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Majority Vote on Educational Standards

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Abstract

The direct democratic choice of an examination standard, i.e., a performance level required to graduate, is evaluated against a utilitarian welfare function. It is shown that the median preferred standard is inefficiently low if the marginal cost of reaching a higher performance reacts more sensitively to ability for high than for low abilities, and if the right tail of the ability distribution is longer than the left tail. Moreover, a high number of agents who choose not to graduate may imply that the median preferred standard is inefficiently low even if these conditions fail.

Keywords: examination, school, drop-outs, democracy, median voter

JEL classification: I21, D72, I28

1 Introduction

Improving educational achievements and promoting excellence is a stated goal in most countries. In the same time, as the periodic PISA studies of the OECD show, many countries struggle to reach this goal. The present paper provides an explication of this dilemma based on the nature of politics in a democracy: Since in a well-functioning democracy, the median voter is decisive, education policy is governed by the middle ground rather than by the highest achievers, and so tends to promote mediocrity rather than excellence. Specifically, the analysis in this paper shows why, and under what conditions, educational standards chosen by a majority of voters tend to be less demanding than would be efficient.

The tension between democratic policy and excellence in education can be illustrated by some observations. For example, the best universities around the world are private

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institutions who owe their existence to large endowments donated by rich individuals, rather than to a political choice by a majority. Even among the public universities, the best ones typically date from monarchical times, whereas newly founded public institutions hardly match the traditional ones in terms of quality. Similarly, many excellent secondary schools are privately run.

Another observation relates to the demands of parents or their organisations in education policy debates. Here, a common complaint, at least in Europe, is that school is too tough for children, and that politics should aim at reducing the stress created by school. Probably reacting to such demands, some German states currently consider abolishing the repetition of grades. Conversely, one hardly sees parents pushing teachers, schools, or politicians to tighten the standards at school.

Finally, in countries where the responsibility for education is assigned at a lower level of government, a typical reaction to insufficient student performance is to call for a transfer of power to a higher layer. However, while this will likely improve performance in the least performing jurisdictions, it is equally likely to reduce achievement in the best performing places, which is why they will typically oppose such a centralisation of education policy.

In the present paper, I provide a theoretical analysis which explains, at a general level, the tension between the preferences of the majority and the quality of education. A model is presented where students of differing abilities decide how much effort to put into schooling. The effort determines whether a student graduates, which requires to reach a certain performance at the examination called the standard. The standard determines the wage earned by graduates and effort is costly, but more able students find it easier to comply with any given standard. For this reason, more able students prefer higher standards than students with lower abilities.

The standard is determined by a majority vote among agents, say the students' parents, who care for the interest of students. In the main results it is analysed whether, starting from the standard preferred by agents with median ability, a marginal increase in the standard improves a utilitarian welfare criterion. Such a result obtains if two kinds of conditions are satisfied. The first type of condition requires that the marginal cost of satisfying a higher standard decreases more steeply in ability when ability is above the median than when it is below the median. The second kind of condition requires that the distribution of abilities is spread out more widely at the high end than at the low end. Intuitively, there is a lot to gain by tougher standards if there are a few students with very high abilities, and if for those students it is very easy to satisfy more demanding standards.

There is an additional force which pushes down the median preferred standard compared to an efficient choice. Students with lowest abilities may find it optimal to forgo graduation and not to put in any effort. Such ‘drop-outs’ are not affected by a marginal increase in standard since they anyway do not bear any effort costs. Consequently, the standard should rise if the number of drop-outs is large. In an example, I illustrate that this effect may cause the democratically chosen standard to be too low even when then general condition on effort cost is not met.

The theoretical literature on examination standards focuses on the choice of standards by schools. In that literature, which was initiated by Costrell (1994, 1997) and Betts (1998), the school trades off a higher wage for graduates, which requires a higher standard, against a larger number of graduates, which calls for a more lenient standard. Building upon this trade-off, more recent contributions such as Chan, Hao, and Suen (2007), Mechtenberg (2009), Popov and Bernhardt (2010), Ostrovsky and Schwarz (2010), and Himmler and Schwager (forthcoming, 2013), analyse the causes and consequences of grade inflation, i.e, the tendency of schools to award good grades which are not justified by student’ performance.

Among all these contributions, only Costrell (1994, p. 963-964) contains a short section about a majority vote on standards. Remarkably, he concludes that the democratically chosen standard is excessively tough, based on an assumption on the ability distribution which is similar to the conditions which in the present model imply an inefficiently low standard. The difference between both approaches is that in Costrell (1994), voters, like schools, are only concerned with wages and hence educational outcomes but do not take students’ effort into account. In the model presented here, such a disutility of learning is a major driver of voters’ decisions, and consequently the chosen standard tends to be lower. In this sense, my model is tailored to parents who take great care to protect their children from stress, whereas Costrell (1994) rather features an aspiring type of parents who push their children to highest performance.

The paper continues in Section 2 with a description of the basic economic structure. Voting decisions, and the assumptions required to establish the median preferred standard as a Condorcet winner, are analysed in Section 3. Based on this, Section 4 provides the main welfare analysis, and Section 5 discusses the role of drop-outs. The final Section 6 offers some policy conclusions.

2 The Economic Model

There is a continuum of agents with mass one. Agents have two roles in the model, as students and as voters. One can interpret agents literally as adult individuals who still are in education, for example at a university, at an age where they have the right to vote. Alternatively, and more broadly, agents can be seen as representing families composed of children in education and parents who use their right to vote so as to promote the interests of their children.

Agents are characterised by their ability $a \in A := [a_o, a_1)$, where $a_o \geq 0$ and a_1 may be infinite. Abilities are distributed according to the c.d.f. $F : A \to [0, 1]$ which is continuous and strictly increasing on the support A. Thus, the density is strictly positive for $a \in A$. The mean and median abilities are denoted by $\overline{a} = \int_A a \, dF(a)$ and $a_m = F^{-1}(1/2)$.

In order to succeed at school, agents must exert effort denoted by $e \geq 0$. An agent with ability a who provides effort e incurs cost $c(e, a)$. The cost function $c : \mathbb{R}_{\geq 0} \times A \to \mathbb{R}_{\geq 0}$ is assumed to be three times continuously differentiable. I denote derivatives by subscripts, so that, for example, $c_e(e, a)$ is the partial derivative of cost with respect to effort.

Assumption 1. *(i)* $c(0, a) = 0$ *and* $\lim_{e\to\infty}[c(e, a)/e] > 1$ *for* $a \in A$.

(ii) $c_e(e, a) > 0$ *and* $c_a(e, a) < 0$ *for* $(e, a) \in \mathbb{R}_{>0} \times A$

(iii) $c_{ee}(e, a) > 0$, $c_{aa}(e, a) > 0$, *and* $c_{ea}(e, a) < 0$ *for* $(e, a) \in \mathbb{R}_{>0} \times A$

According to Assumption 1(i), a student who does not exert any effort does not incur any cost, and for increasing effort, the cost eventually exceeds the effort. Assumption 1(ii) says that cost increases in effort but decreases in ability. Finally, Assumption 1(iii) states that the marginal cost of effort is strictly increasing, that the cost-saving effect of ability becomes weaker (in absolute terms) as ability increases, and that higher ability decreases the marginal cost of effort. While effort is costly for all agents $a \in A$, an agent at the upper bound of the ability distribution may be able to learn without any cost. That is, I do not rule out that $c(e, a_1) := \lim_{a\to a_1} c(e, a) = 0$ for all e.

