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Patent Pools, Litigation and Innovation

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Patent Pools, Litigation and Innovation

Abstract

This paper analyzes patent pools and their effects on innovation incentives. It is shown that the pro-competitive effects of patent pools for complementary patents naturally extend for dynamic innovation incentives. However, this simple conclusion may not hold if we entertain the possibility that patents are probabilistic and can be invalidated in court. In such a case, the licensing fees reflect the strength of patents. Patent pools of complementary patents can be used to discourage litigation by depriving potential licensees of the ability to selectively challenge patents and making them committed to a proposition of all-or-nothing in patent litigation. We show that if patents are sufficiently weak, patent pools with complementary patents reduce social welfare as they charge higher licensing fees and chill subsequent innovation incentives.

JEL-Code: O300, L100, L400, D800, K400.

Keywords: patent pools, probabilistic patent rights, patent litigation, complementary patents.

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1 Introduction

This paper analyzes patent pools and their effects on innovation incentives when patent rights are probabilistic. The existing literature on patent pools mainly focuses on the effects of package licensing on pricing and shows that the procompetitive effects of patent pools depend crucially on the relationship among constituent patents. If they are complementary in nature, patent pools can reduce the overall licensing royalties by internalizing pricing externalities and thus are procompetitive. However, if they are substitute patents, patent pools can be used as a collusive mechanism that eliminates price competition, and thus are anticompetitive (Shapiro, 2001; Lerner and Tirole, 2004).

We consider the dynamic effects of patent pools by investigating the effects of patent pools for subsequent innovations that build on patents in the pools. We show that the procompetitive effects of patent pools for complementary patents naturally extend for dynamic innovation incentives. As patent pools can mitigate the patent thicket problem for the current users, they reduce the royalty rates for subsequent innovations as well. As a result, follow-on innovators are less burdened by the royalty rates and subsequent innovations are promoted. However, this simple conclusion may not hold if we entertain the possibility that patents are probabilistic and can be invalidated in court. In such a case, the royalty rates reflect the strength of patents. If patents are weak, the overall royalty rates can be low with independent licensing. Patent pools of complementary patents can be used as a mechanism to discourage patent litigation by depriving potential licensees of the ability to selectively challenge patents and making them committed to a proposition of all-or-nothing in patent litigation. Patent pools thus can be used as a litigation-deterrent mechanism and enable them to charge higher royalty rates when the demand margin is not binding.

Our paper is motivated by recent trends in high-tech industries. As products become more complex and sophisticated, they tend to encompass numerous complementary technologies. In addition, the innovation process is typically cumulative with new technologies building upon previous innovations (Scotchmer, 1991). To reflect such an environment, we consider a setup in which the development of a new technology requires licensing of multiple complementary patents owned by different firms. With complementary patents, patent pools are considered to be an effective way to mitigate the problem of “patent thicket” and reduce transaction costs. For instance, the Antitrust Guide Lines for the Licensing and

Acquisition of Intellectual Property (1995), jointly published by the U.S. Department of Justice and the Federal Trade Commission, recognizes that inclusion of complementary or essential patents in a patent pool is pro-competitive. We point out that such a sanguine view about patent pools with complementary patents may not be justified if we consider probabilistic patent rights.

To illustrate this, we develop the notion of the “litigation margin” that relates the patent holders’ ability to set license fees to litigation incentives by potential licensees. When the patent holders set their license fees, they need to consider the effects of a price increase on demand and litigation incentives by potential licensees. Since the incentives to litigate and invalidate patents increase inversely with the strength of patents, the litigation margin is the binding constraint for the patent holders when patents are weak. We show that patent pools provide a channel to relax the litigation margin, which leads to elevated license fees. Thus, the welfare effects of patent pools with complementary patents depend on whether the demand margin or the litigation margin is binding. When the demand margin is binding, the conventional result holds and patent pools are welfare-enhancing because they eliminate the pricing externality among patent holders. However, if the litigation margin is binding, which occurs with weak patents, patent pools can be welfare-reducing. Our paper thus formalizes the idea expressed in the *Duplan* case in which the court concluded that “[t]he ... patents in suit were known ... to be weak and, ..., they [the parties] were confident that these patents could be invalidated.” The main purpose of the patent pool in the case was “to protect the parties from challenges to the validity of their patents” in order to gain “the power to fix and maintain prices in the form of royalties which they... exercised thereafter.”¹

The literature on the effects of patent pools on dynamic innovation incentives is sparse. Lerner and Tirole (2004), for instance, build a model of a patent pool in which they provide a necessary and sufficient condition for a patent pool to enhance welfare.² However, their analysis is essentially static and its main focus is on the effects of patent pools on pricing whereas the main focus of this paper is on future innovation incentives. On the surface, we

¹Duplan Corp. v. Deering Milliken, Inc., 444 F. Supp 648 (D.S.C. 1977) at 682, 686. See also Gallini (2011) and Gilbert (2004).

²See Santore, McKee and Bjornstad (2010) for experimental evidence in a laborative setting that documents the efficiency effects of patent pools with complementary patents. Aoki and Nagaoka consider sequential coalition formation to discuss the incentives to form patent pools.

can easily generalize the static framework of complementary patent pools to a dynamic context and show that patent pools can have beneficial effects on future innovation incentives, as shown below. However, such a prediction seems to be in conflict with the recent empirical findings. Lampe and Moser (2010, 2013, forthcoming) and Joshi and Nerkar (2011) provide the first empirical tests of the effects of a patent pool on innovation incentives. More specifically, Lampe and Moser (2010, 2011) study the Sewing Machine Combination (1856-1877), the first patent pool in U.S. history whereas Joshi and Nerkar (2011) study the effects of patent pools in the recent global optical disc industry. In both industries, they find that patent pools inhibit, rather than enhance, innovation by participating firms.³

In particular, Lampe and Moser (2010, forthcoming) show that the pool has discouraged patenting and innovation and attribute the negative incentive effects of patent pools to the fact that patent pools create more formidable entities in court and thus increases the threat of litigation for outside firms. Lampe and Moser (2013) further extend their empirical analysis to examine patent pools in 20 industries in the 1930s. They find a substantial decline in patenting after the formation of a pool and come to the same conclusions as for the sewing machine industry. We develop a dynamic model of innovation in the presence of uncertain patent validity and litigation that is consistent with this empirical evidence on patent pools. In particular, our analysis shows that patent strength is an important consideration in the evaluation of patent pools as it affects the term of licensing when the litigation margin is binding.

Our paper closely relates to Shapiro (2003) and Choi (2010) who also recognize that IPR associated with patents are inherently uncertain or imperfect, at least until they have successfully survived a challenge in court. Shapiro (2003), for instance, proposes a general rule for evaluating proposed patent settlements, which is to require that “the proposed settlement generate at least as much surplus for consumers as they would have enjoyed had the settlement not been reached and the dispute instead been resolved through litigation.” However, his proposal and Choi’s (2010) analysis only consider the static welfare and do not consider the implications of innovation incentives for dynamic efficiency.

Llanes and Trento (2012) consider a dynamic model of sequential innovations with each

³In a related empirical research, Baron and Delcamp (2010) explores the impact of patent pools on firm patenting strategies and show that firms that are already members of a pool are able to include narrower, more incremental and less significant patents than outsiders.

innovation building on all prior innovations made. They assume ironclad patents and find that the probability of innovation eventually approaches to zero with independent licensing as the innovation process progresses and increasingly more patent holders lay claims over part of the revenues generated by subsequent innovations. Patent pools alleviate this problem and are shown to increase the probability of innovation as in our model with ironclad patents. However, they do not consider probabilistic patents and thus litigation incentives are not their focus. Dequiedt and Veraevel (2013) and Kwon *et al.* (2008) also analyze the effects of patent pools on innovation incentives. However, these papers adopt *ex ante* perspectives and study the impact of possible pool formation on the incentives to innovate whereas we consider future development incentives by *outsiders* that arise *ex post*. In addition, they do not consider probabilistic patents and the analysis is devoid of any litigation incentives. Finally, Gilbert (2002) provides a brief history of patent pools and points out that patent pools can be used to protect dubious patents from challenges. This paper provides a theoretical foundation of a mechanism through which dubious patents can be shielded from challenges to the validity of the patents.

The remainder of the paper is organized in the following way. In section 2, we set up the basic model to analyze development incentives for subsequent innovations based on a set of complementary patents. As a benchmark case, we analyze the case of ironclad patents and show that patent pools with complementary patents promote subsequent innovations, echoing the basic presumption in the literature and enunciated in the Antitrust Guide Lines for the Licensing and Acquisition of Intellectual Property (1995). In section 3, we extend the analysis to consider probabilistic patents and explicitly consider strategic incentives to litigate. As a first step, we consider a situation in which only the litigation margin is binding by abstracting from the pricing externalities issue associated with the demand margin. This is to isolate the mechanism through which patent pools deter litigation and elevate royalty rates vis-à-vis independent licensing. In section 4, we analyze the full model that takes account for both the litigation and demand margins. We show that the welfare effects of patent pools crucially hinge on the strength of the complementary patents. In particular, patent pools with weak complementary patents can lead to elevated licensee fees and reduce incentives to develop subsequent innovations as they can be used as a mechanism to harbor weak patents from litigation that could invalidate them. Essentially, package licensing by patent pools deprives potential licensees of the ability to selectively challenge

patents. To address this problem, section 5 considers a public policy that mandates patent pools to engage in individual licensing and its welfare effects. Section 6 expands on the basic model and considers extensions of the model to check the robustness of the main result. The last section concludes.

2 Complementary Patent Pools and Future Development Incentives

We consider a situation of multiple patents with dispersed ownership. For analytical simplicity, assume that there are two complementary patents, A and B , which are owned by two separate firms.⁴ The patents are deemed essential as the commercialization of a new technology or product requires the practice of both patents.⁵ We analyze incentives to form a patent pool by the patent owners and the competitive implications of package licensing. As emphasized by Scotchmer (1991), innovations are cumulative. In order to analyze how the formation of patent pools can affect future incentives to develop new innovations that build on existing patents, we consider the following multi-stage game.

