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Ich danke Frank Heinemann für wertvolle Kommentare. Etwaige Fehler gehen natürlich zu meinen Lasten.

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Abstract

A recent strand of literature (see Morris and Shin 2001) shows that multiple equilibria in models of markets for pegged currencies vanish if there is slightly diverse information between traders. It is known that this approach works only if there is not too precise common knowledge in the market. This has led to the conclusion that central banks should try to avoid making their information common knowledge. We present a model in which more transparency of the central bank means better private information, because each trader utilizes public information according to her own private information. Thus, transparency makes multiple equilibria less likely.

1 Introduction

Attacks on pegged currencies by private traders are a constant cause for concern of monetary authorities responible for a peg. One reason for this is that up to now currency crises are not well understood. What determines the outbreak of a currency attack? On the one hand, it seems clear that only "weak" currencies are in danger of massive short selling. Thus attacks should be determined by fundamentals which are exogenously given to the currency market. On the other hand, the coordination of the trader´s actions on the currency market appears to be an important condition for a successful attack. It is, as many economists would argue, so important that a high coordination can even break the peg of not so weak a currency, which otherwise might well have been kept stable. But how does this coordination come about? An intrigueing answer to this question is given by a new strand of literature which applies the theory of global games to the problem of multiple equilibria on monetary and financial markets (see Morris and Shin 1998 and 2001 and Heinemann and Illing 2002).[1] The central result of this literature is that the fundamentals can uniquely determine whether there will be an attack on the currency or not if there is some (arbitrarily small) amount of private knowledge about the fundamentals. This uniqueness, however, holds only as long as information that is dispersed privately is precise enough relative to common knowledge about the fundamentals. Thus, if indetermined situations or, using the technical term, multiple equilibria are considered as unstable and therefore not desirable, the theory appears to have a paradoxical implication: a transparent information policy seems to be bad, because

1 "Global games are games of incomplete information whose type space is determined by the players each observing a noisy signal of the underlying state." (Morris and Shin 2001, page1)

it makes common knowledge more precise and thus endangers the uniqueness of the equilibrium on the currency markets.

This paper argues that transparency, if only defined properly, will improve private information and will not improve decision-relevant common knowledge. A more transparent information policy is defined as a policy that gives more information to the public about how the central bank has come to its assessment of the fundamental strength of the economy. The intuition of the result is straight forward: a detailed account on the reasons which led to the overall assessment by the central bank does not change the public knowledge about the fundamental strength of the economy, because every trader will draw different conclusions from the additional details made public: depending on the trader´s private information, each trader will be able to update her own information. Therefore, a more transparent policy makes private information more precise. Thus, in the context of global games, a transparent information policy will contribute to more stable markets because it can eliminate the multiplicity of equilibria which exist with an intransparent information policy.

In the following this argument will be stated formally: section 2 sets the formal framework. Section 3 discusses the approach of global games for a baseline case which is adapted from Morris and Shin (1999). Section 4 shows that a transparent information policy can eliminate the multiplicity of equilibria which exist with an intransparent information policy. Section 5 sums up and gives an outlook on possible future research.

2 A formal framework for currency attacks

The basic model of currency attacks used here centers around a parameter θ that represents the assessment of the fundamental strength of the economy by the central bank. The chances of an attack will be good if the assessment is low because the determination of the central bank to defend the peg is low. The logic behind this framework stems from the second generation models of currency crises (see Obstfeld 1996): In the long run, the peg gives the central bank the chance to import monetary stability from abroad, but against this benefit stand the costs of defending the peg if the present fundamentals of the economy are weak, because the contractive monetary policy entailed by holding the peg is the more damaging the weaker the economy is. A higher θ represents a more optimistic assessment about the present fundamentals: in this case the central bank thinks that sticking to the peg makes sense and it will fend the attack off. For a low θ the bank will give up on the peg. For intermediate values of θ, however, the central bank´s behaviour depends on the strength of the attack. This is measured by the share of all traders attacking (all the traders keep the same amount of currency, and they either sell or hold it). A high share means that the central bank will have to sell a lot of foreign reserves in order to defend the peg. These are additional costs which can make a defense unattractive for the central bank. We define the function $a(\theta)$ as the minimum share of traders necessary to induce the central bank to give up on the peg, given the assessment θ. For smaller values, the attack will be fended off. Moreover,

- There is $\underline{\theta}$ such that $a(\theta) = 0$ for $\theta \leq \underline{\theta}$

  (for this comparably low value of θ the peg will be abandoned even if nobody attacks).

