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ABSTRACT

When incorporating differences in household characteristics, the choice of equivalence scale can affect the ranking of income distributions. An alternative approach was pioneered by A.B. Atkinson and F. Bourguignon (G.R. Feiwel (Ed.), Arrow and the Foundation of the Theory of Economic Policy, Macmillan, New York, 1987), who derive a sequential Lorenz dominance criterion for com-paring distributions with an identical population structure. In order to make their approach applicable to international comparisons, we extend their criterion to the case of different marginal distributions of household types, and derive a sequential stochastic dominance criterion that highlights the importance of first order dominance of the marginal distribution of household characteristics for obtaining consistent rankings of income distributions. Comparisons of distributions are made using the Luxembourg Income Study database for a number of countries.

Journal of Economic Literature Classification Number: D31, D63. *Keywords*: Income distributions, Differences in needs, Stochastic dominance, International Welfare Comparisons.

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1. Introduction

When comparing the income distributions of different countries, care has to be taken in order to make the comparison valid. In addition to the obvious issues of currency conversion and data source compatibility, there is also the matter of taking account of differences in population structure. A country with a greater proportion of large families would generally be considered to have a lower level of social welfare than one with few large families and a similar distribution of income. The standard approach to dealing with the heterogeneity of the population in distributional analysis is to define equivalence scale factors which can be used to deflate household income. The distributions of equivalent income can then be compared as if they applied to a homogeneous population using the concept of Lorenz dominance in order to obtain unambiguous welfare rankings based on the seminal work of Atkinson (1970) and developments of that work using the criterion of generalised Lorenz dominance [see Marshall and Olkin (1979), Shorrocks (1983)]. This type of approach has been adopted in international comparisons by for example, Bishop, Formby and Smith (1991) and Atkinson, Rainwater and Smeeding (1995). However, the equivalence scale approach sufiers from two major weaknesses. Firstly, the Pigou-Dalton transfer principle which underpins the Lorenz ordering concerns the transfer of a fixed amount of income between rich and poor. A transfer of a fixed amount of equivalised income is less readily comprehended [see e.g. Ebert and Moyes (2002)]. A second shortcoming with this approach is that there is no consensus on how to define equivalence scales and different approaches give rise to different rankings of income distributions [see Buhmann, Rainwater, Schmaus and Smeeding (1988), Coulter, Cowell and Jenkins (1992)].

An alternative approach to taking account of differences in household size and composition when making welfare comparisons, initiated by Atkinson and Bourguignon (1982), avoids these problems by treating the question as an issue of multidimensional distributions, and using multidimensional criteria to make inequality and welfare comparisons. In a later paper, Atkinson and Bourguignon (1987) introduce the concept of sequential Lorenz dominance and show that this criterion provides an unambiguous welfare ranking of heterogeneous distributions as long as one subscribes to a minimal set of value judgements regarding the way differences in needs affect households' welfare and so long as the marginal distribution of needs is identical in the two situations under comparison. Thus, their approach is well-suited to comparisons of income distributions before and after tax for a given country. But their approach does not apply when the marginal distributions of needs or abilities are different in the two distributions under examination as would generally be the case with international comparisons or for a given country over a long period.

Jenkins and Lambert (1993), and more recently Lambert and Ramos (2002), have extended the work of Atkinson and Bourguignon to the case where the marginal distributions differ. Interestingly, incorporating differences in the distribution of needs or abilities does not greatly modify the sequential Lorenz dominance criteria that are obtained. However, there is a key difference between the assumptions made by Jenkins and Lambert, and those of Atkinson and Bourguignon in that the former constrain further the class of household utility functions by assuming that the utility obtained from the maximum level of income in the distribution is the same whatever the level of household need. At first sight this additional condition would not appear to be particularly stringent. However, due to the multidimensional nature of the dominance criteria, we show that because of this assumption the Jenkins and Lambert approach can give rise to a different set of welfare rankings of income distributions depending on the value of maximum income chosen when undertaking the comparison. In other words, it is possible to find cases in which the distribution of country A dominates that of country B in a pairwise comparison, but when a third country C with a higher maximum income is included in the comparison, it is possible that the criterion proposed no longer ranks A above B in welfare terms. We show that this is not simply a theoretical possibility since situations of this kind are found in practice when making comparisons of income distributions for a number of industrialised countries using data from the Luxembourg Income Study. We go on to show that the assumption made by Jenkins and Lambert does not represent a necessary condition for dominance and as such the restrictions on the class of utility functions for ranking income distributions can be weakened. We therefore propose an alternative basis for undertaking welfare comparisons of income distributions when population structures differ that can be straightforwardly applied to international comparisons and we use the criteria obtained to make international comparisons using the Luxembourg Income Study data.