In the first stage, the two firms decide whether or not to form a patent pool. In the second stage, they set licensing fees that allow other firms to use their technologies without infringing them. If they do not form a patent pool, they set the licensing fees independently. If they form a patent pool, they can offer a package licensing. In the third stage, another firm, C , comes up with a potential innovation that can create a total value of v . The cost to implement the innovation is c . The innovation is assumed to be patentable, but cannot be practiced without consent of the holders of the essential patents. Alternatively, we can think of firm C as a downstream firm that commercializes the patented technologies to the market and c can be considered as a development cost. The development cost c is randomly distributed with a cumulative distribution function $G(\cdot)$ and its corresponding density function $g(\cdot)$. Assume that the reversed hazard rate of $G(\cdot)$, defined by $r(\cdot) = g(\cdot)/G(\cdot)$ is monotonically decreasing in its argument.

We briefly comment on our assumptions about timing and modeling. We allow the possibility of ex ante licensing and analyze the patent holders' incentive to offer their inno-

⁴We discuss the case with $n \geq 2$ patents in Section 6.

⁵For instance, the intellectual property to be licensed are research tools (Schankerman and Scotchmer, 2001) and any final producer needs to get licenses from both patentees.

vation at a fixed price before investments for complementary innovations are made. Such ex ante licensing can serve as a commitment mechanism not to hold up against complementary innovations that may come later.

Suppose that both firms offer ex ante contracts independently. Let f_A and f_B be the fixed licensing fees charged by firm A and firm B , respectively. Then, firm C develops the new innovation only when its development cost is less than $(v - f_A - f_B)$. Thus, given the licensing fees of f_A and f_B , the probability of the new innovation to be developed is given by $G(v - f_A - f_B)$.

Without a patent pool, each firm sets its licensing fee independently. Then firm A solves the following problem given firm B 's royalty rate f_B ,

$$\max_{f_A} f_A G(v - f_A - f_B)$$

The first order condition for firm A 's optimal royalty rate f_A is given by

$$G(v - f_A - f_B) - f_A g(v - f_A - f_B) = 0, \quad (1)$$

which can be rewritten as

$$f_A = \frac{G(v - f_A - f_B)}{g(v - f_A - f_B)}.$$

With the monotone reversed hazard rate condition, it can be easily shown that the first order condition for each patentee's maximization problem satisfies the second order condition. Equation (1) thus implicitly defines firm A 's reaction function $f_A = \Theta(f_B)$. Firm B 's reaction function, $f_B = \Theta(f_A)$, can be derived in a similar way. The Nash equilibrium licensing fees f_A^* and f_B^* are at the intersection of these two reaction functions. The monotone reversed hazard rate assumption guarantees the stability and the uniqueness of the Nash equilibrium in licensing fees. With perfect complementarity and ironclad patents, both firms are in a symmetric position and charge $f_A^* = f_B^* = f^*$. The total royalty rate in the absence of a patent pool is given by $F^* = f_A^* + f_B^*$.

In contrast, if firms A and B form a patent pool and practice package licensing, the optimal royalty rate is derived by solving

$$\max_F F G(v - F)$$

Let F^{**} be the optimal ex ante fixed licensing fee for the pool.⁶ Then, F^{**} satisfies the following first order condition:

$$G(v - F^{**}) - F^{**}g(v - F^{**}) = 0, \quad (2)$$

which can be rewritten as

$$F^{**} = \frac{G(v - F^{**})}{g(v - F^{**})}.$$

Proposition 1 shows that the overall licensing fees are lower when firms form a patent pool. Thus, patent pools promote subsequent innovation incentives when the pool patents constitute blocking patents for future innovations.

Proposition 1 $F^* = f_A^* + f_B^* > F^{**}$. *When firms form a patent pool, total licensing fees are lower and there are more subsequent innovations. Social and private incentives to form a patent pool are perfectly aligned.*

This is a variation of the well-known result that dates back to Cournot's (1927) analysis of the complementary monopoly problem. Without coordination in licensing fees, each patentee does not internalize the increase in the other patentee's profits when the demand for the package is increased by a reduction in its price. Thus, a patent pool can decrease the overall royalty rates for the package and simultaneously increase both patentees' profits and induce more future innovations. Consequently, social welfare also increases. Thus an argument can be made for a lenient treatment of patent pools due to their pro-competitive effects when multiple complementary patents form blocking patents for future innovations. Our analysis thus provides an additional dynamic efficiency justification for allowing patent pools for complementary innovations, which goes beyond those identified for static efficiency.

3 Probabilistic Patent Rights and Litigation with Patent Pools

In the previous section, we have seen that patent pools of complementary technologies have additional salutary effects of promoting subsequent innovations. However, this conclusion hinges crucially on the assumption of iron-clad patents. If we recognize that patent rights are probabilistic and can be invalidated in court when challenged, licensing typically takes

⁶Variables associated with patent pools are denoted with double asterisks.

place in the shadow of patent litigation and the licensing terms will reflect the strength of patents. In this section, we show that if patent pools are used as a mechanism to harbor weak patents and deter patent litigation, patent pools may induce higher royalty rates than would be paid if licenses were sold separately by independent patent holders.

A Model of Probabilistic Patents. To analyze incentives to form patent pools with probabilistic patents, we represent the uncertainty about the validity of the patents by the parameters $p_A = \alpha \geq 0$ and $p_B = \beta \leq 1$, which are the probabilities that the court will uphold the validity of patents A and B , respectively, if they are challenged. Without any loss of generality, we assume that patent B is weakly stronger than patent A , that is, $\alpha \leq \beta$. We assume a symmetric information structure in that α and β are common knowledge. The timing is as follows. First, the upstream firms set license fees. If the downstream firm accepts both licenses, the game ends. With probabilistic patents, firm C has the option to challenge one or both patents rather than paying the license fee imposed by the patentees. When firms go to court to determine the validity of a patent, they each incur a cost $L \geq 0$. For simplicity, the litigation cost is independent of the patent validity parameters. If the court invalidates the patent, the downstream firm can use the technology at no cost. If the patent is validated, the patent holder offers a license fee and firm C decides whether to purchase the license or not.⁷ If firm C does not acquire a license of a validated or unchallenged patent, it is unable to produce and receives a profit of zero. Throughout the analysis we focus on parameter values such that

$$(1 - \alpha)(1 - \beta)v \geq 2L. \tag{A}$$

This condition ensures that firm C prefers litigating against both patentholders to remaining inactive.

As an intermediate step towards deriving the optimal licensing fees with both an active demand and litigation margin, we first consider a game that ignores the demand margin and focuses on the litigation margin. In other words, we assume that firm C always develops (or already has developed) the subsequent innovation and we analyze how litigation considerations influence the patentees' licensing decisions. This approach allows us to abstract

⁷Farrell and Shapiro (2008) make a similar assumption. They assume that if a patent is ruled valid, any licenses already signed remain in force, but that the patent holder negotiates anew with the downstream firm(s) that lack licenses.

from the pricing externalities issue associated with the demand margin. We consider the full game with demand and litigation margin in Section 4. In the following we again look at two different organizations of the upstream firms.⁸ First, we solve for the subgame perfect equilibrium when the upstream firms are independent. Then, we analyze the case when they form a patent pool and practice package licensing.

Licensing and Litigation with Independent Firms. Suppose firms A and B propose license fees, f_A and f_B , respectively. At this point firm C has four strategic options. First consider the case in which firm C litigates against both patentholders. If the court declares both patents invalid, firm C can use both technologies at no cost. If exactly one patent is upheld, its owner charges the monopoly price. If both patents are upheld, there exists a Nash equilibrium in which each patent holder charges $v/2$ and firm C makes no profits. Hence, the downstream firm's expected profit is

$$V_{AB} = (1 - \alpha)(1 - \beta)v - 2L.$$

Under assumption (A), it holds that $V_{AB} \geq 0$, that is, litigating both patent owners always dominates remaining inactive. Next suppose firm C challenges the patent of technology A and buys the license of B . If the patent is upheld, firm A charges $v - f_B$ and firm C receives no profits. If the patent is invalidated, the downstream firm can use technology A at no cost. Hence, the expected payoff is

$$V_A = (1 - \alpha)(v - f_B) - L.$$

Similarly, the expected profits of challenging patent B and purchasing the license for A are

$$V_B = (1 - \beta)(v - f_A) - L.$$

Note that the payoff of litigating exactly one patent, decreases in the license fee paid for the other technology. Finally, if firm C accepts both license offers it gets

$$V_0 = v - f_A - f_B.$$

⁸In Section 5 we also consider the possibility of a patent pool selling individual licenses rather than a package license.

What is the optimal licensing and litigation strategy for firm C? As convention, assume that if the downstream firm is indifferent between two options, it chooses the one that involves less litigation. If the downstream firm is indifferent between litigation against A or B, the firm randomizes and litigates with probability 1/2 against one of the two patentees. It can be shown that firm C purchases both licenses if $V_0 \geq V_A$, that is,

$$f_A \leq \alpha(v - f_B) + L \quad (3)$$

and $V_0 \geq V_B$, which requires

$$f_B \leq \beta(v - f_A) + L. \quad (4)$$

Region 0 in FIGURE 1 below contains all license fee pairs that satisfy these two conditions. Let $(\bar{f}_A, \bar{f}_B)$ denote the license fee pair at which both conditions hold with equality. Alternatively, firm C prefers not to purchase licenses and litigate both patents if $V_{AB} \geq V_A$,

$$f_B \geq \beta v + \frac{L}{1 - \alpha} \quad (5)$$

and $V_{AB} \geq V_B$,

$$f_A \geq \alpha v + \frac{L}{1 - \beta}. \quad (6)$$

These conditions are satisfied in region AB of FIGURE 1. Finally, it is easy to check that there exist license fees that neither satisfy the conditions of region 0 nor those of region AB. For these license fees, the downstream firm is best off buying a license from one patent owner and litigating against the other patent. Firm C prefers to litigate patent A if $V_A \geq V_B$ or

$$f_B \leq \frac{\beta - \alpha}{1 - \alpha} v + \frac{1 - \beta}{1 - \alpha} f_A. \quad (7)$$

If the license fee for patent B is relatively small compared to f_A , then the downstream firm litigates patent A (region A). Otherwise, it contests the validity of patent B (region B). We can thus summarize the downstream firm's optimal litigation and licensing as follows.