- There is $\overline{\theta}$ such that $a(\theta)$ is undefined for $\theta > \overline{\theta}$

  (for these comparably high values of θ the peg will not be abandoned independently of how many traders attack).

- $a(\theta)$ is strictly increasing in θ when $0 < a(\theta) < 1$, and there is a bound b on the slope of $a(\cdot)$, so that $0 < b \leq a'(\theta)$.

  (for intermediate values of θ a higher θ means that a higher share of traders attacking is necessary to induce the central bank to abandon the peg.)

How is the share of attacking traders determined? We assume that every single trader decides about selling or holding the amount of currency she owns. Holding gives a payout of zero. Selling entails a transaction cost of c with $0 < c < 1$. If enough traders sell in order to break the peg, the depreciation of the currency will give a payout of 1 to every trader. Therefore, for intermediate values of θ ($\underline{\theta} < \theta < \overline{\theta}$), multiple equilibria exist if θ is common knowledge. Thus it seems as if a central bank that made its assessment of the economy absolutely transparent at every instant would have to face the problem of multiple equilibria. It has been shown, however, that an arbitrarily small amount of uncertainty of the traders about the assessment may be enough to remove the multiplicity.

It is plausible that perfect common knowledge about the central bank´s assessment can never be achieved, even if the central bank tried to do so. One reason for this is that the central bank can communicate its assessment to the public only now and then.[2] We allow for this fact by constructing a model with two time periods: in period zero the central bank makes its assessment of the strength of the economy,

2 The ECB, for example, does this with help of monthly press conferences after meetings of the Governing Council, monthly bulletins and, twice a year, by projections about economic developments in the near future.

θ_0, public. In period 1 the traders decide about attacking the currency. Whether the attack will be successful, depends on θ_1, the central bank´s assessment in period 1. This is not observable by the public, but traders use the last official assessment θ_0 and their private information to estimate it.

3 A baseline model

In a baseline case, traders know that θ_1 is normally distributed with mean θ_0 and variance δ. Moreover, every trader i has some information about the present assessment: She observes the varable

$$y_i = \theta_1 + \eta_i \tag{1}$$

where η_i is a normal random variable with mean zero and variance $\epsilon\delta$. The distribution of the random variables y_i and θ_1 is multivariate normal. The following facts about the conditional distributions (see Morris and Shin (1999)) can be derived:

- $f(y_i \mid \theta_1)$ is normal with mean θ_1 and variance $\epsilon\delta$.
- $f(\theta_1 \mid y_i, \theta_0)$ is normal with mean $(\frac{\epsilon}{1+\epsilon})\theta_0 + (\frac{1}{1+\epsilon})y_i$ and variance $\frac{\epsilon\delta}{1+\epsilon}$.
  A trader forms her beliefs about the present assessment of the central bank on the basis of the past public assessment, updating it with her own information about the present assessment. The trader weights the two sources of her infromation according to the respective variances: the higher the variance, the smaller the weight of the information.
- The correlation between the private information y_i and y_j of two traders i and j is $\frac{1}{(1+\epsilon)}$.

When rational traders decide about attacking or not they utilize these statistical relations for infering the behaviour of other traders. It can be shown that this strategic thinking restricts the set of equilibrium strategies drastically:

Theorem 1 *For ϵ sufficiently small, there is a number h with the following properties: the currency peg is maintained as long as $\theta > h$, but the peg is abandoned as soon as $\theta_1 \leq h$.*

In particular, a sufficient condition for uniqueness is

$$\sqrt{\frac{\epsilon}{2\delta\pi}} < a'(\psi) \tag{2}$$

with π as the number pi and ψ as the assessment θ_1 of the central bank which induces a share of traders that is just enough to cause the central bank to abandon the peg.