The standard $s \in \mathbb{R}_{\geq 0}$ defines the performance level required to pass the examination. Performance is entirely determined by, and measured in the same units as, effort, subsuming the influence of ability in the cost function $c(e, a)$. Students who exert effort $e \geq s$ graduate, while those with $e < s$ fail and will be referred to as drop-outs.

After leaving school, agents will be employed by firms which operate a constant returns to scale technology transforming one efficiency unit of labour into one unit of a numéraire output. The amount of efficiency units supplied by a worker is given by her examination performance, and hence by the effort level e deployed at school. Firms cannot observe the examination performance of an individual worker but know whether she graduated or not. Therefore, all graduates will obtain the same wage w_s, and all drop-outs will receive the same wage w_o. In a competitive equilibrium on the labour market, the graduate wage w_s (the drop-out wage w_o) must be equal to the expected productivity of all agents who exert effort $e \geq s$ ($e < s$). Ownership of firms is sufficiently widely distributed among the agents that price-taking behaviour is justified, but otherwise need not be specified. The reason is that, because of constant returns to scale, firms earn zero profits whatever the standard or the behaviour of students, so that firm ownership does not change the stakes any individual has in the choice of standards.

For given standard s, a student of ability $a \in A$ chooses effort so as to maximise the expected wage net of effort cost. Conditional on choosing an effort $e \geq s$ sufficient to graduate this payoff is $w_s - c(e, a)$. Since the wage does not depend on effort as long as the constraint $e \geq s$ is met, from $c_e > 0$ the minimal effort $e = s$ dominates all effort levels $e > s$. In the same way, conditional on not graduating ($e < s$), the payoff is $w_o - c(e, a)$ which is maximised by $e = 0$. Thus, students either just meet the standard and graduate, or they do not put in any effort at school and fail. Observing $c(0, a) = 0$ from Assumption 1(i), one sees that graduation (dropping out) is optimal if $w_s - c(s, a) \geq \ (<) \, w_o$.

With this behaviour, equilibrium wages will be $w_s = s$ and $w_o = 0$. Thus, in equilibrium graduation is optimal if

$$s - c(s, a) \geq 0 \,. \tag{1}$$

Figure 1 shows the payoff from graduating on the l.h.s. of (1) as a function of the standard s for several levels of ability. From Assumption 1(i), this payoff is zero at $s = 0$ and eventually becomes negative for high enough s. Moreover, from assumption 1(ii), the payoff's slope $1 - c_e(s, a)$ is positive at $s = 0$ so that for all $a \in A$, at some (possibly low) standard, graduation is worthwhile and a positive payoff can be reached.

Assumptions 1(i,iii) imply that for each $a \in A$ there is a unique positive standard $s_{\max}(a)$, given by the solution to (1) as an equality, which yields a payoff of zero. As is

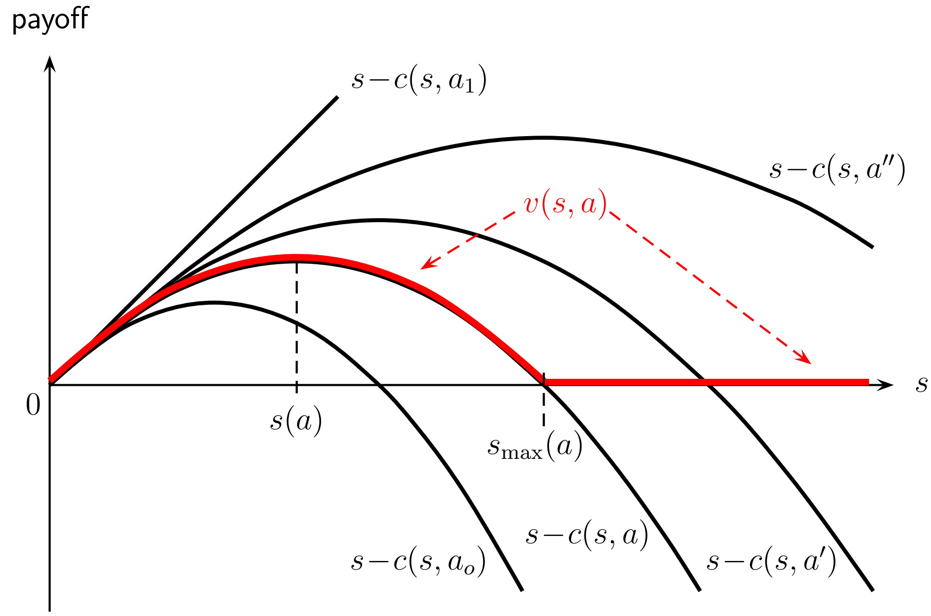

Figure 1: The payoff from graduating and indirect utility.

apparent from Figure 1, an agent with ability a will graduate if $s \leq s_{\max}(a)$ and drop out if $s > s_{\max}(a)$. Thus, $s_{\max}(a)$ is the maximal standard which an agent of ability a is willing to satisfy.

Combining these observations, one finds the indirect utility function $v(s,a) = \max\{0; s - c(s,a)\}$ of an agent $a \in A$. This function, which in Figure 1 is illustrated by a bold red line, relates the standards about which agents vote to the individual agent's utility, anticipating her own effort and graduation choices and the equilibrium wages ensuing from the chosen standard. From assumption 1(iii), the payoff from graduating is strictly concave in s so that for each $a \in A$, there is a unique standard $s(a) = \operatorname{argmax}_s\{v(s,a)\} > 0$ which maximises the payoff of an agent with ability a.

Increasing ability shifts the payoff from graduating upward, since Assumption 1(ii) implies that cost decreases in ability. Moreover, from Assumption 1(iii), the marginal cost of effort decreases when ability rises. As illustrated in Figure 1, this means that both the utility maximising standard $s(a)$ and the highest standard $s_{\max}(a)$ which an agent will satisfy strictly increase in ability a.

Inverting the relationship $s_{\max}(a)$, one can express the decision to graduate or not by defining a minimal ability $a_{\min}(s)$ which an agent must have so as to be willing to satisfy a given standard s. This level is called the graduation threshold for standard s.

To formalise this, one has to observe that for some s, the solution to (1) may not be in the support A of the ability distribution. In such a case, all agents (no agent) will graduate for the standard under consideration, and I define this threshold accordingly to be a_o or a_1. Formally:

Definition 1. *For all $s \in \mathbb{R}_{\geq 0}$:*

$$a_{\min}(s) = \begin{cases} a_o & \text{if} \quad s - c(s,a) > 0 \quad \text{for all } a \in A \\ \tilde{a} & \text{if} \quad s - c(s,\tilde{a}) = 0 \quad \text{for some } \tilde{a} \in A \\ a_1 & \text{if} \quad s - c(s,a) < 0 \quad \text{for all } a \in A \end{cases}$$

Notice that from $c_a < 0$ in Assumption 1(ii), $\tilde{a}$ in the second line of Definition 1 must be unique if it exists and hence $a_{\min}(s)$ is well defined. Moreover, all agents with $a < a_{\min}(s)$ will fail and all agents with $a \geq a_{\min}(s)$ graduate. In the first (third) line of Definition 1, one has the case of a low (high) standard which yields positive (negative) payoff for all agents and hence the graduation threshold is the lower (upper) bound of the ability distribution. Finally, differentiating $s - c(s,a) = 0$ shows that

$$\frac{da_{\min}(s)}{ds} = \frac{1 - c_e(s, a_{\min}(s))}{c_a(s, a_{\min}(s))} > 0\,, \tag{2}$$

where the last inequality follows on $c_a < 0$ and the fact that at $a_{\min}(s)$, the payoff from graduating must be decreasing. Therefore, the graduation threshold is weakly increasing in the standard s, and strictly so if the threshold is in the interior of A.