Lemma 1 *If f_A and f_B are both sufficiently low, firm C buys both licenses. If f_A and f_B are both sufficiently high, the downstream firm litigates both patents. Otherwise, firm C litigates exactly one patent.*

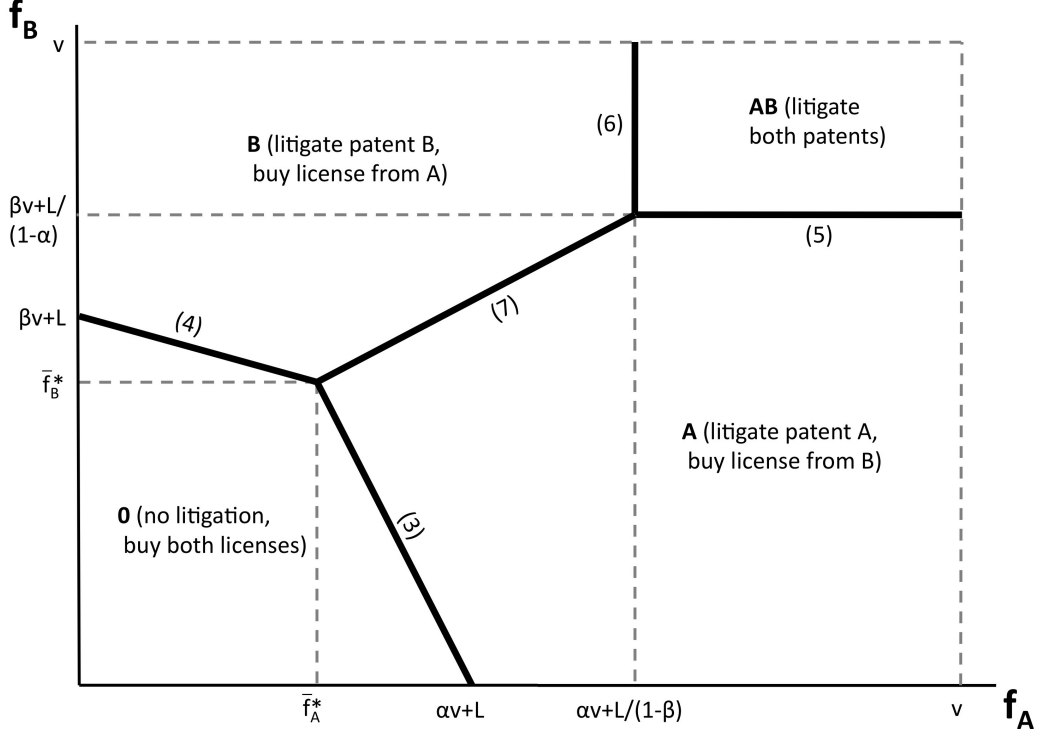


Figure 1: *Litigation Incentives of Downstream Firm*

Before deriving the license fee equilibrium, it is instructive to consider the effect of patent complementarity on the licensing and litigation decision of the downstream firm. Suppose the patents A and B were independent, each offering a market value of $v/2$ for firm C. In this case, the litigation decisions for the two patents are completely independent as firm C would litigate patent i if and only if $f_i \geq p_i v/2 + L$. By contrast, complementary patents introduce a negative externality from the litigated technology to the returns with the other technology. To see this rewrite the profits from litigation against both patents as

$$V_{AB} = (1 - \alpha)v/2 + (1 - \beta)v/2 - 2L - (\alpha + \beta - 2\alpha\beta)v/2.$$

The last term is the negative profit difference between litigation against two complementary and two independent patents, respectively. In the former case, firm C needs to win both litigation cases in order to achieve a positive profit. Hence, a negative litigation outcome with one patent eliminates the returns from the other technology except for the case where both suits are lost. This externality is only one-way when the downstream firm litigates against exactly one patent. Rewriting, the returns from litigating, say technology B, and

buying the license for A, gives

$$V_B = v/2 - f_A + (1 - \beta)v/2 - L - \beta(v/2 - f_A).$$

The last term is again due to the externality with complementary patents as a negative litigation outcome with patent B eliminates the rents from the purchase of license A. The existence of these externalities reduces the profitability of litigation in the presence of complementary patents. Relative to independent patents, there are more license fee pairs for which the downstream firm buys both licenses and less fee pairs at which both patents are litigated. Additionally, there exist total license fee levels such that firm C strictly prefers to litigate against one patent only.

Let us now turn to the equilibrium analysis. In the absence of a patent pool, patentees A and B set their license fees independently and maximize their respective, expected profits. If firm C buys patent i , its owner obtains f_i . This occurs in region 0 as well as for fees where firm C only litigates against the other patent, that is, in region j . If the downstream firm litigates against patent i and buys the license for technology j , patent owner i gets $p_i(v - f_j) - L$. Finally, if firm C litigates against both patentees, patent holder i gets an expected payoff of $p_i p_j v/2 + p_i(1 - p_j)v - 2L$. We now show that each patentee's best response to a license fee of the other patent holder is a limit licensing strategy that avoids litigation from the downstream firm with probability one. The limit licensing strategy for patent holder i is given by

$$f_i = \Lambda(f_j) = \begin{cases} p_i(v - f_j) + L & \text{if } f_j \leq \bar{f}_j, \\ [(1 - p_i)f_j - (p_j - p_i)v]/(1 - p_j) - \epsilon & \text{if } \bar{f}_j < f_j \leq p_j v + L/(1 - p_i), \\ p_i v + L/(1 - p_j) & \text{otherwise,} \end{cases}$$

where $\epsilon > 0$ is an infinitesimally small number. The three segments correspond to the three different limit licensing fees necessary to avoid litigation. In the first segment, patentee i sets f_i such that $V_0 = V_i$, in the second segment the fee is at the highest level such that $V_j > V_i$ and in the third segment the limit fee satisfies $V_j = V_{AB}$. The limit license fee in the first segment, $p_i(v - f_j) + L$, yields strictly more than the patent holder can earn by increasing its fee and inducing litigation against itself. Further note that the limit license

fee increases in f_j in the second segment whereas the litigation profits decrease. Hence, limit pricing is again optimal.⁹ Finally, in the third segment, limit licensing occurs at $p_i v + L/(1 - p_j)$ which, upon simple inspection, always exceeds patentee i 's expected profits when both patents are litigated. It is then easy to check that at the intersection of the best response functions, the unique Nash equilibrium in license fees is given by

$$(\bar{f}_A^*, \bar{f}_B^*) = (\bar{f}_A, \bar{f}_B) = \left(\frac{(1 - \alpha)L + \alpha(1 - \beta)v}{1 - \alpha\beta}, \frac{(1 - \beta)L + \beta(1 - \alpha)v}{1 - \alpha\beta} \right).$$

We thus get the following result.

Proposition 2 *Independent patent holders set limit licensing fees that prevent litigation from the downstream firm. Equilibrium license fees increase in the strength of its own patent and decrease in the strength of the other patent.*

Licensing and Litigation with a Patent Pool and License Packaging. Suppose the two firms form a patent pool and sell the two licenses in a bundle for a fee $\bar{F}$. The patent pool maximizes the joint profits of the patent holders. The downstream firm can either buy the package license, litigate against both patents or remain inactive. Challenging exactly one patent is not enough to invalidate the patent package and is always dominated by remaining inactive. By assumption (A) challenging both patents is superior to remaining inactive. Hence, the downstream firm buys the package license if

$$v - \bar{F} \geq (1 - \alpha)(1 - \beta)v - 2L \tag{8}$$

or

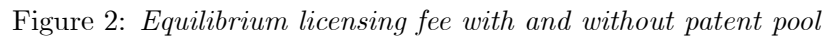
$$\bar{F} \leq [1 - (1 - \alpha)(1 - \beta)]v + 2L \equiv \bar{F}^{**}.$$

Otherwise, it challenges both patents in the pool. The patent pool can either limit license and set the highest fee that avoids litigation or enter litigation. Litigation yields monopoly profits if at least one patent is deemed valid by the court. Thus, expected profits from litigation are

$$[1 - (1 - \alpha)(1 - \beta)]v - 2L.$$

⁹In particular, limit licensing also dominates the license fee that satisfies (7) with equality. At that level a patentee would get an infinitesimally small fee increase at cost of being litigated against with probability 1/2. Hence, the expected profits would be strictly lower.