For the proof of the theorem and equation 2 see Morris and Shin (1999). Here we give an intuitive interpretation for the result that a) the multiplicity of equilibria can vanish with private information about θ_1, and b) that uniqueness is certain for a sufficiently small ϵ, that is, if private information is sufficiently precise relative to common knowledge.

Consider first the strategic situation of a trader i who observes a relatively high value $y_i = \overline{\theta_1}$ pointing to a central bank that is relatively determined to defend the peg. If this signal were public information, attacking were only successful if all other agents also attacked. But because there is a small dispersion of the signals about the determination, a single trader knows that, with high probability, some traders get a signal pointing to an even stronger central bank. The trader knows that with this information it is strictly optimal not to attack, independently of other trader´s strategies, and that therefore these traders will certainly not attack. Therefore, it is neither for trader i optimal to attack. Holding the currency is the dominant strategy, and in the search for equilibrium strategies attacking is eliminated for $y_i = \overline{\theta_1}$. Iterating this principle for ever smaller values of y_i and applying it analogously from "the lower end" of the range $[\underline{\theta_1}; \overline{\theta_1}]$ upwards makes the range of multiple equilibria ever smaller. Indeed, as is shown in the appendix, for small values of ϵ , the multiplicity of equilibria is eliminated: in this case the optimal strategy for a trader i is to attack only for signals about the determination of the central bank $y_i \leq y^*$.

The iterated elimination of dominated strategies will not necessarily lead to a unique equilibrium if, as in the baseline model, there is both private information y_i and some public information θ_0 about the determination of the central bank. The intuition behind this fact is as follows: without common knowledge θ_0, if all traders had the same strategy of attacking only for $y_i \leq y'$, the expected benefit for a trader observing $y', U(y')$, would decline monotonically with the signal about the central bank´s assessment, which guarantees a unique equilibrium threshold value $y' = y^*$ that is defined by the condition $U(y^*) = 0$. This might longer hold if public information is to be taken into account: A high θ_0 suggesting a high θ_1 makes it probable that a low y_i is misleading private information and that most other traders, being more pessimistic about an attack, will leave the attacking trader alone. Therefore, the benefit function $U(y')$ might not be monotonic, and the threshold value y^* might not be unique. Clearly, this effect is stronger if private information is less precise relative to public information, that is if ϵ is large.

4 Transparency of central bank assessments

The result that multiple equilibria are more likely if public information is an important source of information for traders seems to be relevant for the discussion about costs and benefits of transparency of central banks (see e.g. Geraats 2002). The implications, however, are strange: a more transparent central bank would surely, one is inclined to think, lead to better public information about the central bank´s assessment, and might thus be responsible for multiple equilibria. The economy as a whole may end up more intransparent. Hence, Heinemann and Illing (2002) discuss ways of making private information more precise without giving public information:

> One way to achieve this might be giving everybody reliable information on request without announcing it publicly. Then, speculators can never be certain that other agents have the same information at any given moment... Another way might be a decentralized approach: Suppose, there are several sources of excellent but costly information. If each agent uses at least one of these sources, their information is rather precise. But, the probability that two agents use exactly the same sources is sufficiently small to prevent the high degree of common beliefs that is necessary for multiple equilibria.

To the author of this paper it seems dubious whether recommendations of this sort will convince economic policy advisors. Instead, this paper argues that transparency, if only defined properly, will by itself improve private information and might not improve common knowledge at all. We define a more transparent information policy as a policy which gives more information to the public about how the central bank has come to its assessment of the economy´s state. This definition of transparency is precise enough to allow inquiring into its consequences for the question of multiple equilibria in the Morris-Shin framework. The intuition of the result is straight forward: if the declaration of the central bank about its overall assessment has been truthful, a more detailed account on the information which has led to the assessment does not change the public knowledge about the assessment itself. Each trader will, however, privately benefit from the additional information. Depending on her private information, everyone will be able to update her belief in a different way. Therefore, a more transparent policy makes private information more precise. In the context of global games multiple equilibria become less likely.

