Gigliotti, Gary; Sopher, Barry

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AN EXPERIMENTAL TEST FOR STABILITY OF THE TRANSFORMATION FUNCTION IN RANK-DEPENDENT EXPECTED UTILITYTHEORY AND ORDER-DEPENDENT PRESENT VALUE THEORY

Gary Gigliotti
Rutgers University
and
Barry Sopher
Rutgers University

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Abstract

We propose and analyze a generalization of present value maximization, "order-dependent present value (ODPV)," for intertemporal income choice. The model is analagous to the rank dependent expected utility model (RDEU) for choice under risk. The main feature of interest in the model is the "payment transformation function," which operates on proportions of a fixed total of payments just as the probability weighting function in RDEU operates on probabilities. These models can accomodate many choice patterns, for both risky and intertemporal choice, so we conduct experiments in an attempt to (i) measure the structure of preferences over lotteries and intertemporal income streams and (ii) test for stability of the probability and payment transformation functions over different choice sets. The design is based on manipulations of the "probability triangle" and the "intertemporal choice triangle." If, as in many previous studies of the RDEU model, a representative agent approach is taken, then the average preference structure in both the domain of risky choice and the domain of intertermporal choice can be characterized as "homothetic" in the respective choice triangles. This implies a strictly concave transformation function, and is at odds witht the finding of an "inverted S" shaped function that many researchers have suggested for the RDEU model. Individual analysis reveals considerable heterogeneity of preferences. A disaggregated analysis in which we classify subjects according to which transformation function is most consistent with their revealed choice behavior shows that a linear and a strictly concave transformation function are the most common for both risky choice and for intertemporal choice. Direct estimation of the transformation function is consistent with this classification. In particular, there is no evidence of an inverted S-shaped transformation function for choice under risk, contrary to several previous studies. The difference between our results and those of previous studies can be mainly attributed to the choice of functional forms used in estimating the transformation function, or to the limited space of lotteries upon which estimates have been based.

Keywords: Intertemporal choice, present value maximization.
JEL Classifications: C91, D90

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1. Introduction

Anticipated Utilty theory (Quiggin (1982)) and the many “rank dependent” generalizations of expected utility that followed (Chew, Karni and Safra (1985), Segal (1989), Wakker (1990), to name just a few) provides a theoretically coherent and testable structure within which violations of expected utility theory can be interpreted and specific remedies (e.g., the nature of nonlinear weighting of probabilities) can be tested. Emprical studies of the rank dependent model (e.g., Wu and Gonzalez, Prelec, Camerer and Ho) have generally concluded that the best-fitting specification for the probability weighting function is an inverted S-shaped function. These studies all suffer from one shortcoming, however, which is that they assume a representative agent in estimating the probability weighting function. That is, it is assumed that all subjects have the same preferences, but that they make errors in decision making. One contriubution of this paper is to provide a disaggregated analysis of choice under risk without the homogeneity assumption.

The theoretical structure of the rank dependent models can be easily adapted for the analysis of intertemporal choice. Given the recent surge of empricial studies that cast doubt on some of the most basic tenets of rational choice in a dyanmic setting (e.g., Loewenstein and Prelec (1992) and references therein, Gigliotti and Sopher (1997a, 1997b), Chapman (1996a, 1996b)) the time would appear to be ripe for a careful and systematic evaluation of intertemporal choice, with an eye towards testing a specific alternative to conventional theory. In an earlier paper (Gigliotti and Sopher (1997b)), we proposed a new framework for the analysis of intertemporal choice, the intertemporal choice triangle, which is a straightforward adaptation of the well-known Marshak-Machina probability triangle, which has been used to great effect in the analysis of choice under risk. The theory of present value maximization makes predictions for the intertemporal choice triangle that are as sharp and specific as the predicitons of expected utility theory for the probability triangle. In the earlier paper we also showed that it is easy to generate and test generalizations of present value maximization in the intertemporal choice triangle. We proceed with that research program in this paper by developing and testing *Time-Order Dependent Present Value* theory, a generalization of present-value maximization.

In both the analysis of choice under risk and choice over time, our focus is on testing for stability of the relevant transformation function (of probabilities or of payments). Our strategy in each case is to compare choice patterns in two different triangles. Consider choice under risk: In the first triangle, the middle of three monetary prizes is close in size to the largest prize, while in the second triangle the middle prize is close in size to the smallest prize. The assumptions of increasing utilty of money (for risk) or of a positive discount rate for money in the future (for intertemporal choice) leads to a predictable shift in choices between thefirst and second trinangle, namely, choices should move towards the hypotenuse. In a way that we shall make precise, this prediction is due only to the fact that (elementary) utility is increasing in mony or the fact that the disount factor is decreasing with time. The precise location of choice in the relevant triangle can be rationalized by such a utility or discount function and a stable transformation function. We now discuss the intertermporal choice triangle in more detail.