Compare the total limit license fees charged by a patent pool and independent patent holders. Note that along $f_A + f_B = \overline{F}^{**}$ it holds that $V_{AB} = V_0$. In order to prevent litigation, a patent pool sets a package license fee that makes the downstream firm indifferent between litigating both patents or not litigating at all. By contrast, independent patent holders set equilibrium license fees such that the downstream firm is indifferent between litigating each patent separately or not at all. However, due to the litigation externalities with complementary patents, we have that at $V_A = V_B = V_0$ it holds that $V_{AB} < V_0$. Thus, the patent pool is able to increase the license for the patent package further and can charge higher overall fees compared to individual patent holders. This is illustrated in FIGURE 2 below. Put differently, with complementary patents it is always easier to satisfy the condition $V_0 \geq V_{AB}$ rather than the conditions $V_0 \geq V_A$ and $V_0 \geq V_B$ jointly. We therefore



Proposition 3 *A patent pool with a package license sets a limit licensing fee that avoids litigation from the downstream firm. The patent pool always charges higher licensing fees than independent patent holders, i.e. $\overline{F}^* = \overline{f}_A^* + \overline{f}_B^* < \overline{F}^{**}$.*

In the presence of weak patents and litigation, we get the reverse result of Proposition 1. A patent pool issuing package licenses is able to charge higher license fees than independent patent holders. Two arguments explain this result. First, package licensing makes it unprofitable to challenge exactly one patent and imposes an all-or-nothing litigation proposition on the downstream firm. This changes - due to the litigation externalities with complementary patents - the optimal litigation behavior for given overall license fees. If license fees are low to intermediate, that is, (8) holds while (3) and (4) do not hold, the downstream firm would not litigate if it faces a patent pool whereas it would litigate one patent with independent patent holders. This allows the patent pool to charge higher fees without being litigated against. Second, independent patent holders are unable to sustain such high license fees because they are engaged in a Bertrand-type competition with respect to litigation. Suppose individual patent holders set their fees above the equilibrium fees $(\bar{f}_A, \bar{f}_B)$. In this situation, an individual patent holder is always best off reducing its license fee in order to avoid possible litigation against its own patent. This competition externality creates downward pressure on license fees and individual patent holders compete each other down to the limit licensing levels.¹⁰

We have shown that patent pools can elevate the total licensing fees when they are used to shield weak patents from the threat of litigation. However, the elevated licensing fees have no efficiency consequences in the simple model where only the litigation margin is binding. Licensing fees are just a transfer between the patent holders and the downstream firm. The only source of inefficiency is costly litigation, which does not arise in equilibrium. In the next section, we extend our model to allow both the demand and the litigation margin to be binding.

4 The Interplay of the Demand Margin and Litigation Incentives

We have considered two extreme cases where either only the demand margin or only the litigation margin was binding. Now we analyze the full game, in which both considerations figure into the patentee's licensing decisions.

¹⁰In Section 5, we consider a patent pool that issues individual licenses and is able to internalize this licensing fee externality.

The Set-up of the Full Game. To account for strategic litigation incentives, we amend the game analyzed in the previous section by including two additional stages. More specifically, the game proceeds in the following way.

1. Firms A and B decide whether or not to form a patent pool.
2. Firms A and B set licensing fees. If they form a patent pool, they coordinate their license fees. Otherwise, they set licensing fees independently.
3. Firm C draws its innovation/development cost c from a distribution $G(\cdot)$. After realizing its cost, firm C decides whether to incur the cost and engage in the subsequent innovation/development. If Firm C does not engage in the innovation, the game ends.
4. If firm C develops the innovation, it decides for each technology whether to buy the license or whether to litigate the validity of the patent. Alternatively, it can remain inactive which yields zero profits.
5. Litigation outcomes are revealed. If a patent has been challenged and upheld, its holder proposes a new license fee for firm C . If both patents have been challenged and validated, the upstream firms simultaneously choose their license fee.
6. If firm C has a license for all non-invalidated patents, it receives a profit of v .

A few comments on the timing are in order. We assume that litigation takes place after the subsequent innovation. We make this assumption for two reasons. First, if litigation takes place after realizing the cost, but before sinking development cost, the litigation itself conveys private information about the development costs, which unnecessarily complicates the analysis without changing the main qualitative results. Second, and more importantly, firm C may not have legal standing to sue until it has developed any innovation based on the prior technologies and is in a position to be a direct purchaser of licenses.

With ironclad patents, the patent holders' licensing decisions were driven solely by the demand margin, captured by the innovation cost distribution of firm C , which yielded a downward demand function for the licenses as $G(v - f_A - f_B)$. With probabilistic patents, they also need to pay attention to the litigation incentives of firm C because setting too high a licensing fee may trigger litigation by firm C . As will be shown below, the optimal licensing fees will depend on whether the demand or the litigation margin is binding.

Equilibrium Licensing Fees with Independent Licensing. Let us first consider the licensing decisions when both firms set licensing fees independently without forming a patent pool. With the assumption $\alpha \leq \beta$, we have $\bar{f}_B \geq \bar{f}_A$. Three possibilities can arise as a function of whether the litigation or demand margin is binding.

Case 1: Litigation margins not binding. This is the case when both patents are strong such that

$$f^* < \bar{f}_A \leq \bar{f}_B. \quad (9)$$

The downstream firm C has no incentives to litigate when firms set their equilibrium licensing fees from the analysis of section 2 where only the demand margin is binding. Hence, firms behave as if their patent were ironclad and the equilibrium licensing fees are given by $f_A^* = f_B^* = f^*$. Again, licensing fees are symmetric and do not depend on the relative strength of the two patents.

Case 2: Both litigation margins binding. This occurs when each firm's limit litigation fee from section 3 is less than its best response to the rival's limit litigation fee, that is, for $\bar{f}_i \leq \Theta(\bar{f}_j)$. For $\alpha \leq \beta$ both conditions are satisfied if

$$\bar{f}_B \leq \Theta(\bar{f}_A). \quad (10)$$

In this case, the litigation margin is binding for both firms. Given firm j sets $\bar{f}_j$, firm i has no incentive to increase its fee as it would trigger litigation against firm i . Condition (10) ensures that firms have no incentive to decrease their license fee either. Thus, in a subgame perfect equilibrium, each firm sets its licensing fee at the level that deters litigation, $f_A^* = \bar{f}_A^*$ and $f_B^* = \bar{f}_B^*$.

When both patents are of equal strength, $\alpha = \beta$, conditions (9) and (10) coincide. This means that in a subgame perfect equilibrium of the complete game firms are either constrained by the demand margin and price like in section 2 or they are constrained by the litigation margin and set the equilibrium fees of section 3. If patents are asymmetric a third case can arise.

Case 3: Litigation margin only binds for firm A. This is a mixed case where conditions (9) and (10) are both violated. Here, the firm with the weaker patent is constrained by the litigation limit whereas firm B operates on the demand margin. In such situations, a

pure-strategy equilibrium in license fees might not exist. The reason is that, when firm A prices close to its litigation limit where $V_0 = V_A$ (see condition (3) in Figure 1), firm B might profitably increase its license fee and induce the downstream firm to litigate firm A. Such deviations are not possible in the equilibria of Cases 1 and 2 above.¹¹

We delegate the formal proof of our discussion to the appendix and state the main result of this analysis.

Lemma 2 *If $\bar{f}_B \leq \Theta(\bar{f}_A)$, then there exists a unique subgame perfect equilibrium, in which firms A and B set their limit litigation license fees $\bar{f}_A$ and $\bar{f}_B$, respectively.*

Equilibrium License Fees with Patent Pool. Now suppose that firms A and B form a patent pool. Again, the optimal package licensing fee depends on whether the demand or the litigation margin is binding. If $F^{**} < \bar{F}^{**}$, the patent pool can set its package licensing fee as if its patents were ironclad because they are sufficiently strong and there is no threat of litigation by firm C. Otherwise, the litigation margin is binding and the patent pool sets its licensing fee at $\bar{F}^{**}$ to deter litigation. Thus, the patent pool's optimal licensing fee is given by $\min[F^{**}, \bar{F}^{**}]$. Note that F^{**} is completely determined by the demand conditions (that is, cost distribution function $G(\cdot)$) while $\bar{F}^{**}$ is determined by the strength of patents (α and β) and litigation costs (L).

Welfare Effects of Patent Pools. The welfare effects of patent pools depend on whether patent pools elevate or reduce the overall licensing fees paid by the downstream firm. From the above analysis it is clear that in Case 1 where independent firms are not constrained by the litigation margin, the result of the traditional analysis obtains. Since

$$\min \{F^{**}, \bar{F}^{**}\} \leq F^{**} < F^*,$$

patent pools charge lower overall license fees as they avoid royalty stacking. However, when the litigation margin is binding for independent patent holders, we can get the same result as in Section 3 and patent pools are able to extract a higher total license fee. As $\bar{F}^* = \bar{f}_A + \bar{f}_B < \bar{F}^{**}$ it suffices to show that $\bar{F}^*$ can be smaller than the unconstrained license fee of the patent pool F^{**} when the litigation margin is binding in the equilibrium

¹¹We provide sufficient conditions for such non-existence to arise in the appendix to the next lemma.

with independent patent holders. Since $F^{**} = \Theta(0)$ we get that $\bar{F}^* < F^{**}$ if and only if

$$\bar{f}_B \leq \Theta(0) - \bar{f}_A. \quad (11)$$

A necessary condition for (11) to hold is that the litigation margins are binding with independent patent holders, that is, $\bar{f}_B \leq \Theta(\bar{f}_A)$. Furthermore, we can explicitly solve this condition and show that it is satisfied if and only if

$$L \leq \frac{1 - \alpha\beta}{2 - \alpha - \beta} \Theta(0) - \frac{\alpha + \beta - 2\alpha\beta}{2 - \alpha - \beta} v \equiv L'.$$

Upon inspection, we find that if the patent validity parameters are sufficiently small, then there always exists a threshold value $L' > 0$ for the litigation cost below which the total license fee is higher with a patent pool. We thus get the following result.

Proposition 4 *Consider the full game with demand and litigation margin. If patents are sufficiently weak and litigation cost low relative to the value of the innovation, then patent pools hinder subsequent innovations and reduce welfare.*

The condition in Proposition 4 arises when the threat of litigating weak patents is sufficiently strong. In such a case, patent pools can be used for safe-harboring weak patents from litigation in order to elevate the overall licensing fees.