4.1 Formation of assessments and transparency

In this paper a central bank is transparent if in period 0 it gives information on how it comes to its public assessment θ_0. An intransparent central bank does not give that information. We consider the simple case that the overall assessment is the sum of the assessments of only two subsystems of the economy:

$$\theta_0 = x_{0i} + x_{0j} \tag{3}$$

To give a concrete example, θ_0 might be the negative of the inflation rate the central bank expects: a higher inflation makes it more difficult to defend the peg to a stable foreign currency. If inflation is forecast according to the quantity theory of money and the central bank follows a strict rule for money growth, the overall assessment depends on the bank´s estimates for real output growth x_{0i} and for the growth of velocity x_{0j}.[3]

In period 1 these estimates for output and velocity growth have changed:

$$\theta_1 = x_{1i} + x_{1j} \tag{4}$$

The following relation exists between the cenrtal bank´s estimations in period 0 and period 1:

$$x_{1i} = x_{0i} + \gamma_i \tag{5}$$

$$x_{1j} = x_{0j} + \gamma j \tag{6}$$

where γ_i and γ_j are independent, normally distributed random variables with mean 0 and variance α.

If the basis for the traders' decision on whether to attack or not is only common knowledge θ_0, there are, as in the baseline model, multiple equilibria for values of θ_0 between $\underline{\theta}_0$ and $\overline{\theta}_0$.

3 Equation 3 is equivalent to $\theta_0 = -\pi_0 = -(\mu - y_0 + \nu_0)$ with μ as money growth, y_0 as the bank´s estimate of output growth and ν_0 as the bank´s estimate of velocity growth, if μ is normalized to 0, $y_0 = x_{0i}$ and $-\nu_0 = x_{0j}$.

We now assume that there are only two traders: trader i is an expert on the real sector of the economy, while trader j is an expert on the financial sector. Each of them knows the developments of the respective sectors so well that they observe the assessment of the central bank in period one exactly: trader i observes x_{1i} and trader j observes x_{1j} (this is the case, if, for example, it is commonly known that both the central bank and the respective expert are able to observe output and velocity growth in period 1 exactly). Thus, every trader has only to estimate the variable she is not expert of. The basis of this estimation is the information the central bank has given in period zero.

The behaviour of the central bank facing only two traders is a simple case of the behaviour assumed in the baseline model: there is an assessment $\underline{\theta}$ such that $a(\theta) = 0$ for $\theta \leq \underline{\theta}$ (for these comparably low values of θ the bank will give up on the peg even if nobody attacks). Moreover there is a $\overline{\theta}$ such that $a(\theta)$ is undefined for $\theta > \overline{\theta}$ (for these comparably high values of θ the peg will not be abandoned independently of how many traders attack). In addition, we define θ' (with $\underline{\theta} \leq \theta' \leq \overline{\theta}$) as follows: For $\underline{\theta} \leq \theta \leq \theta' : a(\theta) = 1/2$ (the central bank gives up on the peg if at least one trader attacks). For $\theta' \leq \theta \leq \overline{\theta} : a(\theta) = 1$ (the central bank gives up on the peg only if both traders attack).

We consider two cases: a transparent and an intransparent information policy.

4.2 The case of transparency

An information policy is transparent if all the information the bank has is given to the public in period 0: the vector x_0 (this is x_{0i} and x_{0j}) and thus its overall assessment θ_0 as the sum of the two are common knowledge. In period 1 trader i uses x_{0j} and trader j uses x_{0i} according to the central bank´s estimation in period 0 in order to estimate the variable they are not experts of .

The estimation of θ_1 by trader i is y_i:

$$y_i = x_{1i} + x_{0j} = \theta_1 - \gamma_j \tag{7}$$

The estimation of θ_1 by trader j is y_j:

$$y_j = x_{1j} + x_{0i} = \theta_1 - \gamma_i \tag{8}$$

The conditional distributions are now as follows:

- $f(y_i \mid \theta_1)$ and $f(y_j \mid \theta_1)$ are normal with mean zero and variance α.

- $f(\theta_1 \mid y_i)$ (which is the perspective of trader i) is normal with mean y_i and variance α and $f(\theta_1 \mid y_j)$ (the perspective of trader j) is normal with mean y_j and variance α.

