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Academia or the private sector? Sorting of agents into output-maximizing institutions

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Academia or the Private Sector? Sorting of Agents into Output-Maximizing Institutions

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Academia or the Private Sector? Sorting of Agents into Institutions and an Outside Sector*

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Abstract

This paper develops an equilibrium sorting model with utility maximizing researchers who differ in their ability on one side of the market, and on the other side universities and an outside sector. In equilibrium, the top of the ability distribution is allocated to the academic sector, while the bottom of the ability distribution is allocated to the outside sector. For low values of the outside option, the academic sector is unaffected by it and exit from academia happens involuntarily. For higher values of the outside option, the academic sector is constrained by the outside sector and the universities are unable to satisfy their demands for researchers. In this case all exit from academia is voluntary and, furthermore, an increase in the value of the outside option decreases the difference in average quality between the higher and lower ranked universities.

1 Introduction

Individuals with doctoral degrees are not only employed by the academic sector but also by the private sector. Some of those who exit from academia choose to do so voluntarily, while others would have preferred to work in the academic sector. Whether a particular individual is employed by the academic sector or has exited to the outside sector voluntarily or involuntarily typically depends on the individual's ability to do research.

This paper takes these ideas and develops an equilibrium sorting model with utility maximizing agents who differ in their ability on one side of the market, and on the other side institutions and an outside sector. Agents are assumed to care about both peer effects and their relative status within institutions. Subsequently, I use the interpretation of the

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agents as researchers and the institutions as universities. The model is, however, applicable to any setting where peer effects and ranking concerns are relevant. Thus, an alternative interpretation could, for example, be one of the agents as musicians and the institutions as orchestras.

I solve for the allocation of researchers' ability across sectors and within the academic sector as well as for how changes in the outside option affects this allocation. The analysis enables comparative statics analysis between fields with different attractiveness of the outside option.

The most important results are the following: The top of the ability distribution is allocated to the academic sector, while the bottom of the ability distribution is allocated to the outside sector. For low values of the outside option, the academic sector is unaffected by it and exit from academia happens involuntarily. For higher values of the outside option, the academic sector will be affected by it and by changes in its value. In particular, the universities will be unable to satisfy their demands for researchers, all exit from academia is voluntary, and when the value of the outside option increases the difference in average quality between the higher and lower ranked universities decreases.

This paper builds on the work of Damiano, Li, and Suen (forthcoming), who also assume that researchers care about both peer effects and their relative status within institutions. The weight researchers assign to status relative to peer effect need not be interpreted purely as stemming from the researchers' personality, it can also be interpreted to incorporate e.g. technological innovation. For example, if the internet facilitating joint work with researchers at other institutions has lead to the peer effect being less important, it could be interpreted as an increase in the weight on status relative to peer effect.

The sorting equilibrium concept for the allocation of researchers used in this paper is the same as in Damiano et al. (forthcoming), extended to account for the outside option. Damiano et al. do not allow for an outside option, and thus they cannot address a number of the questions addressed by the present paper. This will be discussed further after I present my results.

In the present paper, a researcher derives non-pecuniary utility from being matched to a university, a feature that can also be found in Besley and Ghatak (2005). The importance of peer effects is, for example, demonstrated in Sarpca (2007), who considers peer effects between students. Various aspects of the problems pertaining to universities and researchers have been addressed by other authors, including incentives for researchers (Lazear, 1997), dynamic incentives (Carayol, 2008), the tenure system (Carmichael, 1988), stratification into research vs. teaching institutions (Del Rey, 2001), and the game between administration and faculty (Ortmann and Squire, 2000). These issues are thus not the focus of the present paper. I choose to focus on the effects of the outside option.

The paper is organized as follows. In section 2, I describe the sorting model. In section 3, I characterize the allocation of researchers between the two universities and the outside sector. Section 4 concludes.

2 Model

Consider a sorting model with a continuum of researchers on one side and two universities and an outside sector on the other side. Each researcher seeks to maximize his utility by choosing the job among his offers which is the best match.

There is a continuum of researchers of mass 1. Researchers are characterized by a one-dimensional variable θ , interpreted as ability, which is identically and independently distributed on the interval $[\underline{\theta}, \bar{\theta}]$. Let $F(\theta)$ denote the cumulative distribution function.