The graduation threshold and the indirect utility function guide the voting behaviour of agents to which I now turn.

3 Voting

The analysis of voting outcomes starts with determining how agents of different ability evaluate alternative standards. The upshot of this discussion will be that more able individuals tend to prefer higher standards. To make this statement precise, consider two standards s and s' where s' is more demanding than s, that is, $0 < s < s'$, and observe how the payoff from graduating under these two standards is affected by a marginal increase in ability. From $c_a < 0$ and $c_{ea} < 0$ in Assumptions 1(ii,iii), we have

$$0 < \frac{\partial[s - c(s,a)]}{\partial a} < \frac{\partial[s' - c(s',a)]}{\partial a} \tag{3}$$

Thus, graduation is more rewarding for more able individuals, and an increase in ability procures a larger gain when the standard is higher.

It is useful to consider three cases depending on the preferences of the least (most) able agents. In case [1], $s - c(s,a_o) \leq s' - c(s',a_o)$: Even for an agent with the lowest ability, the payoff from graduating is at least as large with the more demanding standard as with the more lenient standard. Case [2] is defined by $s - c(s,a_o) > s' - c(s',a_o)$ and $s - c(s,a_1) < s' - c(s',a_1)$. Here, an agent with the lowest ability obtains a higher payoff from the smaller standard, whereas an agent with an ability close to the upper bound gains more from the higher standard. Finally, the last case [3] obtains if $s - c(s,a_1) \geq s' - c(s',a_1)$. Here, even agents with the highest abilities reap a larger payoff from graduating under the lower standard than under the higher standard.

From (3), in case [1], one has $s - c(s,a) < s' - c(s',a)$ for all $a \in A, a > a_o$, and in case [3], it follows $s - c(s,a) > s' - c(s',a)$ for all $a \in A$. Therefore, the three cases are mutually exclusive and exhaust all possibilities. Moreover, in case [2], by continuity, there is $\hat{a} \in A$ such that $s - c(s,\hat{a}) = s' - c(s',\hat{a})$. Again from (3), one has $s - c(s,a) > s' - c(s',a)$ for $a < \hat{a}$ and $s - c(s,a) < s' - c(s',a)$ for $a > \hat{a}$, so that $\hat{a}$ is unique. This allows to define:

Definition 2. *For all $s, s' \in \mathbb{R}_{>0}$ with $s < s'$:*

$$\hat{a}(s,s') = \begin{cases} a_o & \text{if } [1]\, s - c(s,a_o) \leq s' - c(s',a_o) \\ \hat{a} & \text{if } [2]\, s - c(s,a_o) > s' - c(s',a_o) \\ & \qquad \text{and } s - c(s,a_1) < s' - c(s',a_1) \\ a_1 & \text{if } [3]\, s - c(s,a_1) \geq s' - c(s',a_1) \end{cases}$$

where in the second line, $\hat{a}$ is the solution to $s - c(s,\hat{a}) = s' - c(s',\hat{a})$. In case [2], the ability $\hat{a}(s,s')$ is the critical ability level where the payoff from graduating is the same from both standards (see Figure 2). In cases [1] and [3], where no such level exists, the critical value is defined to be, respectively, the lowest or the highest ability. In each case, agents with ability below (above) $\hat{a}(s,s')$ obtain a higher payoff from the lower (higher) standard.

I turn now to analysing the preferences of agents in the three cases. Starting with

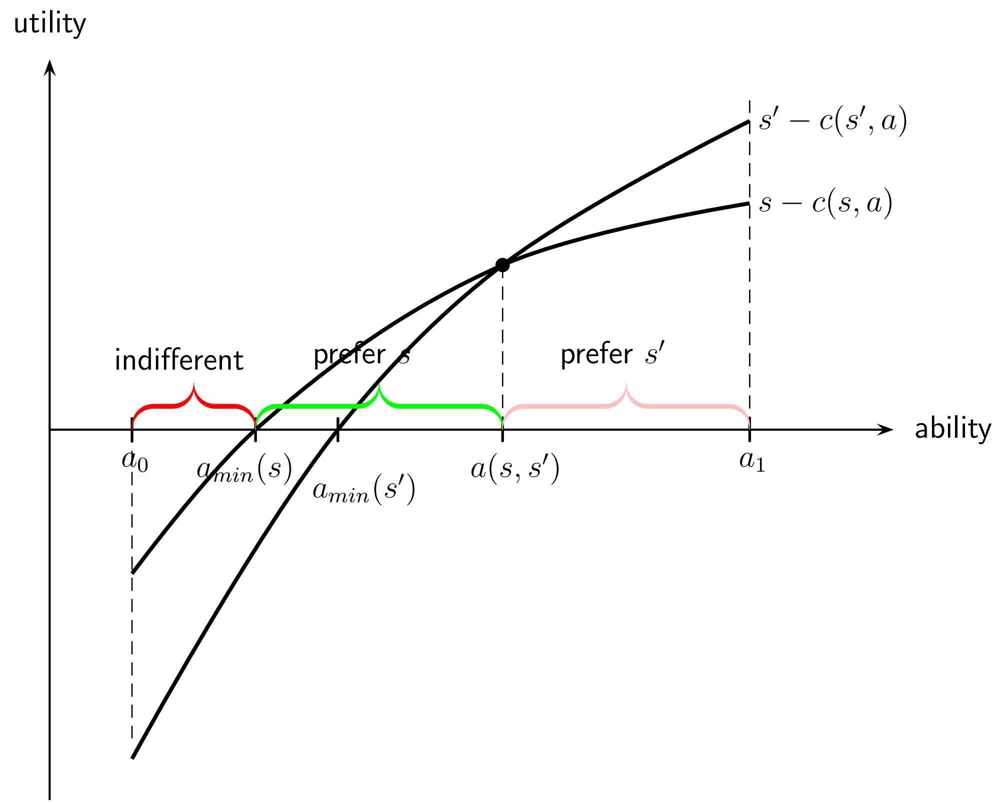

Figure 2: Preferences of agents in a vote between two standards $s < s'$.

case [1], it is apparent from Figure 1 that this case can only occur if the lower standard s is on the increasing part of the payoff curve, so that $s - c(s, a) > 0$ for all $a \in A$, or equivalently, $a_{\min}(s) = a_o$. Therefore, all agents graduate under the standard s, yielding $v(s, a) = s - c(s, a) > 0$. Together with $s - c(s, a) < s' - c(s', a)$, one concludes that in case [1], $v(s, a) = s - c(s, a) < s' - c(s', a) = v(s', a)$ for all $a > a_o$. Hence in this case, all agents except possibly those with the lowest ability strictly prefer the higher standard.

