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# Abuse of forward contracts to semi-collude in volatile markets

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**Abuse of forward contracts to semi-collude in volatile markets**

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**Abstract**

I model the optimal semi-collusive strategy of firms using forward contracts in volatile markets. It has been shown that forward contracts can be used to stabilize a collusive agreement under deterministic (Liski and Montero, 2006) as well as under stochastic market conditions (Aichele, 2012). However, forward trading has a negative effect on the expected profit for collusive firms, since firms have the obligation to fulfill their forward contracts in booms as well as in recessions. Thus, in recessions firms involuntarily sell more than the optimal collusive amount. This profit decreasing effect of forward trading is in contrast to the existing literature, since under certainty forward trading does not alter the collusive profit.

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Keywords: Industrial organization, Energy Markets, Collusion, Forward trading

# 1 Introduction

Commodity markets like that for gas, electricity and oil are characterized by several common market characteristics. Due to high entry costs and the economics of scale oligopolistic market structures often emerge. For most industries these commodities can be seen as homogeneous products. Especially these three market characteristics lead firms into temptation to narrow competition. One popular way to narrow competition is to (tacitly) collude on a price that is higher than in competitive settings. For this reason the markets for gas (e.g. the EU-Commission has opened proceedings against Gazprom on 04/09/2012) and electricity (see the study of Bundeskartellamt (2011) about the german electricity market) are frequently under supervision by antitrust authorities and by the scientific community. Fabra and Toro (2005), for instance, analyzed the Spanish electricity market and found empirical evidence for collusion. In order to analyze collusive behavior from a theoretical perspective, two additional market characteristics should be taken into account: The large forward traded amount and the volatility in demand and cost structure that all market participants are facing.
Liski and Montero (2006) were the first to work out the stabilizing effect of forward trading on a collusive agreement. Therefore, they modeled an infinitely repeated oligopoly game in which firms are allowed to act on the spot as well

as on the forward market. Using a deterministic linear demand function it is shown that forward trading facilitates to commit on a collusive price. Aichele (2012) shows that in volatile markets the stabilizing effect of forward trading on a collusive agreement still holds true.
Looking at the profits colluding firms can achieve, the fact whether market conditions are deterministic or not plays a crucial role. In the model of Liski and Montero (2006) the forward traded amount does not alter the profit firms can achieve, since they use deterministic market conditions. As will be shown in this article, under volatile market conditions the forward traded amount strictly decreases the expected profit of colluding firms. The economic reason is as follows: Firms do not know in advance whether a boom or a recession occurs. Thus, when firms decide about the amount being collusively traded forward, they do not know the optimal collusive amount. In recessions this leads to the problem, that firms involuntarily have traded forward more than the optimal collusive amount, which drops the price below the optimal collusive price. To show this effect I proceed as follows. Firstly, the main assumptions will be presented. Afterwards, the profit for a firm will be derived when it colludes with the other firm and when it decides to deviate from the collusive agreement. These profits will be used to decide whether a collusive agreement is self-enforcing or not, which is, since colluding firms can neither conclude nor enforce contracts, a necessary condition for a collusive agreement. Using this necessary condition the optimal collusive strategy of colluding firms is identified for different scenarios and the expected profit for colluding firms is determined. Afterwards it will be shown that for patient firms, which appreciate actual and future profits the same (firms with a discount factor close to one), the expected collusive per period profit is strictly decreasing in the forward traded amount.
Compared to the findings of Liski and Montero (2006) this are ambivalent news for antitrust authorities. At the one hand forward contracts can still be used by colluding firms to stabilize a collusive agreement. On the other hand when firms use forward contracts in volatile markets to stabilize a collusive agreement this results in lower prices.

# 2 The model

## 2.1 Main Assumptions

The exact outcome of prices, quantities and profits is stochastic and depends on the difference between the reservation price ($a$) and marginal costs ($c$). I do not distinguish between demand and supply shocks. The difference between the reservation price and marginal costs ($\gamma = a - c$) will be called "spread" in the analysis.
The spot and the forward market are connected as follows: In the first period, both firms choose simultaneously the amount of forward contracts they want to trade (forward market period). In the second period, contracts are settled and

firms choose the amount they want to sell additionally on the spot market (spot market period). The forward market opens in the even periods ($t = 0, 2, \dots$) and the spot market in the odd periods ($t = 1, 3, \dots$). For comparability with pure-spot market games the per period discount factor is given by $\sqrt{\delta}$. Alternatively the spot market opens a marginal unit of time right after the forward market and the discount factor is given by $\delta$. The important fact is that the only discounting is between two spot markets, two forward markets or the forward market in $t$ and the spot market in $t+1$. Hence, no discounting takes place between consecutive forward and spot markets. The structure of trading initially on the forward market and settling contracts afterwards as well as meeting residual demand on spot market is infinitely repeated. One can think of firms deciding around Christmas each year about forward contracts delivered in the following year.
Firms compete in prices. Whenever firm $i$ sets a price lower than competing firm $j$, firm $i$ meets the whole spot market demand. When prices are equal they equally split the market. The demand that can be achieved on the spot market for a firm deviating is restricted by already sold future contracts. Each firm has a already secured supply of $f_i$. The secured supply of both firms firms is given by F ($f_i + f_j = F$). This secured supply decreases the reservation price and consequently the accessible demand. This gives (residual) demand function on the spot market as:

$$D_i^R = \begin{cases} (a - F - p_i) & \text{if} \quad p_i < p_j, \\ \frac{1}{2}(a - F - p_i) & \text{if} \quad p_i = p_j, \\ 0 & \text{if} \quad p_i > p_j \end{cases} \tag{1}$$

It is assumed, that speculators do not store the amount they purchased from forward contracts. Thus, the total forward traded amount is always sold on the spot market. This seems to be, especially for the electricity market a valid assumption.
Consider the following trigger strategy: In the first forward market round (period 0), firm i sells $f_i^{0,1}$ and $f_i^{0,l} = 0$ for all $l > 1$. Hence, firms only sell forward contracts that will be settled in the following spot market. In this following spot market period firm $i$ sets the collusive price ($p_i^t = p^{sc}$) if and only if in every period preceding t both firms have set the collusive prices in the spot market and have contracted in the forward market the collusive amount $f_i^{0,1} = f_j^{0,1} = f$ one period ahead. Whenever firm $j$ deviates from this agreement, firm $i$ sets price at marginal cost in the spot market and sells any arbitrary amount of forward contracts forever ("Friedman trigger").
Liski and Montero (2006) do not allow in their model of forward trading and collusion in a deterministic market structure forward contracts to exceed monopoly quantity. However, in a volatile market, firms do not know in any forward market period the demand and the cost structure they will face in the following spot market period. Hence, firms might have traded forward more than the quantity they can sell on spot market with their collusive price. This may happen e.g. for a relatively small realization of the difference of reservation price and marginal costs.

In general two possibilities of deviation could be possible. Firstly, to set the price lower than the collusive price in the spot market. Secondly, to increase forward sales in the forward market. The latter is never profitable since speculators, which are taking the counterpart, immediately realize any deviation from collusion in the forward market and are not willing to pay any higher price than the next period's stock market price, which is given by marginal costs. Hence, profitable deviation is restricted to the spot market and a firm trying to deviate knows the actual state of the economy.
Collusive behavior of firms can occur if and only if there is no incentive for any firm to deviate from the collusive agreement unilaterally. There exists no incentive for any firm to break the collusive agreement unilaterally if the net present value of profits gained by collusion is greater than or equal to the net present value of profits gained by ending collusion. The highest profit that can be earned by colluding firms is in every period given by the monopoly profit. The collusively earned profit is shared equally by both firms. In order to analyze the effects of forward trading on the stability of a collusive agreement, the net present value of profits for a deviating and a colluding firm, have to be compared.

