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**Working Paper**  
**Intel Economics**

IUI Working Paper, No. 524

**Provided in Cooperation with:**

Research Institute of Industrial Economics (IFN), Stockholm

*Suggested Citation:* Segerstrom, Paul S. (1999) : Intel Economics, IUI Working Paper, No. 524, The Research Institute of Industrial Economics (IUI), Stockholm

This Version is available at:

<https://hdl.handle.net/10419/94684>

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# Intel Economics

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Current version: December 12, 1999

**Abstract:** This paper presents a model to explain why both industry leaders and follower firms often invest in R&D and explores the welfare implications of these R&D investment choices. Regardless of initial conditions, the equilibrium path in this model involves gradually convergence to a balanced growth path and R&D subsidies have no effect on the balanced growth rate. Nevertheless, it is always optimal for the government to intervene by subsidizing the R&D expenditures of industry leaders and taxing the R&D expenditures of follower firms. Without government intervention, market forces generate too much creative destruction.

**JEL classification numbers:** O32, O41.

**Key words:** economic growth, R&D.

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**Acknowledgements:** I gratefully acknowledge the financial support of the Research Institute of Industrial Economics (IUI) in Stockholm Sweden, where this paper was largely written.

# 1 Introduction

In 1968, engineers Gordon Moore and Bob Noyce left Fairchild, one of the largest semiconductor corporations in Silicon Valley, to found their own start-up company Intel. Initially, Intel made custom memory chips, but in trying to develop some custom circuits for the Japanese calculator manufacturer Busicom, engineers at Intel made a remarkable discovery. They succeeded in imitating at the chip level the architecture of computers by developing a general purpose programable chip. The Intel 4004 chip introduced in 1971 was the world's first microprocessor and maybe the most significant innovation of the 20th century.

The engineers at Intel did not rest on their past accomplishments but immediately went to work on developing more complex and powerful microprocessors. Whereas the Intel 4004 chip contained only 2,300 transistors, the Intel 8080 introduced in 1974 contained 6,000 transistors and was 10 times as powerful as the 4004. The Intel 8080 was in turn followed by the 8086, 286, 386, 486, Pentium, Pentium Pro, Pentium II and Pentium III chips (see Table 1).<sup>1</sup> From 1971 to the present, the number of transistors on an Intel chip has roughly doubled every 2 years, a trend known as “Moore’s Law” (see Figure 1).<sup>2</sup> Intel microprocessor performance, measured in MIPS (millions of instructions per second), has roughly doubled every 18 months. Most recently in 1998, Intel spent \$2.7 billion on R&D, 10 percent of net revenues.<sup>3</sup> More than 80% of Intel’s revenues—and all of its profits—came from microprocessors.<sup>4</sup> The company ended 1998 with a stock market capitalization of \$194 billion, the third highest in the world (behind only Microsoft and General Electric).<sup>5</sup>

Over the past 30 years, Intel’s experience has been that increasing the speed

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<sup>1</sup>Table 1 sources: <http://www.intel.com/intel/museum/25anniv/hof/tspec.htm>; Winn L. Rosch (1999), pp. 137.

<sup>2</sup>The last chip shown in Figure 1 is the 650 MHz Athlon processor introduced by Advanced Micro Devices in August 1999. It contains approximately 22 million transistors and is currently the fastest microprocessor for x86 computer systems.

<sup>3</sup>Source: <http://www.intel.com/intel/annual98/facts.htm>.

<sup>4</sup>Source: “Reinventing Intel,” by Elizabeth Corcoran, *Forbes Magazine*, May 3, 1999.

<sup>5</sup>Source: “Unbearable lightness of being,” by John Plender, *Financial Times*, December 8, 1998.

of its chips increases the demand for its chips, as more and more applications are developed which utilize Intel microprocessors. The first microprocessor, the Intel 4004, was only used in the Busicom desktop calculator. The 10-times faster Intel 8080 chip introduced in 1974 was the first microprocessor used to power a personal computer—the Altair. But personal computers did not take off in popularity until IBM’s decision in 1980 to make the considerably more powerful Intel 8088 chip the brains of the IBM PC.<sup>6</sup> This decision propelled Intel into the ranks of the *Fortune* 500. The 386 chip introduced in 1985 was the first Intel chip powerful enough to run Microsoft Windows system software at reasonable speeds and the resulting increase in “user friendliness” generated an avalanche of new software applications for personal computers. Most recently, the enhanced graphic capabilities of the Pentium chips have helped fuel the phenomenal growth of the world wide web, making personal computers far more useful for accessing information.

Over the past 30 years, Intel has also found, however, that innovating has become increasingly costly. Faster microprocessors have become progressively more difficult to develop. This trend is apparent right from the very beginning of Intel’s history. It took a team of 4 engineers to develop the relatively simple Intel 4004 chip in 1971 and most of the work was done by Federico Faggin.<sup>7</sup> In contrast, a team of 20 engineers was involved in designing the considerably more complex Intel 8086 chip in 1978.<sup>8</sup> In recent years, Intel R&D expenditures have increased dramatically (see Table 2) and in spite of these increases, the company has been having difficulties maintaining the pace of innovation defined by Moore’s Law (see Figure 1).<sup>9</sup> Summarizing Intel’s history, Malone (1995, pp. 253) writes

“But miracles, by definition, aren’t easy, and in the microprocessor business they get harder all the time. The challenges seem to grow with the complexity of the devices... that is, exponentially.”<sup>10</sup>

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<sup>6</sup>The 8088 was identical to the 8086 except for its 8-bit external bus.

<sup>7</sup>See Malone (1995), pp. 5-12.

<sup>8</sup>See Malone (1995), pp. 155.

<sup>9</sup>Table 2 source: <http://www.intel.com/intel/annual98/facts.htm>.

<sup>10</sup>Looking into the future, as the size of the transistor continues to shrink and millions of additional

It is interesting to contrast Intel’s behavior with typical firm behavior in the endogenous growth models that economists have developed for thinking about technological change. In Romer (1990), Grossman and Helpman (1991b, chap. 3) and Jones (1995), all R&D is directed at developing new horizontally differentiated products (or varieties). Nothing “better” is ever developed and as a result, no product ever becomes obsolete. Firms that become industry leaders remain industry leaders forever without exerting any further R&D effort. The Schumpeterian growth models developed by Segerstrom, et al. (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992) come somewhat closer to matching Intel’s experience in that firms engage in R&D aimed at improving the quality of existing products (or production processes). But in these models, it is not profit-maximizing for industry leaders to devote *any* resources to R&D activities. Firms invest in R&D to become industry leaders but once they succeed, they rest on their past accomplishments and do not try to improve their own products (or production processes).<sup>11</sup>

In this paper, I present a new endogenous growth model that is designed to be roughly consistent with Intel’s experience. In this dynamic general equilibrium model, firms invest in R&D to develop higher quality products (i.e., faster chips) and the rate of technological change is determined by their profit-maximizing investment choices. The model has four key features.

First, industry leaders are assumed to have R&D cost advantages (over other

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transistors are squeezed onto thumbnail-sized silicon wafers, researchers must cope with ever stronger electric fields and waste heat generation. The microscopic features of chips will begin to grow lumpy with molecules and designers will have to deal as much with questions of quantum mechanics as gate architecture. As existing materials reach their physical limits, new materials will need to be used: gallium arsenide, for example, has been suggested as a possible replacement for silicon because of its superior semiconductor properties. But with new materials, new manufacturing methods will need to be developed.

<sup>11</sup>Endogenous growth models developed by Caballero and Jaffe (1993), Stokey (1995), Kortum (1997), Segerstrom (1998) and Howitt (1999) also have this property. One exception is the recent model by Segerstrom and Zolnierrek (1999), where both industry leaders and followers participate in R&D races. However, this model has the scale-effect property and is thus vulnerable to Jones’ (1995) empirical critique of first-generation endogenous growth models.

follower firms) in improving their own products, and as a result, actively participate in R&D races. Thus, the model is capable of accounting for Intel's record of repeatedly developing faster microprocessors over the past 30 years. And Intel's behavior as an industry leader is not unusual. In Table 3, the 1998 net sales and R&D expenditures of several well-known industry leaders are reported.<sup>12</sup> It is clear from this table that not only do many industry leaders devote resources to R&D but their expenditures can be quite substantial. For most of the industry leaders in Table 3, R&D expenditures exceed 6 percent of net sales.

Second, leader firm R&D is assumed to be subject to diminishing returns, so small (follower) firms also participate in R&D races.<sup>13</sup> Thus, the model can account for the discovery of the first microprocessor by a small firm (Intel in 1971). Empirical studies reveal that both small and large firms play important roles in the innovation process. For example, according to Scherer (1984, chap. 11), companies with fewer than 1,000 employees were responsible for 47.3 percent of important innovations and companies with over 10,000 employees were responsible for 34.5 percent of important innovations.

Third, industry demand is assumed to increase when firms develop higher quality products. Thus, the model can account for Intel's growth over time (from a small start-up company in Santa Clara California to one of the most valued firms in the world). By way of contrast, in Grossman and Helpman (1991a) and Segerstrom (1998), industry demand does not change as a result of innovation and at any given point in time, profits are the same for firms in all industries. These models cannot account for any growth in the (relative) values of firms.

Finally, R&D difficulty is assumed to increase in each industry as products improve in quality and become more complex. Thus, firms have to devote ever increasing resources to R&D just to maintain a constant rate of innovation and the model can account for the increase over time in Intel's R&D expenditures. Intel's experience

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<sup>12</sup>Table 3 sources: Company annual reports published on the internet.

<sup>13</sup>Kortum (1993) and Thompson (1996) both report evidence of significant diminishing returns to R&D expenditure at the firm level.

with progressively increasing R&D difficulty appears to be widely shared. At the aggregate level, the number of scientists and engineers engaged in R&D has increased dramatically over time (see Table 4) without generating any upward trend in (per capita) economic growth rates (see Jones (1995)), and the patents-per-researcher ratio has declined significantly over time in many countries (see Table 5).<sup>14 15</sup>

The rest of this paper is organized as follows. In section 2, the model is presented and in section 3, its equilibrium properties are explored. The transitional dynamic properties of the model are also characterized in section 3 and this is useful for understanding why the short-run and long-run effects of R&D subsidies differ. In section 4, the welfare implications of the model are explored and it is shown that R&D expenditures of industry leaders and follower firms should be subsidized (at different rates). However, the optimal control calculations in section 4 do not shed much light on why it is optimal for the government to intervene. Thus, in section 5, all the externalities associated with R&D investment by leader and follower firms are explicitly derived from first principles. Section 6 summarizes the conclusions reached in the paper.