The Intertemporal Choice Triangle

The Marschak-Machina (MM) probability triangle has been put to extensive use in the analysis of choice under risk (Machina (1987)). It is an elegant and useful tool, which gives an intuitive and clear picture of how expected utility theory should function, and how individuals may act in violation of it. In the probability triangle framework, shown in Figure 1, any point on the boundary or interior of the triangle represents a lottery over three prizes. The probabilities of the large and small prizes, p_l and p_s respectively, are measured from 0 to 1 on the vertical and horizontal sides of the triangle, respectively. The probability of the middle-sized prize is expressed as the residual $p_m=1-p_l-p_s$. Constant expected utility contours in the triangle are straight parallel lines whose slopes are determined by the ratio scale of an individual's elementary utility function. Constant expected value contours are also linear, making it easy to determine an individuals' risk aversion or risk preference; a risk averse individual would have expected utility contours flatter than expected value contours, a risk neutral individual would have expected utility contours with the same slope as expected value contours, and a risk preferring individual would have expected utility contours that are steeper than expected value contours.

The probability triangle is based on the assumption that probabilities change in a constrained fashion within the triangle. For example, moving from the origin along a ray towards the hypotenuse, the probability of winning the middle prize rises, and the probability of winning the large and small prizes change in fixed proportions. The utility of dollar outcomes does not change unless the dollar outcomes change. Many experimental studies using the MM triangle have shown that subjects may have variable levels of risk aversion as the probability of winning the middle prize changes, and that simple expected utility theory cannot explain this phenomenon.

A similar triangle-based analysis can be done in intertemporal choice theory, as shown in Figure 2. We assume three payout dates, t_0, t_1, t_2, which may or may not be equally spaced. The legs of the triangle represent not probabilities, but the amount of money received in a given period. The vertical leg measures the amount of money received in the first of three given payout dates, and the horizontal leg measures the amount of money received in the last of the three payout dates. These values range from 0 to B, where the latter represents the total dollars available over all three periods, and $B_0+B_1+B_2 = B$, analogous to the sum $p_1+p_2+p_3=1$ in the MM triangle. As described below, constant present value contours within the triangle will be linear, with the slope value dependent on the spacing of payments. If the payout dates are equally spaced, then the slope of the constant present value contour will equal the discount factor for the middle payout period, $0<d_{t1}<1$.

2. The Intertemporal Choice Triangle and Order-Dependent Present Value

Consider a (T+1)-period income stream S, denoted by $S=[B_0, B_1, \ldots ,B_T]$, with discount function *(t). Since total payments, B, are fixed, we can set

$$B_i \,'\, B\& \mathop{j}_{t\ldots i} B_t$$

for some I. For a three-period income stream we can illustrate and analyze income streams in a triangle. The present value of an income stream, S, is

$$PV(S) \,'\, *(t_0)B_0\%*(t_1)(B\&B_0\&B_2)\%*(t_2)B_2$$

Note that t_0 is the time of the initial payment, and t_1 and t_2 are the times when the subsequent payments occur. Taking the total derivative and setting equal to zero, we can derive the following useful expression:

$$\frac{dB_0}{dB_2} \,'\, \frac{[*(t_1)\&*(t_2)]}{[*(t_0)\&*(t_1)]}$$

This is the (constant) slope of a constant-present-value contour in an intertemporal choice triangle. If there is a constant discount rate so that $*(t)=*^{t}$ then the slope does not depend on the time until the initial payment, t_0 (it can be factored out). If, more generally, the discount rate is not constant (e.g., a hyperbolic discount function), then the slope of constant present-value contours will depend on the initial time as well. Since we are not addressing issues related to whether the discount rate is constant or not in this study, we refer the reader to Gigliotti and Sopher (1997b) for details on the implications of nonconstant discounting for choice in the intertermporal choice triangle. *Order dependent present value* augments the basic present value model with a transformation or weighting function for the payments. Letting $S=[B_0, B_1, \ldots, B_T]$ denote a payment stream in terms of *proportions* of a total, B, the order-dependent present value of S is given by:

$$ODPV(S) \,'\, *(t_0)f(B_0)\%*(t_1)(f(1\&B_2)\&f(B_1))\%*(t_2)(1\&f(1\&B_2))$$

where f is the transformation function. The only conditions placed on f are that f(0)=0, f(1)=1, and f is monotonic. If f is smooth and differentiable, then the slope of the ODPV function in the triangle is given, through total differentiation, by

$$\frac{dB_0}{dB_2} \,'\, \frac{[*(t_1)\&*(t_2)]}{[*(t_0)\&*(t_1)]}\frac{[f^{)}(1\&B_2)]}{[f^{)}(B_0)]}$$

This expression can be used to derive implications for the shape of constant order-dependent present value contours in the intertemporal choice triangle for differnt assumed forms of the transformation function. An illustration of several possible transformation functions and the implied preference maps are shown in Figure 3.