## 2.2 Net present value of deviation

As mentioned before, the possibility of a profitable deviation is restricted to the spot market. Thus, a firm deviating from collusion maximizes its profit over its (deviation) price. This leads to following optimal deviation price ($p^d$), quantity ($q^d$) and profit ($\Pi^d$):

$$\begin{aligned} &\max_{p} \quad \Pi_i = (p_i - c)(a - F - p_i) \\ &p^d = \frac{1}{2}[a + c - F], q^d = \frac{1}{2}(a - F - c), \Pi^d = \frac{1}{4}[a - c - F]^2 \end{aligned} \tag{2}$$

A deviating firm sets the residual monopoly price of $p^d = p^{rm} = \frac{1}{2}[a + c - F]$ and earns the residual monopoly profit of $\Pi^d = \Pi^{rm} = \frac{1}{4}[a - c - F]^2$. When firms are setting a semi-collusive price that is less than the optimal deviation price derived in equation 2, a firm can obviously not set this price in order to deviate from the collusive agreement. In these cases deviation occurs by undercutting the semi-collusive price infinitesimally (see Green and Coq (2010) for more details). This gives deviation prices and profits as:

$$\begin{aligned} &p^d = min\left\{\frac{1}{2}[a - c - F] + c, p^{sc} - \epsilon\right\} \\ &\Pi^d = \begin{cases} \frac{1}{4}[a - c - F]^2 & \text{if} \quad p^d = \frac{1}{2}[a - c - F] + c \\ (a - p^{sc})(p^{sc} - c) - F(p^{sc} - c) & \text{if} \quad p^d = p^{sc} - \epsilon \end{cases} \end{aligned} \tag{3}$$

The profit for a deviating firm only is given by the actual profit, since after a deviation no profits can be earned ("‘grim trigger strategy"’). Thus, equation 3 gives the net present value of profits for a deviating firm.

## 2.3 Net present value of collusion

When a firm considers to deviate from the collusive agreement, it has to take the loss of actual and upcoming collusive profits into account.

**The collusive quantity exceeds the forward traded amount**

When the collusive quantity exceeds the total forward traded amount ($F < q^{sc}$), the demand that can be reached by collusive behavior in this period is restricted by already sold forward contracts. When firms set a collusive price they split residual demand, that is given by $D^R = a - F - p^{sc}$ and earn a per-unit-profit of $\pi^{sc} = p^{sc} - c$. Each firms' collusive profit on spot market can be stated as:

$$\Pi^{sc} = \frac{1}{2} D^R \pi^{sc} = \frac{1}{2}\left(a - F - p^{sc}\right)\left(p^{sc} - c\right) \tag{4}$$

Colluding firms can earn additional profits in further periods. Thus, in general the net present value of collusion is given by:

$$NPV(\Pi_i^{sc}) = \frac{1}{2}\left(a - F - p^{sc}\right)\left(p^{sc} - c\right) + \frac{\delta}{1-\delta} E\left[\Pi_i\right] \tag{5}$$

**The forward traded amount exceeds the collusive quantity**

When the total forward traded amount exceeds the collusive quantity ($q^{sc} < F$), no collusive profits can be earned in this period, since the total demand for the collusive price is already satisfied.

$$NPV(\Pi_i^{sc}) = 0 \tag{6}$$

However, not deviating from collusion promises the expected collusive profit in all upcoming periods.

$$\Pi_{NPV}^{sc} = \frac{\delta}{1-\delta} E\left[\Pi_i\right] \tag{7}$$

## 2.4 Different scenarios

As shown in section 2.2, the profit for a deviating firm depends on the price that is set by colluding firms. The first case occurs, whenever it is optimal to deviate from the collusive agreement by setting the residual monopoly price ($p^d = p^{rm} = \frac{1}{2}\left(a - F - c\right)$). The second case occurs, whenever it is optimal to deviate from the collusive agreement by undercutting the collusive price infinitesimally ($p^d = p^{sc} - \epsilon$).

As has been shown in section 2.3, the profit for a colluding firm depends on the amount, that has been traded forward. Again two different cases have to be distinguished: The first case, when both firms' forward traded amount does not exceed the collusive quantity. The second case, when both firms' forward traded amount exceeds the collusive quantity.
Combining the two cases, that determine the deviation profit, with the two cases that determine the collusive profit, different scenarios can be identified. As it is shown in equation A.1 - A.7 in the Appendix, scenario III occurs for relatively small realizations of the random variable, scenario I occurs for moderate realizations of the random variable and scenario IIa and scenario IIb occur for relatively large realizations of the random variable. Scenario IV never occurs, since it would imply a collusive price below marginal costs.
The different scenarios that may occur depending on the realization of the random variable and the forward contracted amount are summarized in Table 1.

| | $F < q^{sc}$ | $F > q^{sc}$ |
|---|---|---|
| $p^d = \frac{1}{2}(a - c - F) + c$ | Scenario I and Scenario IIa | Scenario III |
| $p^d = p^{sc} - \epsilon$ | Scenario IIb | Scenario IV |

Table 1: Scenarios for colluding firms in volatile markets

**Scenario I:**
**Firms set the monopoly price and the monopoly quantity exceeds the forward traded amount**

When colluding firms set the monopoly price, the net present value of collusive behavior is given by:

$$\begin{aligned} NPV(\Pi_i^{sc}) &= \frac{1}{2}\left(a - F - p^m\right)\left(p^m - c\right) + \frac{\delta}{1-\delta}E\left[\Pi_i\right] \\ &= \frac{1}{8}\left(a - c - F\right)^2 - \frac{1}{8}F^2 + \frac{\delta}{1-\delta}E\left[\Pi_i\right] \\ &= \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F + \frac{\delta}{1-\delta}E\left[\Pi_i\right] \end{aligned} \tag{8}$$

**Scenario II:**
**Firms set a semi-collusive price and the collusive quantity exceeds the forward traded amount**

When colluding firms stabilize the collusive agreement by setting a price that is below monopoly price ($p^{sc} < p^m$), the net present value of collusion is given by:

$$NPV(\Pi_i^{sc}) = \frac{1}{2}(a - F - p^{sc})(p^{sc} - c) + \frac{\delta}{1-\delta}E[\Pi_i] \tag{9}$$

**Scenario III:**
**Firms set a collusive price and the forward traded amount exceeds the collusive quantity**

When the total forward traded amount exceeds the collusive quantity ($q^{sc} < F$), collusive behavior does not lead to collusive spot-market profits of firms, since the total demand for the collusive price is already satisfied. However, not deviating from collusion promises the expected collusive profit in all upcoming periods.

$$\Pi_{NPV}^{sc} = \frac{\delta}{1-\delta}E[\Pi_i] \tag{10}$$

**Scenario IV:**
**Firms set a collusive price below the monopoly price and the forward traded amount exceeds the collusive quantity**

As shown in equation A.7 in the Appendix it is necessary for scenario IV that the firms set a collusive price below marginal costs. This scenario can never be relevant, since this would imply negative profits. The Nash-equilibrium of Bertrand competition, which is always sustainable, would be preferred by firms, since it implies non negative profits.

## 2.5 Collusion vs. Deviation

A collusive agreement is stable if and only if for each firm the net present value of acting collusively is larger than the net present value of a deviation. This leads to following necessary inequality for stable collusion, which will be called the no deviation constrained.

$$NPV(Collusion) > NPV(Deviation) \tag{11}$$

Whether the no deviation constrained is fulfilled or not, depends on the forward traded amount (F), the realization of the random variable ($\gamma$) and firms' discount factor ($\delta$). Looking at the collusive profits as well as on the deviation profits, the stabilizing effect of the forward traded amount on a collusive agreement can easily be seen. When firms set the monopoly price collusively, the deviation profit is given by equation 3 and the collusive profit is given by 8. Taking the first order derivative of the profits respect to the forward traded amount yields:

$$\begin{aligned} \frac{\partial \Pi_i^d}{\partial F} &= -\frac{1}{2}\left(a - c - F\right) \\ \frac{\partial \Pi_i^{sc}}{\partial F} &= -\frac{1}{4}\left(a - c\right) \end{aligned} \tag{12}$$

As long as the forward traded amount does not exceed the monopoly quantity ($F < \frac{1}{2}(a-c)$), the forward traded amount decreases the deviation profit more sharply than the collusive profit, since in absolute values the partial derivative of the deviation profit exceeds the partial derivative of the collusive profit ( $|\frac{\partial \Pi_i^d}{\partial F}| > |\frac{\partial \Pi_i^{sc}}{\partial F}| \quad \forall \quad F < \frac{1}{2}(a-c)$ ) .
When the forward traded amount exceeds the monopoly quantity ($F > \frac{1}{2}(a-c)$), no collusive spot market profits are earned. Hence, just the deviation profit is reduced.
When firms set a price between the residual monopoly price and the monopoly price ($\frac{1}{2}(a-c-F)+c < p^{sc} < \frac{1}{2}(a-c)+c$), the first order derivatives of the deviation profit and the collusive profit respect to the forward traded amount look as follows:

$$\begin{aligned} \frac{\partial \Pi_i^d}{\partial F} &= -\frac{1}{2}\left(a - c - F\right) \\ \frac{\partial \Pi_i^{sc}}{\partial F} &= -\frac{1}{2}(p^{sc} - c) \end{aligned} \tag{13}$$

For all collusive prices that lead to a positive spot market demand ($a - F - p^{sc} > 0$), the forward traded amount decreases the deviation profit more sharply than the collusive profit, since in absolute values the partial derivative of the deviation profit exceeds the partial derivative of the collusive profit ( $|\frac{\partial \Pi_i^d}{\partial F}| > |\frac{\partial \Pi_i^{sc}}{\partial F}| \quad \forall \quad p^{sc} < a - F$ ).
When firms set a price below the residual monopoly price ($p^{sc} < \frac{1}{2}(a-c-F)+c$), the first order derivatives of the deviation profit and the collusive profit respect to the forward traded amount look as follows:

$$\begin{aligned} \frac{\partial \Pi_i^d}{\partial F} &= -\left(p^{sc} - c\right) \\ \frac{\partial \Pi_i^{sc}}{\partial F} &= -\frac{1}{2}\left(p^{sc} - c\right) \end{aligned} \tag{14}$$

For all collusive prices exceeding marginal cost, forward trading decreases the deviation profit more sharply, since in absolute values the partial derivative of the deviation profit exceeds the partial derivative of the collusive profit ( $|\frac{\partial \Pi_i^d}{\partial F}| > |\frac{\partial \Pi_i^{sc}}{\partial F}| \quad \forall \quad p^{sc} > c$ ) .
Thus, in all cases the forward traded amount decreases the deviation profit more than the collusive profit, which makes it more advantageous to collude.

