, n$, with $\sum_{i=1}^n w_i = 1$ such that \succsim_W can be represented by

$$W_{\downarrow}^{\varepsilon}(X) = \begin{cases} \left(\sum_{i=1}^n w_i X_{[i]}^{1-\varepsilon} \right)^{1/(1-\varepsilon)} & \text{for } \varepsilon \neq 1 \\ \prod_{i=1}^n X_{[i]}^{w_i} & \text{for } \varepsilon = 1. \end{cases} \quad (\text{A3})$$

Again the ranks have to be taken into account. The social welfare functions are linearly homogeneous functions defined on the rank ordered income vector.

The principle of progressive transfers imposes additional restrictions:

Proposition 5*

The ordering \succsim_w satisfies $WELF\downarrow$, REL , and PT if and only if \succsim_w can be represented by $W_{\downarrow}^{\varepsilon}(X)$ in (+) where ($\varepsilon > 0$ and $w_i \leq w_{i+1}$) or ($\varepsilon = 0$ and $w_i < w_{i+1}$).

In this case also linear social welfare functions are admitted. But their weights w_i have to be strictly increasing. Then relatively lower incomes (having higher ranks get more weight in the “calculation” of social welfare).

Now let us again consider a variable population size, assuming that \succsim_w^n is represented by

$$W_{\downarrow n}(X) = g_n^{-1} \left(\sum_{i=1}^n w_i^n g_n(X_{[i]}) \right).$$

When we additionally impose the principle of population we obtain an interesting result:

Proposition 6*

The orderings $\{\succsim_w^n\}_{n \geq 3}$ satisfy the properties $WELF\downarrow$, REL , PT , and PP if and only if there $\varepsilon \leq 1$ and a strictly increasing function $f: [0,1] \rightarrow [0,1]$ such that 1) $f(0) = 0$, 2) $f(1) = 1$, 3) f is continuous on $[0,1)$, 4) f is (strictly) convex if $\varepsilon > 0$ ($\varepsilon = 0$), and

$$5) \quad W_{\downarrow n}^{\varepsilon}(X) = \begin{cases} \left(\sum_{i=1}^n \left(f\left(\frac{i}{n}\right) - f\left(\frac{i-1}{n}\right) \right) X_{[i]}^{1-\varepsilon} \right)^{1/(1-\varepsilon)} & \text{for } \varepsilon \geq 0 \\ \prod_{i=1}^n X_{[i]}^{(f(i/n) - f((i-1)/n))} & \text{for } \varepsilon = 1. \end{cases}$$

Then the orderings \succsim_W^n are completely defined by the parameter ε and the function f . The latter determines the weights of the different income classes, taking into account the ranking of incomes, but ignoring the level of income.

In this framework we now introduce a principle of aggregation:

Property RA (Restricted Aggregation)

For all $X \in \mathbb{R}_{++}^n$, $3 \leq k \leq n$, and $n \geq 3$ we have

$$X \sim_W^n \left(\xi^k \left(X_{\downarrow}^k \right), \dots, \xi^k \left(X_{\downarrow}^k \right), X_{[k+1]}, \dots, X_{[n]} \right)$$

where $X_{\downarrow}^k = \left(X_{[1]}, \dots, X_{[k]} \right)$.

This property allows us to replace the k -highest incomes by the corresponding equally distributed income $\xi^k \left(X_{\downarrow}^k \right)$ which is determined by \succsim_W^k . Since this substitution is only allowed for the k -highest incomes (and not for arbitrary subgroups) aggregation is restricted. We obtain

Proposition 7

The orderings $\left\{ \succsim_W^n \right\}_{n \geq 3}$ satisfy the properties $WELF_{\downarrow}$, REL , PT , PP , and RA if and only if \succsim_W^n can be represented by (A3) and $f(t) = t^{\delta}$ where either ($\varepsilon > 0$ and $\delta \leq 1$) or ($\varepsilon = 0$ and $\delta > 1$).

Thus the weights have to possess a specific form. The orderings characterized depend on two parameters, namely on ε and δ .

Finally we impose the property ABS considering absolute changes of incomes. Then we end up with

Proposition 8

The orderings $\left\{ \succsim_W^n \right\}_{n \geq 3}$ satisfy the properties $WELF_{\downarrow}$, REL , PT , PP , RA , and ABS if and only if \succsim_W^n can be represented by

$$W_{\downarrow n}^{\delta}(X) = \sum_{i=1}^n \left(\left(\frac{i}{n} \right)^{\delta} - \left(\frac{i-1}{n} \right)^{\delta} \right) X_{[i]} \text{ for } \delta > 1.$$

This family of social welfare functions will be called Gini functions since the orderings \succsim_W^n for $\delta = 2$ yields the Gini coefficient as inequality measure.

A.4 Comparison and discussion

In summary, we have obtained a characterization of the Atkinson and Gini social welfare functions. It is clear that for the Gini welfare functions the ranking of income has to be taken into account. (IND has to be replaced by $\text{IND}\downarrow$.) Furthermore they satisfy the property ABS. A closer inspection reveals that the Atkinson family also satisfies $\text{IND}\downarrow$ and RA. If we add another aggregation property we get another characterization

Property RA* (Restricted Aggregation*)

For all $X \in \mathbb{R}_{++}^n$, $3 \leq k \leq n$, and $n \geq 3$ we have

$$X \sim_W^n \left(X_{[1]}, \dots, X_{[n-k]}, \xi^k \left[\tilde{X}_{\downarrow}^k \right], \dots, \xi^k \left[\tilde{X}_{\downarrow}^k \right] \right)$$

where $\tilde{X}_{\downarrow}^k = (X_{[n-k+1]}, \dots, X_{[n]})$.