An Example. To illustrate these results further, consider an example with a uniform distribution and symmetric patent strengths $\alpha = \beta$. In particular, let the development cost c be distributed uniformly on $[0, 1]$, with the value of innovation normalized at $v = 1$. In this case, it is always optimal to develop the subsequent innovation. However, with patent rights and licensing, the downstream firm develops the new product only when $c + f_A + f_B < 1$. Thus, the demand function for the joint licenses is given by $(1 - f_A - f_B)$, which is the probability that the development cost satisfies the condition $c + f_A + f_B < 1$ given the uniform distribution of c . It is then easily verified that the optimal licensing fees with demand margins are given by $f^* = 1/3$, $F^* = 2/3 > F^{**} = 1/2$. When the litigation margins are binding, the license fees are determined by the strength of the patents and the cost of litigation. With symmetric patents, we get $\bar{f}_A = \bar{f}_B = (L + \alpha)/(1 + \alpha)$ and $\bar{F}^{**} = \alpha(2 - \alpha) + 2L$. FIGURE 3 below illustrates the resulting equilibrium license fees in the

full game with independent patent holders and a patent pool. Independent patent holders charge the unconstrained license fee of Case 1 if $L > (1 - 2\alpha)/3$. Otherwise, Case 2 applies and firms face binding litigation margins in equilibrium. Similarly, the patent pool charges the limit litigation fee if the cost of litigation is low ($L \leq (1 - 2\alpha(2 - \alpha))/4$). It follows that patent pools increase license fees and reduce welfare if and only if $L \leq (1 - 3\alpha)/4$.

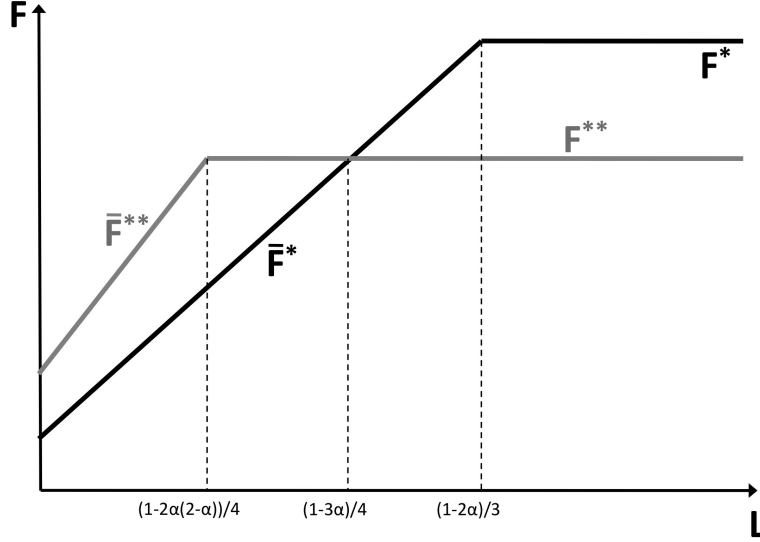


Figure 3: *Overall license fees with independent firms and patent pool*

5 Patent Pool with Individual Licenses

In the previous section, we have shown that patent pools can be anticompetitive, even with complementary patents, once we account for the probabilistic nature of patent rights. By offering package licensing, patent pools deprive the downstream firm of the ability to selectively challenge patents. This allows a patent pool to charge higher licensing fees relative to independent licensing. By contrast, in this section, we discuss the case where the pool offers individual licenses for each patented technology and coordinates pricing. We first characterize the optimal individual license fees for the pool, discuss the incentives to issue individual licenses and give conditions under which mandatory individual licensing increases total welfare.

Profit-Maximizing Individual License Fees. Suppose the patent pool issues individual licenses for each patent charging f_A and f_B , respectively. In this case, the downstream firm's litigation behavior is the same as in the analysis with independent patent holder.

However, the patent pool maximizes the joint profits from both patents. Let us again first consider the case where the downstream firm has already introduced its new product and only the litigation margin is binding. After this, we show how the results relate to the outcome of the set-up where both demand and litigation margin are effective. Three strategic options arise for the optimal license fees of the patent pool. Limit licensing both patents, exactly one patent or inducing litigation against both patents. Limit licensing at $(\bar{f}_A, \bar{f}_B)$ avoids litigation against both patents at the highest possible total license fee. Alternatively, consider the strategy of limit pricing exactly one patent and inducing litigation against the other patent. The highest possible limit license fee for patent i when the downstream firm has an incentive to litigate patent j is $f_i = p_i v + L/(1 - p_j)$. At this license fee, the patent pool makes an expected profit of

$$p_j v + (1 - p_j)[p_i v + \frac{L}{1 - p_j}] - L = [1 - (1 - \alpha)(1 - \beta)]v.$$

Two observations are noteworthy. First, limit licensing exactly one patent yields the same expected payoff independent of which patent is litigated. This implies that setting fees such that the downstream firm is indifferent between litigating A or B yields the same payoff as fees at which firm C strictly prefers litigating one patent. Second, limit licensing one patent always dominates fees that induce litigation against both patents (litigation against both patents yields the above profits minus the cost of litigation of $2L$).

Now compare the patent pool's profits when limit licensing one patent with limit licensing both patents at $(\bar{f}_A, \bar{f}_B)$. Limit licensing exactly one patent is optimal if and only if

$$[1 - (1 - \alpha)(1 - \beta)]v \geq \bar{f}_A + \bar{f}_B = \frac{\alpha + \beta - 2\alpha\beta}{1 - \alpha\beta}v + \frac{[2 - \alpha - \beta]}{1 - \alpha\beta}L$$

or

$$L \leq \frac{\alpha\beta(1 - \alpha)(1 - \beta)}{2 - \alpha - \beta}v.$$

Thus, if the litigation costs are sufficiently small, then a patent pool selling individual licenses is best off with fees such that the downstream firm buys one license and litigates against the other patent.

We have thus far focused on the case where only the litigation margin is binding. However, as we show in the appendix to the next lemma, the analysis and the qualitative results

carry over to the case where both litigation and demand margin are binding. The only difference is that the patent pool's local maximizer in regions A , B and 0 can be interior. Hence, candidate maximizers of the pool's fee setting problem are the interior solution or the limit licensing fees $(\bar{f}_A, \bar{f}_B)$ of region 0 , the interior solution to regions A/B or, as above, the corner solution at $f_i = p_i v + L/(1 - p_j)$. The next lemma gives the optimal license strategy for a patent pool with individual licenses in the presence of demand and litigation margins.

Lemma 3 *Consider a patent pool issuing individual licenses. There exists a threshold value L'' , with $0 < L'' < L'$, such that for $L \leq L''$, the patent pool's optimal license fees induce the downstream firm to buy the license for one patent and litigate against the other patent. For higher litigation costs, $L > L''$, the patent pool charges total licensing fees of $\min\{\bar{f}_A + \bar{f}_B, F^{**}\}$ and no litigation occurs.*

This result is somewhat surprising. If the litigation cost is sufficiently small ($L \leq L''$), litigation arises although the joint profits of upstream and downstream firms are lower compared to licensing arrangements that avoid litigation. The reason for this is that complementary patents lead to negative litigation externalities. If the downstream firm loses the court case for one litigated patent, the pool is able to extract all rents from the other patent. Hence, the more litigation the pool induces, the higher the fee it can charge on patents that are purchased in equilibrium. Obviously, the gains from this fee extraction have to be weighed against the cost of litigation. Thus, if litigation costs are relatively small, the patent pool is better off, selling one license at a high fee and entering litigation against the other patent. The result in Lemma 3 also implies that at $L = L''$, the optimal total licensing fee charged by the patent pool has a discontinuity and jumps downwards to the limit licensing fees $\bar{f}_A + \bar{f}_B$. If $L \geq L'$, the litigation margin is no longer binding and the pool charges F^{**} in total licensing fees.

The above lemma states that for a relatively small cost of litigation, the industry profit-maximizing fees are above the limit licensing level for independent patent holders from Sections 3 and 4 and induce litigation against one patent. As mentioned above, such license fees are not sustainable with independent patent holders as there exists a unilateral incentive to reduce the license fee in order to avoid litigation against the holder's own patent. A patent pool issuing individual licenses allows to internalize this pricing externality and

sustain higher license fees compared to independent patent holders.

Profits, Total Welfare and Patent Pool Policy. First compare the patent pool's profit and total welfare when licenses are sold in a package or individually. From the patent pool's perspective, package licensing allows to harbour weak patents which can be shielded from litigation with relatively high limit licensing fees. Individual licenses make the patents more vulnerable to litigation and command lower limit license fees. If litigation costs are low, individual licenses might lead to higher license fees for one patent but at the cost of litigation against the other patent.

Proposition 5 *Package licensing yields (weakly) higher profits for the patent pool compared to selling individual licenses. If $L \leq L''$, package licensing is welfare superior to individual licenses. For higher values of L , individual licensing yields weakly higher welfare.*

Patent pools strictly prefer package licensing if $L \leq L'$. For higher values of L the litigation margin is not binding and the patent pool charges a total licensing fee of F^{**} with both sale mechanisms. Hence, patent pools weakly prefer package licensing. By contrast, there is an efficiency trade-off between package and individual licenses. Package licensing lead to higher limit licensing fees when litigation is effectively avoided. However, individual licenses may induce litigation against one patent and higher fees for the other patent. Thus, if $L \leq L''$, package licensing is socially efficient as it prevents litigation and excessive fees. For $L'' < L \leq L'$, individual licenses yield higher total welfare as they prevent patent pools from shielding weak patents with high licensing fees.