The paper is organised as follows. In the first section we present the framework for making multidimensional comparisons with heterogeneous populations and spell out the different restrictions placed on utility functions in the tradition of Atkinson and Bourguignon (1987). In the second section, we present Jenkins and Lambert's (1993) extension of this approach to the case of different marginal distributions of needs. We apply their criteria to the Luxemburg Income Study and then highlight a problem with their approach. In the third section, we show how this problem can be avoided by introducing an additional requirement to be met when making welfare comparisons of income distributions with different population structures. We go on to derive a new set of criteria that enable welfare rankings to be made in an unambiguous manner and we use these criteria to perform international comparisons using the Luxembourg Income Study data. We conclude the paper by considering some implementation issues and by suggesting a number of avenues for future work.

2. Previous Work

2.1. Definitions and notation.

We consider *household populations* of fixed size n where each household is endowed with two attributes: *income* and *ability*[1]. We assume that there exists a finite number H ($2 \leq H \leq n$) of distinct abilities or types and we denote as H*:= {1, 2, … ,H} the set of types. The index $h \in H^*$ must be interpreted as an ordinal measure of ability which is a decreasing function of family size for instance: h = 1 distinguishing the least able type of household, h = 2 the second least able, and so on[2]. A *heterogeneous distribution* or *situation* is a partitioned vectors $s \equiv (x;a) := (x_1,...,x_n;a_1,....,a_n)$ *where* $x_i \in D := [\underline{v}, \overline{v}]$ for some $-\infty < \underline{v} < \overline{v} \leq +\infty$[3], and $a_i \in H^*$

[1] We prefer to use ability instead of needs in order to have an argument that increases utility: more able can be interpreted as less needy. The fact that we restrict ourselves with populations of fixed size is made for simplicity and does by no means affect the significance of the results. These extend to the general case where the populations sizes differ by application of the principle of population of Dalton (1920).

[2] This way of reasoning amounts to assimilating family size with a handicap for the household in the same way as a deficient health constitutes a handicap for an individual.

[3] The lower and upper bounds $\underline{v}$ and v are chosen arbitrarily small and large respectively so that all observed incomes belong to D. Although it is theoretically possible to conceive of any such upper and lower bounds for individual incomes, a practical problem might arise about the value of v when implementing the tests of dominance as we will see later on.

are respectively the income and ability of household i and we will let Z represent the set of heterogeneous distributions or situations. The *joint density function* corresponding to situation $s \equiv (x;a) \in Z^*$ is denoted as $f(y,h)$ and

$$(2.1) \qquad F(y,h) := \sum_{r=1}^{h} \int_{\underline{v}}^{y} f(\xi, r) d\xi, \quad \forall h \in H^*, \quad \forall y \in \mathrm{D}$$

is the joint cumulative distribution function[4]. The marginal densities of incomes and abilities are defined in the usual way and denoted as $f(y,H)$ and $f(\bar{v},h)$ respectively. Similarly the *marginal distributions of incomes and abilities* are represented by $F(y,H)$ and $F(\bar{v},h)$ respectively[5]. Given two situations $s^\circ \equiv (x^\circ; a^\circ), s^* \equiv (x^*; a^*) \in Z^*$, we find it convenient to denote as f° and f^* (resp. F° and F^*) the corresponding joint density (resp. distribution) functions. We follow Atkinson and Bourguignon (1987) and consider a utilitarian social welfare function so that

$$(2.2) \qquad W_U(s) := \sum_{i=1}^{n} \left(\frac{1}{n}\right) U(x_i, a_i) \equiv \sum_{h=1}^{H} \int_{\underline{v}}^{\bar{v}} U(y,h) f(y,h) dy$$

represents the social welfare per household in situation $s \equiv (x;a) \in Z^*$, where $U : D \times H^* \to \Re$ is the household's utility function[6].

2.2. Conditions on the utility function.

The utility function is assumed to be continuous and twice differentiable in income. Furthermore,

[4] The situations under consideration being discrete, the distribution function exhibits jumps and the preceding definition is not rigorously correct. It can however be justified on the grounds of arguments presented and discussed by Fishburn and Vickson (1978).

[5] Given a situation $s \in Z^*$, it is worth noting that $F(y,h)=F(\max\{x_i\},h)$ for all $y > \max\{x_i\}$ which in particular implies that $F(\bar{v},h)= F(\max\{x_i\},h)$.

[6] There are two possible ways of modelling social welfare which basically rest on the meaning of the utility function. Here we follow Atkinson and Bourguignon (1987) and interpret *U(y; h)* as the total welfare of a household of type h receiving income y. Another possibility is to assume that *U(y; h)* is the utility of a representative member of a type-h household with income y, in which case *f(y;h)* represents the proportion of individuals living in type-h household with income y.

we assume that the utility function has the same properties as those specified by Atkinson and Bourguignon (1987):

$$(U.1) \qquad U_y(y,h) \geq U_y(y,h+1) \geq 0, \quad \forall y \in D, \quad \forall h \in \{1,2,...H-1\}$$

$$(U.2) \qquad U_{yy}(y,h) \leq U_{yy}(y,h+1) \leq 0, \quad \forall y \in D, \quad \forall h \in \{1,2,...H-1\}$$

where $U_y(y,h)$ and $U_{yy}(y,h)$ refer to the first and second partial derivatives with respect to income respectively. The first condition states that a household's welfare cannot decrease as a result of an increase in income for all income levels, and that the impact on social welfare of a given increase in income is larger the less able the beneficiary household. The second condition concerns the effect of a progressive transfer on social welfare. If income is redistributed among households of a given type, social welfare cannot decrease, and the transfer will have a greater effect on social welfare the less able are the households involved in the transfer.