## 2 The Model

### 2.1 Industry Structure

Consider an economy with a continuum of industries indexed by  $\omega \in [0, 1]$ . In each industry  $\omega$ , firms are distinguished by the quality  $j$  of the products they produce. Higher values of  $j$  denote higher quality and  $j$  is restricted to taking on integer values. At time  $t = 0$ , the state-of-the-art quality product in each industry is  $j = 0$ , that is, some firm in each industry knows how to produce a  $j = 0$  quality product and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry participate in R&D races. In general, when the state-of-the-art quality in an industry is  $j$ , the next winner of a R&D race

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<sup>14</sup>Table 4 sources: National Science Board (1993, 1998).

<sup>15</sup>Table 5 sources: WIPO (1983), WIPO (annual issues).

becomes the sole producer of a  $j + 1$  quality product. Thus, over time, products improve as innovations push each industry up its “quality ladder,” as in Segerstrom et al. (1990) and Grossman and Helpman (1991a).

## 2.2 Consumers and Workers

The economy has a fixed number of identical households that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. The number of members in each family grows over time at the exogenous rate  $n > 0$ , so the supply of labor in the economy at time  $t$  is given by  $L(t) = L_0 e^{nt}$ . Each household is modelled as a dynastic family which maximizes the discounted utility

$$U \equiv \int_0^\infty e^{nt} e^{-\rho t} \ln u(t) dt \quad (1)$$

where  $\rho > n$  is the common subjective discount rate and

$$u(t) \equiv \left[ \int_0^1 \left( \sum_j \lambda^j d(j, \omega, t) \right)^\alpha d\omega \right]^{\frac{1}{\alpha}} \quad (2)$$

is the utility per person at time  $t$ . In equation (2),  $d(j, \omega, t)$  denotes the quantity consumed of a product of quality  $j$  produced in industry  $\omega$  at time  $t$ ,  $\lambda > 1$  measures the size of quality improvements, and  $\alpha \in (0, 1)$  determines the elasticity of substitution between industries  $\sigma = 1/(1 - \alpha)$ . Because  $\lambda^j$  is increasing in  $j$ , (2) captures in a simple way the idea that consumers prefer higher quality products.

Utility maximization involves two steps. First, each household allocates per capita expenditure  $c(t)$  to maximize  $u(t)$  given the prevailing market prices  $p(j, \omega, t)$  at time  $t$ . Solving this optimal control problem yields the per capita demand function

$$d(j, \omega, t) = \frac{\delta^j p(j, \omega, t)^{-(1+\varepsilon)} c(t)}{\int_0^1 [\delta^{j(\omega', t)} / p(j(\omega', t), \omega', t)^\varepsilon] d\omega'}, \quad \varepsilon \equiv \frac{\alpha}{1 - \alpha} \quad \delta \equiv \lambda^\varepsilon \quad (3)$$

for the product  $j(\omega, t)$  in industry  $\omega$  with the lowest quality-adjusted price  $p(j, \omega, t)/\lambda^j$ . The quantity demanded for all other products is zero. To break ties, I assume that when quality adjusted prices are the same for two products of different quality, each

consumer only buys the higher quality product. Second, each household maximizes discounted utility (1) given (2), (3) and the intertemporal budget constraint. Solving this optimal control problem yields the well-known intertemporal optimization condition

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \quad (4)$$

where  $r(t)$  is the instantaneous rate of return at time  $t$ . This differential equation must be satisfied throughout time in equilibrium and implies that a constant per capita expenditure path is optimal only when the market interest rate equals  $\rho$ . A higher market interest rate induces consumers to save more now and spend more later, resulting in increasing per capita consumption over time.

## 2.3 Product Markets

In each industry, firms compete in prices. Labor is the only input in production and there are constant returns to scale. One unit of labor is required to produce one unit of output, regardless of quality. Labor markets are perfectly competitive and the wage is normalized to unity. Consequently, each firm has a constant marginal cost of production equal to one.

Any firm that innovates receives a patent of infinite duration and patent rights are strictly enforced. Thus innovative firms do not have to worry about other firms copying their products. When a new firm innovates and becomes a quality leader, this firm's closest competitor is the previous quality leader in its industry. It is either in the interest of the new industry leader to practice limit pricing (as in Grossman and Helpman (1991a)) or to charge the unconstrained monopoly price, depending on whether the quality difference between the two competing firms is small or large. In either case, the previous quality leader does not sell any goods or earn any profits in equilibrium, now or in the future. Faced with no hope of earning future profits (as a follower firm), I assume following Howitt (1999) that the previous quality leader immediately exits and then cannot threaten to re-enter the industry. Thus, each industry leader charges the unconstrained monopoly price  $p = 1/\alpha$  and earns the

monopoly profit flow

$$\pi(j(\omega, t), t) = \frac{(1 - \alpha)L(t)c(t)}{Q(t)}\delta^{j(\omega, t)}, \quad (5)$$

where  $Q(t) \equiv \int_0^1 \delta^{j(\omega, t)} d\omega$  is the average quality level across industries. Each industry leader's profit flow increases when aggregate consumer expenditure  $L(t)c(t)$  increases, or the firm innovates and the quality of its product  $j(\omega, t)$  increases. However, when firms in other industries innovate and  $Q(t)$  increases, this decreases the demand for the industry leader's product and its profits fall as a consequence.

## 2.4 R&D Races

Labor is the only input used to do R&D in any industry, is perfectly mobile across industries and between production and R&D activities. In each industry, there are two types of firms that can hire R&D workers: the current quality leader and follower firms (all other firms).

There is free entry into each R&D race by follower firms and these firms all have access to the same constant returns to scale R&D technology. A follower firm  $i$  that hires  $\ell_i$  units of R&D labor in industry  $\omega$  at time  $t$  is successful in discovering the next higher quality product  $j(\omega, t) + 1$  with instantaneous probability

$$I_i = A_F \frac{\ell_i}{\delta^{j(\omega, t)}}, \quad (6)$$

where  $A_F > 0$  is a follower firm R&D productivity parameter.<sup>16</sup> Since the denominator is increasing in  $j$ , (6) captures the idea that as products become more complex with each step up the quality ladder, innovating becomes progressively more difficult.

If industry leaders have the same R&D technology as follower firms, then only follower firms choose to participate in R&D races. Firms invest in R&D to become industry leaders but once they succeed, they rest on their past accomplishments and do not try to improve their own products. To avoid this undesirable implication, I

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<sup>16</sup>By instantaneous probability, I mean that  $I_i dt$  is the probability that firm  $i$  will innovate by time  $t + dt$  conditional on not having innovated by time  $t$ , where  $dt$  is an infinitesimal increment of time. Alternatively stated,  $I_i$  is the Poisson arrival rate of innovations by firm  $i$ .

assume that industry leaders have R&D cost advantages over follower firms. Since industry leaders are the only firms using state-of-the-art technologies in their respective industries, it seems natural that industry leaders would have some ideas for improving their own products that are not also possessed by other firms. To be precise, when the industry leader in industry  $\omega$  hires  $\ell_L$  units of R&D labor at time  $t$ , this firm is successful in discovering the next higher quality product with instantaneous probability

$$I_L = A_L \left( \frac{\ell_L}{\delta j(\omega, t)} \right)^\beta, \quad (7)$$

where  $A_L > 0$  is a leader firm R&D productivity parameter and  $\beta < 1$  measures the degree of decreasing returns to leader firm R&D expenditure. The parameter restriction  $\beta < 1$  implies that R&D workers employed by industry leaders are more productive on the margin than R&D workers employed by industry followers (when the scale of R&D operations is not too large).<sup>17</sup>

The returns to engaging in R&D races are independently distributed across firms, across industries, and over time. Thus, the industry-wide instantaneous probability of R&D success in industry  $\omega$  at time  $t$  is simply  $I \equiv I_L + I_F = I_L + \sum_i I_i$  where  $I_F$  is the instantaneous probability of R&D success by all follower firms combined.

## 2.5 R&D Optimization

All firms are assumed to maximize their expected discounted profits by deciding how much to invest in R&D activities. To maximize expected discounted profits, both leaders and followers must solve stochastic optimal control problems where the state variable  $j(\omega, t)$  in each industry  $\omega$  is a Poisson jump process with intensity  $I_L + I_F$  and magnitude  $+1$ . The model is particularly tractable because the value (expected discounted profits) of an industry leader firm  $v_L$  only depends on the quality of its product  $j(\omega, t)$  and not separately on  $t$ ,  $\omega$  or any other state variables. The same holds true for the value of a follower firm  $v_F$ . To simplify notation, I will henceforth

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<sup>17</sup>An alternative way of modelling R&D cost advantages for industry leaders is to assume that  $\beta = 1$  and  $A_L > A_F$ . However this alternative formulation implies that, except for a knife-edge set of parameter values, either industry leaders or follower firms do all the research in the economy.

let  $j_\omega$  denote the state-of-the-art quality level in industry  $\omega$  instead of  $j(\omega, t)$  and leave the functional dependence on  $t$  implicit. Then, the value functions of leader and follower firms can be written compactly as  $v_L(j_\omega)$  and  $v_F(j_\omega)$ , respectively.

For each follower firm  $i$ , the relevant Hamilton-Jacobi-Bellman equation is<sup>18</sup>

$$\begin{aligned} r(t)v_F(j_\omega) = \max_{\ell_i} & -\hat{s}_F\ell_i + I_i[v_L(j_\omega + 1) - v_F(j_\omega)] \\ & + (I_{-i} + I_L)[v_F(j_\omega + 1) - v_F(j_\omega)], \end{aligned} \quad (8)$$

where  $I_{-i} \equiv I_F - I_i$  is the R&D intensity by all other follower firms combined,  $s_F$  is the fraction of each follower firm's R&D costs that are paid by the government (the follower R&D subsidy rate) and  $\hat{s}_F \equiv 1 - s_F$  is the fraction of each follower firm's R&D costs that are paid by the firm. Each follower incurs the R&D cost  $\hat{s}_F\ell_i$  today but earns no profit flow. With instantaneous probability  $I_i$ , the follower innovates, becomes a leader and learns how to produce a  $j_\omega + 1$  quality product. However, with instantaneous probability  $I_{-i} + I_L$ , some other firm innovates (either the current leader or another follower) and the follower continues to be a follower in the next R&D race. Equation (8) states that the maximized expected return on a follower firm's stock must equal the return on an equal-sized investment in a riskless bond.

