Milde, Hellmuth; Riley, John

Working Paper
Signalling in credit markets

Diskussionsbeiträge - Serie A, No. 185

Provided in Cooperation with:
Department of Economics, University of Konstanz

Universität
Konstanz

Fakultät für Wirtschaftswissenschaften und Statistik

Hellmuth Milde and John Riley

Signalling in Credit Markets

Diskussionsbeiträge

Postfach 5560
D-7750 Konstanz

Serie A — Nr. 185
August 1984

SIGNALLING IN CREDIT MARKETS

Hellmuth Milde and John Riley

Serie A - Nr. 185

August 1984

Professor John Riley is professor of economics, University of California, Los Angeles.
Helpful discussions with Ernst Baltensperger, Joseph Stiglitz, and Michael Waldman are gratefully acknowledged.

Serie A: Volkswirtschaftliche Beiträge
Serie B: Finanzwissenschaftliche Arbeitspapiere
Serie C: Betriebswirtschaftliche Beiträge

Stickiness of prices is an essential feature of Keynesian macroeconomics. Thus a major preoccupation of Keynesian theorists has been to provide a satisfactory microfoundation for price stickiness and rationing. One recent contribution to this literature, which has attracted considerable attention, is the analysis of credit rationing by Stiglitz and Weiss [1981]. These authors argue that loan applicants undertaking higher risk projects are more willing to accept higher contractual interest rates because of the higher probability of nonpayment via bankruptcy. As a result the loan market suffers from adverse selection. Therefore, under some circumstances, an increase in the interest rate to reduce excess demand will not be profitable because of the higher attendant risk.

However, this conclusion relies heavily on a special feature of the underlying model: the absence of any technological choices. Once we drop this assumption the stickiness result no longer arises. The purpose of this paper is to characterize equilibrium in a credit market when the project size is no longer exogenously given. Instead the underlying technology is assumed to be neoclassical, with output in each state of nature an increasing function of the size of the loan L. The latter therefore becomes a matter for negotiation between a bank and loan applicant. A loan agreement, $\langle L,R\rangle$, is then a contract to lend a particular amount L at some rate of interest $\underline{R}$. This raises the possibility that lenders can screen applicants by providing a schedule of alternative options.

The paper is organized as follows. In Section I a neoclassical model of the loan market is presented and the equilibrium is characterized under the assumption of symmetric information with respect to firm specific risks. In Section II we assume that loan applicants have private information about loan quality. It is shown that, in equilibrium, applicants with higher quality projects signal this by accepting larger loans.

The positive correlation between loan size and loan quality is not the only possible outcome. In Section III it is shown that, under alternative sets of assumptions about project characteristics, applicants with higher quality projects can signal by accepting smaller loans.

Our central conclusion, therefore, is that under rather broad assumptions, market equilibrium will involve sorting of loan applicants. In contrast Stiglitz and Weiss emphasize the problem of adverse selection and show how it can lead to a pooling equilibrium with rationing.

In Section IV we attempt a partial reconciliation of these different results by outlining a more general model in which sorting takes place but is necessarily incomplete.

I. A Neoclassical Model of the Credit Market

Consider a stylized model of the credit market with a neoclassical production technology. A large number of firms each seek to finance a one period investment project. The gross return, $\tilde{x}$ of a project is stochastic and increasing in the size of the loan L. We assume that the production function has the multiplicative form

$$\tilde{x} = q(\theta,L)\tilde{u}\ , \ \frac{\partial q}{\partial \theta}\ , \ \frac{\partial q}{\partial L} > 0, \tag{1}$$

where θ is a known quality parameter. (Later we shall assume that θ is known only to the applicant.) The size of the stochastic term $\tilde{u}$ is unknown, *ex ante*, to both the loan applicant and the bank.

To simplify the analysis we shall, at certain points, make the following assumptions about the distribution of $\tilde{u}$.

A1: The stochastic term u is nonnegative and has a cumulative distribution function $G(u)$ which is differentiable and strictly increasing wherever $0 < G < 1$.

A2: The hazard rate of G, $G'(u)/(1-G(u))$ increases with u and tends to infinity as $G(u)$ approaches unity.[1]

We shall also appeal to the following restrictions on the form of the production function.

B1: The production function, $q(\theta,L)u$ is a strictly increasing concave function of L, with $q(\theta,0) \geq 0$.[2] Also

$$\lim_{L\to 0} \left(\frac{q(\theta,L)}{L}\right) = \infty \quad \text{and} \quad \lim_{L\to\infty} \left(\frac{q(\theta,L)}{L}\right) = 0$$

Assumption B1 implies that the scale elasticity

$$\varepsilon = \frac{L}{q}\frac{\partial q}{\partial L}$$

satisfies $0 < \varepsilon < 1$.

B2: The scale elasticity, ε, is nondecreasing in θ and nonincreasing in L.

Assumptions B1 and B2 are both relatively weak. They are satisfied, for example, if

$$q(\theta,L) = b(\theta)L^{\beta}\ , \ 0 < \beta < 1$$

---

[1]This assumption is satisfied for a wide range of distribution functions, for example $G(u) = u^{\alpha}$, $\alpha > 0$ and $G(u) = 1 - (1-u)^{\alpha}$, $\alpha > 0$. Actually, all the results in Sections I and II hold under the very weak assumption that

$$\frac{d}{du}\left(\frac{G'(u)}{1-G(u)}\right) \geq -\frac{1}{u^2}$$

[2]If the loan applicant invests some of his own funds in a project, output, even in the absence of a loan, may be positive.

We assume that all lenders and loan applicants are risk neutral. As long as the gross return exceeds the interest cost, a successful loan applicant receives the difference $q(\theta,L)u - RL$. Thus a loan applicant's expected return is

$$A(L,R) = E \max \{q(\theta,L)\tilde{u} - RL, 0\} \tag{2}$$

where R is the contractual interest rate. Defining u^* to satisfy

$$q(\theta,L)u^* - RL = 0 \tag{3}$$

we can rewrite (2) as

$$A(L,R) = \int_{u^*}^{\infty} (q(\theta,L)u - RL)\, dG(u) \tag{4}$$

On the supply side of the market there is a large number of lenders (banks). Assuming that the opportunity cost of funds is I, the expected profit to a bank from the project, if it lends L at an interest rate R is

$$\Pi(L,R) = E \min \{RL, q(\theta,L)u\} - IL$$

Making use of (3) we can rewrite this expression as

$$\Pi(L,R) = (R-I)L + \int_{0}^{u^*} (q(\theta,L)u - RL)\, dG(u) \tag{5}$$

We proceed by analyzing the preference maps of a typical borrower and lender. In the Appendix we derive the following two Propositions.

Proposition 1: Loan Applicant's Indifference Curves

If assumptions A1–B2 hold then indifference curves for a loan applicant, $A(L,R) = \bar{A}$, drawn with L on the horizontal axis have a unique turning point at $L = L^a(R)$ and slope downwards for larger L. Moreover $L^a(R)$ is decreasing in R.