  All information about θ_1 trader i has is included in y_i. But because she knows that trader j does not observe x_{1i}, all information about the beliefs of trader j is included in the old assessment of the central bank θ_0. Therefore:

- Given θ_1, the private information y_i and y_j of the two traders i and j is uncorrelated.

The strategic situation is not a special case of the baseline model of section 3, but it is similar. Again, for suitable parameter values there is a unique equilibrium strategy for currency attacks:

Theorem 2 *For appropriate exogenous values c, x_0, α, θ' and $\overline{\theta}$, the equilibrium in strategies of traders i and j is unique: the currency is attacked by trader i (trader j) if and only if the signal the trader gets is equal to or smaller than a certain threshold value: $x_{1i} \leq x_{0i} + \gamma^*$ ($x_{1j} \leq x_{0j} + \gamma^*$). The equilibrium is unique if one of the two following conditions are fulfilled: either, if the traders had the symmetric strategies to attack for $x_{1i} \leq \theta' - x_{0j}$ (for $x_{1i} \leq \theta' - x_{0j}$), the expected reward for attacking at the threshold signal $x_{1i} = \theta' - x_{0j}$ were strictly positive; or, if the traders had the symmetric strategies to attack for $x_{1i} \leq \overline{\theta} - x_{0j}$ (for $x_{1i} \leq \overline{\theta} - x_{0j}$), the expected reward for attacking at the threshold signal $x_{1i} = \overline{\theta} - x_{0j}$ were strictly negative.*

Proof: see appendix.

The rationale of the theorem is as follows: for suitable values of c, x_0, α, $\theta'$and $\overline{\theta}$, a unique switching strategy is played in equilibrium: trader i attacks if $x_{1i} \leq x_{0i} + \gamma^*$ and trader j attacks if $x_{1j} \leq x_{0j} + \gamma^*$, respectively. The threshold value γ^* has the property that if the trader gets the threshold signal $x_{1i} = x_{0i} + \gamma^*$ (or $x_{1j} = x_{0j} + \gamma^*$), the expected reward of attacking is zero. When will γ^* be unique? It can be shown that the expected reward of attacking at the threshold signal is positive for γ being small, because it is highly probable that the bank will give up on the peg irresectively of how many traders attack, and that the expected reward of attacking at the threshold signal is negative for γ being large, because it is highly probable that the bank will not give up on the peg irresectively of how many traders attack. A unique equilibrium threshold value γ^* were guaranteed if the value decreased monotonically in γ. For a certain range of threshold values, however, this is not the case: in this range, if both traders increase their threshold value γ, the probability of success does not shrink, because the central bank will give up on the peg as long as both traders attack. If the expected reward of an attack in this range is around zero, there might be multiple equilibrium threshold values (see figures at the end of the paper). If, however, for the range of threshold values where both traders are needed for a successful attack, the expected value is either only positive or only negative, there will be only one equilibrium threshold value, and there will be a unique equilibrium strategy for both traders.

4.3 The case of an intransparent policy

We now consider the case of an intransparent policy of information by the central bank. This means that in period 0 it does only publish its overall assessment θ_0, but not its estimation of x_{0i} and x_{0j}. This assessment is the basis of traders´ estimate of the variable unknown to them. But because the traders do not have more information about this variable other than on θ_0, their private knowledge does not help them estimating θ_1:

$$y_i = x_{1i} + E(x_{0j}) = x_{1i} + (\theta_0 - x_{1i}) = \theta_0 \tag{9}$$

$$y_j = x_{1j} + E(x_{0i}) = x_{1j} + (\theta_0 - x_{1j}) = \theta_0 \tag{10}$$

Thus, in case of an intransparent information policy only common knowledge is relevant for the question whether the traders will attack. This means that, as in the baseline model, there are multiple equilibria for values of θ_0 between $\underline{\theta}_0$ and $\overline{\theta}_0$.