The assumption of free entry by follower firms into R&D races implies that the value of a follower firm  $v_F$  must always equal zero in equilibrium. Taking this and (6) into account, the first order condition for an interior solution to the follower firm Bellman equation (8) is  $-\hat{s}_F + A_F v_L(j_\omega + 1)/\delta^{j_\omega} = 0$  which, when solved for  $v_L$  yields

$$v_L(j_\omega) = \frac{\hat{s}_F \delta^{j_\omega - 1}}{A_F}. \quad (9)$$

This equation has a natural economic interpretation. The value of an industry leader  $v_L$  jumps up every time the firm innovates and develops a higher quality product ( $j_\omega$  increases). Also, follower firms respond to either an increase in their R&D subsidy rate  $s_F$  or an increase in their R&D productivity  $A_F$  by innovating more frequently

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<sup>18</sup>See Malliaris and Brock (1982, pp. 123-124) for the application of stochastic dynamic programming techniques to Poisson jump processes, and Thompson and Waldo (1994, pp. 453) for an economic illustration of these techniques.

( $I_F$  increases) and with industry leaders being driven out of business more frequently, the value of being an industry leader naturally falls. As was claimed earlier,  $v_L(\cdot)$  only depends on  $j_\omega$  and not separately on  $\omega$ ,  $t$  or other state variables. Furthermore, it is easily verified that the follower firm Bellman equation (8) is satisfied when (9) holds and  $v_F = 0$ .

For each industry leader, the relevant Hamilton-Jacobi-Bellman equation is

$$r(t)v_L(j_\omega) = \max_{\ell_L} \pi(j_\omega, t) - \hat{s}_L \ell_L + I_L [v_L(j_\omega + 1) - v_L(j_\omega)] + I_F [v_F(j_\omega + 1) - v_L(j_\omega)] \quad (10)$$

where  $s_L$  is the fraction of each leader firm's R&D costs that are paid by the government (the leader R&D subsidy rate) and  $\hat{s}_L \equiv 1 - s_L$  is the fraction of each leader firm's R&D costs that are paid by the firm. Each industry leader earns the monopoly profit flow  $\pi(j_\omega, t)$  today and also incurs the R&D cost  $\hat{s}_L \ell_L$ . With instantaneous probability  $I_L$ , the leader innovates (learns how to produce a  $j_\omega + 1$  quality product) and its value jumps up as a result. However, with instantaneous probability  $I_F$ , some follower firm instead innovates and the leader becomes a follower. Equation (10) states that the maximized expected return on a leader firm's stock must equal the return on an equal-sized investment in a riskless bond.

The first order condition for an interior solution to the leader firm Bellman equation (10) is  $-\hat{s}_L + [v_L(j_\omega + 1) - v_L(j_\omega)] \partial I_L / \partial \ell_L = 0$  which, when solved for  $I_L$  using (7) and (9) yields

$$I_L = A_L \left[ \beta \frac{\hat{s}_F}{\hat{s}_L} \frac{A_L}{A_F} \frac{\delta - 1}{\delta} \right]^{\frac{\beta}{1-\beta}} \quad (11)$$

Thus, the innovation rate for each industry leader firm  $I_L$  is completely pinned down by parameter values and does not change over time or vary across industries. The constancy of  $I_L$  together with (7) implies that each industry leader's R&D employment  $\ell_L$  is constant during an R&D race but jumps up every time the firm innovates. This in turn implies that aggregate R&D employment by industry leaders gradually increases over time as firms (leaders as well as followers) innovate in a wide variety of industries.

Equation (11) has very intuitive implications. Other things being equal, industry leaders are more innovative ( $I_L$  increases) when their R&D workers are more productive ( $A_L$  increases), their R&D expenditures are subsidized to a greater extent ( $s_L$  increases) or innovations represent larger improvements in product quality ( $\lambda$  increases). On the other hand, changes in the structure of the economy that make it more attractive for follower firms to invest in R&D have an adverse effect on the relative R&D effort of industry leaders. Industry leaders are less innovative ( $I_L$  decreases) when follower firm R&D workers are more productive ( $A_F$  increases) or follower firm R&D expenditures are subsidized to a greater extent ( $s_F$  increases).

Solving the leader Bellman equation (10) for  $v_L$ , I obtain

$$v_L(j_\omega) = \frac{\pi(j_\omega, t) - \hat{s}_L \ell_L + I_L v_L(j_\omega + 1)}{r(t) + I_F(\cdot) + I_L} \quad (12)$$

Each industry leader earns the profit flow  $\pi(j_\omega, t)$  from selling the state-of-the-art quality product in its industry and also incurs the R&D cost  $\hat{s}_L \ell_L$ . In determining the value of an industry leader  $v_L(j_\omega)$ , this firm's net profit flow  $\pi - \hat{s}_L \ell_L$  is appropriately discounted using the current market interest rate  $r(t)$  and the instantaneous probability  $I = I_F + I_L$  of further innovation. With instantaneous probability  $I_F$ , the industry leader is driven out of business by follower firm innovation and experiences a total capital loss. However, with instantaneous probability  $I_L$  the industry leader itself innovates and its market value jumps up to  $v_L(j_\omega + 1)$ , which explains the additional term  $I_L v_L(j_\omega + 1)$  in the numerator of (12).

Substituting (5), (7), (9) and (11) into (12), it immediately follows that  $I_F(\cdot)$  does not vary across industries in equilibrium but can possibly change over time. Let  $x(t) \equiv Q(t)/L(t)$  denote the average quality of products relative to the size of the economy and  $R_L \equiv (I_L/A_L)^{1/\beta}$  denote the resources devoted to R&D by each industry leader at time  $t = 0$ . Then the *R&D condition*

$$\frac{\hat{s}_F}{A_F \delta} = \frac{(1 - \alpha) \frac{c(t)}{x(t)} - \hat{s}_L R_L + \hat{s}_F \frac{I_L}{A_F}}{r(t) + I_F(t) + I_L} \quad (13)$$

must be satisfied along any equilibrium path when both industry leaders and follower firms participate in R&D races. Equation (13) is an implication of follower firm R&D

profit maximization and has the standard interpretation that the discounted marginal revenue product of an idea must equal its marginal cost at each point in time.

## 2.6 The Labor Market

All workers are employed by firms in either production or R&D activities. Taking into account in that each industry leader charges the same price  $p = 1/\alpha$  and that consumers only buy goods from industry leaders in equilibrium, it follows from (3) that total employment of labor in production is  $\int_0^1 d(j_\omega, \omega, t) L(t) d\omega = \alpha c(t) L(t)$ . Solving (7) for each industry leader's R&D employment  $\ell_L(\omega, t)$  and then integrating across industries, total R&D employment by industry leaders is  $\int_0^1 \ell_L(\omega, t) d\omega = R_L Q(t)$ . Likewise, solving (6) for individual follower firm R&D employment  $\ell_i(\omega, t)$ , summing over firms to obtain industry-level R&D employment by follower firms  $\ell_F(\omega, t)$  and then integrating across industries, total R&D employment by follower firms is  $\int_0^1 \ell_F(\omega, t) d\omega = I_F(t) Q(t)/A_F$ . Thus, the full employment of labor condition for the economy at time  $t$  is

$$L(t) = \alpha c(t) L(t) + \left[ R_L + \frac{I_F(t)}{A_F} \right] Q(t). \quad (14)$$

An important implication of (14) is that, as the average quality of products  $Q(t)$  increases over time, more workers need to be employed in R&D activities to maintain the innovation rates  $I_L$  and  $I_F$  by leader and follower firms respectively. It is in this sense that innovating becomes progressively more difficult over time.

This completes the description of the model.

## 3 Equilibrium Properties

### 3.1 The Balanced Growth Equilibrium

I now solve the model for a unique balanced growth equilibrium where all endogenous variables grow at constant (not necessarily the same) rates, and both leader and follower firms invest in R&D in each industry.

From the full employment of labor condition (14),  $c(t)$ ,  $I_F(t)$  and  $x(t)$  must all be constants in any balanced growth equilibrium. Differentiating  $Q(t)$  with respect to time yields  $\dot{Q}(t) = \int_0^1 [\delta^{j\omega+1} - \delta^{j\omega}] I(t) d\omega = (\delta - 1)I(t)Q(t)$ . Thus, the constancy of  $x$  implies that  $\dot{x}/x = \dot{Q}/Q - \dot{L}/L = (\delta - 1)I - n = 0$ , which uniquely determines the balanced growth equilibrium innovation rate:

$$I = I_F + I_L = \frac{n}{\delta - 1}. \quad (15)$$

Furthermore, setting  $p = 1/\alpha$  in (3) and then substituting into (2) yields a unique balanced growth equilibrium economic growth rate:

$$g \equiv \frac{\dot{u}(t)}{u(t)} = \frac{1 - \alpha}{\alpha} \frac{\dot{Q}(t)}{Q(t)} = \frac{1}{\varepsilon}(\delta - 1)I = \frac{n}{\varepsilon} \quad (16)$$

The innovation rate  $I$  and the economic growth rate  $g$  are completely determined by three parameter values:  $n$ ,  $\varepsilon$ , and  $\delta$ . Firms innovate more frequently and the economy experiences faster economic growth when the population of workers/potential inventors grows more rapidly ( $n$  increases) or when each innovation is associated with a smaller proportional increase in R&D difficulty ( $\varepsilon$  decreases). When the innovation size parameter  $\delta$  is increased, each innovation is associated with a larger proportional increase in R&D difficulty and the balanced growth equilibrium innovation rate falls as a result. However, an increase in  $\delta$  has no effect on economic growth: although firms innovate less frequently, this is exactly offset by the fact that each innovation represents a bigger improvement in product quality.

Equations (15) and (16) have a remarkable implication that is worth emphasizing. The R&D subsidy rates  $s_L$  and  $s_F$  do not appear at all in these equations and thus R&D subsidies do not have any long-run growth effects. R&D subsidies temporarily increase the rate of technological change and permanently increase the relative size of the R&D sector as will be shown shortly but do not permanently change the economic growth rate. Indeed, since R&D subsidy rates are the only public policy instruments considered in this paper and represent the most direct way of trying to promote R&D investment and economic growth, the correct conclusion to draw from this model is that public policies in general do not have any long run growth effects.

Easterly, Kremer, Pritchett and Summers (1993) find some empirical support for this property of the model. They document the relatively low correlation of economic growth rates across decades, which suggests that differences in economic growth rates across countries may be mostly transitory.