Proposition 2: Bank's Zero Iso-Profit Curve

If Assumption B1 holds, the iso-profit curve $\Pi(L,R) = 0$ is everywhere upward sloping.

Indifference curves for a loan applicant and iso-profit curves for a bank are depicted in Figure 1. For the applicant, a smaller interest rate is strictly preferable hence higher indifference curves are associated with a smaller expected gain. Note that the indifference curve $A(L,R) = A_2$ goes through the origin $\langle 0,0 \rangle$. Thus any contract above this curve would never be accepted by the applicant.

For the bank, larger interest rates are preferred, thus lower iso-profit contours are associated with a smaller expected gain. Since we are interested, here, in examining a competitive banking industry, equilibrium must be on the zero iso-profit contour, $\Pi(L,R) = 0$.

From (5), if $\Pi(L,R) = 0$, it follows that

$$R - I = \int_0^{u^*} - [q(\theta,L)u/L - R]\, dG(u).$$

From (3) the right hand side approaches zero with L. Thus the iso-profit contour $\Pi(L,R) = 0$ goes through the point $\langle 0,I \rangle$, as depicted. Since the bank would never accept any contract below this curve, the set of feasible loan contracts is the shaded region in Figure 1, bounded by $A = A_2$ and $\Pi = 0$.

We next ask which of these contracts is Pareto efficient. Graphically, we seek points $\langle L,R \rangle$ at which the indifference curves for applicant and bank are tangential. Note that the total surplus to be distributed is simply a function of the loan size L, that is, from (4) and (5)

$$A(L,R) + \Pi(L,R) = \int_0^{\infty} q(\theta,L)u\, dG(u) - IL$$

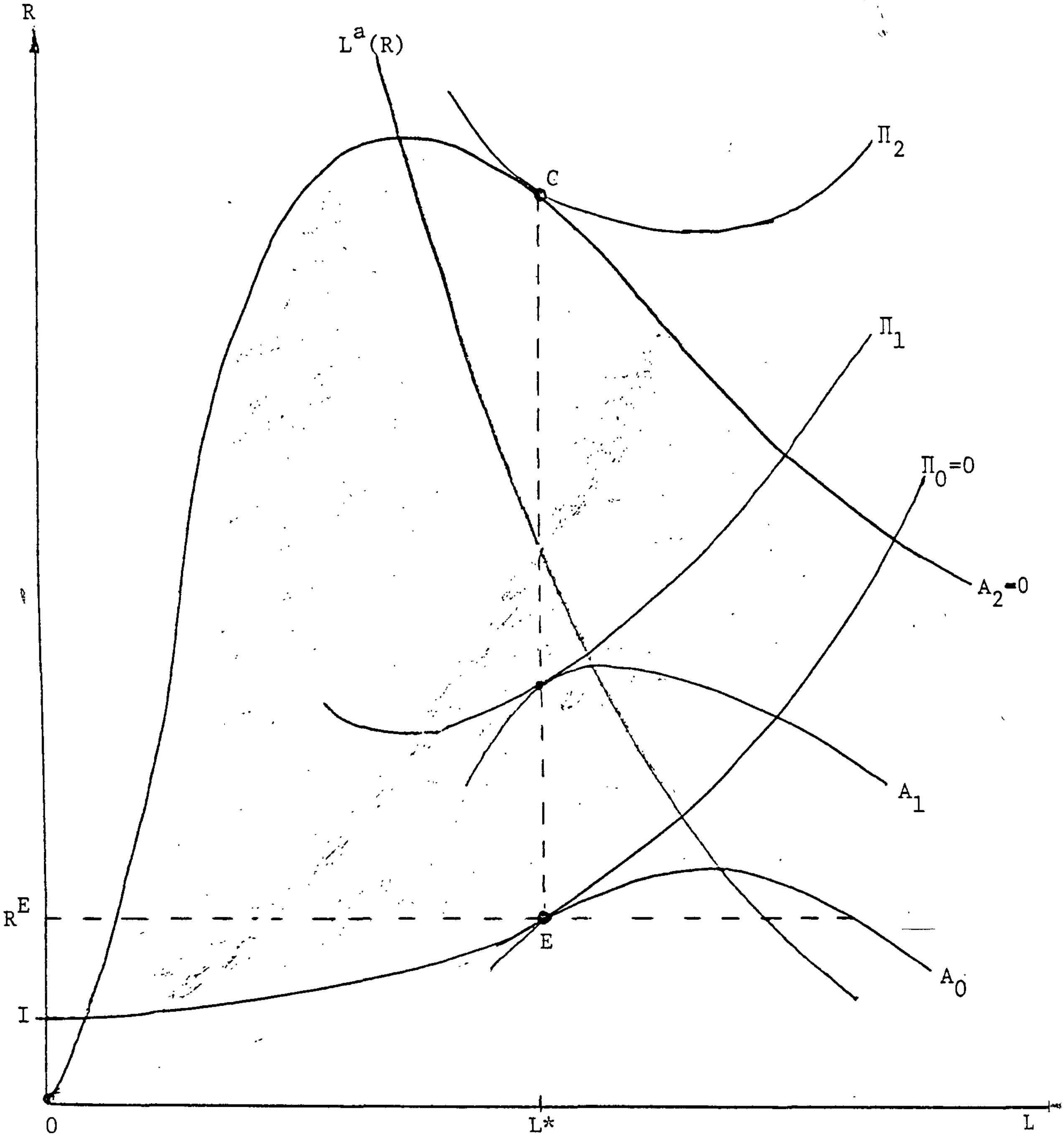

Figure 1: Preferences of Applicant and Bank

$$= q(\theta,L)\bar{u} - IL, \quad \text{where} \quad \bar{u} = Eu.$$

Thus the total gain, $A + \Pi$ is maximized by choosing L^* to satisfy

$$\frac{\partial q}{\partial L}(\theta,L^*)\bar{u} - I = 0 \tag{6}$$

and the contract curve is the vertical line EC.

Perfect competition among banks then results in the equilibrium contract being the point E in Figure 1, where the contract curve intersects $\Pi(L,R) = 0$.

From Proposition 2 the equilibrium contract E is a point at which the applicant's indifference curve has positive slope. Thus, at the equilibrium interest rate R^E, the applicant's optimal loan size, $L^a(R^E)$, exceeds L^*.

While it is perhaps tempting to say that loans are rationed at the equilibrium loan rate, this would be confusing, at best. The curve $L^a(R)$ does not represent the demand for a product of given quality, at different interest rates. Instead, holding R fixed, the quality of the product (the loan) varies as the size of the loan is increased.

To complete the analysis of the equilibrium with symmetric information, we examine the effect of an increase in the parameter θ. In the Appendix we prove:

<u>Proposition 3</u>: If Assumptions B1 and B2 hold, an increase in θ shifts the zero iso-profit contour to the right. Moreover the equilibrium loan $L^*(\theta)$ is strictly increasing in θ.