**Scenario I:**
**Firms set the monopoly price and the monopoly quantity exceeds the forward traded amount**

When colluding firms set the monopoly price, a deviating firm sets the residual monopoly price of $p^d = p^{rm} = \frac{1}{2}(\gamma - F)$ and earns the residual monopoly profit of $\Pi_i^d = \Pi^{rm} = \frac{1}{4}(\gamma - F)^2$. Whenever the monopoly quantity exceeds the forward traded amount$(2F < \gamma)$, colluding firms earn actual as well as future collusive profits of $\Pi_i^{sc} = \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F + \frac{\delta}{1-\delta}E[\Pi_i]$. Thus, the trade-off between deviation and collusion can be stated as:

$$\begin{aligned} \text{NPV(Deviation)} &\leq \text{NPV(Collusion)} \\ \frac{1}{4}(\gamma - F)^2 &\leq \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F + \frac{\delta}{1-\delta}E[\Pi_i] \end{aligned} \tag{15}$$

The upper bound for sustainable collusion in scenario $I$ is found by solving the no deviation constraint for the maximal realization of the random variable $(\gamma)$.

$$\begin{aligned} 0 &\leq -\frac{1}{2}\gamma^2 + \gamma F + 4\frac{\delta}{1-\delta}E[\Pi_i] - F^2 \\ \gamma^* &= F + \sqrt{8\frac{\delta}{1-\delta}E[\Pi_i] - F^2} \end{aligned} \tag{16}$$

Hence, whenever the spread lies above twice the forward traded amount and below the critical spread derived in equation 16 $(2F < \gamma < \gamma^*)$ firms are able to set the monopoly price of $p^m = \frac{1}{2}\gamma + c$ and earn the corresponding spot market profit of $\Pi_i^{sm} = \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F$.
The expected profit on the forward market is given by each firms forward traded amount multiplied by the difference of the forward price and the marginal costs. As mentioned before, the forward market price is given by the expected spot market price, since speculators build rational expectations. Thus, the expected profit on the forward market is given by the expected difference of the spot market price and marginal costs times each firms forward traded amount $(E[\Pi_i^{fm}] = \frac{1}{2}F\left(p^{fm} - c\right) = \frac{1}{2}FE[p - c])$. The spread is distributed according to the distribution $\widehat{F}(\gamma)$ with the density $\widehat{f}(\gamma)$.
This gives following contribution of the corresponding spot and forward market profit in scenario $I$ to the expected collusive profit:

$$\begin{aligned} E[\Pi_i \mid 2F < \gamma < \gamma^*] &= E[\Pi_i^{sm} \mid 2F < \gamma < \gamma^*] + E\left[\Pi_i^{fm} \mid 2F < \gamma < \gamma^*\right] \\ &= E\left[\frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F + \frac{1}{2}F(p^m - c) \mid 2F < \gamma < \gamma^*\right] \\ &= \int_{2F}^{\gamma^*}\left(\frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F\right)\widehat{f}(\gamma)d\gamma + \frac{1}{2}F\int_{2F}^{\gamma^*}\frac{1}{2}\gamma\widehat{f}(\gamma)d\gamma \\ &= \int_{2F}^{\gamma^*}\frac{1}{8}\gamma^2\widehat{f}(\gamma)d\gamma \end{aligned} \tag{17}$$

The increase of the expected forward market profit from forward trading is totally offset by a decrease of the expected spot market profit. Thus, as long as firms are able to collude at the monopoly price the expected collusive profit equals half the expected monopoly profit, irrespective of the forward traded amount.
The most important results for scenario $I$ are summarized in Table 2.

| Scenario $I$ | $2F \leq \gamma < \gamma^*$ |
|---|---|
| Spot market price | $p^m = \frac{1}{2}\gamma + c$ |
| Deviation price | $p^d = \frac{1}{2}(\gamma - F) + c$ |
| Deviation profit | $\Pi^d = \frac{1}{4}(\gamma - F)^2$ |
| Collusive spot market profit | $\Pi^{sc} = \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F$ |
| Contribution to the total expected profit | $E[\Pi_i] = \int_{2F}^{\gamma^*} \gamma^2 \widehat{f}(\gamma) d\gamma$ |
| No deviation constraint | $\frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F + \frac{1}{4}F^2 \leq \frac{\delta}{1-\delta} E[\Pi_i]$ |

Table 2: Prices and profits for scenario $I$

**Scenario II:**
**Firms set a price between marginal costs and the monopoly price and the collusive quantity exceeds the forward traded amount**

When collusion cannot be sustained at the monopoly price, firms can theoretically set any price between marginal costs and the monopoly price to sustain collusion. As long as the collusive agreement is stable for prices above the residual monopoly price, firms face two possibilities to stabilize their collusive agreement.
On the one hand, to set a price between the monopoly price and the residual monopoly price. On the other hand, to set the residual monopoly price. The effect of a certain collusive price on the collusive spot market profit is given by the first order derivative of the collusive spot market profit (equation 4) respect to the collusive price:

$$\frac{\partial \Pi_i^{sc}}{\partial p^{sc}} = \frac{1}{2}(a - 2p^{sc} + c) - \frac{1}{2}F \qquad (18)$$

Hence, both possibilities increase the actual semi-collusive spot market profit, since for any price above the residual monopoly price the first order derivative of the collusive profit is positive ($\frac{\partial \Pi_i^{sc}}{\partial p^{sc}} > 0 \quad \forall\, p > \frac{1}{2}(a - F - c) + c$). It follows, that the highest possible actual spot market profit is earned by setting the residual monopoly price of $p^{rm} = \frac{1}{2}(a - c - F) + c$. For any collusive price between the residual monopoly price and the monopoly price the deviation profit is the same, since the optimal price for a deviating firm is always given by the residual monopoly price.
When looking at the total expected profit, another effect should be taken into

account. The effect on the expected prices and consequently on the expected forward market profits. Setting the residual monopoly price leads to an higher spot market profit. However, this additional profit on the spot market is achieved by lowering prices. This decreases the expected price and consequently the expected profit from forward trading as well. Setting a price above the residual monopoly price leads to a comparably lower spot market profit. However, it leads to a comparably higher spot market price and consequently to a comparably higher profit from forward trading.
Comparing both possibilities' effect on the total expected profit (the sum of expected spot market and forward market profits), leads to the conclusion, that firms profit more from sustaining a higher (expected) price than from earning additional spot market profits. Thus, firms prefer to set a price between the monopoly price and the residual monopoly price. For the detailed derivation, see equation A.11, A.12 and A.13 in the Appendix.
When firms set a price between the residual monopoly price and the monopoly price, firms set the price such that the no deviation constrained is just fulfilled. Thus, the semi-collusive price is found by solving the no deviation constrained for the price. As mentioned before, the deviation profit is unaffected by the collusive price

$$
\begin{aligned}
&\Pi_i^{sc} + \frac{\delta}{1-\delta} E\left[\Pi_i\right] = \Pi_i^d \\
&\frac{1}{2}(a-F-p)(p-c) = \frac{1}{4}(a-c-F)^2 - \frac{\delta}{1-\delta} E\left[\Pi_i\right] \\
&0 = -\frac{1}{2}p^2 + p\left(\frac{1}{2}(a-F-c)+c\right) - \frac{1}{2}c\,(a-F) + \frac{\delta}{1-\delta} E\left[\Pi_i\right] - \frac{1}{4}(a-c-F)^2 \\
&p^{sc} = \frac{1}{2}(a-c-F) + c + \sqrt{2\frac{\delta}{1-\delta} E\left[\Pi_i\right] - \frac{1}{4}(a-c-F)^2}
\end{aligned}
\tag{19}
$$