2.3. The welfare ranking of situations.

Given this framework, the difference in social welfare between two situations is given by:

$$(2.3) \qquad \Delta W_U := W_U(s^*) - W_U(s^\circ) = \sum_{h=1}^{H} \left\{ \int_{\underline{y}}^{\bar{y}} U(y,h) \Delta f(y,h) dy \right\}$$

where $\Delta f(y,h) \equiv f^*(y,h) - f^\circ(y,h) \quad \text{for all } h \in H^* \text{ and for all } y \in \mathrm{D}$. Integrating (2.3) by parts twice, we have:

$$(2.4) \qquad \Delta W_u = \sum_{h=1}^{H} U(\bar{y},h) \int_{\underline{y}}^{\bar{y}} \Delta f(y,h) dy - \sum_{h=1}^{H} U_y(\bar{y},h) \int_{\underline{y}}^{\bar{y}} \left[\int_{\underline{y}}^{y} \Delta f(\xi,h) d\xi \right] dy$$
$$+ \sum_{h=1}^{H} \left\{ \int_{\underline{y}}^{\bar{y}} U_{yy}(y,h) \left[\int_{\underline{y}}^{y} \int_{\underline{y}}^{\xi} \Delta f(\vartheta,h) d\vartheta d\xi \right] dy \right\}$$

Applying Abel's decomposition rule to every summation in the preceding formula, and noting that $\Delta F(y,h) \equiv F^*(y,h) - F^\circ(y,h)$, we obtain:

$$(2.5) \qquad \Delta W_u = -\sum_{h=1}^{H-1}\left[U(\bar{v},h+1) - U(\bar{v},h)\right]\Delta F(\bar{v},h) + U(\bar{v},H)\Delta F(\bar{v},h)$$
$$+\sum_{h=1}^{H-1}\left[U_y(\bar{v},h+1) - U_y(\bar{v},h)\right]\int_{\underline{v}}^{\bar{v}}\Delta F(y,h)dy - U(\bar{v},H)\int_{\underline{v}}^{\bar{v}}\Delta F(y,h)dy$$
$$-\int_{\underline{v}}^{\bar{v}}\left\{\sum_{h=1}^{H-1}\left[U_{yy}(y,h+1) - U_{yy}(y,h)\right]\int_{\underline{v}}^{y}\Delta F(\xi,h)d\xi - U_{yy}(y,H)\int_{\underline{v}}^{y}\Delta F(\xi,h)d\xi\right\}dy$$

In the first line of (2.5), the second term is zero by definition, since $F^*(\bar{v},H) = F^\circ(\bar{v},H) = 1$. In the second line, the term in square brackets is negative by condition U.1 and the term in square brackets in the third line is positive by condition U.2. Atkinson and Bourguignon (1987) consider the case where the marginal distribution of abilities is identical in the two situations so that $F^*(\bar{v},h) = F^\circ(\bar{v},h),$ for all h. In this case the first line of (2.5) is zero, and the overall difference in welfare will be non-negative if:

$$(T.1) \qquad \int_{\underline{v}}^{y} F^*(\xi,h)d\xi \quad \leq \quad \int_{\underline{v}}^{y} F^\circ(\xi,h)d\xi \quad \forall y \in D, \quad \forall h \in H$$

Using the definition of a conditional density function $f(y \mid h) = f(y,h)/f(\bar{v},h)$, this criterion is equivalent to:

$$(2.6) \qquad \sum_{r=1}^{h} f^*(\bar{v},r)\int_{\underline{v}}^{y} F^*(\xi \mid r)d\xi \quad \leq \quad \sum_{r=1}^{h} f^\circ(\bar{v},r)\int_{\underline{v}}^{y} F^\circ(\xi \mid r)d\xi \quad \forall y \in D, \quad \forall h \in H$$

Since the marginal distributions of ability are identical i.e. $f^*(\bar{v},r) = f^\circ(\bar{v},r) = f(\bar{v},r)$ for all r, (2.6) reduces to the condition that F^* dominates F° in the sense of the sequential generalised Lorenz criterion. This path-breaking result by Atkinson and Bourguignon (1987) brings out an important issue when making welfare comparisons of multidimensional distributions. Condition

T.1 is essentially a test of second order stochastic dominance in income but it has to be applied sequentially. One first tests for dominance in income for the least able group. If one distribution dominates, the exercise is continued by adding in the second least able group and testing for dominance for the two groups combined. If one distribution still dominates, the next least able group is added and so forth, until all groups i.e. the total population, are included in the comparison. If at any stage in the sequence there is no longer dominance or if the ranking is reversed, then neither situation is said to dominate the other in welfare terms. Since the condition is expressed in terms of the conditional distribution functions, it is possible to undertake comparisons using generalised Lorenz curves instead of the integrals in condition T.1.