The effects of increasing θ are depicted in Figure 2. While the equilibrium for the preferred loan applicants, E_2, lies to the right of E_1, so that $L^*(\theta_2)$ exceeds $L^*(\theta_1)$, the equilibrium contractual interest rate, $R^E(\theta)$, may rise or fall with θ, depending on the precise form of the

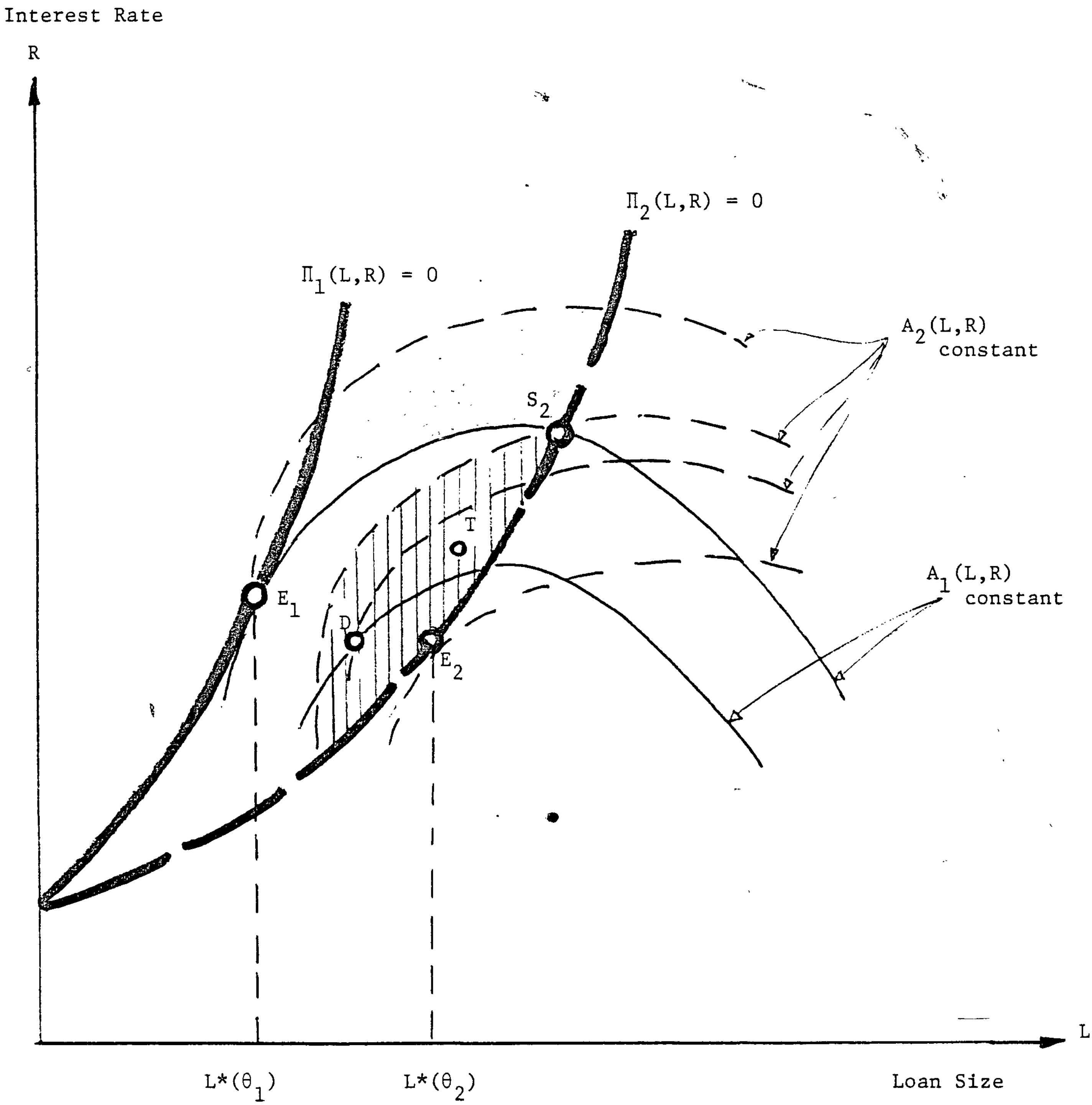

Figure 2: Credit Market Equilibrium

production function and the c.d.f. $G(u)$.

II. Equilibrium With Private Information - Loan Size as a Signal

We now depart from the assumption that information is imperfect but symmetric, and assume instead that each loan applicant has inside information about the quality of his project. Otherwise the bank itself could operate the project. This informational difference between applicant and bank is captured by the parameter θ. Each applicant is assumed to know his own θ, while banks know only the general underlying technology and the probability distribution of θ, $F(\theta)$.

To simplify the exposition we consider the case in which θ takes only 2 values, θ_1 and θ_2 with $\theta_2 > \theta_1$. From Figure 2 it is clear that $\{E_1, E_2\}$, the equilibrium pair of loan contracts, when θ_1 and θ_2 are observable, is not an equilibrium when information is private. Rather than accept the contract E_1, any applicant with a lower quality project is strictly better off accepting the contract E_2. But then E_2 results in losses for the bank.

To separate out the two types, the bank must therefore attempt to exploit differences in the shapes of their preference maps. In the Appendix we prove:

Proposition 4: Marginal Willingness to Pay for a Larger Loan

Given Assumptions, A1, A2 and B1 the increase in interest rate that a loan applicant is willing to accept in order to receive a loan is greater for more desirable projects (higher θ). Formally,

$$\frac{\partial}{\partial \theta} \frac{dR}{dL}\bigg|_A = \frac{\partial}{\partial \theta} \left(- \frac{\partial A/\partial L}{\partial A/\partial R}\right) > 0$$

This difference in the willingness to pay for a larger loan is illustrated in Figure 2. At each point of intersection of an unbroken indifference curve $(\theta = \theta_1)$ and a dashed indifference curve $(\theta = \theta_2)$ the latter has a greater slope. As a result, there is a set of contracts (the shaded region in Figure 2), each member of which (i) is strictly preferred over E_1 only by applicants with high quality projects and (ii) yields expected profits to a bank. Of these, the contract S_2 yields the greatest gains to the high quality applicants. Indeed it can be shown that, of all pairs of contracts which separate the two types, and individually at least break even, the pair $\{E_1, S_2\}$ is Pareto efficient.

While this paper is not the place for an extensive discussion of the theoretical debate about equilibrium in signalling models (see, in particular, Wilson [1977], Riley [1979a, 1984] and Stiglitz and Weiss [1983]) a few remarks are in order.