Thus, when firms cannot sustain collusion using the monopoly price, firms set a price between the residual monopoly price and the monopoly price of $p^{sc} = \frac{1}{2}(\gamma - F) + c + \sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}$ to sustain collusion.
A deviating firm undercuts the collusive price by setting the residual monopoly price of $p^d = \frac{1}{2}(\gamma - F)$ and earns the residual monopoly profit of $\Pi_i^d = \frac{1}{4}(\gamma - F)^2$.
Of course, the surcharge on the residual monopoly price has to be a positive integer value. Thus, scenario $II$a ends, when the surcharge is equal to zero and firms set the residual monopoly price to sustain collusion. As easily can be seen, this condition is equivalent to the no deviation constraint, when the deviation profit is given by the residual monopoly profit and the collusive profit is given by half the residual monopoly profit. The upper bound for sustainable collusion

in scenario $II$a is given by:

$$\begin{aligned}\frac{\delta}{1-\delta}E\left[\Pi_i\right] &= \frac{1}{8}(\gamma-F)^2\\ \gamma^{**} &= F+\sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right]}\end{aligned} \tag{20}$$

Colluding firms earn a actual collusive profit of exactly the difference between the profit of deviation and the expected upcoming profits:

$$\begin{aligned}\Pi_i^{sc} &= \Pi_i^d - \frac{\delta}{1-\delta}E\left[\Pi_i\right]\\ &= \frac{1}{4}(\gamma-F)^2 - \frac{\delta}{1-\delta}E\left[\Pi_i\right]\end{aligned} \tag{21}$$

This leads to a contribution of scenario $II$a to the expected total collusive profit of:

$$\begin{aligned}&E\left[\Pi_i \mid \gamma^* < \gamma < \gamma^{**}\right] = E\left[\Pi_i^{fm} \mid \gamma^* < \gamma < \gamma^{**}\right] + E\left[\Pi_i^{sm} \mid \gamma^* < \gamma < \gamma^{**}\right]\\
&= \frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}}\left(p^{sc}-c\right)\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\gamma^{**}}\Pi_i^{sc}\widehat{f}(\gamma)d\gamma\\
&= \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{4}F(\gamma-F)+\frac{1}{2}F\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{4}(\gamma-F)^2}\right)\widehat{f}(\gamma)d\gamma\\
&\quad + \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{4}(\gamma-F)^2-\frac{\delta}{1-\delta}E\left[\Pi_i\right]\right)\widehat{f}(\gamma)d\gamma\\
&= \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{4}\gamma(\gamma-F)+\frac{1}{2}F\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{4}(\gamma-F)^2}-\frac{\delta}{1-\delta}E\left[\Pi_i\right]\right)\widehat{f}(\gamma)d\gamma\end{aligned} \tag{22}$$

This contribution to the total expected collusive profit is below the profit contribution, when firms would set the monopoly price. A detailed derivation is given in equation A.8, A.9 and A.10 in the Appendix. The most important results for scenario $II$a are summarized in Table 3.

When the no deviation constrained cannot be fulfilled by setting a price between the residual monopoly price and the monopoly price, firms can sustain collusion by setting a price between marginal costs and the residual monopoly price. Then a deviating firm is not able to set the residual monopoly price, since it is above the semi-collusive price. Therefore, a deviating firms undercuts the semi-collusive price infinitesimally and earns twice the semi-collusive profit instead of the single semi-collusive profit. This is the same mechanism as semi-collusion in cases where no forward trading occurs. Thus, the optimal semi-collusive price is found by solving the following no deviation constraint for

| Scenario $II$a | $\gamma^* \leq \gamma < \gamma^{**}$ |
|---|---|
| Spot market price | $p^{sc} = \frac{1}{2}(\gamma - F) + c + \sqrt{\frac{2\delta}{1-\delta} E[\Pi_i] - \frac{1}{4}(\gamma - F)^2}$ |
| Deviation price | $p^d = \frac{1}{2}(\gamma - F) + c + \sqrt{\frac{2\delta}{1-\delta} E[\Pi_i] - \frac{1}{4}(\gamma - F)^2} - \epsilon$ |
| Deviation profit | $\Pi^d = \frac{1}{4}(\gamma - F)^2$ |
| Coll. spot mark. prof. | $\Pi_i^{sc} = \frac{1}{4}(\gamma - F)^2 - \frac{\delta}{1-\delta} E[\Pi_i]$ |
| Contr. tot. exp. prof. | $E[\Pi_i] = \int_{\gamma^*}^{\gamma^{**}} \Big( \frac{1}{4}\gamma(\gamma - F) - \frac{\delta}{1-\delta} E[\Pi_i]$ $+ \frac{1}{2} F \sqrt{2\frac{\delta}{1-\delta} E[\Pi_i] - \frac{1}{4}(\gamma - F)^2} \Big) \widehat{f}(\gamma) d\gamma$ |
| No deviation constraint | $\Pi_i^{sc} + \frac{\delta}{1-\delta} E[\Pi_i] = \Pi_i^d$ |

Table 3: Prices and profits for scenario $II$a $(\gamma^* < \gamma < \gamma^{**})$

the price:

$$\begin{aligned}
\Pi_i^d &= \Pi_i^{sc} + \frac{\delta}{1-\delta} E[\Pi_i] \\
(a - F - p^{sc})(p^{sc} - c) &= \frac{1}{2}(a - F - p^{sc})(p^{sc} - c) + \frac{\delta}{1-\delta} E[\Pi_i] \\
0 &= -p^2 + p[a - F - c + 2c] - c(a - F) - 2\frac{\delta}{1-\delta} E[\Pi_i] \\
p &= \frac{1}{2}(a - F - c) + c - \sqrt{\frac{1}{4}(a - F - c)^2 - 2\frac{\delta}{1-\delta} E[\Pi_i]}
\end{aligned} \tag{23}$$

Of course, inserting this price into the no deviation constraint leads for each firm to a collusive profit and a deviation profit of:

$$\Pi_i^{sc} = \frac{\delta}{1-\delta} E[\Pi_i] \qquad\qquad \Pi_i^d = 2\frac{\delta}{1-\delta} E[\Pi_i] \tag{24}$$

The collusive forward market profit is given by each firms' forward traded amount multiplied by the expected difference of the price and marginal costs. The collusive forward market profit and the collusive spot market profit lead to following contribution of scenario $II$b to the expected total profit:

$$\begin{aligned}
&E[\Pi_i \mid \gamma^{**} < \gamma < \infty] = E\left[\Pi_i^{fm} \mid \gamma^{**} < \gamma < \infty\right] + E[\Pi_i^{sm} \mid \gamma^{**} < \gamma < \infty] \\
&= \frac{1}{2} F \int_{\gamma^{**}}^{\infty} (p^{sc} - c) \widehat{f}(\gamma) d\gamma + \int_{\gamma^{**}}^{\infty} \Pi_i^{sc} \widehat{f}(\gamma) d\gamma \\
&= \int_{\gamma^{**}}^{\infty} \left( \frac{1}{4} F(\gamma - F) - \frac{1}{2} F \sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta} E[\Pi_i]} + \frac{\delta}{1-\delta} E[\Pi_i] \right) \widehat{f}(\gamma) d\gamma
\end{aligned} \tag{25}$$

The most important results for scenario $II$b are summarized in Table 4.

| Scenario $II$b | $\gamma^{**} \leq \gamma < \infty$ |
|---|---|
| Spot market price | $p^{sc} = \frac{1}{2}(\gamma - F) + c - \sqrt{\frac{1}{4}(\gamma - F)^2 - \frac{2\delta}{1-\delta} E[\Pi_i]}$ |
| Deviation price | $p^d = \frac{1}{2}(\gamma - F) + c - \sqrt{\frac{1}{4}(\gamma - F)^2 - \frac{2\delta}{1-\delta} E[\Pi_i]} - \epsilon$ |
| Deviation profit | $\Pi^d = 2\frac{\delta}{1-\delta} E[\Pi_i] - \epsilon$ |
| Coll. spot mark. prof. | $\Pi_i^{sc} = \frac{\delta}{1-\delta} E[\Pi_i]$ |
| Contr. tot. exp. prof. | $E[\Pi_i] = \int_{\gamma^*}^{\gamma^{**}} \Big( \frac{1}{4} F(\gamma - F) + \frac{\delta}{1-\delta} E[\Pi_i]$ |
| | $- \frac{1}{2} F \sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta} E[\Pi_i]} \Big) \widehat{f}(\gamma) d\gamma$ |
| No deviation constraint | $\Pi_i^{sc} + \frac{\delta}{1-\delta} E[\Pi_i] = \Pi_i^d$ |