In this case the k -lowest incomes can be replaced by the corresponding EDEI. It turns out that the aggregation properties PP, RA, and RA* are ‘incompatible’: it is then no longer possible to take into account the ranking of incomes. We obtain

Proposition 8*

The orderings $\{\succsim_W^n\}_{n \geq 3}$ satisfy the properties $\text{WELF}\downarrow$, REL, PT, PP, RA, and RA* if and only if there is exactly one parameter $\varepsilon > 0$ such that the \succsim_W^n can be represented by $W_n^\varepsilon(X)$ defined in (A1) and (A2).

Therefore the differences between the Atkinson and Gini family of social welfare functions can also be described as follows: The respective orderings \succsim_W^n satisfy $\text{WELF}\downarrow$, REL, PT, PP, and RA. The Gini functions satisfy ABS in addition, the Atkinson family additionally fulfills RA*.

A.5 Inequality measures

The ethical measurement of inequality is based on social welfare orderings. Given a welfare ordering \succsim_w we are able to derive a cardinal measure of inequality. It is defined by

$$I(X) := \frac{\mu(X) - \xi(X)}{\mu(X)} = 1 - \xi(X)/\mu(X)$$

and represents the welfare loss due to inequality. It is equal to the share of total income which is not necessary to attain the level of social welfare implied by X . If each individual receives the EDEI $\xi(X)$ we have $\xi(X)\mathbf{1} \sim_w X$. This construction goes back to Kolm (1969), Atkinson (1970), Sen (1973). The measure is therefore called AKS measure. The inequality measure characterized by Proposition 7 is given by

$$I_{\delta,\varepsilon}(X) = 1 - \left(\sum_{i=1}^n \left[\left(\frac{i}{n} \right)^\delta - \left(\frac{i-1}{n} \right)^\delta \right] \left(\frac{X_i}{\mu(X)} \right)^\varepsilon \right)^{1/\varepsilon} \text{ for } \delta \geq 1, \varepsilon \geq 0, \delta + \varepsilon > 1.$$

This two-parameter family comprises the Atkinson family $I_{1,\varepsilon}(X)$ for $\delta = 1$:

$$I_{1,\varepsilon}(X) = 1 - \left(\frac{1}{n} \sum_{i=1}^n \left(\frac{X_i}{\mu(X)} \right)^{1-\varepsilon} \right)^{1/(1-\varepsilon)}$$

and the Gini family $I_{\delta,0}(X)$ for $\varepsilon = 0$

$$I_{\delta,0}(X) = 1 - \left(\sum_{i=1}^n \left[\left(\frac{i}{n} \right)^\delta - \left(\frac{i-1}{n} \right)^\delta \right] \frac{X_i}{\mu(X)} \right).$$

It turns out that there is a simple relationship between the EDEI and the corresponding ethical inequality measure. We obtain

$$\xi(X) = \mu(X)(1 - I(X)) \text{ or } \xi(X) = \mu(X)E(X)$$

where

$$E(X) = 1 - I(X) = \xi(X)/\mu(X)$$

denotes the corresponding measure of equality.

Appendix B: LIS-program

The program is based on the SPSS programs for a computation of the Atkinson measure ($\varepsilon = 0.5$) and the conventional Gini coefficient provided by LIS (2004). It was rewritten and generalized to compute the Atkinson-Gini welfare function and the corresponding AKS inequality measure for $\delta \geq 1$ and $\varepsilon \geq 0$.

```
** user=

** password=

** package=spss

** project=lis

define run ()

** avoid double counting of cases.

** select cases where dpi is filled.

select if d5 ne 3 and dpi ne 0 and dpi ne -1.

** replace weight if hweight=0.

if hweight= 0 hweight=1.

** create person weight as hweight times number of persons.

compute pweight=d4*hweight.

** if the number of equivalent adults is chosen as weight:

** compute pweight=d4**.5*hweight.

weight by pweight.

** set equivalence scale as square root of number of persons.

compute y=dpi/d4**.5.

compute group = 1.

**** aggregate to get mean ei.

agg out=outmn/pre/break=group/meanei=mean(y).

*****use rank to get median dpi.
```

rank dpi by group/ ties=high/rfraction into medn/PRINT=NO.

compute half=0.

if medn ge .5 half=1.

temporary.

select if half=1.

agg out=outmed/pre/break=group/medndpi=min(dpi).

match files file=*/table=outmn/by group.

match files file=*/table=outmed/by group.

** topcoding at ten times unequivalized median.

if dpi gt (10*medndpi) y=(10*medndpi)/d4** .5.

** bottomcoding at one percent equivalized mean.

if y lt (.01*meanei) y=.01*meanei.

sort cases by y (D).

compute oldcum=oldcum+0.

compute delta=1.0

compute epsilon=.1

** Computation of EDEI

compute power=1. - epsilon.

** cumulative number of (equivalent) individuals

compute cumwgt=cumwgt + pweight.

** weight in social welfare function

compute coeff=cumwgt**delta - oldcum**delta.

compute term=coeff*y**power/pweight.

compute oldcum=cumwgt.

```
leave cumwgt,oldcum,delta .

aggregate outfile= *.

    /presorted.

    /break=group.

    /SWF=SUM(TERM).

    /meany= mean(y).

    /DENOM=MAX(CUMWGT).

    /n=n.

*****

compute delta=1.0

compute epsilon=.1

*****

compute power=1. - epsilon

** Normalization

compute swf=swf/denom**delta.

** EDEI

compute swf=swf**(1./power)

** AKS-measure

compute index=(meany-swf)/meany.

formats index (f10.4).

list var=index meany delta epsilon.

!enddefine.
```