In case [2], one sees from Figure 1 that at the critical ability $\hat{a}(s, s')$ the payoff from both standards must be positive, i.e., $s - c(s, \hat{a}(s, s')) = s' - c(s', \hat{a}(s, s')) > 0$. Since the payoff is increasing in a, the graduation thresholds for both standards and the critical ability must then be ordered according to $a_{\min}(s) \leq a_{\min}(s') < \hat{a}(s, s') < a_1$, as displayed in Figure 2. Thus, there are some agents willing to satisfy both standards. However, both $a_{\min}(s)$ and $a_{\min}(s')$ may be equal to or larger than a_o. If $a_o < a_{\min}(s)$, then agents with $a \in [a_o, a_{\min}(s))$ will fail under both standards and obtain the same indirect utility $v(s, a) = v(s', a) = 0$. Agents with $a \in (a_{\min}(s)), \hat{a})$ will graduate under s, whether $a_o = a_{\min}(s)$ or $a_o < a_{\min}(s)$, procuring them a positive utility. Since for such a, the payoff from graduating is higher under the lower standard s than under s', one has $v(s,a) = s - c(s,a) > v(s',a) = \max\{0; s' - c(s',a)\}$, regardless of whether the agent would graduate under the more demanding standard s' or not. Finally, agents with ability $a > \hat{a}(s,s')$ graduate under both standards, but obtain a higher payoff when the more severe standard is chosen, $v(s,a) = s - c(s,a) < s' - c(s',a) = v(s',a)$. To summarise, in case [2] agents with ability below the graduation threshold of the more lenient standard, if such agents exist, are indifferent between both standards since they do not plan to graduate under either of them. Agents with intermediate ability between the graduation threshold for s but below the critical value $\hat{a}(s,s')$ strictly prefer the lower standard, and agents with ability above $\hat{a}(s,s')$ strictly prefer the higher standard.

In case [3], it may arise that no agent is willing to satisfy s' or even s, so that $a_{\min}(s) = a_1$ is possible. Whether or not this is the case, agents with ability $a < a_{\min}(s)$ will again fail under both standards and obtain $v(s,a) = v(s',a) = 0$. If there exist agents with ability $a \in (a_{min}(s), a_1)$, they will graduate under s and obtain utility $v(s,a) = s - c(s,a) > v(s',a) = \max\{0; v(s',a)\}$. Altogether, in case [3], agents with ability below the graduation threshold for the lower standard are indifferent between both standards, and agents whose ability exceeds this threshold, if they exist, strictly prefer the lower standard.

Using Definitions 1 and 2, the following Lemma, which is illustrated in Figure 2, summarises the preceding discussion:

Lemma 1. *For any two standards $s, s' \in \mathbb{R}_{>0}$ with $s < s'$:*

(i) If $a_o \leq a < a_{\min}(s)$, then $v(s,a) = v(s',a)$.

(ii) If $a_{\min}(s) < a < \hat{a}(s,s')$, then $v(s,a) > v(s',a)$.

(iii) If $\hat{a}(s,s') < a \leq a_1$, then $v(s,a) < v(s',a)$.

For completeness, note that for $a = a_{\min}(s)$ or $a = \hat{a}(s,s')$, one has $v(s,a) = v(s',a)$ as long as $a_o < a < a_1$. If the graduation threshold or the critical ability level are a_o, then only weak inequalities can be stated. If $a_o = a_{\min}(s) = \hat{a}(s,s')$ (case [1]), then $v(s,a_o) \leq v(s',a_o)$. If $a_o = a_{\min}(s) < \hat{a}(s,s')$, then $v(s,a_o) \geq v(s',a_o)$ (cases [2], [3]).

The standard is determined by the agents in a series of pairwise votes. It is assumed that agents vote sincerely so that voting for s in a vote against s' is optimal for an agent with ability a if $v(s,a) \geq v(s',a)$. There is an open agenda in the sense that any previously decided standard may be challenged in a new vote by some other standard.

A standard will be democratically chosen if it collects a majority of votes against any other standard. This is captured by the following

Definition 3. *Standard $s \in \mathbb{R}_{\geq 0}$ is a weak Condorcet winner if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s,a) \geq v(s',a)\}} dF(a) > 1/2 \,.$$

In Definition 3, it is worth noting that in order to prevail, a standard must be weakly but not necessarily strictly preferred to any other standard by a majority of agents. Thus, agents behave optimally in each vote, but ties are broken in a way which supports the equilibrium, like in a mixed strategy Nash equilibrium. For this reason, I prefer to call it a *weak* Condorcet winner.

Lemma 2. *The standard $s_m := s(a_m)$ preferred by agents with median ability is a weak Condorcet winner.*

Proof: See Appendix. ∎

Lemma 2 differs from a standard median voter result in that the median preferred standard s_m is only shown to be a *weak* Condorcet winner, and that the Lemma does not claim uniqueness. This is because the opportunity to drop out transforms the payoff from graduating $s - c(s, a)$, a single-peaked function which strictly rises when the standard is moved towards the optimum, into the indirect utility $v(s, a)$ which stays equal to zero for all standards where the agent fails to graduate. If such flat parts are allowed in the indirect utility function (as, for example, in Persson and Tabellini, 2002, Definition 2, p. 22), the indifference of voters must be resolved in a suitable way for a median voter theorem to hold.

Specifically, in a vote between s_m and a higher standard, the majority for s_m must be secured by low ability agents. If some of these drop out under s_m, i.e., if $a_{\min}(s_m) > a_o$, they will also drop out under the alternative, higher standard. These agents are then indifferent and so might vote together with high ability agents in favour of the more demanding standard. Thus, with an arbitrary tie breaking rule, s_m might fail to win such a vote and hence might not be a Condorcet winner. By requiring only that agents have to vote for *some* optimal standard, Definition 3 allows to assign indifferent low ability drop-outs to the camp of supporters of the median preferred standard, establishing s_m as a weak Condorcet winner. Definition 3 does not, however, ensure uniqueness: It is not ruled out that, by attributing votes of indifferent agents in a different way, one can support other standards as weak Condorcet winners as well. Conversely, it becomes clear from this discussion that without drop-outs, s_m is the unique Condorcet winner in the usual sense, defined by replacing, in Definition 3, the "$\geq$" in the condition $v(s, a) \geq v(s', a)$ by "$>$".

Since drop-outs are a relevant issue in education and in particular when it comes to determine examination standards, I do not want to restrict the analysis by excluding them. Alternatively, a suitable tie-breaking assumption which ensures that no other standard can win a majority against s_m consists in requiring that agents who drop out under both standards on the ballot support the lower one of these standards. Since such agents are located at the lower tail of the ability distribution, this kind of behaviour is rather plausible. For example, from earlier experiences these agents might have developed general reservations against a tough educational regime, or they might feel compelled to mimic the voting behaviour of their peers with slightly higher ability, who strictly prefer the lower standard.

More substantively, this kind of tie-breaking rule can also be supported by refining the Condorcet equilibrium in the spirit of trembling hand perfection. To make this precise, I define an ϵ-education model where for every standard $s \in \mathbb{R}_{\geq 0}$ each of the two choices '$e = s$' and '$e = 0$' will be chosen with probability of at least $\epsilon > 0$, where ϵ is a small number. Thus, with probability ϵ the agent will make an 'error' in her graduation decision. While such an error is conceivable in any strategic situation, modeling a deviation from planned behaviour is particularly appealing if one interprets agents as families where education decisions are taken by children. Here, ϵ measures the possibility that children do not follow the educational course which their parents deem optimal for them.