Table 4: Prices and profits for scenario $II$b $(\gamma^{**} < \gamma < \infty)$

**Scenario III:**
**Firms set a collusive price for which the forward traded amount exceeds the collusive quantity**

When the forward traded amount exceeds the collusive quantity no collusive profits are earned in the corresponding period. The profit of a deviation is given by the residual monopoly profit, irrespective of the exact collusive price. This gives the no deviation constrained as:

$$\frac{1}{4}(\gamma - F)^2 \leq \frac{\delta}{1-\delta} E[\Pi_i] \tag{26}$$

The price on the spot market is solely determined by speculators' behavior, since colluding firms sell additional to their forward obligation no quantities on the spot market. Speculators will bring the total forward traded amount to the market, since by assumption they do not store the commodity. Hence, the price on the spot market is given by $p^{sm} = a - F$, which is below the monopoly price and above the residual monopoly price ($p^{rm} = \frac{1}{2}(a - c - F) + c < p^{sm} = a - F < p^m = \frac{1}{2}(a - c)$). Colluding firms do not earn any profits on the spot market. However, they earn a profit from the amount, that they have traded forward. Thus, the contribution of scenario $III$ to the expected total profit is given by the profit from forward trading:

$$\begin{aligned} E[\Pi_i \mid 0 < \gamma < 2F] &= E[\Pi_i^{sm} \mid 0 < \gamma < 2F] + E\left[\Pi_i^{fm} \mid 0 < \gamma < 2F\right] \\ &= 0 + \frac{1}{2} F \int_0^{2F} (\gamma - F) \widehat{f}(\gamma) d\gamma \end{aligned} \tag{27}$$

This contribution to the expected total profit is smaller than the contribution to the expected profit in scenario I. For a detailed derivation, see equation A.21 in the Appendix.
When the no deviation constrained for scenario $III$ (equation 26) is not fulfilled,

which implies a very low discount factor, firms cannot sustain collusion at the given price of $p^m = a - F$. Then firms have to set the residual monopoly price of $p^{rm} = \frac{1}{2}(a - F - c) + c$, which is below the market price in scenario $III$, to sustain collusion. This leads directly to the collusive prices and profits of scenario $II$.
The most important results for scenario $III$ are summarized in Table 5.

| Scenario $III$ | $\gamma < 2F$ |
|---|---|
| Spot market price | $p^{sm} = a - F$ |
| Deviation price | $p^d = \frac{1}{2}(\gamma - F) + c$ |
| Deviation profit | $\Pi^d = \frac{1}{4}(\gamma - F)^2$ |
| Collusive spot market profit | $\Pi^{sc} = 0$ |
| Contribution to the total expected profit | $E[\Pi_i] = \frac{1}{2}F\int_0^{2F}(\gamma - F)\widehat{f}(\gamma)d\gamma$ |
| No deviation constraint | $\frac{1}{4}(\gamma - F)^2 \leq \frac{\delta}{1-\delta}E[\Pi_i]$ |

Table 5: Prices and profits for scenario $III$ and for scenario $I$

## 2.6 The expected collusive profit

**The expected collusive profit on the spot market**

The expected profit on the spot market is found by summing up the expected collusive spot market profits of all scenarios.

$$E[\Pi_i^{sm}] = \int_{2F}^{\gamma^*}\left(\frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F\right)\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{4}(\gamma - F)^2 - \frac{\delta}{1-\delta}E[\Pi_i]\right)\widehat{f}(\gamma)d\gamma + \frac{\delta}{1-\delta}E[\Pi_i]\int_{\gamma^{**}}^{\infty}\widehat{f}(\gamma)d\gamma \tag{28}$$

**The expected collusive profit on the forward market**

The expected difference of the spot market price and marginal costs is found by summing up the expected difference of the spot market price and marginal costs for each scenario. The expected collusive profit on the forward market is given by:
See equation A.15 in the Appendix for a detailed derivation.

$$E[\Pi_i^{fm}] = \frac{1}{2}F\left[\int_0^{2F}(\gamma - F)\widehat{f}(\gamma)d\gamma + \int_{2F}^{\gamma^*}\frac{1}{2}\gamma\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\infty}\frac{1}{2}(\gamma - F)\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\gamma^{**}}\sqrt{2\frac{\delta}{1-\delta}E[\Pi_i] - \frac{1}{4}(\gamma - F)^2}\widehat{f}(\gamma)d\gamma - \int_{\gamma^{**}}^{\infty}\sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta}E[\Pi_i]}\widehat{f}(\gamma)d\gamma\right] \tag{29}$$

**The total expected collusive profit**

The total expected collusive profit is found by either summing up the contributions to the expected profit from all scenarios or by summing up the expected collusive profit on the spot market and the expected collusive profit on the forward market.
See equation A.16 in the Appendix for a detailed derivation

$$\begin{aligned}
E\left[\Pi_i\right] = & \frac{1-\delta}{1-\delta\left(2-2\widehat{F}(\gamma^{**})+\widehat{F}(\gamma^{*})\right)}\Bigg[-\frac{1}{4}F^2\left(1+2\widehat{F}(2F)-\widehat{F}(\gamma^{**})\right) \\
& +\frac{1}{2}F\int_0^{2F}\gamma\widehat{f}(\gamma)d\gamma+\frac{1}{8}\int_{2F}^{\gamma^*}\gamma^2\widehat{f}(\gamma)d\gamma+\frac{1}{4}\int_{\gamma^*}^{\gamma^{**}}\gamma^2\widehat{f}(\gamma)d\gamma-\frac{1}{4}F\int_{\gamma^*}^{\gamma^{**}}\gamma\widehat{f}(\gamma)d\gamma \\
& +\frac{1}{4}F\int_{\gamma^{**}}^{\infty}\gamma\widehat{f}(\gamma)d\gamma+\frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}}\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{4}(\gamma-F)^2}\widehat{f}(\gamma)d\gamma \\
& +\frac{1}{2}F\int_{\gamma^{**}}^{\infty}\sqrt{\frac{1}{4}(\gamma-F)^2-2\frac{\delta}{1-\delta}E\left[\Pi_i\right]}\widehat{f}(\gamma)d\gamma\Bigg]
\end{aligned} \tag{30}$$

In order to illustrate the profit decreasing effect of excessive forward trading, I calculate the total expected profit for very patient firms. Afterwards the exponential distribution ($\widehat{F}(\gamma)=1-e^{-\lambda\gamma}$, $\widehat{f}(\gamma)=\lambda e^{-\lambda\gamma}$) is used to specify this total expected profit.
When firms make no distinction whether profits are earned in the actual or in further periods, technically spoken when firms have a discount factor close to one, the semi-collusive scenario IIa and IIb are not relevant and the total expected collusive profit becomes:
($\gamma^* \to \gamma^{**} \to \infty,\ \widehat{F}(\gamma^*) \to \widehat{F}(\gamma^{**}) \to 1$)

$$E\left[\Pi_i\right] = -\frac{1}{2}F^2\widehat{F}\left(2F\right)+\frac{1}{2}F\int_0^{2F}\gamma\widehat{f}(\gamma)d\gamma+\frac{1}{8}\int_{2F}^{\infty}\gamma^2\widehat{f}(\gamma)d\gamma \tag{31}$$

Using the exponential distribution to specify the total expected profit, the profit looks as follows:
(See equation A.17, A.18 and A.19 in the Appendix for a detailed derivation)

$$E\left[\Pi_i\right] = \frac{1}{2}\frac{F}{\lambda}-\frac{1}{2}F^2+\frac{1}{4}\frac{1}{\lambda^2}e^{-2\lambda F} \tag{32}$$

The effect of forward trading on the expected collusive profit can easily be found by taking the first and second order derivatives respect to the forward traded amount

$$\begin{aligned}
\frac{\partial E\left[\Pi_i\right]}{\partial F} &= \frac{1}{2}\frac{1}{\lambda}\left[1-e^{-2\lambda F}\right]-F \quad < 0 \quad \forall\ F>0 \\
\frac{\partial^2 E\left[\Pi_i\right]}{\partial F^2} &= -1+e^{-2\lambda F} \qquad\qquad < 0 \quad \forall\ F>0
\end{aligned} \tag{33}$$

Thus, the total expected profit for patient firms is concavely decreasing in the contracted amount. Suppose, the colluding firms trade the total expected monopoly quantity forward ($F = \frac{1}{2}\frac{1}{\lambda}$). This leads to a profit of only about 87% of the profit compared to a situation, where firms do not trade any forward contracts.