5 Conclusions

This paper aimed at contributing to the understanding of models of currency attacks based on the theory of global games. It showed, for a specific example, that a more transparent infomation policy can eliminate multiple equilibria on a currency market. This result is of interest because in the standard global games models a more precise common knowledge relative to private information diminishes the range of parameters for which multiple equilibria can be excluded. In our setting, however, a detailed account on the components of the overall assessment does not change public knowledge itself, because each trader draws different conclusions from the details made public. Instead, a transparent policy helps traders to make use of private information.

Clearly, the setting chosen is somewhat special. Generalization would be a natural step further. In particular, traders could be given some a priori information about the variable they are not experts of. In this case, private information of traders would be useful to them even if the central bank published only its overall assessement. The general insight, however, should be robust against such generalizations: a more transparent information policy makes the private information of traders more valuable.

A Proof of the Theorem in Section 4.2

Note that the strategic situation of the two traders are symmetric. Thus, what we conclude for one trader i is valid accordingly to the other as well.

First we analyse the reward function of attacking the peg. We define the strategy of trader j of attacking the peg or not, depending on her prior information x_0 and her private signal γ_j, as $s(\gamma_j, x_0)$. The expected reward of trader i is then given by:

$$u(x_0, \gamma_i, s) = \int_{A(s)} f(\gamma_j \mid x_0) dx_j - c \tag{11}$$

with $A(s)$ as the set of γ_j for which the attack will be successful. The condition of success is that the number of attacking traders is large enough to induce the central bank to give up on the peg: $(1 + s(\gamma_j, x_0))/2 \geq a(x_{0i} + \gamma_i + x_{0j} + \gamma_j)$. We specify the strategy s as I_k: trader j attacks only if $x_{0j} + \gamma_j \leq k$. Thus the expected reward of trader i can be written as:

$$u(x_0, \gamma_i, s) = u(x_0, \gamma_i, I_k) = \int_{A(I_k)} f(\gamma_j \mid x_0) dx_j - c \tag{12}$$

We now assume that trader i expects trader j to have the strategy $I_{x_{0j}+\gamma_i}$, that is to attack only for signals equalt to or smaller than the signal of trader i: $x_{1j} \leq x_{0j} + \gamma_i$. Furthermore, we define $\gamma_j^*(\gamma_i)$ as the smallest γ_j with the property: $(1 + s(\gamma_j, x_0))/2 \geq a(x_{0i} + \gamma_i + x_{0j} + \gamma_j)$.

Thus, the expected reward becomes:

$$U(x_0, \gamma_i) = \int_{-\infty}^{\gamma_j^*(\gamma_i)} f(\gamma_j \mid x_0) dx_j - c = \Phi(\gamma_j^*(\gamma_i), x_0, \sqrt{\alpha}) - c \tag{13}$$

Now we look at the function $\gamma_j^*(\gamma_i)$, which has a piecemeal definition.

For $\gamma_i < (\theta' - x_{0i} - x_{0j})/2$, the attack will be successful as long as $x_{0i} + \gamma_i + x_{0j} + \gamma_j < \theta'$ (even if only trader i attacks). Thus $\gamma_j^* = \theta' - x_{0i} - x_{0j} - \gamma_i$ and $\partial\gamma_j^*/\partial\gamma_i = -1$ for $\gamma_i < (\theta' - x_{0i} - x_{0j})/2$.

For $(\theta' - x_{0i} - x_{0j})/2 \leq \gamma_i \leq (\overline{\theta} - x_{0i} - x_{0j})/2$ the attack will be successful as long as $\gamma_j \leq \gamma_i$, because in this case trader j joins the attack (per assumption), and therefore, the central bank will give up on the peg. Thus $\gamma_j^* = \gamma_i$ and $\partial\gamma_j^*/\partial\gamma_i = 1$ for $\gamma_i < (\theta' - x_{0i} - x_{0j})/2$.

For $(\overline{\theta} - x_{0i} - x_{0j})/2 \leq \underline{\gamma_i}$ the attack (of both traders) will be successful as long as $x_{0i} + \gamma_i + x_{0j} + \gamma_j \leq \overline{\theta}$. Thus $\gamma_j^* = \overline{\theta} - x_{0i} - x_{0j} - \gamma_i$ and $\partial\gamma_j^*/\partial\gamma_i = -1$ for $(\overline{\theta} - x_{0i} - x_{0j})/2 \leq \gamma_i$.