Given (15), a balanced growth equilibrium where both leaders and followers participate in R&D races only exists when  $I_F > 0$  or

$$I_L < \frac{n}{\delta - 1}. \quad (17)$$

Equation (11) implies that the inequality (17) holds when the R&D productivity of industry leaders parameter  $A_L$  is relatively low and I will assume that this is the case. Otherwise, when  $A_L$  is relatively high, then only industry leaders choose to do R&D in equilibrium.

In any balanced growth equilibrium, the constancy of  $c$  over time and (4) together imply that the market interest rate  $r$  must equal  $\rho$ . With  $I_L$  given by (11),  $I_F$  then determined by (15),  $r(t) = \rho$ , and  $x$  constant over time, (13) yields a *balanced growth R&D condition*

$$\frac{\hat{s}_F}{A_F \delta} = \frac{(1 - \alpha) \frac{c}{x} - \hat{s}_L R_L + \hat{s}_F \frac{I_L}{A_F}}{\rho + I_F + I_L} \quad (18)$$

and dividing both sides of (14) by  $L(t)$  yields a *balanced growth resource condition*

$$1 = \alpha c + \left[ R_L + \frac{I_F}{A_F} \right] x. \quad (19)$$

Both balanced growth conditions are illustrated in Figure 2. The vertical axis measures consumption per capita  $c$  and the horizontal axis measures relative R&D difficulty  $x$ . The R&D condition is upward-sloping in  $(x, c)$  space, indicating that when R&D is relatively more difficult, consumer expenditure must be higher to justify the R&D effort levels chosen by firms (that sustain the innovation rates  $I_L$  and  $I_F$ ). The resource condition is downward-sloping in  $(x, c)$  space, indicating that when R&D is relatively more difficult and more resources are used in the R&D sector to maintain the balanced growth innovation rates  $I_L$  and  $I_F$ , less resources are available to produce goods for consumers, so individual consumers must buy less. The unique intersection between the R&D and resource conditions at point  $A$  determines the balanced growth values of consumption per capita  $c^*$  and relative R&D difficulty  $x^*$ .

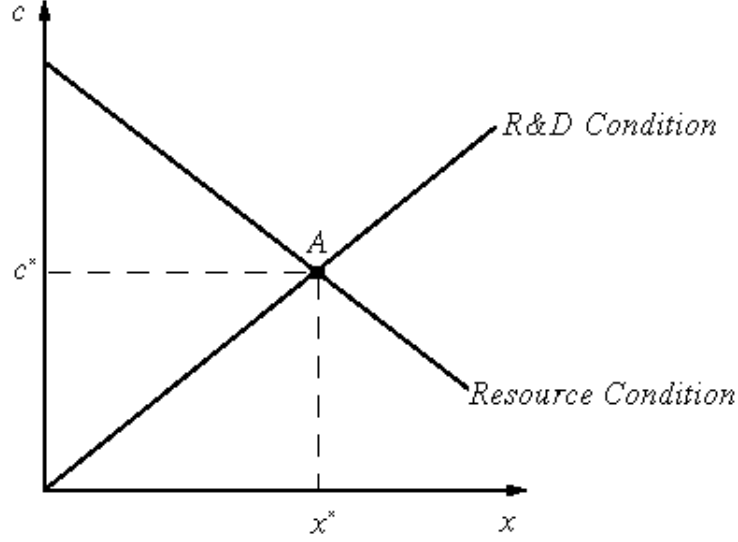


Figure 2: The unique balanced growth equilibrium

If  $x = x^*$  at time  $t = 0$ , then an immediate jump to the balanced growth path can occur. Otherwise, it is imperative to investigate the transitional dynamic properties of the model.

### 3.2 Transitional Dynamics

Differentiating relative R&D difficulty  $x(t) \equiv \frac{Q(t)}{L(t)}$  with respect to time yields  $\frac{\dot{x}(t)}{x(t)} = (\delta - 1) [I_F(t) + I_L] - n$ . Substituting into this expression for  $I_F(t)$  using the resource condition (14) yields one differential equation that must be satisfied along any equilibrium path for the economy:

$$\frac{\dot{x}(t)}{x(t)} = (\delta - 1) \left[ A_F \left( \frac{1 - \alpha c(t)}{x(t)} \right) - A_F R_L + I_L \right] - n \quad (20)$$

Since the RHS of (20) is decreasing in both  $x$  and  $c$ ,  $\dot{x}(t) = 0$  defines the downward-sloping curve in Figure 3. Starting from any point on this curve, an increase in  $x$  leads to  $\dot{x} < 0$  and a decrease in  $x$  leads to  $\dot{x} > 0$ , as is illustrated by the horizontal arrows in Figure 3.

Solving (13) for  $r(t)$  and then substituting for  $I_F(t)$  using (14) yields

$$r(c, x) \equiv \frac{A_F}{x} \left[ \frac{(1 - \alpha)c\delta}{\hat{s}_F} - (1 - \alpha c) \right] + A_F R_L \left[ 1 - \frac{\hat{s}_L}{\hat{s}_F} \delta \right] + I_L(\delta - 1). \quad (21)$$

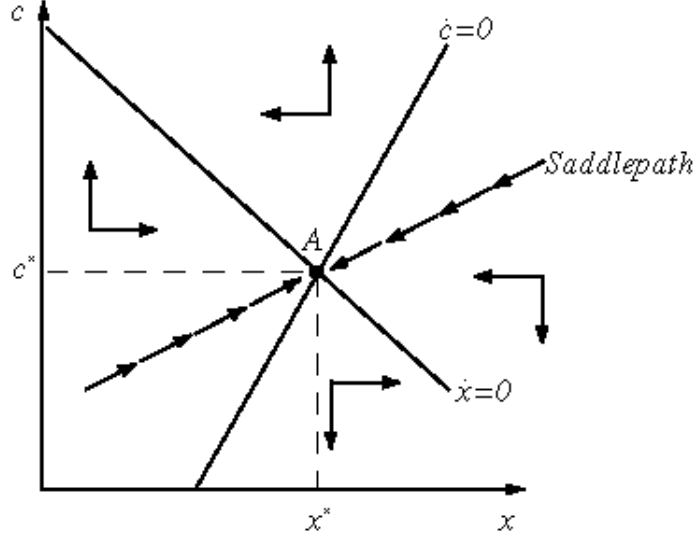


Figure 3: Stability of the balanced growth equilibrium

The consumer optimization condition (4) then yields a second differential equation that must be satisfied along any equilibrium path for the economy:

$$\frac{\dot{c}(t)}{c(t)} = r(c(t), x(t)) - \rho. \quad (22)$$

Since  $\partial r / \partial c$  is unambiguously positive, the slope of the  $\dot{c}(t) = 0$  curve hinges on the sign of  $\partial r / \partial x$ . When  $A_L$  is small, (11) implies that both  $I_L$  and  $I_L / A_L$  are close to zero and then  $\partial r / \partial x$  is negative in a neighborhood of the balanced growth equilibrium. Thus, the  $\dot{c}(t) = 0$  curve is locally upward-sloping in  $(x, c)$  space when  $A_L$  is relatively small and  $\partial r / \partial x < 0$  (this case is illustrated in Figure 3). Starting from any point on this curve, an increase in  $x$  leads to  $\dot{c} < 0$  and a decrease in  $x$  leads to  $\dot{c} > 0$ , implying that there exists an upward-sloping saddlepath. When  $A_L$  is increased holding other parameter values fixed, it is possible that, in a neighborhood of the balanced growth equilibrium point  $A$ , the sign of  $\partial r / \partial x$  switches from negative to zero to positive. If so, then the  $\dot{c}(t) = 0$  curve switches from being locally upward-sloping, to horizontal, to downward-sloping. However, the uniqueness of the balanced growth equilibrium and the continuity properties of the model guarantee that the balanced growth equilibrium is always locally saddlepath stable. By jumping onto this saddlepath and staying on it forever, convergence to the balanced growth equilibrium occurs, just like in the neoclassical growth model.

The above-developed phase-diagram analysis is useful for thinking about the effects of R&D subsidies. Suppose that the economy is in a balanced growth equilibrium and that the general R&D subsidy rate  $s = s_L = s_F$  is permanently increased. Equations (11) and (15) imply that the balanced growth equilibrium values of  $I$  and  $I_L$  are unaffected and thus, it follows from (20) that the  $\dot{x} = 0$  curve remains unchanged. If  $\partial r / \partial x < 0$  in a neighborhood of the balanced growth equilibrium, then (21) and (22) imply that the  $\dot{c} = 0$  curve shifts to the right when  $s$  increases. This case is illustrated in Figure 4. In response to the permanent (and unanticipated) R&D subsidy increase,

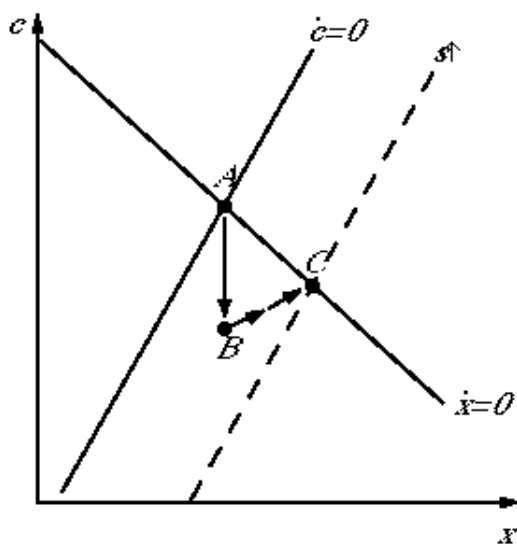


Figure 4: A Permanent Increase in the R&D Subsidy Rate

per capital consumption  $c$  immediately drops, R&D employment correspondingly increases and the innovation rate  $I$  jumps up in each industry. This is illustrated in Figure 4 by the movement from point  $A$  (the initial balanced growth equilibrium) to point  $B$  (which lies on the new saddlepath). After the initial jump up in  $I$ , the resulting above normal rate of technological change leads to a gradual increase over time in  $x = Q/L$  as  $\dot{Q}/Q = (\delta - 1)I$  implies that the average quality of products  $Q$  grows faster than the normal rate: the population growth rate  $\dot{L}/L = n$ . However, with firms improving their products at a faster than normal rate and with R&D difficulty increasing with each step up each industry's quality ladder, the innovation rate in each industry  $I$  gradually falls over time down to its original level as the economy

converges to the new balanced growth equilibrium. This is illustrated in Figure 4 by the movement along the new saddlepath from point  $B$  to point  $C$  (the new balanced growth equilibrium). If  $\partial r/\partial x > 0$  in a neighborhood of the balanced growth equilibrium, then the new saddlepath is downward-sloping instead of upward-sloping but the same adjustment process (an initial jump down in  $c$  followed by a gradual rise over time in  $x$ ) occurs.