3. Jenkins and Lambert's Extension

3.1. An additional condition on the utility function.

A problem arises however for this dominance criterion for distributions where the marginal distribution of abilities is not the same in the situations to be evaluated as in the case of international comparisons. In this case $F^*(\bar{v},h) \neq F^\circ(\bar{v},h)$ for at least one $h \in \{1, 2,..., H-1\}$, and the first line in (2.5) no longer cancels. Jenkins and Lambert (1993) derive essentially the same sequential second order stochastic dominance criterion (given in condition T.1) by adding the further assumption that:

$$(U.3) \qquad U(\bar{v},h) = U(\bar{v},h+1) \geq 0, \qquad \forall h = 1, 2,..., H-1$$

This condition states that all household types would derive the same level of welfare from the maximum level of income. Together with condition U.1, which states that $U_y(y,h) \geq U_y(y,h+1) \geq 0$ for all $y \in D$, it is interesting to note that condition U.3 implies that

$$(3.1) \qquad U(y,h+1) \geq U(y,h) \qquad \forall y < \bar{v}, \qquad \forall h = 1, 2,..., H-1$$

or that less able households derive less welfare from a given level of income than more able households. It is clear from (2.5) that if the marginal distribution of abilities is different in the two distributions, so that $F^*(\bar{v},h)-F^{\circ}(\bar{v},h)\neq 0$ for some h, then Jenkins and Lambert's condition U.3 ensures that the first line is still equal to zero and the sequential second order dominance criterion T.1 applies. We emphasize that the sign of the term $F^*(\bar{v},h)-F^{\circ}(\bar{v},h)$, has no role to play in establishing a welfare ranking of situations due to condition U.3.

3.2. An application with the Luxembourg Income Study database.

In order to illustrate these results, we compare the distributions of income and family size for a selection of eight OECD countries using data from the Luxemburg Income Study for the years 1994 and 1995, namely: United States (US), France (FR), United Kingdom (UK), Germany (GR), Italy (IT), Netherlands (NL), Canada (CAN) and Australia (AUS).We group households into three groups by increasing level of ability: married couples with children (the least able group), married couples without children and single persons with no children. Income is defined to include total household income after tax plus transfers, and is converted into dollars using purchasing power parity exchange rates. Consumer price indices are also used to convert 1995 incomes to 1994 levels where applicable. Using these data, we construct the joint distribution function for each household type, using household sampling weights. It is important to make a distinction between (i) the theoretical income range $D:=[\underline{v},\bar{v}]$, and (ii) the income range $\overline{D}(S)$ in practice, where $S\subset Z$ is the subset of situations we want to compare. For each situation $s\equiv(x;a)$ we then calculate the integral of the joint distribution function for each type and for each country using the following approximation for discrete data [see Anderson (1996)]:

$$(3.2)\qquad \int_{\underline{v}}^{y_j} F(\xi\mid h)d\xi \cong 0.5[F(y_j\mid h)+\sum_{i=1}^{j-1}(d_i+d_{i+1})F(y_i\mid h)] \qquad \forall y_j\in\overline{D}(\{s\}),\quad \forall h\in H$$

where $\overline{D}(\{s\}):=[\min\{x_i\},\max\{x_i\}]$, is the income range chosen for checking dominance and d_j is the length of the jth income interval so that $\overline{D}(\{s\})=\bigcup_{j=1}^{q} d_j$ for some q ≥ 2. Comparisons are made in the following way. For each country, the above integral is calculated over the union of the income domains of the country in question and that of the United States, which in all cases contains the highest income for each ability group. For each pair of countries, sequential dominance is examined by comparing the value of the countries' integrals using the deciles of the United States income distribution (for each group of households in the sequential procedure). In this way, all pairwise comparisons are made at the same dollar income levels. The results of the sequential procedure proposed by Jenkins and Lambert (1993) are presented in Tables 3.1 and 3.2. The country abbreviation that figures in the cells of the tables indicates which country dominates in a pairwise comparison. A question mark indicates that no country dominates and a question mark with a '#' means that two integrals cross in the highest decile of the income distribution only.

On the basis of Jenkins and Lambert's sequential criterion, we are able to obtain a ranking in 12 out of 28 possible pairwise rankings. If attention had been confined to married couples, more rankings would have been possible, but the sequential nature of the dominance criteria means that when the most able households (single persons) are added in, there is no longer overall dominance. We will provide further more detailed discussion of the results in a later section.