$$\frac{E\left[\Pi_i | F = \frac{1}{2}\frac{1}{\lambda}\right]}{E\left[\Pi_i | F = 0\right]} = \frac{1}{2} + e^{-1} \approx 0.8679 \tag{34}$$

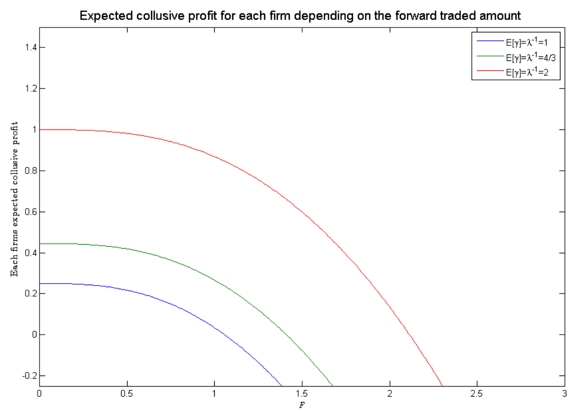


Figure 1: Effect of the forward traded amount on the collusive profit for $\delta \to 1$

Figure 1 shows up the collusive profit for firms depending on the forward traded amount, when they could sustain a full collusion at any price ($\delta \to 1$). For a exponential distributed spread the expected monopoly quantity is given by $E[q^m] = \frac{1}{2}\frac{1}{\lambda}$. Thus,in Figure 1 the expected collusive per period profit is drawn for a expected monopoly quantity of $E[q^m] = \frac{1}{2}$, $E[q^m] = \frac{2}{3}$ and $E[q^m] = 1$. For moderate amounts traded forward the profit decreasing effect of forward trading is rather small. This can mainly be explained by two reasons. Firstly, when firms just trade a moderate amount forward, the probability, that the forward traded amount exceeds the collusive monopoly quantity is rather small. Secondly, even if the forward traded amount exceeds the collusive monopoly quantity, only rather small monopoly profits on the spot market are crowded out by forward trading. Higher realizations of the random difference between the reservation price and marginal costs, which contribute much more to the expected profit, are not affected. The opposite is true for excessive amounts traded forward. Then, it becomes rather likely, that the forward traded amount exceeds the monopoly quantity and even relatively large realization of the spread are affected. This graphically illustrates the fundamental finding, that is in contrast to the deterministic market conditions modeled by Liski and Montero (2006): Stabilizing a collusive agreement using forward contracts is costly in volatile markets.

# 3 Conclusion

When firms trade forward on a volatile market, they do not know in advance the demand and cost structure they will face at the date of delivery. For colluding firms this always leads to the problem of involuntarily having contracted more than the optimal collusive amount. When firms have contracted more than the optimal collusive amount, solely the speculators decide about the price on the spot market. This leads to a lower price than the collusive price, since speculators do not store any amount. This lower price leads to a decrease of the forward price, since the forward price is determined on the basis of rational expectations. As a consequence, the expected profit from trading forward a certain amount is beneath the expected profit from selling the same amount on the spot market. Therefore, the total expected value of the profit for each colluding firm is decreased by forward trading. The more forward contracts are sold, the more severe is the reduction of collusive profit by (additional) forward contracts.

The main result of this article can be stated as follows: Yes, forward contracts can be used in deterministic as well as in volatile markets to stabilize a collusive agreement. However, in volatile markets forward trading strictly decreases the expected total profit of colluding firms. Thus, firms will choose the lowest forward traded amount necessary to sustain their collusive agreement, since excessive forward has its price.

Further research should be done, since there still seem to be several interesting questions concerning forward trading and collusion. For antitrust authorities the question for suitable regulation policy has to be answered. This regulation policy has to balance the legal desire of firms to hedge risk and the anti-competitive effect of forward trading. To answer this question properly, risk aversion should be incorporated. In the above presented model firms trade on the forward market as well as on the spot market simultaneously. To allow additionally for sequential (forward or spot market) trading one could use the work of Mouraviev and Rey (2011) as a good starting point. An important characteristic of financial markets seems to be the imperfect observability of spot and/or forward market positions. It is known from pure spot market games, that imperfect observability leads to totally different strategic implications (Green and Porter (1984) and Sannikov and Skrzypacz (1997)). Thus, modeling imperfect observability seems to be an important step to fully understand the strategic implications of forward trading.

For commodity markets in general, and in particular for electricity markets capacities and convex cost structures play a crucial role. For competing firms Adilov (2012) modeled the strategic implications of forward contracts and capacity constraints. For pure spot market games, the strategic effects of a convex cost structure has e.g. modeled by Dastidar (1995). Both articles might be used as a good starting point to gain a deeper insight into the collusion stabilizing effects of forward trading on commodity markets.

# 4 Appendix

## 4.1 Different scenarios

Scenario I:

a) The semi-collusive quantity exceeds the forward traded amount($F < q^{sc}$)
b) Deviation occurs by setting the residual monopoly price ($\frac{1}{2}(a-c-F)+c < p^{sc}$)
The condition given by a) is obviously given by $F < a - p^{sc}$
The condition given by b) can be brought to:

$$\begin{aligned} p^{sc} &> \frac{1}{2}(a-c-F)+c \\ F &> a+c-2p^{sc} \end{aligned} \tag{A.1}$$

For all collusive prices exceeding marginal costs condition a) is larger than condition b), since $a+c-2p^{sc} < a-p^{sc}$ is equivalent to $c < p^{sc}$ Thus, for scenario I the forward traded amount is bounded on the left by $a+c-2p^{sc} < F$ and on the right by $F < a - p^{sc}$:

$$\begin{aligned} \text{Condition for scenario I :} \qquad & a+c-2p^{sc} < F < a-p^{sc} \\ \text{At monopoly price the condition becomes :} \quad & 2F < a-c \wedge 0 < F \end{aligned} \tag{A.2}$$

Thus, the necessary condition for scenario I is the spread exceeding the twice the forward traded amount $2F < \gamma$ Scenario IIa:

a) The semi-collusive quantity exceeds the forward traded amount ($F < q^{sc}$)
b) Firms set a price between the residual monopoly price and the monopoly price $\frac{1}{2}(a-c-F)+c < p^{sc} < \frac{1}{2}(a-c)+c$
The condition given by a) is again obviously given by $F < a - p^{sc}$
The only reason for profit maximizing colluding firms to set a price below the monopoly price is to sustain collusion. The critical spread, above which firms set a price below the monopoly price ($\gamma^*$), is determined by the no deviation constraint for scenario I (equation 16). The critical spread, above which firms set a price below the residual monopoly price ($\gamma^{**}$), is determined by the no deviation constraint for scenario IIa (equation 20).
The lower bound for scenario IIa lies above the lower bound of scenario I, since:

$$\begin{aligned} 2F &< F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2} \\ F &< 2\sqrt{\frac{\delta}{1-\delta}E\left[\Pi_i\right]} \end{aligned} \tag{A.3}$$

Thus, for reasonable forward traded amounts scenario IIa is bounded by:

$$\text{Condition for scenario IIa :} \quad \gamma^* < \gamma < \gamma^{**} \tag{A.4}$$

Scenario IIb:

a) The semi-collusive quantity exceeds the forward traded amount ($F < q^{sc}$)
b) Firms set a price between marginal costs and the residual monopoly price $c < p^{sc} < \frac{1}{2}(a - c - F) + c$
The condition given by a) is again obviously given by $F < a - p^{sc}$
A price decrease below the residual monopoly price is done by firms, when collusion cannot be sustained in scenario IIa. The critical spread, above which firms set a price below the residual monopoly price($\gamma^{**}$), is determined by the no deviation constraint for scenario IIa (equation 20). This gives the lower bound for scenario IIb. A upper bound does not exist.
Thus scenario IIb lies in the interval:

$$\text{Condition for scenario IIb} \quad \gamma^{**} < \gamma < \infty \tag{A.5}$$

Scenario III:

a) The semi-collusive quantity does not exceed the forward traded amount ($q^{sc} < F$)
b) Deviation occurs by setting the residual monopoly price ($\frac{1}{2}(a-c-F)+c < p^{sc}$)
The condition given by a) is again obviously given by $F > a - p^{sc}$
The condition given by b) is given by $a + c - 2p^{sc} < F$
Both conditions give a lower bound to the contracted amount. However, condition a) is more restrictive for all $c < p^{sc}$, since $a+c-2p^{sc} < a-p^{sc}$ is equivalent to $c < p^{sc}$

$$\begin{aligned} \text{Condition for scenario III} \quad & a - 2p^{sc} < F \\ \text{At monopoly price the condition becomes} \quad & a - c < 2F \end{aligned} \tag{A.6}$$

Scenario IV:

a) The semi-collusive quantity does not exceed the forward traded amount ($q^{sc} < F$)
b) Deviation occurs by undercutting the collusive price infinitesimally ($\frac{1}{2}(a - c - F) + c > p^{sc}$)
The condition given by a) is again obviously given by $F > a - p^{sc}$
The condition given by b) is given by $F < a + c - 2p^{sc}$
These conditions can never be fulfilled at the same time for a collusive price exceeding marginal costs, since this would mean:

$$\begin{aligned} & a - p^{sc} < F < a + c - 2p^{sc} \\ \Rightarrow & a - p^{sc} < a + c - 2p^{sc} \\ \Rightarrow & p^{sc} < c \end{aligned} \tag{A.7}$$

Thus, scenario IV can never be relevant!