Although a permanent general R&D subsidy increase does not have any long-run growth effects, the R&D subsidy increase does contribute to permanently increasing R&D employment as a fraction of total employment. To see this, first note from (19) that the relative size of the R&D sector is  $L_I/L \equiv [R_L + (I_F/A_F)]x$ . Solving (18) for  $c$  and then substituting into (19) yields the equilibrium value of  $x$ . Thus, the relative size of the R&D sector in the balanced growth equilibrium is

$$\frac{L_I}{L} = \left[ 1 + \frac{\frac{\hat{s}_F}{A_F \delta} (\rho + I) + \hat{s}_L R_L - \hat{s}_F \frac{I_L}{A_F} \varepsilon}{R_L + \frac{I_F}{A_F}} \right]^{-1}. \quad (23)$$

Since an increase in the general R&D subsidy rate  $s = s_F = s_L$  has no effect on  $\hat{s}_F/\hat{s}_L$ , (11) and (15) imply that a general R&D subsidy increase has no long-run effect on  $I_L$ ,  $I$  or  $I_F$ . However, an increase in  $s$  does directly decrease the denominator in (23), and thus, an increase in  $s$  does increase the long-run relative size of the R&D sector  $L_I/L$ , as was earlier claimed.

## 4 Welfare Analysis

I now explore the properties of the model when all allocation decisions are made by a social planner. I assume that the social planner's objective is to maximize the discounted utility of the representative family. Solving for the welfare-maximizing path for the economy is a four step procedure.

The first step is to determine how given production resources are allocated across industries to maximize individual consumer utility at a point in time  $t$ , that is, I solve

$$\max_{d(\cdot)} \int_0^1 \lambda^{\alpha j \omega} d(\omega, t)^\alpha d\omega \text{ subject to } \int_0^1 d(\omega, t) d\omega = \bar{L} \quad (24)$$

where  $d(\omega, t)$  is the per capita quantity consumed of a leading-edge quality good in industry  $\omega$  at time  $t$  and  $\bar{L}$  is the amount of labor devoted to producing goods for the representative consumer. Solving this optimal control problem yields the welfare maximizing allocation of production resources across industries at a point in time:

$$d(\omega, t) = \frac{\bar{L}}{Q(t)} \delta^{j_\omega} \quad (25)$$

It is desirable for industry leaders to produce more output in industries where the state-of-the-art quality  $j_\omega$  is higher. On an equilibrium path for the economy at time  $t$ , (14) implies that  $\bar{L} = \alpha c(t)$ . Substituting this expression for  $c(t)$  and  $p = 1/\alpha$  into (3) and solving also yields (25). Thus, the equilibrium and welfare maximizing allocations of production resources across industries at a point in time coincide.

The second step is to determine how given R&D resources are allocated within an industry  $\omega$  to maximize the industry's innovation rate  $I = I_L + I_F$  at a point in time  $t$ , that is, I solve

$$\max_{\ell_L, \ell_F} A_L \left( \frac{\ell_L}{\delta^{j_\omega}} \right)^\beta + A_F \frac{\ell_F}{\delta^{j_\omega}} \text{ subject to } \ell_L + \ell_F = \bar{L}, \quad (26)$$

where  $\ell_F \equiv \sum_i \ell_i$  denotes the total R&D employment by follower firms and  $\bar{L}$  now represents the amount of labor devoted to R&D (in industry  $\omega$  at time  $t$ ). The first order condition for an interior solution yields  $\ell_L = [A_F/(A_L\beta)]^{1/(\beta-1)} \delta^{j_\omega}$  and substituting this expression back into (7) yields the welfare maximizing innovation rate by leader firms in each industry

$$I_L = A_L \left( \frac{A_L\beta}{A_F} \right)^{\frac{\beta}{1-\beta}}. \quad (27)$$

It is optimal for each industry leader to employ enough R&D workers to generate the innovation rate  $I_L$  given by (27), with all remaining R&D workers employed by follower firms. Comparing (11) with (27), the equilibrium leader innovation rate coincides with the optimal leader innovation rate if and only if

$$\frac{\hat{s}_F}{\hat{s}_L} = \frac{\delta}{\delta - 1}. \quad (28)$$

Equation (28) has striking implications. Even though R&D subsidies do not have long-run growth effects, a laissez faire public policy ( $s_F = s_L = 0$ ) is never welfare-maximizing. It is always optimal for the government to subsidize the R&D expenditures of industry leaders more than the R&D expenditures of follower firms ( $s_L > s_F$ ) and in the absence of government intervention, market forces generate too much creative destruction in each industry ( $I_F/I_L$  is too high when  $s_F = s_L = 0$ ). I will assume that (28) holds in the remainder of the welfare analysis.

The third step is to determine how given R&D resources  $\bar{L}$  are allocated across industries to maximize the economy's rate of technological change  $\dot{Q}(t)$  at a point in time  $t$ , that is, I solve

$$\max_{\ell_F(\cdot), \ell_L(\cdot)} \int_0^1 I(\omega, t) [\delta^{j_\omega+1} - \delta^{j_\omega}] d\omega \quad \text{s. t.} \quad \int_0^1 [\ell_F(\omega, t) + \ell_L(\omega, t)] d\omega = \bar{L}. \quad (29)$$

Substituting for  $I_F$  using (6) and for  $\ell_L$  using (7), this problem can be rewritten as

$$\begin{aligned} & \max_{\ell_F(\cdot)} \int_0^1 \left[ \frac{A_F \ell_F(\omega, t)}{\delta^{j_\omega}} + I_L \right] (\delta - 1) \delta^{j_\omega} d\omega \\ & \text{subject to} \quad \int_0^1 \ell_F(\omega, t) d\omega + Q(t) R_L = \bar{L} \end{aligned} \quad (30)$$

where  $I_L$  is a constant pinned down by (27). Any feasible  $\ell_F(\cdot)$  function solves the problem (30) and thus, the equilibrium  $\ell_F(\cdot)$  function does as well. I conclude that the equilibrium allocation of R&D resources across industries at any point in time is optimal when the R&D subsidy rates satisfy (28).

The fourth and final step is to determine the mix of production and R&D employment over time that maximizes the discounted utility of the representative family. This mix is essentially determined by the innovation rate function  $I(t) = I_F(t) + I_L$  since  $I_L$  is a constant given by (27), the equilibrium innovation rate by follower firms  $I_F(t)$  does not vary across industries, and this is also optimal. Substituting (2) and (3) into (1) using the equilibrium price  $p = 1/\alpha$ , the discounted utility of the representative family simplifies to  $U = \int_0^\infty e^{-(\rho-n)t} \{\ln[\alpha c(t)] + \frac{1}{\varepsilon} \ln Q(t)\} dt$ . Equation (14) can then be solved for  $\alpha c(t)$  and here it is convenient to exploit the following algebraic trick: since (27) implies that  $R_L = I_L \beta / A_F$ , it immediately follows that

$$\frac{I_L}{A_F} - R_L = \frac{(1 - \beta) I_L}{A_F}. \quad (31)$$

I can also substitute into the integral for  $\ln Q(t)$  using the definition of the state variable  $x(t) \equiv Q(t)/L(t)$ . Thus, the optimal control problem facing the social planner is

$$\max_{I(\cdot)} \int_0^\infty e^{-(\rho-n)t} \left\{ \ln \left( 1 - \frac{I(t) - (1-\beta)I_L}{A_F} x(t) \right) + \frac{1}{\varepsilon} \ln x(t) \right\} dt \quad (32)$$

subject to the state equation  $\dot{x}(t) = x(t)[(\delta-1)I(t) - n]$  and the initial condition  $x(0) = 1/L_0$ . In (32), I have left out the constant term  $\int_0^\infty e^{-(\rho-n)t} (1/\varepsilon) \ln L(t) dt$  which plays no role in determining the optimal control.

The current value Hamiltonian for the social planner's optimal control problem is given by

$$\mathcal{H} \equiv \ln \left( 1 - \frac{I - (1-\beta)I_L}{A_F} x \right) + \frac{1}{\varepsilon} \ln x + \theta x[(\delta-1)I - n] \quad (33)$$

Solving for an interior solution yields the first order condition

$$\frac{\partial \mathcal{H}}{\partial I} = \theta x(\delta-1) - \frac{x}{A_F} \left( 1 - \frac{I - (1-\beta)I_L}{A_F} x \right)^{-1} = 0 \quad (34)$$

and the costate equation

$$\begin{aligned} \dot{\theta} = (\rho-n)\theta - \frac{\partial \mathcal{H}}{\partial x} = (\rho-n)\theta - \left\{ \frac{1}{\varepsilon x} + \theta[(\delta-1)I - n] \right. \\ \left. - \left( \frac{I - (1-\beta)I_L}{A_F} \right) \left( 1 - \frac{I - (1-\beta)I_L}{A_F} x \right)^{-1} \right\}. \end{aligned} \quad (35)$$

Solving (34) for  $I$  and substituting into (35) and the state equation, one obtains a nonlinear, autonomous differential equation system:

$$\dot{\theta} = \theta(\rho - \gamma I_L) - \frac{1}{\varepsilon x}, \quad (36)$$

$$\dot{x} = A_F(\delta-1) - \frac{1}{\theta} - x(n - \gamma I_L) \quad (37)$$

where  $\gamma \equiv (\delta-1)(1-\beta)$ . The phase diagram corresponding to this system is illustrated in Figure 5. The inequality (17) together with the parameter restrictions  $\rho > n$  and  $0 < \beta < 1$  imply that the bracketed expressions in (36) and (37) are both strictly positive. Thus, as illustrated, the  $\dot{x} = 0$  curve is upward sloping and the  $\dot{\theta} = 0$  curve is downward sloping. There is a unique steady state given by point  $A$  and this steady state is a saddle-point equilibrium of the differential equation system.