3. Design of the Experiment

4. Empirical Results

Table 1 contains a summary statistic for the choices in the experiment, the mean *normalized choice* on each chord in each triangle. The normalized choice ranges from 0 to 1, with 0 indicating a choice on one of the axes of a triangle, 1 indicating a choice on the hypotenuse, and numbers strictly between 0 and 1 indicating choices strictly in the interior of a triangle. The lines in the table labeled “horizontal dom.” and “vertical dom.” refer to the first two choices in the experiment, each of which has a dominant choice. The horizontal dominance question calls for a choice of 0, since any other choice involves a direct trade of probability of the largest prize for probability of the smallest prize (for Risk) or a direct trade of a payment in the earliest period for a payment in the latest period (Time). The vertical dominance question calls for a choice of 1, since any other choice involves trading probability of the largest for probability of the middle prize, or payment in the eariest period for payment in the middle period. The other lines in the table refer to the chords described in the design section above, and illustrated in Figure 3.

The main thing to note in this table is that there is generally an increase in the normalized choice from Triangle I to Triangle II in each experiment, consistent with the RDEU or ODPV, respectively. That is, the average choices are consistent with the expression for the slope of constant preference contours that is separable into a part due to the utilty or discount function and a part due to the transformation function.

Table 2 contains a detailed classification of choice patterns. Specifically, the data is organized into triples of choices, with each triple containing choices along chords of the same slope within a given triangle. For example, column Ia refers to the chord choices in Triangle I with a slope of 1/3, Ib refers to chord choices in Triangle II with a slope of 1, and Ic refers to chord choices in Triangle I with a slope of 3. Each triple of choices is classified into one of 27 patterns according to whether the choices in the tripe were at the axis end of a chord, in the interior of the chord, or at the hypotenuse of the chord. In the "description" column of the table there is a triple of numbers for each possible pattern, (#1, #2, #3), where each entry is either a 1 (axis choice), a 2 (interior choice) or a 3 (hypotenuse choice). A choice is classified as at the end of a chord if it was within .01 of the length of the chord from that end. The first eight patterns (1-8) in the table all have at least 2 end choices at the same end of the chord and at most one interior choice. The next 12 patterns (9-20) all have at least one end choice at each end of a chord, and at most one interior choice. The last seven patterns (21-27) all have at least two interior choices. In the next part of the analysis we aggregate choice patters into the just described three groups of patterns, and refer to them as "linear," "inflected," and "concave" patterns, with the name of each group indicating the shape of transformation function consistent with such a choice pattern. The main thing to note in Table 2 is the prominance of the strictly linear (patterns 1 and 5) and the strictly concave (pattern 27) choice patterns. Together these three pattern account for just under 50% of choices in the Risk Experiment and just over 50% of choices in the Time Experiment.

Tables 3 and 4 contains direct comparisons of choice patterns over chord triples of the same slope between the two triangles in each Experiment. For example, the matrix at the top of Table 3 compares choice patterns over the chord choices with slope 1/3 in Triangle I of the Risk Experiment with choice patters over the same chord choices in Triangle II. The Test statistics reported beside each matrix are various tests of the hypothesis that the distribution of choice patterns is unchanging from Triangle I to Triangle II. We interpret these tests as evidence of stability (or lack thereof) of the transformation function. If choice patterns in Triangle I and Triangle II are consistent with the same general sort of transformation function (linear, infllected, or concave), then we call the transformation function stable. This hypothesis is never rejected statistically. More interesting, perhaps, than the stability of the transformation function is what the distribution of transformation functions implies. In the Risk Experiment, the preponderance of choice patterns are consistent with a linear or concave transformation function, and with very few inflected patterns. Conventiional wisdom has it that the probability transformation function is inflected, with a crossing point around p=.35, but there is simply no evidence in support of this in our experiment. In the Time Experiment, there is a more uniform distribution of patterns consistent with each type of transformation functiion, but the inflected patterns are still the least prevalent.