Furthermore, we can state the following properties of the function $U(x_0, \gamma_i)$:

U is positive for small γ_i (tends to $1 - c$)

U is negative for large γ_i (tends to $-c$)

U is continuous in γ_i.

Therefore, U crosses the horizontal axis at least once.

Next, we show that any equilibrium strategies will have upper and lower bounds with the property $U(x_0, \gamma_i) = 0$ (for trader i; the same holds for trader j.). In particular, we show:

Lemma 3 $\min\{\gamma_k \mid U(x_0, \gamma_k) = 0\} = \underline{\gamma}_k; \max\{\gamma_k \mid U(x_0, \gamma_k) = 0\} = \overline{\gamma}_k$

with k as either i or j. The first equation tells us that the smallest γ_k, at which at least one trader does not attack, $\underline{\gamma_k}$, equals the minimum of γ_k with the property: $U(x_0, \gamma_k) = 0$. The second equation says that the largest γ_k, at which at least one trader attacks, $\overline{\gamma_k}$, equals the maximum of γ_k with the property: $U(x_0, \gamma_k) = 0$.

Proof of Lemma 1:

First note that the expected reward of attacking for i is larger if j attacks than if j does not attack, because the attack of both traders might be necessary to induce the central bank to give up on the peg. Second, note that for $\underline{\gamma_i}$ the expected reward of an attack is $u(x_0, \underline{\gamma_i}, \pi) \leq 0$ (because otherwise everybody would attack). Therefore the reward of an attack for i at $\underline{\gamma_i}$ if j does not attack is not positive. Thus $U(x_0, \underline{\gamma}_i) \leq 0$. This implies that the smallest γ_i with $U(x_0, \gamma_i) = 0$ is smaller or equal than $\underline{\gamma}_{1i}$ (because for a higher γ_i, $U(x_0, \gamma_i)$ will fall even further):

$$\min\{\gamma_i \mid U(x_0, \gamma_i) = 0\} \leq \underline{\gamma}_i \tag{14}$$

Meanwhile, we can construct the following equilibrium in switching strategies: a trader attacks if she gets a signal γ_k which is larger than $\min\{\gamma_k \mid U(x_0, \gamma_k) = 0\}$. This is an equilibrium because $U(x_0, \gamma_k)$ is decreasing in $\min\{\gamma_k \mid U(x_0, \gamma_k) = 0\}$. Therefore, by definition of $\underline{\gamma_k}$,

$$\min\{\gamma_i \mid U(x_0, \gamma_i) = 0\} \geq \underline{\gamma}_i \tag{15}$$

Thus:

$$\min\{\gamma_i \mid U(x_0, \gamma_i) = 0\} = \underline{\gamma}_i \tag{16}$$

By analogous reasoning we get:

$$\max\{\gamma_i \mid U(x_0, \gamma_i) = 0\} = \overline{\gamma}_i \tag{17}$$

If $U(x_0, \gamma_k) = 0$ is unique (thus $\min\{\gamma_k \mid U(x_0, \gamma_k) = 0\} = \max\{\gamma_k \mid U(x_0, \gamma_k) = 0\}$), both traders attack for $\gamma_k < \gamma_k \mid U(x_0, \gamma_k) = 0$, and nobody attacks for $\gamma_k > \gamma_k \mid U(x_0, \gamma_k) = 0$.

What are the conditions for $\gamma_k \mid U(x_0, x_{1k}) = 0$ to be unique? We know that $\partial U / \partial \gamma_k < 0$ for $\gamma_k < (\theta' - x_{0i} - x_{0j})/2$ and for $(\overline{\theta} - x_{0i} - x_{0j})/2 \leq x_{1k}$. But $\partial U / \partial x_{1k} > 0$ for $(\theta' - x_{0i} - x_{0j})/2 \leq \gamma_i \leq (\overline{\theta} - x_{0i} - x_{0j})/2$. Thus:

Lemma 4 $\gamma_k \mid U(x_0, \gamma_k) = 0$ *is unique if* $U(x_0, \gamma_k = (\theta' - x_{0i} - x_{0j})/2) > 0$ *or if* $U(x_0, \gamma_i = (\overline{\theta} - x_{0i} - x_{0j})/2) < 0$.