## 4.2 Proof that the monopoly profit exceeds the profit of semi-collusion in scenario $IIa$

When the spread equals exactly the critical spread for collusion at the monopoly price ($\gamma = \gamma^* = F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}$) and the firms are setting the monopoly price the contribution to the total expected collusive profit is given by:

$$\begin{aligned}\Pi_i(p = p^m) &= \frac{1}{8}\left(F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}\right)^2 \\ &= \frac{\delta}{1-\delta}E\left[\Pi_i\right] + \frac{1}{4}\sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}\end{aligned} \tag{A.8}$$

When the spread equals exactly the critical spread for collusion at the monopoly price ($\gamma = \gamma^* = F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}$) and the firms are setting the optimal price between the residual monopoly price and the monopoly price the contribution to the total expected collusive profit monopoly profit is given by:

$$\begin{aligned}&\Pi_i\left(p = p^{rm} + \sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\right) \\ &\quad = \frac{1}{4}\left(F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}\right)\left(F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2} - F\right) \\ &\quad - \frac{\delta}{1-\delta}E\left[\Pi_i\right] + \frac{1}{2}F\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}\left(F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2} - F\right)^2} \\ &= \frac{\delta}{1-\delta}E\left[\Pi_i\right] + \frac{1}{4}F\sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}\end{aligned} \tag{A.9}$$

Thus, for $\gamma = \gamma^* = F + \sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right] - F^2}$ both profits are the same.
Comparing the partial derivatives of both profit contributions leads to:

$$\begin{aligned}&\frac{\partial\Pi_i(p = p^m)}{\partial\gamma} > \frac{\partial\ \ \Pi_i\left(p = p^{rm} + \sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\right)}{\partial\gamma} \\ &\frac{1}{4}\gamma > \frac{1}{2}\gamma - \frac{1}{4}F + \frac{1}{4}F\left(2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2\right)^{-\frac{1}{2}}\left(-\frac{1}{2}\gamma + \frac{1}{4}F\right) \\ &2 < F\left(2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2\right)^{-\frac{1}{2}} \quad \Leftrightarrow \quad 0 < \gamma^2 - 2\gamma F + 2F^2 - 8\frac{\delta}{1-\delta}E\left[\Pi_i\right] \\ &\gamma^{crit} = F + \sqrt{8\frac{\delta}{1-\delta} - F^2}\end{aligned} \tag{A.10}$$

Thus, the contribution to the collusive profit always is higher, when firms are setting the monopoly price, since for $\gamma = \gamma^*$ both profits are equal and the monopoly profit is increasing faster in $\gamma$ in the corresponding area, since $\gamma > \gamma^*$ in scenario IIa. Therefore, firms would prefer to set the monopoly price, but are forced to set a lower price in order to sustain the collusive agreement.

## 4.3 Optimal collusive price when collusion at the monopoly price fails

In cases where $\gamma$ lies between the critical spread for collusion at monopoly price ($\gamma^*$) and the critical spread for collusion at residual monopoly price ($\gamma^{**}$) firms either set a collusive price between the residual monopoly price and the monopoly price or exactly the residual monopoly price.
When firms set a price between the residual monopoly price and the monopoly price, the contribution to the expected total profit is as follows:

$$\begin{aligned}
&E\left[\Pi_i\left(p = p^{rm} + \sqrt{\ldots}\right)\right] = E\left[\Pi_i^{FM}\left(p = p^{rm} + \sqrt{\ldots}\right)\right] + E\left[\Pi_i^{SM}\left(p = p^{rm} + \sqrt{\ldots}\right)\right] \\
&= \frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}} (p^{rm} + \sqrt{\ldots} - c)\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\gamma^{**}} \Pi_i^{sc}\widehat{f}(\gamma)d\gamma \\
&= \frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}} \left(\frac{1}{2}(\gamma - F) + \sqrt{\frac{2\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\right)\widehat{f}(\gamma)d\gamma \\
&\quad + \int_{\gamma^*}^{\gamma^{**}} \left(\frac{1}{4}(\gamma - F)^2 - \frac{\delta}{1-\delta}E\left[\Pi_i\right]\right)\widehat{f}(\gamma)d\gamma \\
&= \int_{\gamma^*}^{\gamma^{**}} \left(\frac{1}{4}\gamma(\gamma - F) - \frac{\delta}{1-\delta}E\left[\Pi_i\right] + \frac{1}{2}F\sqrt{\frac{2\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\right)\widehat{f}(\gamma)d\gamma
\end{aligned} \tag{A.11}$$

When firms set the residual monopoly price, the contribution to the expected total profit is as follows:

$$\begin{aligned}
&E\left[\Pi_i\left(p = p^{rm}\right)\right] = E\left[\Pi_i^{FM}\left(p = p^{rm}\right)\right] + E\left[\Pi_i^{SM}\left(p = p^{rm}\right)\right] \\
&= \frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}} (p^{rm} - c)\widehat{f}(\gamma)d\gamma + \frac{1}{2}\int_{\gamma^*}^{\gamma^{**}} \Pi_i^{rm}\widehat{f}(\gamma)d\gamma \\
&= \int_{\gamma^*}^{\gamma^{**}} \frac{1}{4}F\left(\gamma - F\right)\widehat{f}(\gamma)d\gamma + \int_{\gamma^*}^{\gamma^{**}} \frac{1}{8}\left(\gamma - F\right)^2\widehat{f}(\gamma)d\gamma \\
&= \int_{\gamma^*}^{\gamma^{**}} \frac{1}{8}(\gamma^2 - F^2)\widehat{f}(\gamma)d\gamma
\end{aligned} \tag{A.12}$$

To decide which price is set by the firms, the contribution of each price to expected profits is compared. Setting the residual monopoly price would be favorable if

$$
\begin{aligned}
&E\left[\Pi_i\left(p=p^{rm}\right)\right]>E\left[\Pi_i\left(p=p^{rm}+\sqrt{\ldots}\right)\right]\\
&\frac{1}{8}(\gamma^2-F^2)\geq\frac{1}{4}\gamma(\gamma-F)-\frac{\delta}{1-\delta}E\left[\Pi_i\right]+\frac{1}{2}F\sqrt{\frac{2\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{4}(\gamma-F)^2}\\
&\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{8}(\gamma-F)^2\geq\frac{\sqrt{2}}{2}F\sqrt{\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{8}(\gamma-F)^2}\\
&-\frac{1}{8}(\gamma-F)^2+\frac{\delta}{1-\delta}E\left[\Pi_i\right]-\frac{1}{2}F^2\geq 0\\
&-\gamma^2+2\gamma F-5F^2+8\frac{\delta}{1-\delta}E\left[\Pi_i\right]\geq 0\\
&\tilde{\gamma}=F+\sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right]-F^2}
\end{aligned}
\tag{A.13}
$$

This means, that setting the residual monopoly price would be favorable for a spread below $\tilde{\gamma}$. This can never be optimal, since the necessary condition for a semi-collusion is given by a spread above $\gamma^*$ ($\tilde{\gamma}=\gamma^*=F+\sqrt{8\frac{\delta}{1-\delta}E\left[\Pi_i\right]-F^2}$). Thus, in scenario $IIa$ it is always optimal to set the highest price between residual monopoly price and monopoly price, that leads to a sustainable collusion.