As emphasized in the early papers by Riley [1975] and Rothschild and Stiglitz [1976], there may be no Nash equilibrium if, as assumed here, the less informed agents (banks) move first, announcing a schedule of loan offers. This point is easily made for the simple case of high and low quality loan applicants. Suppose that banks initially offer the Pareto efficient separating pair $\{E_1, S_2\}$, as depicted in Figure 2. Consider the alternative D in the vertically shaded region. Since D lies above $\Pi_2(L,R) = 0$ and below the indifference curve of high quality applicants through S_2, these applicants would be attracted by such an offer and would generate profits. But D would also attract low quality applicants and these would generate losses (since D lies below $\Pi_1(L,R) = 0$). If the proportion of low quality loan applicants is sufficiently large, the overall profit is negative and so $\{E_1, S_2\}$ is a Nash equilibrium. However, this is no longer an equilibrium if the proportion of low quality applicants is small, for then the overall profit from the alternative offer, D, is positive.

Despite the potential gains from introducing the offer D, we wish to argue that banks are likely to be deterred from such a "defection" from the loan schedule $\{E_1, S_2\}$. The reason is that, with one bank offering D, another can exploit the fact that high quality applicants have steeper indifference curves through D and react with an offer such as T, also depicted in Figure 2. Note that T "skims the cream", attracting high quality (profitable) applicants, leaving only low quality applicants choosing D. As a result, the original "defecting" bank ends up losing money.

Moreover, and this is a crucial point, since the reaction, T, generates profits on each applicant accepting, there can be no further reactions by other banks which result in losses. At worst, offers superior to T bid applicants away and profits are zero.

As long as the defector recognizes that there is no risk of loss associated with a reaction such as T, his incentive to offer D disappears. The original pair of contracts $\{E_1, S_2\}$ is then a "reactive equilibrium" (Riley [1979a]).

More formally we have the following definition

Reactive Equilibrium

A set of loan contracts, C, is a reactive equilibrium if, for any additional set D which is profitable when $D \cup C$ is offered, there is a further set T such that, when $T \cup D \cup C$ is offered, D generates losses and each loan contract signed in T is strictly profitable.

As argued in Riley [1979a], or, under weaker assumptions by Engers and Fernandez [1984] we have

Proposition 5: Existence of a Reactive Equilibrium

Suppose there are n types of project and Assumptions A1-B2 hold. Then the set of loan contracts which is Pareto efficient for the loan applicants, among those sets of contracts which individually at least break even and separate the n types, is the unique Reactive equilibrium.[3]

III. Signalling With Loan Size and Quality Inversely Related

Certainly the strongest of the assumptions made in the previous sections is that the production function $x(\theta,L,u)$, should take the multiplicative form

$$x = q(\theta,L)u$$

While it seems highly plausible that qualitatively similar results will hold under much weaker assumptions, it is not enough to assume that $x(\theta,L,u)$ is a strictly increasing concave function.

In fact, as we shall now show, it is quite possible that applicants with lower quality projects are willing to pay a larger interest rate premium for a larger loan. In such a world applicants with higher quality projects can signal by accepting a smaller rather than a larger loan.

Suppose that the production technology is of the form

$$\tilde{x} = \beta(L)L + \tilde{u}L + \theta \tag{7}$$

We assume that the net return $\tilde{x} - IL$ is concave and decreasing for L sufficiently large. Therefore the expected net return is maximized with a

---

[3]Wilson [1977] and Miyasaki [1977] have suggested alternative equilibrium concepts, each of which involve some form of anticipation by banks of the responses to their actions. For an introductory discussion of the different concepts see Riley [1979b]. For the class of models considered here, there is a unique equilibrium of each type.

loan size, L** satisfying

$$\frac{\partial}{\partial L}\{E\tilde{x} - IL\} = \frac{\partial}{\partial L}\{\beta(L)L - \bar{u}L - IL + \theta\}$$

$$= \{1 + \varepsilon(\beta, L^{**})\}\ \beta(L^{**}) + \bar{u} - I$$

$$= 0$$

Note that L** is independent of the loan quality parameter θ.

Arguing almost exactly as in the previous section, we can establish that the zero iso-profit contour shifts rightwards as θ increases. Thus the equilibrium, when both banks and applicants know θ, is as depicted in Figure 3.

We also have the following counterpart of Proposition 4.

Proposition 6: Marginal Willingness to Accept a Smaller Loan

Given Assumptions A1 and A2 and the production technology, (7), the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for more desirable projects (greater θ).

In graphical terms, the indifference curve for a low quality loan applicant, at any point <L,R> is steeper than the indifference curve for a high quality applicant. As in the previous section, $\{E_1, E_2\}$, the equilibrium with θ observable, is no longer feasible since all loan applicants prefer E_2 to E_1. However, given the difference in the preference maps there is again a set of loan contracts which, if offered in conjunction with E_1, separate out the two types of applicant. This is the shaded region in Figure 3. Then the best pair of separating contracts, which also at least break even, is the pair $\{E_1, S_2\}$. From Proposition 5 this is the unique Reactive equilibrium. Moreover, if the proportion of low quality applicants