Table 3.1. Sequential Second Order Stochastic Dominance

a. Couples with Children

	US	FR	UK	GE	IT	NL	CAN
FR	?						
UK	?#	FR					
GE	?	?#	GE				
IT	US	FR	UK	GE			
NL	?#	FR	?	GE	NL		
CAN	?#	?#	CAN	?#	CAN	CAN	
AUS	US	FR	?	GE	AUS	NL	CAN

b. All Couples

	US	FR	UK	GE	IT	NL	CAN
FR	?#						
UK	US	FR					
GE	?	?	GE				
IT	US	FR	UK	GE			
NL	US	FR	?	GE	NL		
CAN	?	?#	CAN	?#	CAN	CAN	
AUS	US	FR	?	GE	AUS	?	CAN

c. All Families

	US	FR	UK	GE	IT	NL	CAN
FR	?						
UK	?	?					
GE	?	?	?				
IT	US	FR	UK	GE			
NL	US	FR	?	?	NL		
CAN	?	CAN	CAN	CAN	CAN	CAN	
AUS	US	FR	UK	?	?	?	CAN

Table 3.2. Comparisons of Countries on the Basis of Living Standards [Condition T.1]

	US	FR	UK	GE	IT	NL	CAN
FR	?						
UK	?	?					
GE	?	?	?				
IT	US	FR	UK	GE			
NL	?	FR	?	?	NL		
CAN	?	?	CAN	?	CAN	CAN	
AUS	US	FR	?	?	?	?	CAN

3.3. A problem with the Jenkins and Lambert's approach.

Like Atkinson and Bourguignon, their criterion is also applied sequentially beginning with the least able group. However, the sign of the difference $F^*(y,h) - F°(y,h)$ plays no role in their dominance criterion, and this can give rise to an anomaly. Recall that $F^*(\bar{v},h)$ and $F°(\bar{v},h)$ represent the marginal cumulative distribution functions of ability in situations s* and s° respectively. When testing for second order stochastic dominance for households with levels of ability less than or equal to m over the income domain $\overline{D}(\{s^*,s°\}) := \left[\min\{x_i^*,x_i^o\},\max\{x_i^*,x_i^o\}\right]$, the slope of the integral for $y \geq \max\{x_i^*,x_i^o\}$ is equal to $F(\max\{x_i^*,x_i^o\},m)$, the value of the marginal cumulative distribution function for ability level m. Thus if the integral of *F*(y,m)* is beneath the integral of *F°(y,m)* over the domain $\overline{D}(\{s^*,s°\})$, but there is a smaller proportion of households with ability less than or equal to m in situation s° so that $F^*(\max\{x_i^*,x_i^o\},m) > F°(\max\{x_i^*,x_i^o\},m)$, then at some income level v* with $\max\{x_i^*,x_i^o\} <$ v* the dominance criterion will be violated. It is important to note that this situation arises only in the multidimensional case with different population structures.

In the unidimensional case the slope of the integral after the highest income level is equal to unity for both distributions. More precisely given two income distributions $x^* := (x_1^*,...,x_n^*)$ and $x° := (x_1^o,...,x_n^o)$ dominance over the domain $\overline{D}(\{x^*,x°\})$ implies dominance over any domain V such that $\overline{D}(\{s^*,s°\}) \subset V$. In the multidimensional context, the same applies to the Atkinson-Bourguignon criterion since their approach is confined to the case where the marginal distribution of ability is identical in the distributions under consideration so that $F^*(\max\{x_i^*,x_i^o\},m) = F°(\max\{x_i^*,x_i^o\},m)$. In this case the slope of the integral for incomes y > $\max\{x_i^*,x_i^o\}$ is identical and the two integrals are parallel beyond that level of income. The same is not true of the Jenkins and Lambert criterion: if there are different marginal distributions, the two integrals can either diverge (in which case the dominance condition is robust) or they can intersect which is the possibility pointed out above. A consequence of this condition on the marginal distribution not being satisfied is that the choice of upper limit to the income domain over which distributions are compared can influence the ranking of distributions as the following example shows. Consider the three situations

$$s^1 \equiv (x^1; a^1) = \begin{bmatrix} 3 & 1 \\ 3 & 1 \\ 2 & 2 \end{bmatrix}, \quad s^2 \equiv (x^2; a^2) = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 3 & 2 \end{bmatrix}, \quad s^3 \equiv (x^3; a^3) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 5 & 2 \end{bmatrix}$$

where the first column contains income levels and the second ability levels. Suppose we wish to compare s1 and s2 and that we apply condition T.1 over the domain

$$(3.3) \qquad \overline{D}\left(\left\{s^1, s^2\right\}\right) = [2, 3]$$

Then we conclude that situation s^1 dominates situation s^2. Suppose we now introduce s^3 into the comparison so that

$$(3.4) \qquad \overline{D}\left(\left\{s^1, s^2, s^3\right\}\right) = [1, 5]$$

Then we check that

$$(3.5) \qquad \int_{\underline{y}}^{y} F^1(\xi, 1) d\xi \quad < \quad \int_{\underline{y}}^{y} F^2(\xi, 1) d\xi \quad \forall y \in (1, 4)$$

with equality for y = 4, while the opposite sign occurs for $y \in (4; 5]$. In this case, s^1 no longer dominates s^2, although both situations actually dominate s^3 over the interval $\overline{D}\left(\left\{s^1, s^2, s^3\right\}\right)$. This example shows how the ranking between two countries is altered by the addition of a third which changes upper limit to the income domain.