In an ϵ-education model, the payoff from standard s for an agent with ability a will be $v(s, a; \epsilon) = (1 - \epsilon)[s - c(s, a)]$ if $s - c(s, a) \geq 0$ and $v(s, a; \epsilon) = \epsilon[s - c(s, a)]$ if $s - c(s, a) < 0$. Adapting Definition 3 and Lemma 2 to the ϵ-education model, one obtains

Definition 4. *Standard $s \in \mathbb{R}_{\geq 0}$ is a weak Condorcet winner in the ϵ-education model if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s, a; \epsilon) \geq v(s', a; \epsilon)\}} dF(a) > 1/2 .$$

Lemma 3. *For all $0 < \epsilon < 1$, the standard s_m preferred by agents with median ability is the unique weak Condorcet winner in the ϵ-education model.*

Proof: From $1 - \epsilon > 0$, $v(s, a; \epsilon) = (1 - \epsilon)[s - c(s, a)]$ is strictly increasing (strictly decreasing) in s for $0 \leq s < s(a)$ (for $s(a) < s < s_{\max}(a)$). From $\epsilon > 0$, $v(s, a; \epsilon) = \epsilon[s - c(s, a)]$ is still strictly decreasing in s for $s > s_{\max}(a)$. Therefore, in a vote among s_m and a lower (higher) alternative standard $s < s_m$ ($s' > s_m$), all agents with $a > \hat{a}(s, s_m)$ ($a < \hat{a}(s_m, s')$) strictly prefer s_m over s (over s'). As seen in the proof of Lemma 2, these agents constitute more than half of the electorate. Therefore, s_m beats every alternative standard, and no alternative standard can attract a majority against s_m. ■

The key difference between Lemmas 2 and 3 is that in the ϵ-education model, the median preferred standard is the *unique* Condorcet winner. This arises from the fact that the 'trembling hand' assumption blurs the decision to drop out from school. While the standard $s_{\max}(a)$ still defines the cut-off above which an agent of ability a does not plan to graduate anymore, she may still do so erroneously and thus experience the (negative) payoff from graduating with probability $\epsilon > 0$. This breaks the indifference of low ability agents in favour of the less demanding standard, making the indirect utility function $v(s, a; \epsilon)$ single-peaked.

According to the idea of trembling hand perfectness, a Condorcet winner in the original model is only reasonable if it is robust against the possibility of small errors. This is captured by the following definition:

Definition 5. *A standard $s \in \mathbb{R}_{\geq 0}$ is a strong Condorcet winner if*

(i) s is a weak Condorcet winner, and

(ii) there is a sequence $\{s_n\}_{n=1,2,\ldots}$ such that $s_n \to s$ and for all n, s_n is a weak Condorcet winner in an ϵ_n-education model, where $\epsilon_n \in (0, 1)$ and $\epsilon_n \to 0$.

Thus, a weak Condorcet winner is called 'strong' if it is the limit of a sequence of weak Condorcet winners in ϵ-education models the error probabilities of which converge to zero. One immediately concludes from Lemmas 2 and 3:

Proposition 1. *The standard s_m preferred by agents with median ability is the unique strong Condorcet winner.*

To summarise, there are three ways to establish a median-voter theorem in the present model. First, the median-preferred standard s_m is the unique Condorcet winner in the usual sense, i.e., it is strictly preferred by a majority of agents in any pairwise vote, if all agents graduate under this standard, $a_{\min}(s_m) = a_o$. Second, even if there are some dropouts, s_m is a weak Condorcet winner in the sense that a majority of agents weakly prefers this standard in any pairwise vote. Third, s_m is the only strong Condorcet winner, that is, it is the only voting outcome which is robust against small errors in the education decision.

In the following Sections 4 and 5, the welfare properties of the median preferred standard are examined.

4 Welfare Analysis

Welfare is defined by a utilitarian criterion, aggregating the indirect utility of all agents:

Definition 6. *For any given standard s, welfare is*

$$W(s) = \int_{a_o}^{a_1} v(s,a)\, dF(a)\,.$$

For agents who graduate, utility is the wage earned net of effort cost, and for drop-outs, utility is zero. Therefore, welfare is given by $W(s) = \int_{a_{\min}(s)}^{a_1} \big[s - c(s,a)\big]\, dF(a)$. Differentiating this equation w.r.t. s and using the definition of $a_{\min}(s)$, one finds that an increase in the standard changes welfare by

$$W'(s) = \int_{a_{\min}(s)}^{a_1} \big[1 - c_e(s,a)\big]\, dF(a)\ . \tag{4}$$

To understand (4), notice that a change in the standard affects both the graduation threshold $a_{\min}(s)$ and the utilities $v(s,a)$. The first effect cancels, however, since the utility of an agent at the threshold is zero by definition. Since drop-outs anyway receive a utility of zero, the second effect is relevant only for those agents who will graduate under the original standard. For these individuals, raising the standard by one unit increases the wage by one unit, since wage and standard are normalised to be equal. On the other hand, in order to satisfy the higher standard, students have to incur additional effort cost so that, for an agent with ability a, the net gain from increasing the standard is $1 - c_e(s,a)$.

The goal of this section is to examine under what conditions welfare will increase if the standard is raised above the standard chosen by the majority. That is, I provide sufficient conditions for $W'(s_m) > 0$. Only a local welfare analysis is offered since any second order conditions ensuring a global maximum will necessarily require assumptions on the shape of the density $F'(a)$, which are likely to be either very strong or difficult to interpret.

The starting observation in this analysis is that, because s_m is optimal, agents with median ability are indifferent to an increase in standard, $1 - c_e(s_m, a_m) = 0$. Since marginal cost of effort is strictly decreasing in ability, agents with above-median ability will gain from an increase in standard, i.e., $1 - c_e(s_m, a) > 0$ for all $a > a_m$. Agents with below-median ability will lose, $1 - c_e(s_m, a_m) < 0$, as long as they still graduate. Therefore, the net welfare effect of an increase in the standard hinges on the relative sizes of aggregate gains and losses by high and low ability agents respectively. These aggregate amounts in turn are determined by three features: the shape of the marginal cost function c_e, the distribution function $F(\cdot)$, and the graduation threshold $a_{\min}(s_m)$. In this section, I provide two results highlighting the role of the first two features, whereas the importance of the graduation threshold is taken up in Section 5.

The properties of the cost and distribution functions used in these results are described by two pairs of conditions. The first one of these are

Condition 1. $c_{eaa}(s_m, a) \leq 0$ *for* $a \in [a_o, a_1)$.

Condition 2. $\overline{a} \geq a_m$.

Condition 2 simply states that mean ability exceeds median ability. Condition 1, which is illustrated in Figure 3, requires that the marginal cost of effort c_e is a concave function of ability. In Figure 3, ability is depicted on the horizontal axis and marginal cost and benefits of an increase in standard are measured on the vertical axis. The marginal cost of effort evaluated at the median preferred standard, $c_e(s_m, a)$, decreases according to Assumption 1(iii), and cuts the marginal benefit of 1 at the median ability a_m. As illustrated in this figure, if the cost function satisfies Condition 1, the marginal cost curve should become steeper as ability increases. Thus, the effort-enhancing effect of ability increases in ability.

Alternatively, I consider the following pair of conditions:

Condition 3. *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2}c_e(s_m, a_m - x) + \frac{1}{2}c_e(s_m, a_m + x) \leq 1 \, .$$

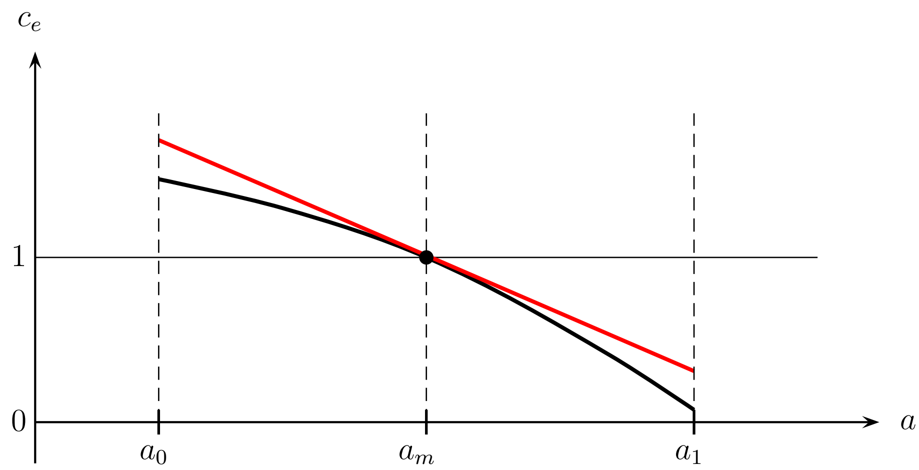

Figure 3: Marginal cost of effort is concave in ability (Condition 1).