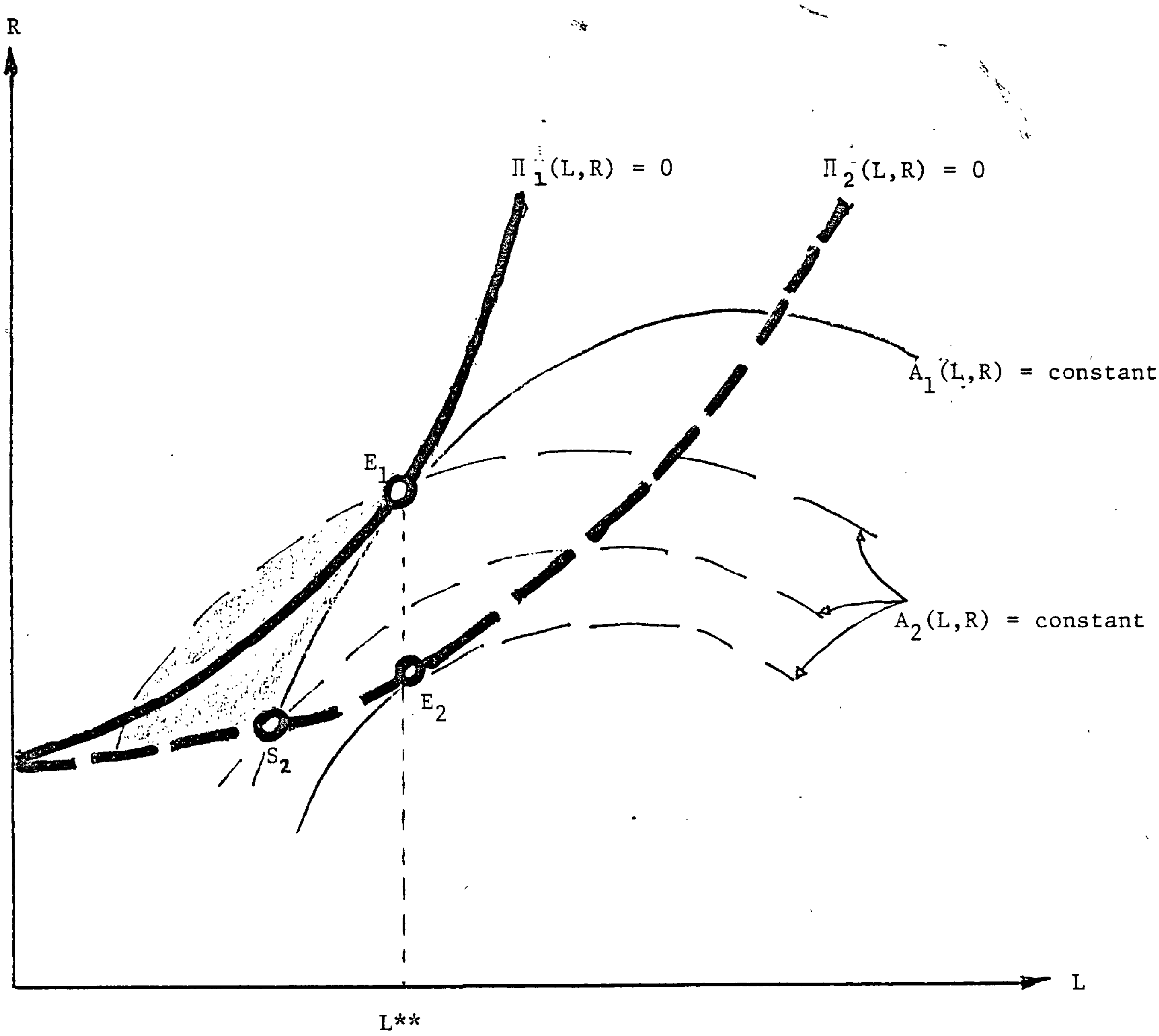

Figure 3: Credit market equilibrium with smaller loans signalling higher quality.

is sufficiently high, $\{E_1, S_2\}$ is also the unique Nash equilibrium.

In both of the above examples, better risks are better in the sense of first order stochastic dominance. We now provide an example in which both projects have the same mean but the first is more risky in the sense of second order stochastic dominance. This is precisely the assumption made by Stiglitz and Weiss.

As these authors argued, the concavity of the bank's profit function $\pi(x) = \min\{x, RL\} - IL$, implies that the less risky loan will be the preferred loan.

We return to a special case of the original formulation of the model with output,

$$\tilde{x}_\theta = q(L)\tilde{v}_\theta, \quad \theta = 1,2$$

where $\tilde{v}_\theta$ has a cumulative distribution function $F_\theta(v)$. In addition to assuming that $F_1(v)$ is a mean preserving spread of $F_2(v)$ we also assume that the expectation $E\{v \mid v > x\}$ is, for positive x, higher for the more risky loan.[4]

We can then derive

<u>Proposition 7</u>: Suppose $F_1(v)$ is a mean preserving spread of $F_2(v)$.

Suppose, also, that the conditional mean

$$E\{v \mid v > x\} = \int_x^\infty \frac{v dF_\theta(v)}{1-F_\theta(x)}$$

[4]This additional restriction is easily satisfied. For example it holds for the family of uniform distributions

$$F_\theta(v) = \theta(v-\bar{v}) + \frac{1}{2}; \quad |v-\bar{v}| < \frac{1}{2\theta}$$

is for all $x > 0$, higher for $\theta = 1$ than $\theta = 2$. Then if Assumptions A1, B1 and B2 hold, the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for the project which is more desirable for the bank $(\theta=2)$.

Comparing Propositions 6 and 7 it follow that the signalling equilibrium is essentially as depicted in Figure 3.

IV. Concluding Remarks

In the model analyzed by Stiglitz and Weiss, banks identify a set of projects which have equal fixed borrowing requirements $(L=L_1)$ and equal expected returns per dollar invested $(E\tilde{x} = \mu_1)$. Since a loan applicant's return, $\max\{x-RL_1, 0\}$ is a convex function of x, while a bank's profit, $\min\{RL_1, x\} - IL_1$ is a concave function of x, less risky loans (i) are preferred by banks, and (ii) yield lower expected returns to loan applicants. Thus, if the nominal cost of borrowing, R, is raised, it is the preferred applicants who drop out. As a result, beyond some interest rate R_1, depicted in Figure 4, the expected gross yield $\rho_1(R)$, declines as R increases.

Suppose the cost of loanable funds is $\bar{I}$. While bank profits are maximized by charging a nominal interest rate an excess of $\bar{R}$, competition for borrowers forces the nominal rate to this level (ignoring banking costs). Then, given the inverse loan demand curve $R_1(L)$, the derived demand for funds by the banking industry is $D_1(\bar{I})$. For $I < I_1$, $D_1(I)$ declines smoothly as I increases. However, at $I = I_1$, the derived demand drops to zero. The derived demand for funds is then depicted as in Figure 5.

Also depicted in Figure 5 is the derived demand for funds from some other set of projects with lower mean return μ_2. Under weak assumptions the yield