The issue of upper income limit however is not one of purely conceptual importance since using the Luxembourg Income Study, there are a number of cases in which pairwise dominance is violated when including a third country with a higher maximum income in the comparison in two cases. Continuing with the example presented above in Tables 3.1 and 3.2, for each ability level, the highest income is found in the United States sample. Thus where Jenkins and Lambert's criterion indicates higher welfare in country A than in country B in a pairwise comparison, when the comparison set is enlarged by including the United States and the distributions are compared over the union of the three income domains, country A no longer has higher welfare than country B. The choice of income domain therefore influences the outcome in practice. This occurs in certain cases for particular ability groups, such as Canada compared to the Netherlands for couples with and without children. Here Canada dominates over the union of the two countries' income domains, but fails to dominate the Netherlands in a three-way comparison with the United States.

4. An Alternative Approach

In conceptual terms, as Jenkins and Lambert point out, their normative assumption concerns the income level at which needs cease to influence household welfare. While this would not at first sight appear to be a stringent condition, when making comparisons of income distributions in a multidimensional setting in practice, an upper income level will have to be specified. If, as in the empirical example provided above, this is chosen to be the highest income level observed in the sample survey, then including another distribution may lead to a different choice of domain and a different ranking. This is clearly an unsatisfactory basis for making comparisons of income distributions with heterogeneous populations.

4.1. Independence with respect to the income range.

In order to assure consistency in comparisons of multidimensional distributions, it would seem appropriate to require that the definition of the income domain over which comparisons are made should not influence the outcome. We will call this requirement domain independence and it is akin to the independence of irrelevant alternatives condition in social choice theory. More precisely we state:

DOMAIN INDEPENDENCE [DI]. We will say that there is *domain independence* if, for all $s^*, s^\circ \in Z^*$

$$(4.1) \qquad \int_{\underline{v}}^{y} F^*(\xi,h)d\xi \quad \leq \quad \int_{\underline{v}}^{y} F^{\circ}(\xi,h)d\xi \qquad \forall h \in H$$

holds for all $y \in \overline{D}(\{s^o, s^*\})$ implies that it is also true for all V such that $\overline{D}(\{s^o, s^*\}) \in V$.

As we have seen above, for there to be domain independence we need to ensure that there is first order dominance in the marginal distribution of abilities is a necessary condition for there second order stochastic dominance in the cumulative distribution of income for each ability threshold[7]. If this requirement is not met then the ranking of distributions becomes manipulable. In technical terms, the issue posed by the multidimensional nature of the comparisons being made can be seen in terms of the first line of equation (2.5):

$$(4.2) \qquad -\sum_{h=1}^{H-1} \left[U(\bar{v},h+1) - U(\bar{v},h)\right] \Delta F(\bar{v},h)$$

In the Atkinson-Bourguignon framework, this term is zero due to the assumption of identical marginal distributions of needs, so that $\Delta F(\bar{v},h) = 0$, for all h. In the Jenkins-Lambert approach, the term is zero because the term in square brackets is equal to zero by dint of the assumption that there are no differences in welfare for households of different types at the maximum income stated as condition U.3 above. If we add the requirement of domain independence, so that for welfare to be higher in situation s* compared to situation s°, there must be no more less able households in the former situation compared to the latter, then $\Delta F(\bar{v},h) \leq 0$, for all h.

4.2. A new dominance result.

The only way of ensuring that the condition expressed in equation (4.2) is non-negative and at the same time satisfies the requirement of domain independence is that $U(y,h+1) - U(y,h) \geq 0$ for all $h = 1, 2, ..., H-1$. In other words, if we retain the axioms used by Atkinson and Bourguignon, and wish to extend their approach to population with different marginal distributions of ability

7 Actually first order dominance in the marginal distribution of abilities is a necessary and sufficient condition for domain independence [see Bazen and Moyes (2002)].

without arbitrarily defining the domain over which comparisons are made, it is necessary to assume that more able households obtain no less welfare from a given income level than less able households:

$$(U.4) \qquad U(y,h) \le U(y,h+1) \qquad \forall y \in D, \qquad \forall h \in \{1, 2, ..., H-1\}$$

As is clear from equation (2.5), if the second order stochastic dominance condition expressed in condition T.1 is satisfied, and since domain independence requires $\Delta F(y,h) = F^*(y,h) - F°(y,h) \le 0$, for all h, then conditions U.1, U.2 and U.4 are sufficient for welfare to be no lower in situation s* compared to s°.