Condition 4. *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2} - F(a_m - x) \geq F(a_m + x) - \frac{1}{2}\,.$$

In Condition 3, two agents are considered whose abilities exceed and fall short of the median ability by the same amount x. The condition requires that the average marginal cost of these two individuals does not exceed the marginal benefit. Thus, on average, these two agents gain from raising the standard. Figure 4 gives a geometric intuition for this property, which is based on splitting the graph of the marginal cost curve c_e in the two parts corresponding to the domains of below and above median abilities. Condition 3 requires that, when one of these parts is mirrored at the point $(a_m, 1)$, the image should be located below the other part.

Finally, also Condition 4 (see Figure 5) starts from considering two ability levels which are located symmetrically around the median. The condition requires that the mass of agents with abilities between the lower one of these values and the median is at least as large as the mass of agents with abilities between the median and the higher one of these values.

To summarize, Conditions 1 and 3 represent the idea that the impact of ability on marginal effort cost should be stronger on the high side of the ability distribution than on the low side. Put differently, this means that academic performance is very sensitive to ability when one compares good and very good students, whereas below the median ability, differences in ability do not matter much. It is an empirical issue whether such a property holds in reality. A priori, it seems plausible to me because, on the one hand, weak students mostly can reach a satisfactory performance with sufficient training,

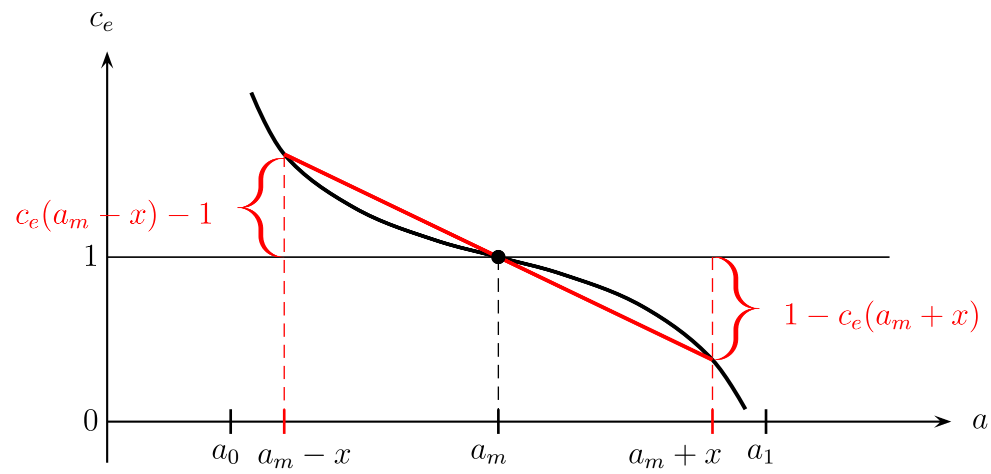

Figure 4: On average, two agents with abilities symmetric to the median gain by a marginal increase in standard (Condition 3).

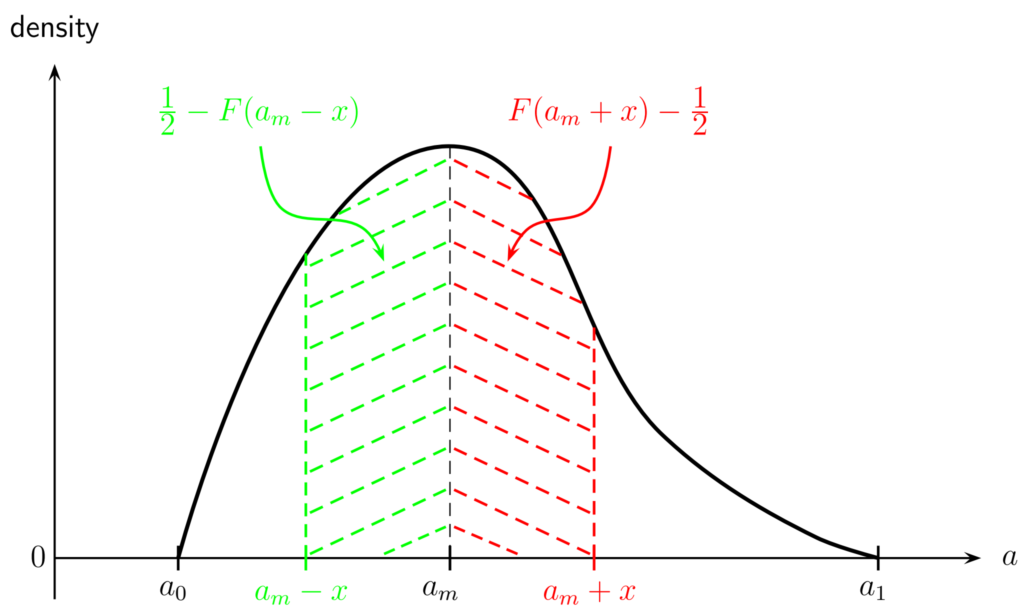

Figure 5: Higher abilities are more spread out than lower abilities (Condition 4).

whereas, on the other hand, really excellent achievements are out of reach except for the very brightest.

According to Conditions 2 and 4, the distribution of abilities is more 'spread-out' at the upper end of the support than at the lower end. This can arise, for example, by the presence in the economy of a few agents with very high ability, who raise the mean, whereas a large mass of agents is concentrated at moderately low ability levels. This corresponds to the empirical fact that income distributions, which at least partially reflect distributions of productivity or ability, are typically right-skewed.

Proposition 2. *If Conditions 1 and 2 hold, then* $W'(s_m) \geq 0$*. If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$*, then* $W'(s_m) > 0$*.*

Proof. See Appendix. ∎

Proposition 3. *If Conditions 3 and 4 hold, then* $W'(s_m) \geq 0$*. If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$*, then* $W'(s_m) > 0$*.*

Proof. See Appendix. ∎

Propositions 2 and 3 show that democratic choice leads to an inefficiently low examination standard if the cost function and the distribution function satisfy one of the pairs of Conditions 1 and 2, or 3 and 4. Intuitively, an increase in standard is beneficial if the gain conferred this way to agents whose abilities exceed the median by a certain amount outweighs the loss incurred by agents whose abilities fall short of the median by a similar amount. This is the case if the marginal cost of effort decreases fast once ability is raised above the median but rises only slowly when ability falls below the median, as required by Conditions 1 or 3. Moreover, the aggregate gain (loss) is large (small) if the mass of agents with very high (low) ability is relatively large (small), as postulated in Conditions 2 or 4.

Looking closer at the pairs of conditions required in each proposition, one notices that Condition 1 implies Condition 3, and that Condition 4 implies Condition 2. Therefore, there is a substitutive relationship between the properties of the cost function and the distribution function in the sense that it is possible to weaken one of them if one strengthens the other.