We are now in a position to assess the social efficiency of a patent pool with package or individual licenses relative to a situation with independent patent holders. From Proposition 5 we know that a patent pool - when it forms - prefers to use package licensing. From our analysis in Section 4 follows that if $L \leq L''$, a patent pool with package licenses always charges total license fees that are closer to the industry profit maximizing level of F^{**} compared to independent patent holders. Hence, a patent pool would form for those values of the litigation cost and it would optimally use package licenses. By contrast, for $L \leq L''$, total welfare is maximized with independent patent holders. One policy option is, thus, to block patent pool formation, in situation where the threat of litigation is large, that is, when litigation costs are small relative to the value of the innovation. Short of prohibiting patent pools, our analysis also suggests that for intermediate values of the litigation cost,

$L' < L \leq L''$, a patent pool with individual licenses charges the same limit licensing fees as individual patent holders. Hence, in those situations, a policy that mandates patent pools to offer individual licenses increases welfare and implements the same efficiency outcome as with individual patent holders. This discussion is summarized in the next proposition.

Proposition 6 *For litigation cost such that $L \leq L''$, patent pools increase upstream profits and lower total welfare relative to independent patent holders. For litigation cost such that $L' < L \leq L''$, mandatory individual licensing for patent pools increases welfare and implements the same outcome as with independent patent holders.*

6 Extensions

Licensing and litigation with more than two patents. In this extension we show that the qualitative results of our above analysis hold when there are more than two complementary patents. For this purpose, consider $n \geq 2$ complementary technologies that are necessary to develop the final product. Each technology is covered by a probabilistic patent of strength α and each patent is owned by a different firm. It is again useful to first consider the model when only the litigation margin is binding. We compare the case of n independent patent holders with the scenario where the n firms form a patent pool and market the package license jointly. We then show that our result also applies when the demand margin is binding.

Suppose the patent holders offer a license for their patent $i \in \{1, 2..n\}$ at a fee f_i . The optimal licensing and litigation strategy of the downstream firm can be characterized as follows.

Lemma 4 *There exists a $l^* \in \{0..n\}$ such that the downstream buys l^* of the (weakly) cheapest licenses and litigates against the remaining $n - l^*$ patents. The optimal number of patent litigations increases in the overall licensing fee.*

For a given number of litigation cases, the downstream firm always prefers to litigate against a patent that involves a higher license fee and buy a license of lower priced technology. The incentive to litigate depends on the overall licensing fee and its distribution. Litigating against the marginal patent implies the risk of losing this case and the net returns from

buying the infra-marginal licenses. Hence, the higher the license fees, the lower the loss from litigation and the higher the number of patent litigations.

Now consider the best response function of an individual patent holder for a given fee profile of the other patent holders. Suppose patent holder j 's license fee is ranked between 1 and $l^* - 1$. In this case, slightly increasing its own license fee is always optimal until the patent at rank l^* is litigated against. Now assume that owner j 's patent is the marginal patent at rank l^* . The downstream firm prefers not to litigate against patent j if

$$(1 - \alpha)^{n-l^*} (v - \sum_{r=1}^{l^*-1} f_r - f_j) - (n - l^*)L \geq (1 - \alpha)^{n-l^*+1} (v - \sum_{r=1}^{l^*-1} f_r) - (n - l^* + 1)L$$

which holds if

$$f_j \leq \frac{L}{(1 - \alpha)^{n-l^*}} + \alpha(v - \sum_{r=1}^{l^*-1} f_r). \quad (12)$$

This is the n -firm equivalent of condition (3) in Section 3. Again we can show that patent holder j has no incentive to violate this condition and prefers to limit license in order to avoid litigation. If the patent holder j charges a higher fee, firm C litigates against his patent. In this case, the patent holder only receives a return if his patent is upheld by the court. His share of the total upstream profit is determined by how many other patents are upheld. Let $\Pr\{k|n-l^*\}$ denote the probability that k out of the $n-l^*$ remaining litigated patents are upheld. Then the expected profit from inducing litigation is

$$\alpha \sum_{k=0}^{n-l^*} \frac{\Pr\{k|n-l^*\}}{k+1} (v - \sum_{i \in \mathcal{L}} f_i) - L = \frac{1 - (1 - \alpha)^{n-l^*+1}}{n - l^* + 1} (v - \sum_{i \in \mathcal{L}} f_i) - L.$$

From the fact that patent holder j 's expected market share is always less than α , it follows that limit licensing always dominates. Hence, in the unique symmetric fee setting equilibrium, firms charge $f_i = \bar{f}^*(n)$ such that (12) holds with equality. The resulting Nash equilibrium in license fees is given by

$$\bar{f}^*(n) = \frac{L + \alpha v}{1 + \alpha(n-1)}.$$

The equilibrium individual license fee is decreasing in n . More patents increase the total infra-marginal license fee and the incentive to litigate. Thus, the limit license fee is decreasing. Note, however, that the total licensing fee for the downstream firm, $n\bar{f}^*(n)$ is increasing in the number of patents.

Now consider a patent pool offering a package license for all n patents at a fee F . The downstream firm buys the package license if and only if

$$v - F \geq (1 - \alpha)^n v - nL$$

or

$$F \leq (1 - (1 - \alpha)^n)v + nL \equiv \bar{F}^{**}(n).$$

The probability of invalidating all patents in court is decreasing in the number of patented technologies. Thus, the limit license fee for a patent pool with license packaging increases in n .

We are now in a position to compare the total licensing fees when the litigation margins hold for both independent patent holders and a patent pool. Since $\bar{F}^{**}(n)$ increases exponentially and $\bar{F}^{**}(n = 2) > 2\bar{f}^*(2)$, we can show in the appendix to the next proposition that

$$\bar{F}^{**}(n) > n\bar{f}^*(n). \quad (13)$$

Hence, the result from Section 3 holds for any $n \geq 2$, that is, while litigation margins are binding, a patent pool increases total licensing fees relative to independent patent holders. It remains to consider the effect of the number of patents on the interplay between demand and litigation margin. Let $f^*(n)$ denote the Nash equilibrium license fee with n independent patent holders when only the demand margin is binding.¹² By contrast, the patent pool's optimal license without the litigation margin is independent of the number of patents and given by F^{**} . Hence, the total licensing fees with individual patent holders is $n \min \{f^*(n), \bar{f}^*(n)\}$ whereas the patent pool charges $\min \{\bar{F}^{**}(n), F^{**}\}$. From $F^{**} < n\bar{f}^*(n)$ and (13) follows that individual patent holders charge lower fees if $n\bar{f}^*(n) < F^{**}$. Under this condition, a patent pool with $n \geq 2$ complementary patents reduces welfare and hinders subsequent innovation. Since $n\bar{f}^*(n)$ is increasing and approaching v as n becomes large while $F^{**} < v$ is not affected by the number of patents, this condition must fail to hold when n is sufficiently large. Hence, there must exist an upper bound on the number of patents, above which the litigation limit for independent patent holders is unable to restrict the fees below the monopoly level such that patent pools are welfare improving. We can therefore summarize

¹²The demand margin analysis with n patents can be found in the appendix to Proposition 1.

as follows.

Proposition 7 *If $n\bar{f}^*(n) \leq F^{**}$, then patent pools with $n \geq 2$ patents charge higher total license fees and reduce total welfare relative to independent patent holders. This condition is harder to satisfy, the higher the number of complementary patents. There exists a finite upper bound $\bar{n}$ on the number of patents, above which patent pools increase total welfare.*

Sequential litigation. In our analysis up to this point, we assume that the downstream firm litigates against both patents simultaneously. This is a good description of many situations, in which a short lead time to commercialisation is crucial. In some situations, however, the downstream firm might be able to consider a sequential litigation strategy instead of litigating both patents simultaneously. While sequential litigation has the same success probability of invalidating both patents, it might save the cost of the second litigation if the first litigation is unsuccessful. Suppose the downstream firm first litigates against patent i and then, if successful, litigates against patent j . The expected profit of this strategy is

$$(1 - p_i)[(1 - p_j)v - L] - L = (1 - p_i)(1 - p_j)v - 2L + p_iL.$$

Hence, the optimal sequential strategy is to litigate the stronger patent B first as it allows for a higher probability of litigation cost savings in case of an unsuccessful first litigation. The payoff with this strategy is

$$V_{AB}^s = (1 - \alpha)(1 - \beta)v - 2L + \beta L = V_{AB} + \beta L,$$

that is, the strategy to litigate against both patents is now more profitable. It follows that litigating both patents sequentially dominates litgating against patent A only if $V_{AB}^s > V_A$ or

$$f_A > \alpha v + L,$$

while it is superior to litigating against patent B if $V_{AB}^s > V_B$ or

$$f_B > \beta v + \frac{1 - \beta}{1 - \alpha}L. \tag{14}$$

The remainder of the analysis of the downstream firm's litigation and licensing behavior is the same as in Section 3. Now consider the best response functions. As long as (14)

is not satisfied, firm A faces the same situation as in Section 3 and limit licenses to avoid litigation. When (14) holds, firm A prefers limit licensing to sequential litigation if

$$\begin{aligned}\alpha v + L &\geq \beta v/2 + \alpha(1 - \beta)v - L \\ 2L + \beta(\alpha - 1/2)v &\geq 0.\end{aligned}$$

Hence, if the litigation cost is small and its patent weak, firm A prefers to induce sequential litigation. By contrast, firm B always prefers limit licensing to sequential litigation since

$$\beta v + \frac{1 - \beta}{1 - \alpha}L \geq \beta v/2 - L.$$

As a consequence, the unique Nash equilibrium in license fees with independent patent holder is the same as in Section 3, that is $(\bar{f}_A, \bar{f}_B)$.

A patent pool selling a package license practices limit pricing at $V_{AB}^s = V_0$ with a limit license fee of

$$\bar{F}^s = [1 - (1 - \alpha)(1 - \beta)]v + 2L - \beta L < \bar{F}^{**}.$$

Due to the potential litigation cost savings for the downstream firm C, the limit license fee is lower with sequential litigation. Finally, check that the total licensing fees are higher in the presence of a patent pool if and only if $\bar{F}^s > \bar{f}_A + \bar{f}_B$ or

$$\frac{\alpha(1 - \beta)}{1 - \alpha\beta}[(1 - \beta)L + (1 - \alpha)\beta v] > 0$$

which always holds. Thus, sequential litigation increases the value of litigation for firm C and reduces the limit licensing fee for the patent pool. However, the qualitative nature of the results in Section 3 remains unchanged.