Letting U* denote the class of utility functions that satisfy conditions U.1, U.2 and U.4, this result can be formally stated as follows:

PROPOSITION 4.1. Consider two situations s°, s* ∈ Z. Then $W_U(s^*) \ge W_U(s°)$, for all U ∈ U*, if and only if:

$$(T.1) \qquad \int_{\underline{y}}^{y} F^*(\xi,h)d\xi \quad \le \quad \int_{\underline{y}}^{y} F°(\xi,h)d\xi \quad \forall y \in D, \quad \forall h \in H$$

$$(T.2) \qquad F^*(\bar{v},h) \le F°(\bar{v},h) \qquad \textit{for all } h = 1, 2, ..., H-1$$

PROOF:

SUFFICIENCY: By definition $\Delta F(\bar{v},h) = 0$ and conditions T.1 and T.2 guarantee that (2.5) is non negative, for all U ∈ U*.

NECESSITY: Let us first suppose that condition T.2 is not fulfilled and let h* be the smallest $h \in \{1, 2, ..., H-1\}$ such that $F°(\bar{v},h) > F°(\bar{v},h)$. Choose U such that *U(y, h)*:= 0, for all y ∈ D and all h = 1, 2,…, h*, and *U(y; h)* := υ > 0, for all y ∈ D and all h = h* + 1, h* + 2,…,H. While U ∈ U*, we have $\Delta W_U < 0$. Suppose next that condition T.1 does not hold and let (yfi; hfi) be the smallest couple (y, h) (in the lexicographic sense) such that

$$(4.3) \qquad \int_{\underline{y}}^{y} F^*(\xi,h)d\xi \quad > \quad \int_{\underline{y}}^{y} F°(\xi,h)d\xi$$

Consider then the piecewise linear, non-decreasing and concave function

$$(4.4) \qquad \psi(y) \equiv \begin{cases} y - y^* & for \quad \underline{v} \leq y < y^*, \\ 0 & for \quad y^* \leq y \leq \overline{v} \end{cases}$$

and let ψ^* be an approximation of ψ with non-negative first derivatives and non-positive second derivatives [see Fishburn and Vickson (1978, p. 76)]. Consider the utility function U defined by $U(y, h) := \psi^*(y)$, for all $y \in D$ and all $h = 1, 2, \ldots, h^*$, and $U(y, h) := 0$, for all $y \in D$ and all $h = h^* + 1, h^* + 2, \ldots, H$. By definition $U \in U^*$, but $\Delta W_U < 0$, hence a contradiction.□

Conditions T.1 and T.2 define dominance criteria which constitute the counterpart to sequential generalised Lorenz dominance when the situations being compared have different marginal distributions of ability which generally be the case when making international comparisons. Condition T.2 is in fact first order dominance in ability and requires that there be no more less able households in the dominating situation than in the dominated situation, whatever the level of ability. It is this condition that guarantees domain independence. However, it also implies that no matter how well-ofi a country is in income terms, there can be no overall dominance in welfare terms if the country has more less able households than another. Condition T.1 is the sequential second order stochastic dominance criterion derived by Jenkins and Lambert (1993).

Proposition 4.1 is obtained by modifying one of the assumptions made by Jenkins and Lambert (1993). However, since their key assumption in respect of non-identical marginal distributions implies the condition presented in (3.1), our assumption U.4 contains their condition as a special case. Furthermore, condition U.4 is no more than an ordinal and very general statement of the rationale underlying the use of equivalence scales, namely that the welfare of a large household is no greater than that of a small household for a given level of income[8].

[8] Choose the household with ability H - the household with the smallest family size e.g., the single adult - as the reference type. By definition of the equivalence scale λ_H, we have $U(y, h) = U(y/\lambda_H(h);H)$, for all $y \in D$. Invoking U.4, we obtain $U(y, h) = U(y/\lambda_H(h);H) \leq U(y/\lambda_H(h+1);H) = U(y, h+1)$, which upon inverting gives $\lambda_H(h+1) \leq \lambda_H(h)$ for all $h < H$ and all $y \in D$ [see e.g. Ebert and Moyes (2002)].

4.3. International comparisons with the Luxembourg Income Study database revisited.

This new criterion is applied to the comparisons of distributions using the same data as above from the Luxembourg Income Study. Table 4.1 indicates the marginal distribution of family types (or equivalently abilities) for each country in the sample. A first remark is that the distributions of types differ substantially from one country to another. Table 4.2 indicates which countries dominate the others in terms of the marginal distribution of abilities. The additional condition concerning the marginal distribution of abilities precludes for example the United States from dominating France and Germany, but both of the latter countries can in principle dominate the United States depending on condition T.1. No ranking will possible between the United States, and the United Kingdom, the Netherlands and Canada since the marginal distribution functions of abilities cross. Italy does not dominate any other country on the basis of condition T.2, and therefore cannot dominate any country in welfare terms.

Condition T.1 is the same as that derived by Jenkins and Lambert (1993) and so the pattern of dominance in incomes is that given in Table 3.2. Combining this information with the rankings based on the condition T.2 in Table 4.2, the overall welfare ranking of countries is presented in Table 4.3. At first sight, the results are somewhat disappointing: out of 28 pairwise comparisons, in only 9 are we able to establish dominance on the basis of conditions T.1 and T.2. The overall ranking obtained reveals that social welfare is higher in the United States and France than in Italy and Australia. The same is true for the United Kingdom, Germany, the Netherlands and Canada compared to Italy, and social welfare is higher in Canada compared to Australia. If we were to totally ignore differences in family type, we would establish dominance in 17 pairwise comparisons as indicated in Table 3.1.c.