As mentioned in the introduction, Costrell (1994) proves a result which appears to be contrary to Propositions 2 and 3. In his model, the standard chosen by majority vote is inefficiently high if the distribution of preferred standards is symmetric unimodal. This contrasts with Conditions 2 and 4, both of which are satisfied with equality if the distribution of abilities is symmetric. The main difference in my set-up and the analysis by Costrell (1994) lies in the objective function of voters: In Costrell (1994), voters care only about academic performance and hence try to maximise the productivity of students, but do not take effort cost into account. In contrast, in the present analysis, it is assumed that parents will vote for reducing the standard if they feel that their children suffer too much from the effort required in school. Not surprisingly then, an education system where effort cost of students is politically important is likely to be less demanding than a system which only aims at raising educational outcomes.

In Propositions 2 and 3 the existence of agents who do not graduate under the median preferred standard figures only as a tie-breaking device in case both of the respective conditions are just satisfied as equalities. In the following Section 5, I show that the presence of a substantial number of drop-outs independently contributes to an insufficiently high median preferred standard. For this reason, the pairs of conditions used in each proposition are not at all necessary.

5 The Role of Dropouts

As long as one considers only agents who graduate under the median preferred standard, increasing marginal effort cost for given ability will clearly reduce the net benefit of a higher standard. Geometrically, if one bends the c_e-function in Figures 3 and 4 upwards while keeping the point $(a_m, 1)$ fixed, so that the median preferred standard does not change, then it is apparent that losses of an increase in standard increase and gains shrink. From this effect, one concludes that the higher the marginal cost for low ability agents, the less likely it is that the median preferred standard is too lenient. Intuitively, a high standard hurts the agents with below median ability ever more if they suffer more and more form learning, and so society should increasingly protect these students from being pushed to high effort.

This argument however misses the fact that students have the opportunity to avoid such costly effort by not graduating. Moreover, from the individual education decision, students are increasingly likely to do so when marginal effort cost rises. Therefore, the presence of drop-outs creates a countervailing effect of rising marginal effort cost which may well overcompensate the increasing loss of those who graduate.

In the following, I will illustrate this effect by means of an example. In this example, ability is uniformly distributed on $A = [a_o, a_1] = [0, 2]$ so that $a_m = \overline{a} = 1$. Moreover, effort cost is given by a family of functions

$$c(e,a;\gamma) = \frac{e^2}{2}\Big[1 + (a_m - a) + \gamma(a_m - a)^2\Big]\,, \tag{5}$$

where the parameter is restricted to $0 \le \gamma \le 1/2$ to ensure that $c(e,a;\gamma)$ satisfies Assumption 1. Computing $s_m = 1$ and

$$\begin{aligned} c_e(s_m, a; \gamma) &= 2 - a + \gamma(1-a)^2\,, \\ c_{ea}(s_m, a; \gamma) &= -1 - 2\gamma(1-a)\,, \\ c_{eaa}(s_m, a; \gamma) &= 2\gamma\,, \end{aligned}$$

one sees that γ determines the curvature of the marginal cost of effort. Specifically, for $\gamma > 0$ the example violates both Conditions 1 and 3.

The graduation threshold $a_{\min}(s_m;\gamma)$ solves the equation $c(s_m, a;\gamma) = s_m$, or equivalently, $1+(a_m-a)+\gamma(a_m-a)^2 = 2/s_m$. With $a_m = s_m = 1$ it follows that for all admissible γ, the marginal cost of effort at the graduation threshold is $c_e(s_m, a_{\min}(s_m;\gamma);\gamma) = 2$. The change in welfare induced by a marginal increase in the standard can be computed from (4) as

$$\frac{\partial W(s_m;\gamma)}{\partial s_m} = -\frac{1}{2}\int_{a_{\min}(s_m;\gamma)}^{2} \left[(1-a) + \gamma(1-a)^2\right] da\,.$$

From these equations, one derives:

Proposition 4. *If the cost of effort is given by (5) and ability is distributed uniformly on* $[0,2]$*, then* $\partial W(s_m;\gamma)/\partial s_m > 0$ *for all* $0 < \gamma < 1/2$.

Proof. Computations done with Mathematica, and available from the author upon request, reveal that the only two values of $\gamma \in [0,1/2]$ with $\partial W(s_m;\gamma)/\partial s_m = 0$ are $\gamma = 0$ and $\gamma = 1/2$, and that $\partial W(s_m;\gamma)/\partial s_m$ is increasing in γ at $\gamma = 0$. From this, it follows $\partial W(s_m;\gamma)/\partial s_m > 0$ for $0 < \gamma < 1/2$. ■

The logic of Proposition 4 is illustrated in Figure 6. Here, the downward sloping straight line is the marginal cost of effort for the lowest admissible value $\gamma = 0$, which leads to the graduation threshold $a_{\min}(s_m;0) = a_o = 0$. When γ rises above zero, the marginal cost of effort bends upwards and becomes strictly convex, as seen in the green curve. The highest possible $\gamma = 1/2$ finally results in the highest curve, painted red. With uniform distribution of ability, the aggregate losses and gains of an increase in

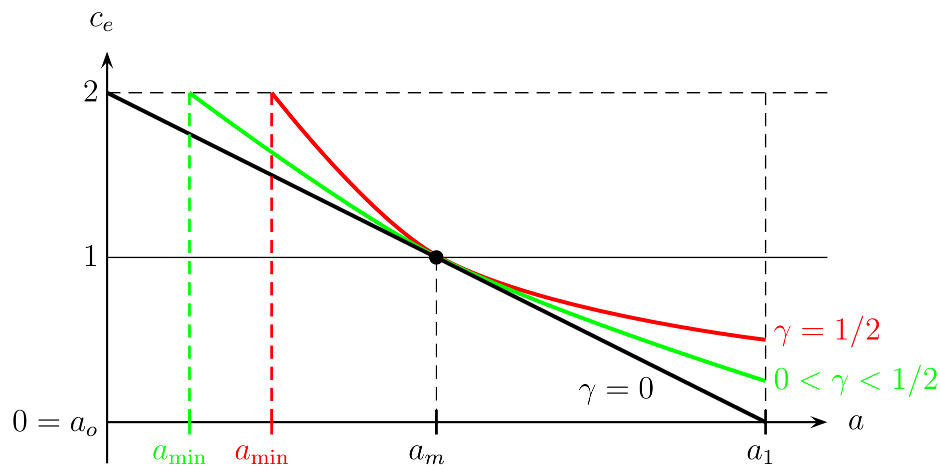

Figure 6: If marginal effort cost bend upwards, the graduation threshold rises.

standard are directly measured by the areas between these curves and the marginal benefit of 1. It is apparent that the net gain would decrease in γ if the graduation threshold remained at $a_o = 0$. However, the threshold moves to the right as γ increases, so that the area representing the loss, which is bounded below by the vertical line at $a_{\min}(s_m; \gamma)$, shrinks.

6 Conclusion

It is difficult to derive an immediate policy recommendation from the inefficiency shown in the present paper, since one obviously would not argue against democracy. Nevertheless, it is worthwhile to discuss some effects not included in the model which may possibly counteract the tendency of democratic education policy towards overly lenient standards. The first such feature is the fact that turnout is generally lower among low income voters than in the general electorate. Inasmuch as income and ability are correlated, this otherwise deplorable fact tends to raise median ability among voters and hence works in favor of a higher standard. Second, a tax-transfer scheme may give low ability agents, who would be recipients of transfers, a stake in higher standards since these will raise wages and tax revenues. Finally, the present analysis suggests that there is a role for the private sector in education, if only in order to promote excellence.