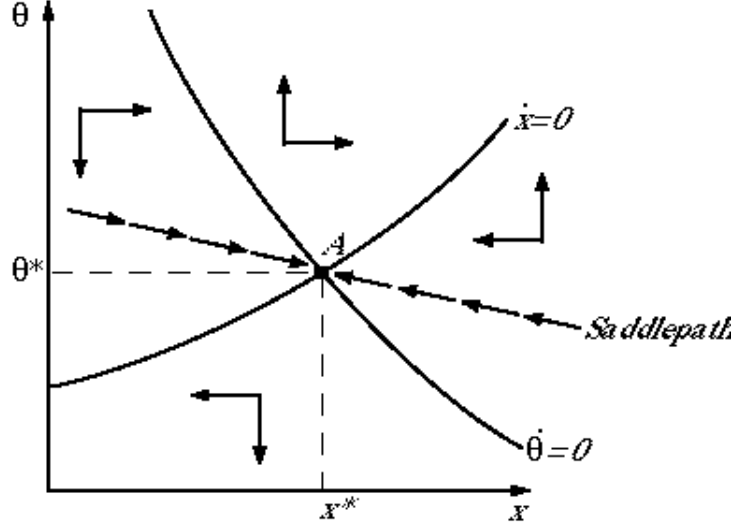


Figure 5: Stability of the optimal balanced growth path

Solving (34) for  $I$  and substituting back into (33), the “maximized Hamiltonian”  $\mathcal{H}^0 \equiv (1/\varepsilon) \ln x + A_F(\delta - 1)\theta - \ln[A_F(\delta - 1)\theta] - 1 - n\theta x + I_L\gamma\theta x$  is clearly a strictly concave function of  $x$  for given  $\theta$ . Thus by Proposition 10 in Arrow and Kurz (1970, pp.51), jumping onto the saddlepath at time  $t = 0$  and staying on the saddlepath forever represents an optimal path for the economy.

Having established that there exists a unique optimal balanced growth path (given by point  $A$  in Figure 5) and that it is saddlepath stable, I now solve for the defining characteristics of this optimal balanced growth path. Substituting the steady state condition  $\dot{x} = 0$  back into the state equation yields the optimal balanced growth innovation rate

$$I = \frac{n}{\delta - 1}. \quad (38)$$

From (19), the relative size of the R&D sector is  $L_I/L \equiv [R_L + (I_F/A_F)]x$ . Solving  $\dot{x} = 0$  and  $\dot{\theta} = 0$  simultaneously using (36) and (37) yields  $x^*$ , from which it follows [using (31)] that the optimal balanced growth R&D ratio is

$$\frac{L_I}{L} = \left[ 1 + \frac{\rho - \gamma I_L}{n - \gamma I_L} \varepsilon \right]^{-1}. \quad (39)$$

Interestingly, the optimal industry-level innovation rate  $I$  [given by (38)] coincides with the equilibrium industry-level innovation rate  $I$  [given by (15)] regardless of what R&D subsidy rates are chosen. However, it does not follow that a laissez faire public

policy ( $s_L = s_F = 0$ ) is welfare-maximizing. As has already been established, the optimal industry leader innovation rate  $I_L$  coincides with the corresponding equilibrium industry leader innovation rate only when (28) holds, and it immediately follows from the identity  $I = I_L + I_F$  that the optimal follower firm innovation rate  $I_F$  then coincides with the corresponding equilibrium follower firm innovation rate. Thus, non-zero R&D subsidy rates are in general needed to ensure an optimal mix of R&D effort between leader and follower firms in each industry.

R&D subsidies are also needed to ensure that the fraction of resources devoted to R&D along the equilibrium balanced growth path is optimal. Comparing (39) with (23) using (28) to substitute for  $s_F$  and (31) to simplify terms, the optimal leader R&D subsidy rate  $s_L$  is

$$s_L = \frac{I_F}{I_F + \rho - \gamma I_L} \quad (40)$$

Since  $I_F > 0$  and  $\rho - \gamma I_L > 0$ ,  $s_L$  is strictly positive. Equations (28) and (40) together imply that the optimal follower R&D subsidy  $s_F$  is

$$s_F = \frac{\frac{n-\rho}{\delta-1} - \beta I_L}{I_F + \rho - \gamma I_L}. \quad (41)$$

Since  $\rho > n$ ,  $s_F$  is strictly negative. Thus, it is always optimal to subsidize the R&D expenditures of industry leaders and it is always optimal to tax the R&D expenditures of industry followers (all other firms).

It is interesting to compare these welfare findings with those derived in the earlier literature. In Grossman and Helpmans' (1991a) "quality ladders" endogenous growth model where all R&D is done by follower firms in equilibrium, either R&D subsidies or R&D taxes can be welfare maximizing depending on parameter values. Stokey (1995) has studied a more general version of this model and found that R&D taxes are only optimal under extreme circumstances; for all empirically plausible parameter values, it is desirable to subsidize all R&D activities by follower firms. In this model by contrast, it is optimal to tax the R&D expenditures of follower firms for all parameter values. The conclusion that it is always optimal to subsidize the R&D expenditures of industry leaders is also found in Thompson and Waldo (1994). However, in Thompson and Waldo's model of trustified capitalism, only industry leaders participate in R&D

aces (by assumption) and thus they do not find that follower firm R&D taxes are also necessary for welfare maximization.

## 5 Identifying The External Effects

Although the welfare analysis of the preceeding section completely characterizes the optimal R&D subsidies, the derivation of (40) and (41) sheds little light on why it is optimal to subsidize leader R&D and tax follower R&D. To understand what is driving the welfare results, it is helpful to solve for all the external effects of R&D investment and that is the focus of this section. The general procedure used for identifying the external effects is the same as in Grossman and Helpman (1991b, pp.110-111) and Segerstrom (1998) but involves some new twists since there are now two types of firms that participate in each R&D race (industry leaders and followers).

To solve for the external effects, it is useful to introduce a new state variable for the economy. Let  $\Phi(t) \equiv \int_0^t I(\tau) d\tau$  denote the expected number of R&D successes in the typical industry before time  $t$ . The fundamental theorem of calculus implies that  $\dot{\Phi}(t) = I(t)$ . Solving the first order linear differential equation  $\dot{Q}(t) - I(t)(\delta - 1)Q(t) = 0$  with the boundary condition  $Q(0) = 1$  then yields

$$Q(t) = e^{(\delta-1)\Phi(t)}. \quad (42)$$

Using (42) and  $E(t) = c(t)L(t)$ , the discounted utility of the representative family  $U = \int_0^\infty e^{-(\rho-n)t} \{(1/\varepsilon) \ln Q(t) + \ln [\alpha c(t)]\} dt$  can be rewritten as

$$U = \int_0^\infty e^{-(\rho-n)t} \left\{ \frac{\delta-1}{\varepsilon} \Phi(t) + \ln \left( \frac{\alpha E(t)}{L(t)} \right) \right\} dt. \quad (43)$$

### 5.1 Innovation by Leader Firms

Suppose now that the economy is on a balanced growth equilibrium path and consider the marginal innovation by an industry leader at time  $t = 0$  (innovation by follower firms will be considered in subsection 5.2). I perturb the market equilibrium by  $d\Phi$  at each moment in time after  $t = 0$  (thereby preserving the initial path of innovation) and

compute the impact on the welfare of agents other than the industry leader responsible for the marginal innovation. Ignored in this calculation are the R&D costs incurred (and the profits earned) by the “deviant” industry leader since this firm’s expected discounted profits are not affected by a marginal increase in R&D effort at  $t = 0$  (given that expected discounted profits were being maximized by the equilibrium R&D effort choice). Because the goal is to assess whether or not the industry leader has the right R&D incentives in the absence of government intervention, I assume that  $s_L = 0$ . Also, since a balanced growth equilibrium with  $s_L = 0$  can only possibly be optimal when (28) holds, I assume that  $s_F = -1/(\delta - 1) < 0$ , that is, follower R&D is being taxed in the balanced growth equilibrium. The external effects of the marginal leader innovation are found by differentiating (43) with respect to  $\Phi$ . This yields

$$\frac{dU}{d\Phi} = \int_0^\infty e^{-(\rho-n)t} \frac{\delta - 1}{\varepsilon} dt + \int_0^\infty e^{-(\rho-n)t} \frac{1}{E(t)} \frac{dE(t)}{d\Phi(t)} dt. \quad (44)$$

The first integral in (44) represents the *consumer surplus effect* of the marginal innovation. Every time a firm innovates, consumers benefit because they can buy a higher quality product at the same price that they used to pay for a lower quality substitute. Furthermore these consumer benefits last forever because future innovations build on all the innovations of the past. Carrying out the integration, this positive external effect of the marginal innovation measures  $\frac{\delta-1}{\varepsilon(\rho-n)}$  in terms of the utility metric given by (1). Because individual R&D firms do not take into account the external benefits to consumers that innovations generate in their profit-maximization calculations, this external effect represents one reason why firms may under-invest in R&D activities from a social perspective. Consumers benefit more from the marginal innovation when innovations represent larger improvements in product quality (large  $\delta$ ), consumers are more patient (small  $\rho$ ), and there are more consumers in the future to benefit from the marginal innovation (large  $n$ ). Consumers also benefit more from the marginal innovation when there is a higher elasticity of substitution between products in different industries (large  $\varepsilon$ ), because then the industry in which the marginal innovation occurs experiences a larger increase in demand.<sup>19</sup>

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<sup>19</sup>The last property, that  $f(\varepsilon) \equiv (\lambda^\varepsilon - 1)/\varepsilon$  is increasing in  $\varepsilon$ , is not obvious. To prove this property,

The second integral in (44) reflects the effect of the marginal innovation on aggregate spending by agents other than the owners of the innovating firm. Since the marginal innovation reduces the demand for products in other industries, the owners of the industry leaders in other industries suffer a loss in profit income and aggregate consumer expenditure correspondingly falls. Also, since the marginal innovation makes future innovations costlier to discover, more resources must go into R&D investment to maintain the steady-state innovation rate and less resources are left for producing consumer goods, which represents another reason why aggregate consumer expenditure falls. Industry leaders take into account in their profit-maximization calculations that R&D success today increases R&D difficulty in the future but not the effect of this increase in R&D difficulty on aggregate consumer expenditure. Thus, the second integral in (44) combines the *across-industry business stealing effect* and the *intertemporal R&D spillover effect*. I will now solve for these two negative external effects.

Spending equals total income minus savings and savings equals investment in a closed economy. Thus (14) implies that in a balanced growth equilibrium

$$E(t) = L(t) + \Pi(t) - \left( R_L + \frac{I_F}{A_F} \right) Q(t) \quad (45)$$

where  $L(t)$  is wage income (since the wage rate equals one) and  $\Pi(t)$  is aggregate profit income. Differentiating (45) with respect to  $\Phi$  using (31) and (42) then yields

$$\frac{dE(t)}{d\Phi(t)} = \frac{d\Pi(t)}{d\Phi(t)} - \frac{I - (1 - \beta)I_L}{A_F} (\delta - 1) Q(t). \quad (46)$$

The first term on the right-hand side of (46) represents the across-industry business stealing effect and the second term represents the intertemporal R&D spillover effect.