Table 4.1. Marginal Distribution of Abilities

	US (1994)	FR (1994)	UK (1995)	GE (1994)	IT (1995)	NL (1994)	CAN (1994)	AUS (1994)
Couples with children	0.316	0.283	0.290	0.257	0.382	0.270	0.311	0.331
All couples	0.635	0.597	0.663	0.581	0.699	0.645	0.651	0.664
All families	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4.2. Comparisons of Countries on the Basis of Ability [Condition T.2]

	US	FR	UK	GE	IT	NL	CAN
FR	FR						
UK	?	FR					
GE	GE	GE	GE				
IT	US	FR	UK	GE			
NL	?	?	NL	GE	NL		
CAN	?	FR	?	GE	CAN	NL	
AUS	US	FR	UK	GE	AUS	NL	CAN

Table 4.3. Comparisons of Countries on the Basis of Living Standards [Conditions T.1 and T.2]

	US	FR	UK	GE	IT	NL	CAN
FR	?						
UK	?	?					
GE	?	?	?				
IT	US	FR	UK	GE			
NL	?	?	?	?	NL		
CAN	?	?	?	?	CAN	?	
AUS	US	FR	?	?	?	?	CAN

Violations occur in certain cases for reasons related to the fact that there is an additional condition to be met when comparing distributions with different marginal distributions of ability. For example, dominance in income over the observed range of incomes is in conflict with the dominance in the marginal distributions of abilities in the case of Canada compared with the United Kingdom and the Netherlands, and that of France compared with the Netherlands. Canada and France dominate the Netherlands only in income terms and therefore will no longer dominate at some income level beyond the range of observed incomes as pointed out in Section 3. In other cases, there is dominance right through the sequence of tests except when single persons - the most able group of households - are added into the comparison. This occurs particularly in comparisons involving Germany which would dominate Australia, the United Kingdom and the Netherlands if attention was confined to married couples with and without children. The same outcome is found for France which nearly dominates the United Kingdom and for Italy when compared with Australia. It is likely that cultural differences such as whether young, single persons live alone or with their family, and the propensity to remain in full-time education play a role here. The tax treatment of married couples and children along with the payment of child-related transfers could also influence international comparisons of welfare (at the margin) when applied sequentially. For example, the United Kingdom dominates Australia except in the income distribution for married couples with children.

5. Concluding Remarks

5.1. Implementation issues.

Several implementation issues arise when applying the criteria proposed here. In the example we use, we have deliberately simplified the ranking of household types. This is because it is necessary for there to be unanimity concerning the ordinal ranking of ability, and the grouping chosen is more likely to command such unanimity. If we had included single parent families, would we rank them above or below married couples with an identical number of children? Furthermore, a finer partition of family types substantially increases the risk of not being able to rank distributions. A second set of issues arises because the criteria derived here are based on second order stochastic dominance in the income distribution and when applied are subject to the same shortcomings as in the standard case. In particular, the distribution that has the lowest minimum income recorded cannot dominate the other no matter what happens higher up in the distribution. Bottom-coding, the elimination of negative incomes and the presence of minimum income guarantees can all influence the outcome. Thirdly, and particularly important in the light of the discussion in Section 3 above, there will be data differences that could affect the ranking, notably the top-coding of incomes.

5.2. Summary and extensions.

In this paper we have proposed a consistent criterion for comparing income distributions where the population structure differs by extending the approach initiated by Atkinson and Bourguignon (1987). It is particularly appropriate when undertaking international comparisons of income distributions since countries have different distributions of household types. It is also worth emphasising that while we have considered one particular non-income characteristic of household welfare albeit of major importance, the approach we have described can be applied to distributions where the additional dimension is health status or age, for example. One of the main findings concerns the importance of the definition of the income domain over which this type of multidimensional comparison is made and the critical role played by the marginal distribution of ability in assuring that rankings are consistent. The dominance criteria proposed give rise to a relatively small number of unambiguous welfare rankings in the international comparisons undertaken using a fairly simple illustration based on the Luxemburg Income Study data. More powerful results could of course be obtained in practice by selecting an equivalence scale and applying the usual techniques for analysing unidimensional income distributions. However, as we stressed at the outset, the approach proposed by Atkinson and Bourguignon and extended in this paper seeks precisely to avoid the arbitrariness associated with the use of equivalence scales. Extensions of the approach adopted here and future research could therefore concentrate on how a more complete ranking of distributions could be obtained. One possible avenue would be to introduce further restrictions on the utility function by allowing for instance the possibility of comparing differences of utility between households with different abilities. Another more extreme possibility would be to specify a particular utility function from the class examined in this paper to represent household welfare and to supplement our stochastic quasi-ordering by single figure indices of the type used in the unidimensional case (see e.g. Ebert (1995)).

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