## 4.4 Expected spot market profit, forward market profit and total profit

The expected profit on the spot market is found by summing up the expected collusive spot market profits of all scenarios.

$$
\begin{aligned}
E\left[\Pi_i^{sm}\right]=&\int_0^{2F}0\widehat{f}(\gamma)d\gamma+\int_{2F}^{\gamma^*}\left(\frac{1}{8}\gamma^2-\frac{1}{4}\gamma F\right)\widehat{f}(\gamma)d\gamma+\int_{\gamma^*}^{\gamma^{**}}\frac{1}{8}\left(\gamma-F\right)^2\widehat{f}(\gamma)d\gamma\\
&+\int_{\gamma^{**}}^{\infty}\frac{\delta}{1-\delta}E\left[\Pi_i\right]\widehat{f}(\gamma)d\gamma\\
=&\frac{1}{8}\int_{2F}^{\gamma^{**}}\gamma^2\widehat{f}(\gamma)d\gamma-\frac{1}{4}F\int_{2F}^{\gamma^{**}}\gamma\widehat{f}(\gamma)d\gamma+\frac{1}{8}F^2\int_{\gamma^*}^{\gamma^{**}}\widehat{f}(\gamma)d\gamma+\frac{\delta}{1-\delta}E\left[\Pi_i\right]\int_{\gamma^{**}}^{\infty}\widehat{f}(\gamma)d\gamma
\end{aligned}
\tag{A.14}
$$

The expected profit on the forward market is found by summing up the expected collusive forward market profits of all scenarios.

$$
\begin{aligned}
E[\Pi_i^{fm}] &= \frac{1}{2}FE\left[p-c\right] = \frac{1}{2}FE\left[p-c \mid \gamma < 2F\right] + \frac{1}{2}FE\left[p-c \mid 2F < \gamma < \gamma^*\right] \\
&\quad + \frac{1}{2}FE\left[p-c \mid \gamma^* < \gamma < \gamma^{**}\right] + \frac{1}{2}FE\left[p-c \mid \gamma^{**} < \gamma < \infty\right] \\
&= \frac{1}{2}F\Bigg[\int_0^{2F}\left(\gamma - F\right)\widehat{f}(\gamma)d\gamma + \int_{2F}^{\gamma^*}\frac{1}{2}\gamma\widehat{f}(\gamma)d\gamma \\
&\quad + \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{2}\left(\gamma - F\right) + \sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\right)\widehat{f}(\gamma)d\gamma \\
&\quad + \int_{\gamma^{**}}^{\infty}\left(\frac{1}{2}\left(\gamma - F\right) - \sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta}E\left[\Pi_i\right]}\right)\widehat{f}(\gamma)d\gamma\Bigg]
\end{aligned}
\tag{A.15}
$$

The total expected profit is found by summing up all (four) contributions from the three scenarios to the total expected profit. This leads to:

$$
\begin{aligned}
E\left[\Pi_i\right] &= E\left[\Pi_i|0 < \gamma < 2F\right] + E\left[\Pi_i|2F < \gamma < \gamma^*\right] + E\left[\Pi_i|\gamma^* < \gamma < \gamma^{**}\right] \\
&\quad + E\left[\Pi_i|\gamma^{**} < \gamma < \infty\right] \\
&= \frac{1}{2}F\int_0^{2F}(\gamma - F)\widehat{f}(\gamma)d\gamma + \int_{2F}^{\gamma^*}\frac{1}{8}\gamma^2\widehat{f}(\gamma)d\gamma \\
&\quad + \int_{\gamma^*}^{\gamma^{**}}\left(\frac{1}{4}\gamma F(\gamma - F) + \frac{1}{2}F\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2} - \frac{\delta}{1-\delta}E\left[\Pi_i\right]\right)\widehat{f}(\gamma)d\gamma \\
&\quad + \int_{\gamma^{**}}^{\infty}\left(\frac{1}{4}F(\gamma - F) - \frac{1}{2}F\sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta}E\left[\Pi_i\right]} + \frac{\delta}{1-\delta}E\left[\Pi_i\right]\right)\widehat{f}(\gamma)d\gamma \\
E\left[\Pi_i\right] &= \frac{1-\delta}{1-\delta\left(2 - 2\widehat{F}(\gamma^{**}) + \widehat{F}(\gamma^*)\right)}\Bigg[-\frac{1}{4}F^2\left(1 + 2\widehat{F}(2F) - \widehat{F}(\gamma^{**})\right) \\
&\quad + \frac{1}{2}F\int_0^{2F}\gamma\widehat{f}(\gamma)d\gamma + \frac{1}{8}\int_{2F}^{\gamma^*}\gamma^2\widehat{f}(\gamma)d\gamma + \frac{1}{4}\int_{\gamma^*}^{\gamma^{**}}\gamma^2\widehat{f}(\gamma)d\gamma - \frac{1}{4}F\int_{\gamma^*}^{\gamma^{**}}\gamma\widehat{f}(\gamma)d\gamma \\
&\quad + \frac{1}{4}F\int_{\gamma^{**}}^{\infty}\gamma\widehat{f}(\gamma)d\gamma + \frac{1}{2}F\int_{\gamma^*}^{\gamma^{**}}\sqrt{2\frac{\delta}{1-\delta}E\left[\Pi_i\right] - \frac{1}{4}(\gamma - F)^2}\widehat{f}(\gamma)d\gamma \\
&\quad + \frac{1}{2}F\int_{\gamma^{**}}^{\infty}\sqrt{\frac{1}{4}(\gamma - F)^2 - 2\frac{\delta}{1-\delta}E\left[\Pi_i\right]}\widehat{f}(\gamma)d\gamma\Bigg]
\end{aligned}
\tag{A.16}
$$

For derivation of the total expected profit for a exponential distributed spread it is separated into part A, part B and part C

$$E[\Pi_i] = \underbrace{\frac{1}{2}F\int_0^{2F}\gamma\widehat{f}(\gamma)d\gamma}_{A} + \underbrace{-\frac{1}{2}F^2\widehat{F}(2F)}_{B} + \underbrace{\frac{1}{8}\int_{2F}^{\infty}\gamma^2\widehat{f}(\gamma)d\gamma}_{C}$$

The first part (A) can be brought to:

$$\begin{aligned} A &= \frac{1}{2}F\lambda\int_0^{2F}\gamma e^{-\lambda\gamma}d\gamma = \frac{1}{2}F\lambda\left[-2F\frac{1}{\lambda}e^{-2F\lambda} + 0 + \frac{1}{\lambda}\int_0^{2F}e^{-\lambda\gamma}d\gamma\right] \\ &= -F^2e^{-2F\lambda} + \frac{1}{2}\frac{F}{\lambda}\left[1 - e^{-2F\lambda}\right] \end{aligned} \tag{A.17}$$

The second part (B) can be brought to:

$$B = -\frac{1}{2}F^2\left[1 - e^{-2F\lambda}\right] \tag{A.18}$$

The third part (C) can be brought to:

$$\begin{aligned} C &= \frac{1}{8}\lambda\int_{2F}^{\infty}\gamma^2 e^{-2F\lambda}d\gamma = \frac{1}{8}\lambda\left[\frac{1}{\lambda}4F^2e^{-2F\lambda} + 2\frac{1}{\lambda}\int_{2F}^{\infty}e^{-\lambda\gamma}d\gamma\right] \\ &= \frac{1}{2}F^2e^{-2F\lambda} + \frac{1}{4}\left[\frac{1}{\lambda}e^{-2F\lambda}2F + \frac{1}{\lambda}\int_{2F}^{\infty}e^{-\lambda\gamma}d\gamma\right] \\ &= \frac{1}{2}F^2e^{-2F\lambda} + \frac{1}{2}\frac{F}{\lambda}e^{-2F\lambda} + \frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda} \end{aligned} \tag{A.19}$$

Summing up the first (A), the second (B) and the third part (C) yields:

$$\begin{aligned} E[\Pi_i] &= -F^2e^{-2F\lambda} + \frac{1}{2}\frac{F}{\lambda}\left[1 - e^{-2F\lambda}\right] - \frac{1}{2}F^2\left[1 - e^{-2F\lambda}\right] + \frac{1}{2}F^2e^{-2F\lambda} + \frac{1}{2}\frac{F}{\lambda}e^{-2F\lambda} \\ &\quad + \frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda} = \frac{1}{2}\frac{F}{\lambda} - \frac{1}{2}F^2 + \frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda} \end{aligned} \tag{A.20}$$

**Comparison of the collusive forward market profit in scenario $III$ and half the monopoly profit**

$$\begin{aligned} \frac{1}{2}F(\gamma - F) &\leq \frac{1}{8}\gamma^2 \Leftrightarrow 0 \leq \frac{1}{8}\gamma^2 - \frac{1}{2}F\gamma + \frac{1}{2}F^2 \\ \gamma &= \frac{-\frac{1}{2}F \pm \sqrt{\frac{1}{4}F^2 - 4 * \frac{1}{8}\frac{1}{2}F^2}}{\frac{2}{8}} = 2F \end{aligned} \tag{A.21}$$

Thus, the contribution to the total expected profit in scenario III (coming solely from forward trading) always is below the contribution to the total expected profit in scenario I(given by half the monopoly profit).

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