To determine the size of the business stealing effect, it is useful to first calculate the steady-state value  $v_L(j_\omega)$  of being an industry leader. I will calculate the steady-state value of being an industry leader at time  $t = 0$  when  $j_\omega = 0$  for all  $\omega$ . Equation

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it is helpful to compare the functions  $g(x) \equiv \ln x$  and  $h(x) \equiv 1 - x^{-1}$ . Since  $f(1) = g(1) = 0$ ,  $f'(1) = g'(1) = 1$ , and  $f'(x) = 1/x > g'(x) = 1/x^2$  for all  $x > 1$ , it follows that  $\ln x > 1 - x^{-1}$  for all  $x > 1$ . Substituting into this inequality  $x = \lambda^\varepsilon > 1$ , I obtain that  $\ln \lambda^\varepsilon > (\lambda^\varepsilon - 1)/\lambda^\varepsilon$ , from which it follows that  $f'(\varepsilon) = [\lambda^\varepsilon \ln \lambda^\varepsilon - (\lambda^\varepsilon - 1)]/\varepsilon^2 > 0$ .

(5) implies that  $\pi(0, 0) = (1 - \alpha)E(0)/Q(0)$ , (9) implies that  $v_L(1) = \delta v_L(0)$  and  $v_L(0) = \hat{s}_F/(A_F\delta) = 1/(A_F(\delta - 1))$ , and then (7) implies that  $\ell_L = A_F(\delta - 1)R_L v_L(0)$ . Substituting these expressions back into the leader Bellman equation (10) yields

$$\rho v_L(0) = (1 - \alpha) \frac{E(0)}{Q(0)} - A_F(\delta - 1)R_L v_L(0) + (\delta - 1)I_L v_L(0) - I_F v_L(0),$$

and solving this equation for  $v_L(0)$  using (31), I obtain

$$v_L(0) = \frac{(1 - \alpha) \frac{E(0)}{Q(0)}}{\rho - \gamma I_L + I_F}. \quad (47)$$

Now since  $x(t) \equiv Q(t)/L(t)$  is constant in a balanced growth equilibrium, both  $Q(t)$  and  $L(t)$  grow over time at the same rate  $n$  and  $E(0)/Q(0) = E(t)/Q(t)$  for all time  $t$ . Thus (47) implies that the expected discounted profits from being an industry leader at time  $t = 0$  are equivalent to earning the certain profit flow

$$\hat{\pi}(t) \equiv (1 - \alpha) \frac{E(t)}{Q(t)} e^{(\gamma I_L - I_F)t} \quad (48)$$

for all  $t \geq 0$ .

By innovating at time  $t = 0$ , the deviant industry leader increases  $Q(t)$  for all  $t \geq 0$ , which reduces the demand for all other industry leaders' products and contributes to lower profits for these firms. There is also a multiplier effect because the lower profits earned by industry leaders mean lower income for the owners of these firms, which in turn implies that consumer expenditure falls and the profits earned by industry leaders fall further as a consequence. In the balanced growth equilibrium, it follows from (5) that the aggregate profit flow earned by industry leaders at time  $t$  is  $\int_0^1 \pi(j(\omega, t), t) d\omega = (1 - \alpha)E(t)$ . Thus, the aggregate change in profit income at time  $t$  due to the marginal innovation at time 0, including the multiplier effect, is

$$\frac{d\Pi(t)}{d\Phi(t)} = \frac{dQ(t)}{d\Phi(t)} \frac{d\hat{\pi}(t)}{dQ(t)} + (1 - \alpha) \frac{dE(t)}{d\Phi(t)}. \quad (49)$$

Substituting (49) back into (46) and simplifying using (42) and (48) yields

$$\begin{aligned} \frac{dE(t)}{d\Phi(t)} &= -(1 - \alpha)(\delta - 1) \frac{E(t)}{Q(t)} e^{(\gamma I_L - I_F)t} \\ &\quad - (\delta - 1) \frac{I - (1 - \beta)I_L}{A_F} Q(t) + (1 - \alpha) \frac{dE(t)}{d\Phi(t)}. \end{aligned} \quad (50)$$

Dividing both sides of (50) by  $\alpha E(t)$  and simplifying further using the steady-state conditions  $Q(t) = e^{(\delta-1)It} = e^{nt}$  and  $E(t) = cL(t) = cL_0 e^{nt}$  yields

$$\frac{1}{E(t)} \frac{dE(t)}{d\Phi(t)} = -\frac{\delta-1}{\varepsilon} e^{(\gamma I_L - I_F - n)t} - \frac{\delta-1}{\alpha c L_0} \left( \frac{I - (1-\beta)I_L}{A_F} \right) \quad (51)$$

Taking into account that  $x \equiv Q(t)/L(t) = 1/L_0$ ,  $s_L = 0$  and  $1 - s_F = \delta/(\delta-1)$  in the balanced growth equilibrium at time  $t = 0$ , the R&D condition (18) can be rewritten using (31) as

$$\alpha c L_0 = \frac{\varepsilon}{(\delta-1)A_F} [\rho - \gamma I_L + I_F]. \quad (52)$$

Substituting for  $\alpha c L_0$  in (51) using (52), the integration in (44) can now be completed and yields

$$\frac{dU}{d\Phi} = \frac{\delta-1}{\varepsilon(\rho-n)} - \frac{(\delta-1)/\varepsilon}{\rho - \gamma I_L + I_F} - \frac{\delta-1}{\varepsilon(\rho-n)} \frac{n - \gamma I_L}{\rho - \gamma I_L + I_F}, \quad (53)$$

where the terms on the right-hand side are the consumer surplus, across-industry business stealing and intertemporal R&D spillover effects, respectively.

Every time a leader firm innovates, it takes some demand away from industry leaders in other industries<sup>20</sup> and lowers the profits earned by industry leaders in these other industries.<sup>21</sup> The lower profits earned by industry leaders in turn imply that consumer income and expenditure fall, and the profits earned by industry leaders fall further as a consequence.<sup>22</sup> Because industry leaders do not take into account in their profit-maximization calculations the losses incurred by other firms from the marginal innovation, these losses represent one reason why firms may over-invest in R&D activities from a social perspective. These losses are captured by the across-industry business stealing effect, which measures  $\frac{(\delta-1)/\varepsilon}{\rho - \gamma I_L + I_F}$  in terms of the utility metric given by (1). The across-industry business stealing effect is larger when innovations reduce demand more in other industries ( $\delta$  large), industry leaders earn higher per unit profit margins ( $\frac{1}{\varepsilon} = \frac{1}{\alpha} - 1 = p - MC$  large), future profits are lightly discounted ( $\rho$  small),

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<sup>20</sup>Substituting the price  $p = 1/\alpha$  charged by other industry leaders into (3), the demand facing each industry leader is given by  $d(j, \omega, t)L(t) = \delta^j p(j, \omega, t)^{-(1+\varepsilon)} E(t)/(\alpha^\varepsilon Q(t))$ , which decreases when  $Q(t)$  increases.

<sup>21</sup>Equation (5) implies that  $\pi(j(\omega, t), t)$  decreases when  $Q(t)$  increases.

<sup>22</sup>Equation (5) implies that  $\pi(j(\omega, t), t)$  decreases when  $E(t) = c(t)L(t)$  decreases.

industry leader firms expect to be in business for a long time ( $I_F$  small, which occurs when  $n$  small), and industry leaders earn higher profits from doing R&D themselves ( $I_L$  large, which occurs when  $A_L$  large or  $A_F$  small).

Also, every time a leader firm innovates, innovating becomes more difficult in the firm's industry.<sup>23</sup> Future innovations become costlier to discover and more resources must go into the R&D activities to maintain the steady-state innovation rate  $I = n/(\delta - 1)$ . That means that less resources are left for producing consumer goods and consumer expenditure decreases as a consequence.<sup>24</sup> The decrease in consumer expenditure implies that the profits earned by all industry leaders decrease and since these profits represent income for consumers, consumer expenditure falls further as a consequence. Industry leaders take into account in their profit-maximization calculations that R&D success today increases R&D difficulty in the future but not the negative effect of this increase in R&D difficulty on consumer expenditure. Thus, there is a negative intertemporal R&D spillover effect present in the model and this represents a second reason why firms may over-invest in R&D activities from a social perspective. The intertemporal R&D spillover effect measures  $\frac{\delta-1}{\varepsilon(\rho-n)} \frac{n-\gamma I_L}{\rho-\gamma I_L+I_F} = \frac{\delta-1}{\rho-n} \frac{L_R(t)}{L_P(t)}$  in terms of the utility metric given by (1), where  $L_R(t)$  represent total employment of labor in R&D activities and  $L_P(t)$  represent total employment of labor in production activities. The intertemporal R&D spillover effect is larger when a larger fraction of the economy's resources are devoted to R&D activities ( $L_R/L_P$  large), and consumers place a higher weight on the future ( $\rho$  small). For most changes in basic parameter values, the direction of change in the size of the intertemporal R&D spillover effect is theoretically ambiguous.

It is straightforward to verify that the optimal R&D subsidy condition (40) is equivalent to

$$\frac{\delta-1}{\varepsilon(\rho-n)} \hat{s}_L = \frac{(\delta-1)/\varepsilon}{\rho-\gamma I_L+I_F} + \frac{\delta-1}{\varepsilon(\rho-n)} \frac{n-\gamma I_L}{\rho-\gamma I_L+I_F}, \quad (54)$$

which confirms the correctness of the external effects calculations. The optimal R&D subsidy for industry leaders  $s_L$  is strictly positive because the consumer surplus effect

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<sup>23</sup>Equations (6) and (7) imply that both  $I_L$  and  $I_F$  are decreasing functions of  $j(\omega, t)$ .

<sup>24</sup>Equation (19) implies that any increase in  $x$  must be matched by a corresponding decrease in  $c$ .

always outweighs the combined across-industry business stealing and intertemporal R&D spillover effects.

## 5.2 Innovation by Follower Firms

To identify the external effects associated with the marginal innovation by a follower firm at time  $t = 0$ , I perturb the market equilibrium by  $d\Phi$  at each moment in time after  $t = 0$  (thereby preserving the initial path of innovation) and compute the impact on the welfare of agents other than the follower firm responsible for the marginal innovation. Because the goal is to assess whether or not follower firms has the right R&D incentives in the absence of government intervention, I assume that  $s_F = 0$ . Also, since a balanced growth equilibrium with  $s_F = 0$  can only possibly be optimal when (28) holds, I assume that  $s_L = 1/\delta > 0$ , that is, leader firm R&D is subsidized in the balanced growth equilibrium.