7 Concluding Remarks

This paper analyzes the effects of patent pools with complementary patents on incentives to develop subsequent innovations. We find that the effects of patent pools depend on the strength of patents included in the pool. If patents are relatively strong, then the conventional result holds that pools with complementary patents mitigate the double marginalization problem and reduce overall licensing fees, which promotes subsequent innovations.

However, if patents are relatively weak, patent pools can be used as a mechanism to deter litigation that would invalidate the patents in the pool. Package licensing of complementary patents imposes an all-or-nothing proposition in litigation on downstream firms. This allows patent pools to safe-harbour weak patents which would be targeted in litigation if the licenses would be sold independently. Our analysis shows that if patents are sufficiently weak, patent pools reduce social welfare as they raise total licensing fees and hinder subsequent innovations. This conclusion is robust to extensions of our analysis, which allow for more than two patents and sequential litigation strategies. We further explore the policy implications of mandated individual licenses to make the pool patents more vulnerable to litigation and command lower limit license fees. We find that the welfare effects of such policy mandates crucially depend on the size of the litigation cost relative to the value of the innovation. Hence, overall, our analysis suggests that a blanket approval of patent pools based on the complementary nature of the included patents is not warranted and a more cautious approach that takes into account the strength of patents and incentives to litigate is called for.

Appendix

Proof of Proposition 1. As we require this analysis with $n \geq 2$ patents in Section 6, we prove the result for more than two firms at this point. The first-order condition for patent holder $i \in \{1..n\}$ is

$$G(v - \sum_{j=1}^n f_j) - f_i g(v - \sum_{j=1}^n f_j) = 0.$$

Hence the equilibrium license fees $(f_1^*..f_n^*)$ satisfy

$$nG(v - \sum_{j=1}^n f_j^*) - \sum_{j=1}^n f_j^* g(v - \sum_{j=1}^n f_j^*) = 0.$$

Evaluate the first order condition (2) for the patent pool at $F^* = \sum_{j=1}^n f_j^*$, which yields

$$G(v - F^*) - F^* g(v - F^*) = -(n-1)G(v - F^*) < 0.$$

This implies the desired result that $F^* > F^{**}$. ■

Proof of Lemma 2. Let $\Pi_i^k(f_i, f_j)$, $i \neq j$, denote firm i 's profits in region $k \in \{0, A, B, AB\}$:

$$\begin{aligned} \Pi_i^k(f_i, f_j) &= G(V_k)f_i \text{ for } k \in \{0, j\}, \Pi_i^i(f_i, f_j) = G(V_i)(p_i(v - f_j) - L) \text{ and} \\ \Pi_i^{AB}(f_i, f_j) &= G(V_{AB})(p_i p_j v / 2 + p_i(1 - p_j)v - L). \end{aligned}$$

First, consider firm i 's best response function for $0 \leq f_j \leq \bar{f}_j$. Check that $\Pi_i^0(f_i, f_j) > \Pi_i^i(f_i, f_j)$ when $V_0 = V_i$ since

$$\begin{aligned} \Pi_i^0(p_i(v - f_j) + L, f_j) &= G(v - p_i(v - f_j) - L - f_j)(p_i(v - f_j) + L) \\ &= G((1 - p_i)(v - f_j) - L)(p_i(v - f_j) + L) \\ &> G((1 - p_i)(v - f_j) - L)(p_i(v - f_j) - L). \\ &= \Pi_i^i(p_i(v - f_j) + L, f_j). \end{aligned}$$

Since $\Pi_i^i(f_i, f_j)$ is independent of f_i it follows that the best response function for $0 \leq f_j \leq \bar{f}_j$ is continuous and given by

$$\Psi_i(f_j) = \min \{ \Theta(f_j), p_i(v - f_j) + L \}.$$

Hence, for $\bar{f}_i \leq \Theta(\bar{f}_j)$, there exists a Nash equilibrium in which firms charge $\bar{f}_A$ and $\bar{f}_B$, respectively.

Next assume $\bar{f}_j < f_j \leq p_j v + L/(1 - p_i)$. Define the local maximizer in region j as

$$\hat{f}_i^j \equiv \arg \max_{f_A} \Pi_i^j(f_i, f_j) = f_i G(V_j) = f_i G((1 - p_j)(v - f_i) - L).$$

This maximizer satisfies the first-order condition

$$\hat{f}_i^j = \frac{G(V_j)}{(1 - p_j)g(V_j)}.$$

Note that the maximizer in this region does not depend on f_j . Further note that for (f_i, f_j) such that $V_0 = V_j$, it holds that $\Pi_i^0(f_i, f_j) = \Pi_i^j(f_i, f_j)$. Verify that for values (f_i, f_j) such that $V_i = V_j$, we get $\Pi_i^j(f_i, f_j)|_{V_i=V_j} > \Pi_i^i(f_i, f_j)|_{V_i=V_j}$ if and only if $f_i > p_i(v - f_j) - L$ or

$$\begin{aligned} \frac{p_i - p_j}{1 - p_j} v + \frac{1 - p_i}{1 - p_j} f_j &> p_i(v - f_j) - L \\ \Leftrightarrow f_j &> \frac{p_j - p_i}{1 - p_i p_j} v - \frac{1 - p_j}{1 - p_i p_j} L \end{aligned}$$

which always holds for any

$$f_j > \bar{f}_j = \frac{p_j - p_i}{1 - p_i p_j} v + \frac{1 - p_j}{1 - p_i p_j} L.$$

The above profit inequality implies that firm i always prefers to price slightly below the fee that yields $V_i = V_j$ rather than setting f_i such that $V_i = V_j$ or $V_i > V_j$. Undercutting yields $\Pi_i^j(f_i, f_j)$ whereas the two latter price points give $\Pi_i^i(f_i, f_j)/2 + \Pi_i^j(f_i, f_j)/2$ and $\Pi_i^i(f_i, f_j)$, respectively. Hence, if $\bar{f}_i \leq \Theta(\bar{f}_j)$, then firm i 's best response is either such that $V_0 = V_j$ or strictly interior in region j . From the concavity of $\Pi_i^j(f_i, f_j)$ in f_i follows that $\Psi_i(f_j)$ is continuous. In particular, if $\hat{f}_i^j < \bar{f}_i$, then

$$\Psi_i(f_j) = \max \left\{ v + L/p_j - f_j/p_j, \hat{f}_i^j \right\};$$

otherwise,

$$\Psi_i(f_j) = \min \left\{ \frac{p_i - p_j}{1 - p_j} v + \frac{1 - p_i}{1 - p_j} f_j - \epsilon, \hat{f}_i^j \right\}.$$

Finally, consider $f_j > p_j v + L/(1 - p_i)$. Check that $\Pi_i^j(f_i, f_j)|_{V_j=V_{AB}} > \Pi_i^i(f_i, f_j)|_{V_j=V_{AB}}$ if

and only if $f_i > p_i p_j v / 2 + p_i(1 - p_j)v - L$ or

$$\begin{aligned} p_i v + L / (1 - p_j) &> p_i p_j v / 2 + p_i(1 - p_j)v - L \\ \Leftrightarrow p_i p_j v / 2 + L / (1 - p_j) - L &> 0 \end{aligned}$$

which is always satisfied. Hence, the best response is continuous and lies in region j ,

$$\Psi_i(f_j) = \min \left\{ p_i v + L / (1 - p_j), \hat{f}_i^j \right\}.$$

Since, for $f_j > \bar{f}_j$, firm i 's best response function is in region j or where $V_0 = V_j$, no further equilibrium exists. Finally, check that $\bar{f}_A \leq \Theta(\bar{f}_B)$ follows from $\bar{f}_B \leq \Theta(\bar{f}_A)$. Since $\partial\Theta/\partial f > -1$, we have $\bar{f}_B - \bar{f}_A > \Theta(\bar{f}_A) - \Theta(\bar{f}_B)$ or $\Theta(\bar{f}_B) - \bar{f}_A > \Theta(\bar{f}_A) - \bar{f}_B$. Thus, if $\Theta(\bar{f}_A) - \bar{f}_B \geq 0$, then $\Theta(\bar{f}_B) - \bar{f}_A > 0$. The lemma follows.