Appendix

Proof of Lemma 2

Consider s_m in a vote against some standard $s < s_m$. I first show that $\hat{a}(s, s_m) < a_m$. If case [1] applies, this is immediate from $\hat{a}(s, s_m) = a_o < a_m$. In case [2], it exists $\hat{a}(s, s_m) \in A$ such that $s - c(s, \hat{a}(s, s_m)) = s_m - c(s_m, \hat{a}(s, s_m))$. Since the payoff from graduating $s - c(s, \cdot)$ is strictly concave in s, for the standard $s(\hat{a}(s, s_m))$ which maximises the utility of an agent with ability $\hat{a}(s, s_m)$, it must hold $s < s(\hat{a}(s, s_m)) < s_m$. Now $\hat{a}(s, s_m) < a_m$ follows from the second inequality and the fact that the optimal standard $s(a)$ is strictly increasing in ability. Finally, case [3] would imply $s - c(s, a_m) > s_m - c(s_m, a_m)$, contradicting the fact that s_m is optimal for the median. Hence case [3] is ruled out, establishing $\hat{a}(s, s_m) < a_m$. Therefore, the mass of agents with ability $a \in (\hat{a}(s, s_m), a_1)$ exceeds 1/2. From Lemma 1(iii), these agents strictly prefer s over s_m so that $\int_{\{a \in A | v(s_m, a) > v(s, a)\}} dF(a) > 1/2$ follows.

Consider now s_m in a vote against some standard $s' > s_m$. By an argument analogous to the one laid out in the previous paragraph, one derives $a_m < \hat{a}(s_m, s')$. From Lemma 1(i), all agents with ability a such that $a_o \leq a < a_{\min}(s_m)$ are indifferent between both standards. From Lemma 1(ii), all agents with ability a such that $a_{\min}(s_m) < a < \hat{a}(s_m, s')$ strictly prefer the lower standard s_m. From $a_m < \hat{a}(s_m, s')$, these subsets of agents together make up more than half of the electorate so that $\int_{\{a \in A | v(s_m, a) \geq v(s', a)\}} dF(a) > 1/2$ is proved. ■

Proof of Proposition 2

(i) For brevity, define $\beta := -c_{ea}(s_m, a_m) > 0$. With this, from Condition 1, one has

$$c_e(s_m, a) \leq c_e(s_m, a_m) + \beta(a_m - a) \tag{A.1}$$

for all $a \in A$ (see Figure 3), and, since $c_e(s_m, a_m) = 1$, it follows $c_e(s_m, a) \leq 1 + \beta(a_m - a)$ for all $a \in A$. Inserting into (4), one obtains

$$\begin{aligned} W'(s_m) &\geq \beta \int_{a_{\min}(s_m)}^{a_1} (a - a_m)\, dF(a) \qquad \text{(A.2)} \\ &= \beta\big[1 - F(a_{\min}(s_m))\big] \cdot \big[E\big(a | a > a_{\min}(s_m)\big) - a_m\big] \ , \end{aligned}$$

where $E\big(a|a > a_{\min}(s_m)\big) = \int_{a_{\min}(s_m)}^{a_1} a\, dF(a) / \big[1 - F(a_{\min}(s_m))\big]$ is the expected ability of graduates, which is well defined since under the median preferred standard, a positive mass of agents will graduate. Clearly, $E\big(a|a > a_{\min}(s_m)\big) \geq \overline{a}$, and hence (A.2) implies

$$W'(s_m) \geq \beta\big[1 - F(a_{\min}(s_m))\big] \cdot [\overline{a} - a_m] \; . \tag{A.3}$$

From this and Condition 2, it follows $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 1 is strict, then (A.1), and by consequence (A.2) hold with strict inequality. If the inequality in Condition 2 is strict, then the right-hand-side of (A.3) is strictly positive. If $a_{\min}(s_m) > a_o$, then $E\big(a|a > a_{\min}(s_m)\big) > \overline{a}$ so that in (A.2), the right-hand-side is strictly positive. In all three cases, it follows $W'(s_m) > 0$. ∎

Proof of Proposition 3

(i) Splitting the integral in (4) at the median and writing $a_m - x = a$ for $a < a_m$ and $a_m + x = a$ for $a > a_m$, one obtains

$$\begin{aligned} W'(s_m) = &- \int_{a_{\min}(s_m)}^{a_m} \big[c_e(s_m, a_m - x) - 1\big]\, dF(a_m - x) \\ &+ \int_{a_m}^{a_1} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m + x) \, . \end{aligned} \tag{A.4}$$

From Condition 4, it must hold $a_m - a_o \leq a_1 - a_m$ so that Condition 3 implies

$$c_e(s_m, a_m - x) - 1 \leq 1 - c_e(s_m, a_m + x) \tag{A.5}$$

for all $x \in (0, a_m - a_o]$. Using (A.5) in (A.4), one concludes

$$\begin{aligned} W'(s_m) \geq &- \int_{a_{\min}(s_m)}^{a_m} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m - x) \\ &+ \int_{a_m}^{a_1} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m + x) \, . \end{aligned} \tag{A.6}$$

Define the two distribution functions $G(x) := 1 - 2F(a_m - x)$ for $x \in [0, a_m - a_o]$ and $H(x) := 2F(a_m + x) - 1$ for $x \in [0, a_1 - a_m]$. ($G(x)$ resp. $H(x)$ is the probability that ability is at most a distance x away from the median, conditional on being below resp. above the median.) One has $dG(x) = -2dF(a_m - x)$ and $dH(x) = 2dF(a_m + x)$.

Using G and H in (A.6), adjusting the integration bounds appropriately and reversing the order of integration in the first integral, one arrives at

$$\begin{aligned} W'(s_m) \geq &-\frac{1}{2}\int_0^{a_m - a_{\min}(s_m)} \big[1 - c_e(s_m, a_m + x)\big]\, dG(x) \\ &+ \frac{1}{2}\int_0^{a_1 - a_m} \big[1 - c_e(s_m, a_m + x)\big]\, dH(x)\,. \end{aligned}$$

Since $a_{\min(s_m)} \geq a_o$ and $1 - c_e(s_m, a_m + x) > 0$ for $x > 0$, it follows furthermore

$$\begin{aligned} W'(s_m) \geq &-\frac{1}{2}\int_0^{a_m - a_o} \big[1 - c_e(s_m, a_m + x)\big]\, dG(x) \\ &+ \frac{1}{2}\int_0^{a_1 - a_m} \big[1 - c_e(s_m, a_m + x)\big]\, dH(x)\,. \end{aligned} \tag{A.7}$$

Now observe that the two integrals in (A.7) have the form of expected utilities, with $1 - c_e(s_m, a_m + x)$ as the utility function which strictly increases in the random variable x because of Assumption 1(iii). Moreover, Condition 4 implies that the distribution $H(x)$ first order stochastically dominates the distribution $G(x)$. Since a decision-maker with monotonic preferences will prefer the dominating to the dominated lottery (see Yildiz, 2010), the second integral must be at least as large as the first one. This implies $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 3 is strict, then (A.5) and hence (A.6) hold as strict inequalities. If the inequality in Condition 4 is strict, the dominance of the second over the first integral in (A.7) is strict, implying that the right-hand-side of this inequality is strictly positive. Finally, if $a_{\min}(s_m) > a_o$, extending the integration in (A.7) to the values $x \in (a_m - a_{\min}(s_m), a_m - a_o]$ adds a positive mass of strictly negative values, so that (A.7) holds as a strict inequality. In all three cases, it follows $W'(s_m) > 0$. ∎

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