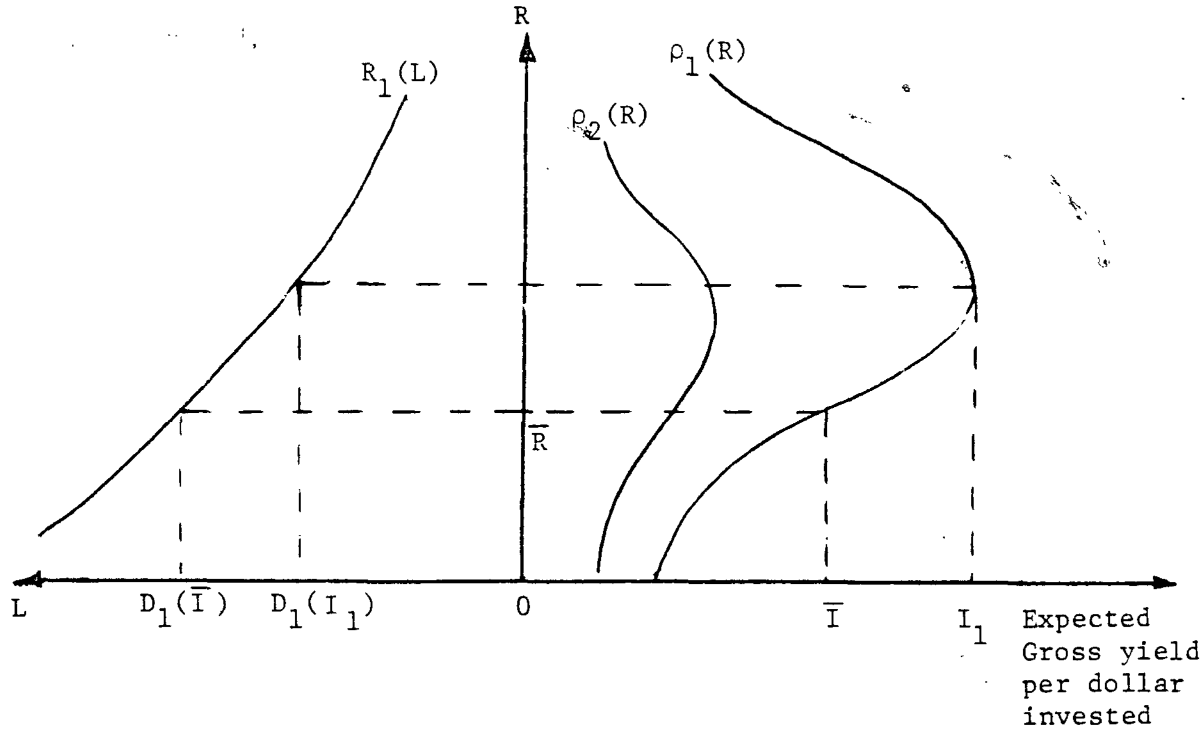

Figure 4: Deriving The Banking Industry's Demand For Loanable Funds

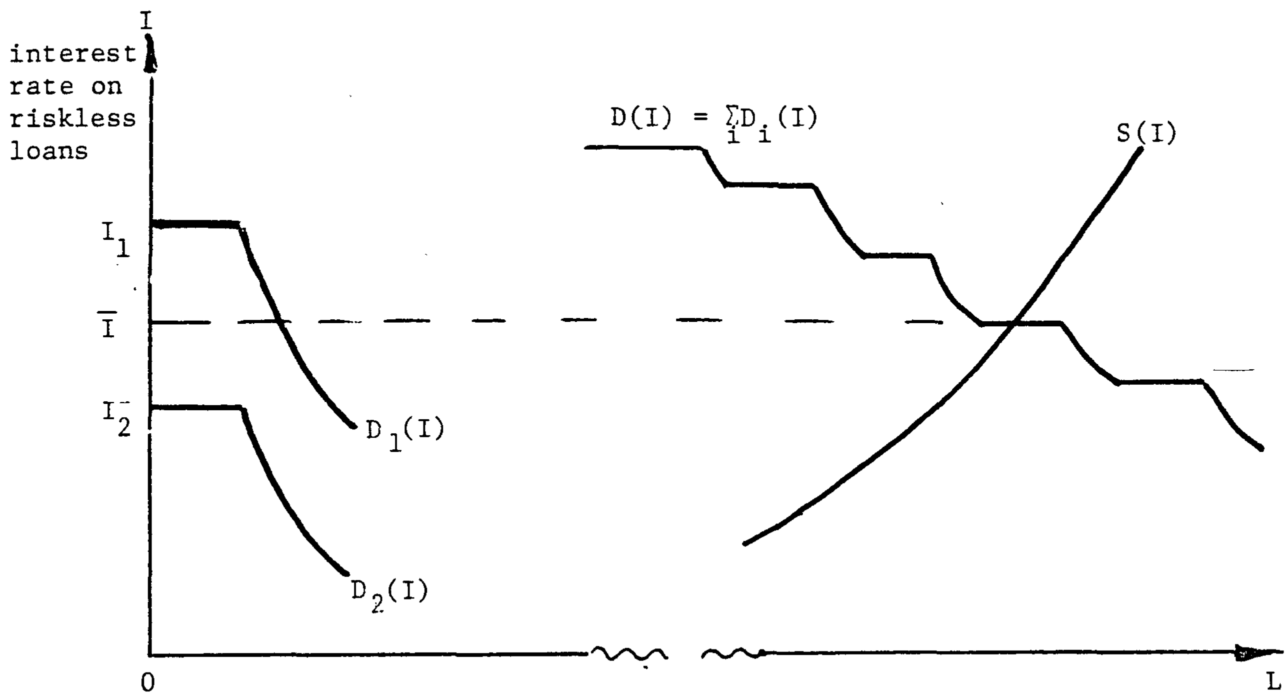

Figure 5: Equilibrium In The Market For Loanable Funds

curve for this second group, $\rho_2(R)$ lies to the left of $\rho_1(R)$ and hence I_2 is less than I_1. Aggregating over a large number of subsets of projects we obtain the banking industry's aggregate derived demand for funds $D(I) = \sum_i D_i(I)$. This is also depicted in Figure 5 along with the deposit supply curve $S(I)$ and the equilibrium cost of borrowing $\bar{I}$.[5] If, as depicted, the two curves intersect on a flat segment of the aggregate loan demand curve, there is rationing. But note that it is only the marginal subset of funded projects which are rationed. It seems reasonable to infer, therefore, that the number of projects which are not funded due to rationing will be a very small fraction of the number of funded projects. Given this, in combination with the unusual nature of their production technology, we conclude that the Stiglitz-Weiss analysis does not, by itself, provide a plausible explanation of rationing.[6]

One puzzling feature of the Stiglitz-Weiss model, at least to a Bayesian, is the assumption that banks can form beliefs about mean returns without simultaneously forming beliefs about higher moments as well. Surely it is more plausible that informational asymmetry will involve uncertainty about all moments of the distribution. As we have seen, it is easy to construct models in which such uncertainty can be overcome with the introduction of a schedule

[5]Note that an upward shift in the supply curve, and hence an increase in $\bar{I}$ effects not just the marginal group of applicants, but inframarginal groups as well. This is clear from Figure 4. Any increase in $\bar{I}$ leads to a higher equilibrium nominal interest rate for the group and hence further adverse selection.

[6]Of course the nominal interest rates offered to preferred applicants groups would be happily accepted by a group with a low mean return, μ_2. However, because this group has a lower mean return, the expected gross yield curve (depicted in Figure 4) is closer to the vertical axis. Thus, for any cost of borrowing, I, the nominal interest rate that banks can offer and still break even is higher.

of alternative loan opportunities. Instead of applicants with preferred projects withdrawing from the market, banks are able to exploit differences in the opportunity cost of a larger loan and so sort out different risk classes.

However, in all the models presented, differences in projects are captured by a single parameter θ. A more general model would allow for a wider range of differences. For example, suppose the random gross return on a project

$$\tilde{x}(\theta,\alpha) = \theta q(L)\tilde{u}_{\alpha}$$

where $\hat{\alpha} > \alpha$ implies that $F_{\alpha}(u)$ is a mean preserving spread of $F_{\hat{\alpha}}(u)$. From the analysis above it is clear that some sorting will take place. But it should also be clear that the single instrument, loan size, cannot signal two unobservable characteristics. Thus sorting is necessarily incomplete.