Following the same procedure as in subsection 5.1 but using different R&D subsidy rates, it is straightforward to verify that equations (44), (45), (46), (47), and (48) still hold. However, (49) now becomes

$$\frac{d\Pi(t)}{d\Phi(t)} = -\hat{\pi}(t) + \frac{dQ(t)}{d\Phi(t)} \frac{d\hat{\pi}(t)}{dQ(t)} + (1 - \alpha) \frac{dE(t)}{d\Phi(t)}, \quad (55)$$

where the additional term  $-\hat{\pi}(t)$  reflects the fact that when a follower firm innovates, it drives the previous industry leader out of business, destroying the previous industry leader's profits. Taking into account the additional  $-\hat{\pi}(t)$  term in (55), equation (53) becomes

$$\frac{dU}{d\Phi} = \frac{\delta - 1}{\varepsilon(\rho - n)} - \frac{(\delta - 1)/\varepsilon}{\rho - \gamma I_L + I_F} - \frac{\delta}{\varepsilon(\rho - n)} \frac{n - \gamma I_L}{\rho - \gamma I_L + I_F} - \frac{1/\varepsilon}{\rho - \gamma I_L + I_F}, \quad (56)$$

where the terms on the right-hand side are the consumer surplus, across-industry business stealing, intertemporal R&D spillover, and within-industry business stealing external effects, respectively. The first three external effects have the same properties as in subsection 5.1, so I will focus on discussing the fourth external effect.

Every time a follower firm innovates, it drives the previous industry leader out of business. The previous industry leader forfeits a stream of monopoly profits and

the owners of this firm experience a windfall loss. By itself, this loss in profit income contributes to lower aggregate consumer expenditure and hence, lower profits for all industry leaders. Because follower firms do not take into account in their profit-maximization calculations the windfall losses that are incurred by industry leaders, these losses represent an additional reason why follower firms may over-invest in R&D activities from a social perspective. These losses are captured by the within-industry business stealing effect, which measures  $\frac{1/\varepsilon}{\rho - \gamma I_L + I_F}$  in terms of the utility metric given by (1). The within-industry business stealing effect is larger when industry leaders earn higher per unit profit margins ( $\frac{1}{\varepsilon} = \frac{1}{\alpha} - 1 = p - MC$  large), future profits are lightly discounted ( $\rho$  small), and industry leaders expect to be in business for a long time ( $I_L$  large, which occurs when  $A_L$  large or  $A_F$  small).

It is straightforward to verify that the optimal R&D subsidy condition (41) is equivalent to

$$\frac{\delta - 1}{\varepsilon(\rho - n)} \hat{s}_F = \frac{\frac{\delta-1}{\varepsilon} + \frac{1}{\varepsilon}}{\rho - \gamma I_L + I_F} + \frac{\delta}{\varepsilon(\rho - n)} \frac{n - \gamma I_L}{\rho - \gamma I_L + I_F}, \quad (57)$$

which confirms the correctness of the external effects calculations. It is optimal to tax follower firm R&D expenditures ( $s_F < 0$ ) because the consumer surplus effect is always outweighed by the combined across-industry business stealing, within-industry business stealing and intertemporal R&D spillover effects.

## 6 Conclusions

This paper presents a model to explain why both industry leaders and follower firms often invest in R&D and explores the welfare implications of these R&D investment choices. Industry leaders are assumed to have R&D cost advantages over other firms in improving their own products and as a result, industry leaders do not rest on their past accomplishments, but invest in R&D to maintain their leadership positions over time. Industry leader R&D expenditure is also assumed to be subject to diminishing returns, so small (follower) firms participate in R&D races as well. The model is designed to roughly match Intel's experience over time: aggressively investing in

R&D as an industry leader for the past 30 years and developing the world's first microprocessor in 1971 when it was a small startup company (a follower firm). Intel represents a convenient symbol of today's information economy and thus the paper is titled "Intel Economics."

In the model, firms engage in R&D aimed at improving the quality of products in each industry. When firms innovate and become industry leaders, they earn temporary monopoly profits as a reward for their R&D efforts. Due to positive growth in the population of consumers and the increases in industry demand that quality improvements generate, the reward for innovating grows over time in the typical industry. However, counterbalancing these two considerations, innovating becomes progressively more difficult in each industry as products improve in quality and become more complex. The rate at which the economy grows is determined by the profit-maximizing R&D decisions of both industry leader and follower firms.

Regardless of initial conditions, equilibrium behavior in this model involves gradual convergence to a balanced-growth path where the innovation rate in each industry is constant over time and the economy grows at a constant rate. Starting from this balanced-growth path, a permanent unanticipated increase in the R&D subsidy leads to an increase in the rate of technological change. However, this normal effect is only temporary. With firms devoting more resources to R&D, R&D difficulty also increases more rapidly and the rate of technological change gradually falls back to the balanced-growth rate, as in Segerstrom (1998). A permanently higher R&D subsidy permanently increases the fraction of workers doing research but only temporarily stimulates the rate of technological change. Since public policies like R&D subsidies do not have any long-run growth effects, the model provides an explanation for the relatively low correlations in economic growth rates across decades documented in Easterly, Kremer, Pritchett and Summers (1993).

Even though R&D subsidies do not have long-run growth effects, it does not follow that a laissez-faire public policy is welfare maximizing. In fact, it is always optimal for the government to intervene by subsidizing/taxing R&D expenditures.

For industry leaders, there are three external effects associated with R&D activ-

ities: the consumer surplus effect (each innovation reduces a quality-adjusted price), the across-industry business stealing effect (each innovation reduces the demand for products in other industries) and the intertemporal R&D spillover effect (each innovation makes future innovations costlier to discover). Because the positive consumer surplus effect always dominates the combined across-industry business stealing and intertemporal R&D spillover effects, it is always optimal for the government to subsidize the R&D expenditures of industry leaders.

For follower firms, there is an additional external effect associated with R&D expenditures: the within-industry business stealing effect (every time a follower firm innovates, it drives the previous industry leader out of business). Because the combined within-industry business stealing, across-industry business stealing and intertemporal R&D spillover effects together dominate the positive consumer surplus effect, it is always optimal for the government to tax the R&D expenditures of followers. Without government intervention, follower firms overachieve given their innovative abilities, industry leaders underachieve given their innovative abilities and market forces generate too much creative destruction.

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Table 1: History of Intel Microprocessors

| Intel<br>Chip | Intro.<br>Date | MIPS<br>(est.) | Number of<br>Transistors | Addressable<br>Memory |
|---------------|----------------|----------------|--------------------------|-----------------------|
| 4004          | 11/71          | 0.06           | 2,300                    | 640 bytes             |
| 8080          | 4/74           | 0.64           | 6,000                    | 64 KB                 |
| 8086          | 6/78           | 0.75           | 29,000                   | 1 MB                  |
| 286           | 2/82           | 1.6            | 134,000                  | 16 MB                 |
| 386           | 10/85          | 5.5            | 275,000                  | 4 GB                  |
| 486           | 4/89           | 20             | 1,200,000                | 4 GB                  |
| Pentium       | 3/93           | 112            | 3,100,000                | 4 GB                  |
| Pentium Pro   | 11/95          | 450            | 5,500,000                | 64 GB                 |
| Pentium II    | 5/97           | –              | 7,500,000                | 64 GB                 |
| Pentium III   | 2/99           | –              | 9,500,000                | 64 GB                 |

Table 2: Intel R&D Expenditures  
(in millions of dollars)

| Year | R&D exp. |
|------|----------|
| 1989 | \$365    |
| 1990 | \$517    |
| 1991 | \$618    |
| 1992 | \$780    |
| 1993 | \$970    |
| 1994 | \$1,111  |
| 1995 | \$1,296  |
| 1996 | \$1,808  |
| 1997 | \$2,347  |
| 1998 | \$2,674  |

Table 3: 1998 Net Sales and R&D Expenditures  
of Select Industry Leaders (in billions of dollars)

| Industry<br>Leader | Net<br>Sales | R&D<br>Expenditure | R&D as a<br>% of Sales |
|--------------------|--------------|--------------------|------------------------|
| Boeing             | \$56.1       | \$1.9              | 3.4%                   |
| DuPont             | \$24.8       | \$1.3              | 5.2%                   |
| Eastman Kodak      | \$13.4       | \$0.9              | 6.7%                   |
| General Electric   | \$100.5      | \$1.9              | 1.9%                   |
| Hewlett Packard    | \$47.0       | \$3.4              | 7.2%                   |
| IBM                | \$81.7       | \$5.0              | 6.1%                   |
| Intel              | \$26.3       | \$2.7              | 10.3%                  |
| Johnson & Johnson  | \$23.7       | \$2.3              | 9.7%                   |
| Merck              | \$26.9       | \$1.8              | 6.7%                   |
| Microsoft          | \$14.5       | \$2.5              | 17.2%                  |
| Motorola           | \$29.4       | \$2.9              | 9.9%                   |
| 3M                 | \$15.0       | \$1.0              | 6.7%                   |
| Nokia              | \$15.6       | \$1.3              | 8.3%                   |
| Pfizer             | \$13.5       | \$2.3              | 17.0%                  |
| Xerox              | \$19.4       | \$1.0              | 5.2%                   |

Table 4: Scientists And Engineers Engaged In R&D  
(in thousands)

|      | United |       | West    |        | United  |
|------|--------|-------|---------|--------|---------|
| Year | States | Japan | Germany | France | Kingdom |
| 1965 | 494.2  | 117.6 | 61.0    | 42.8   | 49.9    |
| 1975 | 527.4  | 225.2 | 103.7   | 65.3   | 80.5    |
| 1985 | 801.9  | 380.3 | 143.6   | 102.3  | 131.0   |
| 1989 | 924.2  | 457.5 | 176.4   | 120.4  | 133.0   |
| 1993 | 962.7  | 526.5 | NA      | 145.9  | 140.0   |

Table 5: Average Annual Growth Rate  
in the Patents-Per-Researcher Ratio

| Country        | Time Period | Growth Rate |
|----------------|-------------|-------------|
| United States  | 1965-1993   | -2.18%      |
| France         | 1965-1993   | -6.07%      |
| Japan          | 1965-1993   | -0.11%      |
| Sweden         | 1971-1993   | -6.26%      |
| United Kingdom | 1969-1993   | -5.74%      |