5. Conclusions

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Table 1: Normalized Choices in Risk and Time Experiments

Means of Normalized Choices:	RISK		TIME	
Triangle Chord	I	II	I	II
horizontal dom.	.05		.23	
vertical dom.	.86		.78	
a1	.25	.23	.45	.51
a2	.25	.30	.44	.60
a3	.29	.51	.52	.74
b1	.16	.22	.39	.79
b2	.21	.43	.47	.73
b3	.24	.42	.71	.82
c1	.11	.25	.32	.64
c2	.13	.22	.38	.59
c3	.35	.50	.55	.64

Table 2: Choice Patterns in Risk and Time Experiments

	Frequencies of Choice Patterns:	RISK						TIME					
Pattern #	Description	Ia	IIa	Ib	IIb	Ic	IIc	Ia	IIa	Ib	IIb	Ic	IIc
1 (linear x)	1,1,1	12	6	7	1	2	1	9	1	9	3	4	0
2 (alx1)	1,1,2	1	5	1	3	1	0	0	2	0	0	0	0
3 (alx2)	1,2,1	1	1	2	1	0	0	0	0	0	1	0	0
4 (alx3)	2,1,1	1	0	0	0	1	0	0	0	0	0	0	0
5 (linear h)	3,3,3	0	0	0	0	1	1	1	7	2	5	6	8
6 (alh1)	3,3,2	1	0	1	0	1	0	0	0	1	2	1	1
7 (alh2)	3,2,3	0	0	0	0	1	1	0	1	1	0	0	0
8 (alh3)	2,3,3	0	0	0	0	0	1	0	1	0	1	0	2
9 (lnpv1)	1,1,3	0	0	0	0	1	1	0	0	0	0	1	0
10 (lnpv2)	1,3,1	0	0	1	0	0	0	1	5	2	1	2	3
11 (lnpv3)	3,1,1	0	0	0	0	0	1	4	0	1	1	1	2
12 (lnpv4)	3,3,1	0	0	0	0	0	0	0	1	0	2	1	2
13 (lnpv5)	3,1,3	0	0	0	0	0	0	0	0	1	1	0	0
14 (lnpv6)	1,3,3	0	0	0	0	0	1	2	0	1	2	3	1
15 (alnpv1)	1,3,2	0	1	0	2	0	0	0	1	0	1	1	0
16 (alnpv2)	3,1,2	1	0	1	1	0	0	0	0	0	0	0	0
17 (alnpv3)	1,2,3	0	0	0	0	0	0	0	2	0	1	0	0
18 (alnpv4)	3,2,1	1	0	0	0	0	0	0	0	0	0	0	0
19 (alnpv5)	2,1,3	0	0	0	0	0	0	0	0	0	0	0	0
20 (alnpv6)	2,3,1	0	0	0	0	0	1	0	0	0	0	0	0
21 (acon1)	2,2,3	0	0	0	0	1	3	0	1	0	0	1	1
22 (acon2)	2,3,2	0	0	0	2	0	0	1	0	0	0	2	0
23 (acon3)	3,2,2	1	1	2	1	1	3	2	3	1	2	0	0
24 (acon4)	2,2,1	2	1	2	3	2	0	1	0	1	0	0	0
25 (acon5)	2,1,2	1	0	1	0	4	0	0	0	0	0	0	0
26 (acon6)	1,2,2	2	2	3	5	4	0	0	0	1	0	0	0
27 (concave)	2,2,2	3	10	6	8	7	13	7	4	7	5	5	8
TOTAL		27	27	27	27	27	27	28	28	28	28	28	28

Table 3: Test for Stability of the Probability Weighting Function

Risk: Pattern Ia vs Pattern IIa

After Before	Linear	Inflecte d	Concave	Total
Linear	11	1	4	16
Inflected	0	0	2	2
Concave	1	0	8	9
Total	12	1	14	27

Risk: Pattern Ib vs Pattern IIb

After Before	Linear	Inflecte d	Concave	Total
Linear	4	3	4	11
Inflected	0	0	2	2
Concave	1	0	13	14
Total	5	3	19	27

Risk: Pattern Ic vs Pattern IIc

After Before	Linear	Inflecte d	Concave	Total
Linear	2	1	4	7
Inflected	0	1	0	1
Concave	2	2	15	19
Total	4	4	19	27

Table 4: Test for Stability of the Payment Weighting Function

Time: Pattern Ia vs Pattern IIa

After Before	Linear	Inflecte d	Concave	Total
Linear	6	3	1	10
Inflected	3	4	0	7
Concave	2	2	7	11
Total	11	9	8	28

Time: Pattern Ib vs Pattern IIb

After Before	Linear	Inflecte d	Concave	Total
Linear	6	6	1	13
Inflected	3	2	0	5
Concave	3	1	6	10
Total	12	9	7	28

Time: Pattern Ic vs Pattern IIc

After Before	Linear	Inflecte d	Concave	Total
Linear	7	3	1	11
Inflected	2	5	2	9
Concave	2	0	6	8
Total	11	8	7	28