Unfortunately, there is, as yet, no formal theory of equilibrium signalling with multiple unobservable characteristics.[7] Despite this, it seems reasonable to suppose that, as with a single unobservable characteristic, there will be an equilibrium loan offer schedule $\langle L,R(L)\rangle$. With multiple unobservable characteristics, each element of this schedule will be selected by a heterogeneous group of applicants. As Stiglitz and Weiss emphasize in their analysis, heterogeneity can lead to adverse selection. It thus seems plausible that the equilibrium of a more general model will involve partial screening and also adverse selection. This then suggests the possibility that a more convincing explantion of rationing might emerge from such a model.

[7]Engers [1984] proves the existence and uniqueness for the n-signal, n characteristic case, but only under strong simplifying assumptions.

One final remark is in order concerning the possible role of collateral. A straightforward extension of the model developed in Section I can be used to establish that the reduction in interest rate necessary to make an applicant willing to offer greater collateral is equal to the odds of bankruptcy (see also Bester [1984]).[8] Thus a very general model of the loan market will include not only loan size but also collateral as a potential signal of loan quality.

[8]With production function $\tilde{x} = q(L)\tilde{v}$ and collateral level C, a loan applicant's expected return can be expressed as

$$A(L,R,C) = -C + \int_{v^*}^{\infty} [vq(L) - RL + C]dF(v)$$

where a firm is bankrupt if and only if $v < v^*$, that is,

$$q(L)v^* = RL - C$$

Therefore,

$$\left.\frac{dC}{dR}\right|_A = - \frac{\partial A}{\partial C} \Big/ \frac{\partial A}{\partial R} = F(v^*)/(1-F(v^*)L$$

APPENDIX

Lemma 1: If Assumption A2 holds, that is the hazard rate of G, $G'/(1-G)$ is increasing, then for any x such that $G(x) < 1$,

$$(8) \qquad H(x) \equiv \frac{\int_x^{\infty} (1-G(u))du}{1 - G(x)},$$

is a decreasing function of x.

Proof: Let $\hat{x}$ be the upper support of $G(u)$. By l'Hôpital's rule

$$H(\hat{x}) = \lim_{x \to \hat{x}} \frac{-(1-G(\hat{x}))}{-G'(\hat{x})} = 0,$$

since, by Assumption A2, $G'/(1-G)$ tends to infinity as $x \to \hat{x}$.

Differentiating (8) by x we obtain

$$(9) \qquad H'(x) = -1 + \frac{G'(x)}{1-G(x)} H(x)$$

Differentiating again

$$(10) \qquad H''(x) = \frac{d}{dx} \left(\frac{G'}{1-G}\right) H(x) + \left(\frac{G'}{1-G}\right) H'(x)$$

By hypothesis, $G'/(1-G)$ is increasing. Therefore, whenever $H(x)$ is positive the first term on the right hand side of equation (10) is positive. Then

$$H'(x) = 0 \Rightarrow H''(x) > 0.$$

Thus any turning point of $H(x)$ is a minimum. But $H(x) > 0$ and $H(1) = 0$. Thus there can be no turning points and hence $H(x)$ is decreasing everywhere.

Q.E.D.

Proposition 1: Loan Applicant's Indifference Curves

If assumptions A1–B2 hold then indifference curves for a loan applicant, $A(L,R) = \bar{A}$, drawn with L on the horizontal axis, have a unique turning point at $L = L^a(R)$ and slope downwards for larger L. Moreover $L^*(R)$ is decreasing in R.

Proof: From equation (4)

$$A(L,R) = q(\theta,L) \int_{u^*}^{\infty} u\,dG(u) - (1-G(u^*))RL \tag{11}$$

where, from (3)

$$u^* = \frac{RL}{q(\theta,L)} \tag{12}$$

Since q is concave q/L declines with L and hence u^* is an increasing function of L. Also since q is increasing in θ u^* is a decreasing function of θ. Summarizing, we have

$$\frac{\partial u^*}{\partial L} > 0 \ , \ \frac{\partial u^*}{\partial \theta} < 0 \tag{13}$$

From (11)

$$\left.\frac{dR}{dL}\right|_A = \frac{-\frac{\partial A}{\partial L}}{\frac{\partial A}{\partial R}} = \frac{\frac{\partial q}{\partial L} \int_{u^*}^{\infty} u\,dG - (1-G(u^*))R}{(1-G(u^*))L} \tag{14}$$

$$= \frac{R}{L}\left[\frac{L}{q}\frac{\partial q}{\partial L}\frac{\int_{u^*}^{\infty} u\,dG}{u^*(1-G(u^*))} - 1\right]$$

Also, by assumption B1

$$\lim_{L\to\infty} q(\theta,L)/L = 0$$

Hence $\lim_{L\to\infty} u^* = \infty$

Integrating by parts

$$\int_{u^*}^{\infty} u dG = u^*(1-G(u^*)) + \int_{u^*}^{\infty} (1-G(u)du$$

Substituting this expression into (14) we therefore obtain

$$\left.\frac{dR}{dL}\right|_A = \frac{R}{L} \left[\frac{L}{q} \frac{\partial q}{\partial L} (1 + \frac{H(u^*)}{u^*}) - 1 \right] \tag{15}$$

where H(.), defined in Lemma 1 is a decreasing function.

By hypothesis the scale elasticity $\varepsilon = \frac{L}{q} \frac{\partial q}{\partial L}$ is nonincreasing in L. Moreover, from (13) u^* is increasing in L and hence $H(u^*)/u^*$ is decreasing in L. Therefore, from (15)

$$\left.\frac{dR}{dL}\right|_A = 0 \implies \frac{\partial}{\partial L} \left(\left.\frac{dR}{dL}\right|_A\right) < 0$$

Thus, drawn with L on the horizontal axis, a loan applicant's indifference curves are strictly quasi concave.

From Lemma 1 $H(u^*) = 0$ for u^* sufficiently large. Therefore, for L sufficiently large it follows from (15) that

$$\left.\frac{dR}{dL}\right|_A = \frac{R}{L} \left[\frac{L}{q} \frac{\partial q}{\partial L} - 1 \right]$$

Thus by Assumption B1, the slope is negative for sufficiently large L.

Finally, from (12), u^* is increasing in R and hence $H(u^*)/u^*$ is decreasing in R. Therefore, from (15),

$$\left.\frac{dR}{dL}\right|_A = 0 \implies \frac{\partial}{\partial R} \left(\left.\frac{dR}{dL}\right|_A\right) < 0$$

Thus $L^a(R)$ is decreasing in R

Q.E.D.

Proposition 2: Bank's Zero Iso-Profit Curve

If Assumption B1 holds, the iso-profit curve $\Pi(L,R) = 0$ is everywhere upward sloping.