Non-existence of pure-strategy equilibria in Case 3. Suppose the conditions for Case 3 in the main text are satisfied. Further, assume $f^* < \alpha(v - f^*) + L$, which implies that firm A's best response for $f_B \leq \bar{f}_B$ is $f_A = \alpha(v - f_B) + L$ such that $V_A = V_0$. Since firm i 's best response is never in region j , a pure-strategy equilibrium (if it exists) has to satisfy $V_A = V_0$ and $f_B \leq \bar{f}_B$. Since $\bar{f}_B > \Theta(\bar{f}_A)$, there exists a unique pair (f_A^c, f_B^c) that satisfies $f_A = \alpha(v - f_B) + L$, $f_B = \Theta(f_A)$ such that $f_B^c < \bar{f}_B$. However, at this point, firm B has a profitable deviation since $\hat{f}_B^A > f_B^c$ and $\Pi_B^A(f_B)$ is continuous and concave in f_B . To show that $\hat{f}_B^A > f_B^c$, let $\tilde{f}_B(f_A) = v + (L - f_A)/\alpha$ define the value such that $V_0 = V_A$. Further, let f'_A denote the value such that $\tilde{f}_B(f_A) = \hat{f}_B^A$. Consider the first-order condition for $\hat{f}_B^A$, $G(V_A)/g(V_A) = (1 - \alpha)f_B$, and for $\Theta(f_A)$, which is $G(V_0)/g(V_0) = f_B$. Since at f'_A , we have $\tilde{f}_B(f_A) = \hat{f}_B^A$ and G/g is increasing it holds that $G(V_0)/g(V_0) > G(V_A)/g(V_A)$ for any $f_B < \tilde{f}_B(f'_A)$. It follows that $\Theta(f'_A) < \hat{f}_B^A$. Since $\Theta(f_A)$ is decreasing, we get the result that $\hat{f}_B^A > f_B^c$. ■

Proof of Lemma 3. The patent pool's profits with individual licenses in the four regions of the license fee space from section 3 are given by

$$\begin{aligned} \Pi^0 &= G(V_0)(v - V_0), \Pi^{AB} = G(V_{AB})(v - 4L - V_{AB}), \\ \Pi^k &= G(V_k)(v - 2L - V_k), \text{ for } k \in \{A, B\}. \end{aligned}$$

It follows straight from the discontinuity at $V_k = V_{AB}$ that license fees in region AB are never

optimal. Let $V_0^* = v - F^{**}$ denote the argument that maximizes Π^0 . It then follows from the definition of the profits that if $F^{**} \leq \bar{f}_A + \bar{f}_B$, then any $f_A + f_B = F^{**}$ maximizes the pool's global profits. Further let $V_i^* = (1 - p_i)(v - \hat{f}_j) - L$ denote the argument that maximizes Π^i . This implies that $V_A^* = V_B^*$ such that $\hat{f}_A$ and $\hat{f}_B = (\beta - \alpha)v/(1 - \alpha) + (1 - \beta)\hat{f}_A/(1 - \alpha)$ are a license fee pair that - if interior - maximizes the pool's profit in region A and B. The maximizer in regions A and B is thus either interior $(\hat{f}_A, \hat{f}_B)$ or at the boundary $(\alpha v + L/(1 - \beta), \beta v + L/(1 - \alpha))$. The next step is to show that if $F^{**} > \bar{f}_A + \bar{f}_B$, that is the local maximizer in region 0 is at $(\bar{f}_A, \bar{f}_B)$, then $\hat{f}_A > \bar{f}_A$. From profit maximization follows that $V_0^* > V_B^*$ or

$$\hat{f}_A > \frac{F^{**}}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \frac{\bar{f}_A + \bar{f}_B}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \bar{f}_A.$$

Hence, for any $F^{**} > \bar{f}_A + \bar{f}_B$, we get $\hat{f}_A > \bar{f}_A$. This means that if $F^{**} > \bar{f}_A + \bar{f}_B$, there are two potential global maximizers, $(\bar{f}_A, \bar{f}_B)$ or the maximizer in region A and B. Check that at $L = 0$ the global profit function is continuous at $V_0 = V_A = V_B$. Thus, it follows from $F^{**} > \bar{f}_A + \bar{f}_B$, $\hat{f}_A > \bar{f}_A$ and the concavity of the profit functions that there exists a $L' > 0$ such that if $L \leq L'$, then the global maximizer is the local maximizer of regions A and B. Finally, $L'' < L'$ is implied by the fact that $\bar{F}^* \leq F^{**}$ holds if $L \leq L'$. ■

Proof of Proposition 5. (i) *Profit ranking:* If $L \geq L'$, then $F^{**} < \bar{f}_A + \bar{f}_B$ and the patent pool's total license fee is F^{**} independent of whether it sells package or individual licenses. Both arrangements yield the same profit. Now consider $L'' < L < L'$. With package licensing the pool charges $\min\{F^{**}, \bar{F}^{**}\}$, whereas with individual licenses it charges $\bar{f}_A + \bar{f}_B < \min\{F^{**}, \bar{F}^{**}\}$. Without litigation, the pool's profits are maximized at $F = F^{**}$. Hence, package licensing strictly dominates. Finally, suppose $L \leq L'$. Further suppose $\bar{F}^{**} \geq F^{**}$ such that with package licensing, the pool charges F^{**} . In the case where a pool with individual licenses charges fees at the corner solution $(\alpha v + L/(1 - \beta), \beta v + L/(1 - \alpha))$, the downstream firm gets V_{AB} and package licensing strictly dominates since

$$G(V_0^*)(v - V_0^*) > G(V_{AB})(v - 2L - V_{AB})$$

which holds due to the fact that V_0^* maximizes $G(V)(v - V)$. In the case where a pool with individual licenses charges $\hat{f}_A$ and $\hat{f}_B$, the downstream firm gets $V_B^* < V_0^*$ and package

licensing dominates since

$$G(V_0^*)(v - V_0^*) > G(V_B^*)(v - 2L - V_B^*).$$

Finally, consider $F^{**} > \bar{F}^{**}$ such that with package licensing, the pool charges $\bar{F}^{**}$ and the downstream firm gets V_{AB} . From our analysis in the proof of Lemma 3 we know that the interior maximizer in regions A and B satisfies

$$\hat{f}_A > \frac{F^{**}}{1 - \beta} - \frac{\beta v + L}{1 - \beta}.$$

Since $F^{**} > \bar{F}^{**}$ the minimum value the RHS can take is

$$\frac{\bar{F}^{**}}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \frac{(1 - (1 - \alpha)(1 - \beta))v + 2L}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \alpha v + \frac{L}{1 - \beta}.$$

Hence, $\hat{f}_A > \alpha v + L/(1 - \beta)$ and the boundary solution in regions A and B holds. This implies that package licensing dominates since

$$G(V_{AB})(v - V_{AB}) > G(V_{AB})(v - 2L - V_{AB}).$$

(ii) *Total welfare ranking:* Suppose $0 \leq L \leq L''$. Total welfare with package licensing is welfare superior to individual licenses if

$$G(\max\{V_{AB}, V_0^*\})v \geq G(\max\{V_B^*, V_{AB}\})(v - 2L)$$

which always holds due to $V_0^* > V_B^*$. Consider $L'' < L \leq L'$ where a pool with individual licensing charges $\bar{F}^* = \bar{f}_A + \bar{f}_B$. Individual licensing is welfare superior if

$$G(V_0(\bar{F}^*))v \geq G(\max\{V_{AB}, V_0^*\})v = G(\max\{V_0(\bar{F}^{**}), V_0^*\})v$$

which always holds since $\bar{F}^* \leq \min\{\bar{F}^{**}, F^{**}\}$. Finally, for $L > L'$ a patent pool charges F^{**} in total licensing fees both with individual and package licenses. Thus, since there is no litigation in both cases, total welfare is the same. ■

Proof of Lemma 4. Rank all license fee offers in increasing order. Buying the l lowest ranked licenses and litigating against the remaining $n - l$ patents yield

$$V(l) = (1 - \alpha)^{n-l} (v - \sum_{r=1}^l f_r) - (n - l)L.$$

Litigating patent i and buying license of patent j is never optimal when $f_j > f_i$. It involves the same litigation cost and results in higher expected license fees. In order to show that $V(l)$ is concave in l , we prove that (i) if $V(i) \geq V(i + 1)$, then $V(i + 1) \geq V(i + 2)$ and (ii) if $V(i) \geq V(i - 1)$, then $V(i - 1) \geq V(i - 2)$. Check that $V(i) \geq V(i + 1)$ if

$$av \leq f_{i+1} + \alpha \sum_{r=1}^i f_r - \frac{L}{(1 - \alpha)^{n-i-1}}$$

and $V(i + 1) \geq V(i + 2)$ if

$$av \leq f_{i+2} + \alpha \sum_{r=1}^{i+1} f_r - \frac{L}{(1 - \alpha)^{n-i-2}}.$$

The first condition implies the second since

$$f_{i+2} - f_{i+1} + \alpha f_{i+1} + \frac{L}{(1 - \alpha)^{n-i-1}} - \frac{L}{(1 - \alpha)^{n-i-2}} > 0.$$

Next verify that $V(i) \geq V(i - 1)$ if

$$av \geq f_i + \alpha \sum_{r=1}^{i-1} f_r - \frac{L}{(1 - \alpha)^{n-i}}$$

and $V(i - 1) \geq V(i - 2)$ if

$$av \leq f_{i-1} + \alpha \sum_{r=1}^{i-2} f_r - \frac{L}{(1 - \alpha)^{n-i+1}}.$$

Again the first condition implies the second since

$$f_i - f_{i-1} + \alpha f_{i-1} + \frac{L}{(1 - \alpha)^{n-i+1}} - \frac{L}{(1 - \alpha)^{n-i}} > 0.$$

From this the lemma follows. ■

Proof of Proposition 7. To show that condition (13) always holds, re-write it as

$$v(1 - (1 - \alpha)^n) + nL \geq n \frac{L + \alpha v}{1 + \alpha(n - 1)}.$$

The LHS increases faster in L than the RHS. If this condition holds for $L = 0$, then it must hold for all $L \geq 0$. At $L = 0$, this condition holds if

$$\alpha \leq \frac{1 - (1 - \alpha)^n}{1 + (1 - \alpha)^n(n - 1)} \equiv \Upsilon(\alpha).$$

Check that $\Upsilon(0) = 0$, $\Upsilon(1) = 1$ and

$$\frac{\partial \Upsilon}{\partial \alpha}(\alpha = 0) = \frac{(1 - \alpha)^{n-1} n^2}{[1 + (1 - \alpha)^n(n - 1)]^2} = 1.$$

Furthermore, we have

$$\frac{\partial^2 \Upsilon}{(\partial \alpha)^2} = \frac{(1 - \alpha)^{n-2} n^2 (n - 1)}{[1 + (1 - \alpha)^n(n - 1)]^3} [(n + 1)(1 - \alpha)^n - 1].$$

It thus holds that there exists an α' , with $0 < \alpha' < 1$ such that Υ is convex in α for $\alpha \leq \alpha'$ and concave otherwise. It follows that $\Upsilon(\alpha) \geq \alpha$ for all $\alpha \in [0, 1]$. ■

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