The conditions of the lemma imply that either the expected reward of attacking is positive for the range of threshold signals $x_{1k} = x_{0k} + \gamma_k$ which induce the central bank to give up on the peg only if both traders attack, or that the expected reward of attacking is negative for this range (see the figure in section 4.2).

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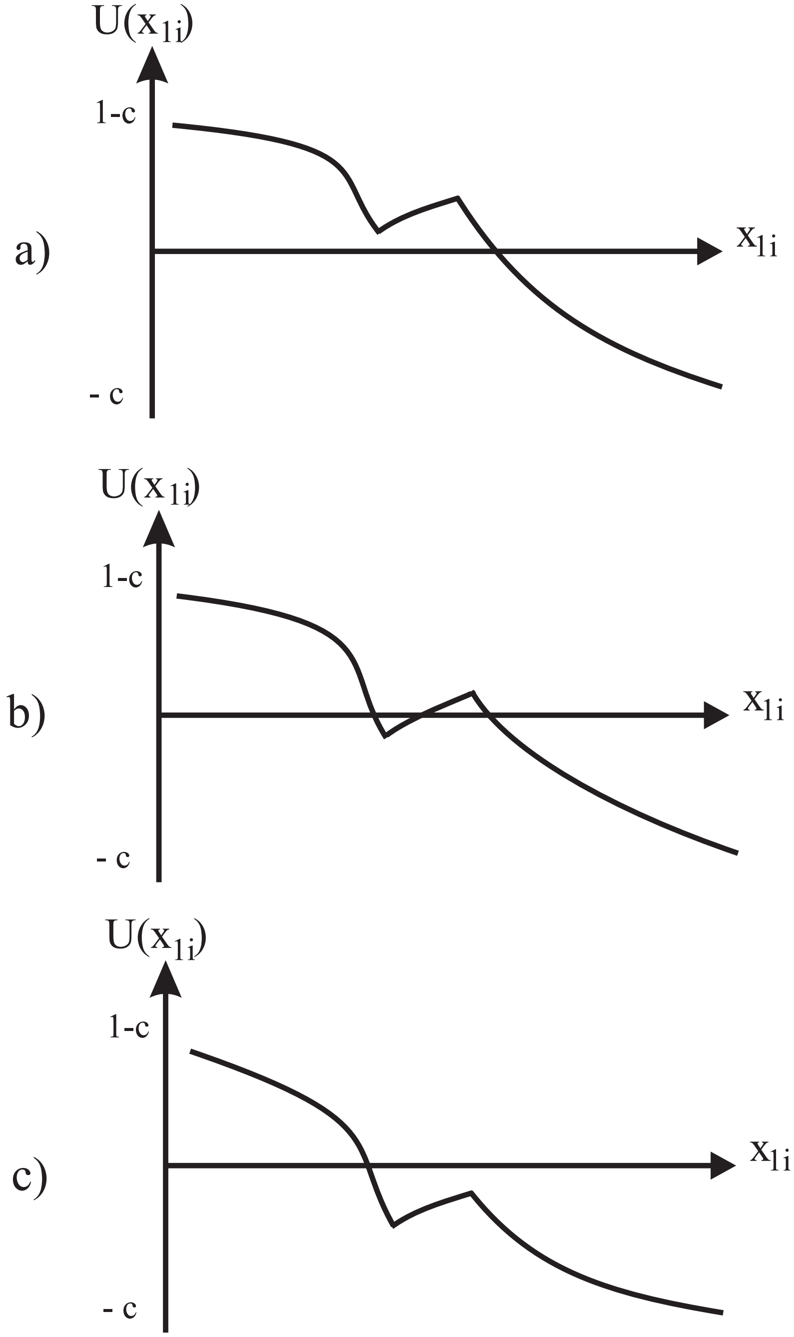

Figures: unique equilibrium in a) and c) at $U(x_{1,i})=0$,
multiple equilibria in b)