Proof: From equation (5)

$$\left.\frac{dR}{dL}\right|_{\Pi} = - \frac{\frac{\partial \Pi}{\partial L}}{\frac{\partial \Pi}{\partial R}} = - \frac{R-I + \int_0^{u^*} [\frac{\partial q}{\partial L} u - R] dG(u)}{L(1-G(u^*))}$$

$$= - [\frac{(R-I)L + {}_0\int^{u^*} (\varepsilon q(\theta,L)u - RL) dG(u)}{L^2(1-G(u^*))}]$$

$$= \frac{- \Pi(L,R) + (1-\varepsilon)\, {}_0\int^{u^*} qudG(u)}{L^2(1-G(u^*)}$$

By Assumption B1 the scale elasticity ε is less than unity. Therefore

$$\Pi(L,R) = 0 \Rightarrow \left.\frac{dR}{dL}\right|_{\Pi} > 0$$

Q.E.D.

Proposition 3: If Assumptions B1 and B2 hold, an increase in θ shifts the zero iso-profit contour to the right. Moreover the equilibrium loan $L^*(\theta)$ is strictly increasing in θ.

Proof: We first show that, for any R, the zero iso-profit curve shifts to the right. Totally differentiating

$$\Pi(L,\bar{R},\theta) = 0$$

by θ we obtain

$$\frac{\partial \Pi}{\partial L}\frac{dL}{d\theta} + \frac{\partial \Pi}{\partial \theta} = 0 \tag{16}$$

But, from Proposition 2

$$\left.\frac{dR}{dL}\right|_{\Pi=0} = - \frac{\frac{\partial\Pi}{\partial L}}{\frac{\partial\Pi}{\partial R}} > 0$$

Then, since $\partial\Pi/\partial R$ is positive, $\partial\Pi/\partial L$ is negative. From (5) $\partial\Pi/\partial\theta$ is positive and so, from (16) $dL/d\theta$ is positive.

To complete the proof we must show that the efficient loan size $L^*(\theta)$ is increasing. This follows directly from (6), since, from Assumption B2, $\frac{\partial}{\partial\theta}(\frac{\partial q}{\partial L}) > 0$. Q.E.D.

Proposition 4: Marginal Willingness to Pay for a Larger Loan

Given Assumptions A1, A2 and B1 the increase in interest rate that a loan applicant is willing to accept in order to receive a loan is greater for more desirable projects (higher θ). Formally,

$$\frac{\partial}{\partial\theta}\left.\frac{dR}{dL}\right|_{A} = \frac{\partial}{\partial\theta}\left(-\frac{\partial A/\partial L}{\partial A/\partial R}\right) > 0$$

Proof: From (15)

$$\left.\frac{dR}{dL}\right|_{A} = \frac{R}{L}\left[\varepsilon\left(1 + \frac{H(u^*)}{u^*}\right) - 1\right]$$

From (13) u^* declines as θ increases. Therefore, since $H(.)$ is a decreasing function, $H(u^*)/u^*$ increases as θ increases. Finally, given Assumption B2, the scale elasticity, ε, is nondecreasing in θ. Therefore

$$\frac{\partial}{\partial\theta}\left(\left.\frac{dR}{dL}\right|_{A}\right) > 0$$

Q.E.D.

Proposition 6: Marginal Willingness To Accept A Smaller Loan

Given Assumptions A1 and A2 and the production technology, (7), the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for more desirable projects (greater θ).

Proof: From (7), the expected gain to a loan applicant is

(17) $$\bar{A}(L,R) = \int_{\hat{u}}^{1} (\beta(L)L + uL + \theta - RL)\, dG(u)$$

where

(18) $$\beta(L)L + \hat{u}L + \theta - RL = 0$$

Then

$$\left.\frac{dR}{dL}\right|_{\bar{A}} = -\frac{\frac{\partial \bar{A}}{\partial L}}{\frac{\partial \bar{A}}{\partial R}} = \int_{\hat{u}}^{1} \frac{[\beta + L\beta' + u - R]\, dG(u)}{(1-G(\hat{u}))L}$$

$$= [\beta + L\beta' - R + \frac{\int_{\hat{u}}^{1} u dG(u)}{(1-G(\hat{u}))L}]/L$$

$$= [\beta + L\beta' - R + \hat{u} + H(\hat{u})] \,/\, L$$

Hence

$$\frac{\partial}{\partial \hat{u}} \left(\left.\frac{dR}{dL}\right|_{\bar{A}}\right) = \frac{1 + H'(\hat{u})}{L}$$

From (18) an increase in θ lowers $\hat{u}$. From (9), $1 + H'(\hat{u})$ is positive. Therefore

$$\frac{\partial}{\partial \theta} \left(\left.\frac{dR}{dL}\right|_{\bar{A}}\right) < 0.$$

Q.E.D.

Proposition 7: Suppose $F_1(v)$ is a mean preserving spread of $F_2(v)$. Suppose, also, that the conditional mean

$$E\{v \mid v > x\} = \int_{x}^{\infty} \frac{vdF_{\theta}(v)}{1-F_{\theta}(x)}$$

is for all $x > 0$, higher for $\theta = 1$ than $\theta = 2$. Then if Assumptions A1, B1 and B2 hold, the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for the project which is more desirable for the bank $(\theta = 2)$.

Proof:

$$E(v \mid v > x) = \frac{\int_x^\infty v dF_\theta(v)}{1-F_\theta(x)}$$

$$= x + \frac{\int_x^\infty (1-F_\theta(v))dv}{1-F_\theta(x)}$$

$$= x + H_\theta(x)$$

By hypothesis the left hand side is larger for $\theta = 1$, and $x > 0$, therefore, $H_1(x) > H_2(x)$ with a strict inequality for $x > 0$. That $\left.\frac{dR}{dL}\right|_A$ is larger for $\theta = 1$ then follows directly from (15).

Q.E.D.

References

Bester, H., "Screening Versus Rationing in Credit Markets With Imperfect Information," University of Bonn Discussion Paper (1984).

Engers, M., "Equilibrium With Asymmetric Information," unpublished dissertation UCLA (1984).

__________, and L. Fernandez, "On the Existence and Uniqueness of Signalling Equilibrium," mimeo (1984).

Miyazaki, H., "The Rat Race and Internal Markets," Bell Journal of Economics, 8, (1977), 394-418

Riley, J.G., "Competitive Signalling," J. Econ. Theory, 10 (1975), 174-186.

__________, "Informational Equilibrium," Econometrica, 47 (1979a), 331-360.

__________, "Noncooperative Equilibrium and Market Signalling," American Economic Review Papers and Proceedings, 69 (1979b), 303-307.

__________, "Competitive Signalling Reconsidered," UCLA Working Paper #294 (revised March 1984).

Rothschild, M. and J.E. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 80 (1976), 628-649.

Spence, A.M., Market Signalling: Information Transfer in Hiring and Related Processes, (Cambridge, MA: Harvard University Press, 1973).

Stiglitz, J.E. and A. Weiss, "Credit Rationing in Markets With Imperfect Information," American Economic Review, 71 (1981), 393-410.

__________, "Sorting Out the Differences Between Screening and Signalling Models," Bell Laboratories Working Paper, August (1983).

Wilson, C.A., "A Model of Insurance Markets With Incomplete Information," Journal of Economic Theory, 16 (1977), 167-207.