The two universities are named A and B . A researcher has utility function

$$V_i(\theta) = \alpha r_i(\theta) + m_i + w,$$

where $i = A$ if the researcher works for university A and $i = B$ if the researcher works for university B , m_i is the average ability of those employed by university i , $r_i(\theta)$ is the rank within university i of a researcher of type θ , and w is the market wage in academia in the researcher's area of research. The utility of taking a job outside of academia is independent of the researcher's type and equals the wage in the non-academic sector. It is given by $V_C(\theta) = c$. Note that this outside option depends on the researcher's field. It is assumed that researchers always have the possibility of choosing the outside option.

Researchers choose among available jobs in academia and the outside option. Thus, if a researcher of type θ has an offer from both A and B , he will choose A if $V_A(\theta) \geq V_B(\theta)$ and $V_A(\theta) \geq V_C(\theta)$, B if $V_B(\theta) \geq V_A(\theta)$ and $V_B(\theta) \geq V_C(\theta)$, and otherwise leave academia. If the researcher has an offer from only university i , he will accept the offer if $V_i(\theta) \geq V_C(\theta)$.

Let $H_A(\bar{\theta})$ and $H_B(\bar{\theta})$ be the size of university A respectively B , and let $H_C(\bar{\theta})$ denote the number of researchers who leave academia. Without loss of generality, let A denote the best university, i.e. the university with the highest average ability.

I assume that a researcher of type θ gets an offer from university i if either i has not yet reached its desired size, or θ is higher than the lowest type at i . In the latter case, I assume that i can lay off its lowest ranked researcher so as to still satisfy its budget constraint. Given this assumption, in equilibrium no researcher with the option of changing workplace should strictly prefer to do so. In this sense, the equilibrium is stable as defined in Gale and Shapley (1962).

3 Allocation of researchers

I now solve for the allocation of researchers between the universities and the outside sector given the demands of the universities and the wages in the academic and outside sectors. A researcher's rank within university i , $r_i(\theta)$, is given by the cumulative distribution function of types of researchers within university i . Since the size of university i is $H_i(\bar{\theta})$, I can define the function

$$H_i(\theta) = r_i(\theta)H_i(\bar{\theta}), \tag{1}$$

and abusing vocabulary I call this function $H_i(\theta)$ the non-normalized type distribution for university i . A feasible allocation is then a triple (H_A, H_B, H_C) of non-normalized type distributions such that $H_A(\theta) + H_B(\theta) + H_C(\theta) = F(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Note in particular that $H_A(\bar{\theta}) + H_B(\bar{\theta}) + H_C(\bar{\theta}) = 1$. If both H_i and H_j , $i \neq j$, are strictly increasing in a neighborhood of θ , then types around θ are split between workplaces i and j . Let T_i be the support set of workplace i , defined as the closure of the set of types at which H_i is strictly increasing.

Following Damiano et al. (forthcoming) I apply the following equilibrium concept:

Definition 1 (Sorting equilibrium). *A sorting equilibrium is a feasible allocation (H_A, H_B, H_C) such that for each $i, j = A, B, C$ and $i \neq j$, if $\theta \in T_i$ and $\theta > \inf T_j$, then $V_i(\theta) \geq V_j(\theta)$.*

To determine whether a sorting equilibrium is stable, let t denote the difference in average types between the universities, i.e.

$$t = m_A - m_B,$$

and define also

$$D(t) = m_A(t) - m_B(t).$$

A stationary point has $D(t) = t$. By Theorem 6.5 in Stokey and Lucas (1989), a stationary point is stable if the derivative of $D(t)$ is less than one in absolute value.

The researchers face a tradeoff between rank and peer effect, the latter measured by the average type m_i in the researcher's institution. If a researcher works for the better university A the peer effect m_A will be higher than if he works for university B , but the researcher's rank $r_A(\theta)$ will be lower than if he works for university B . Thus, there can be a range of researchers who are indifferent between working for the two universities, and this illustrates the intuition behind the following definition of an overlapping interval allocation:

Definition 2 (Overlapping interval allocation). *There exist types s, z, v, y, x , and t with $s \leq v \leq z$, $y \leq x \leq t$, and $v \leq x$, such that the support sets for A , B , and C are $[y, t]$, $[v, x]$, and $[s, z]$, respectively.*

Figure 1 illustrates an overlapping interval allocation with $s = \underline{\theta}$ and $t = \bar{\theta}$. Here, researchers with $\theta \in [\underline{\theta}, v)$ work exclusively in the non-academic sector, researchers with $\theta \in [v, z]$ are indifferent between the non-academic sector and university B , researchers with $\theta \in (z, y)$ work exclusively for B , researchers with $\theta \in [y, x]$ are indifferent between A and B , and researchers with $\theta \in (x, \bar{\theta}]$ work exclusively for A .

Proposition 1 below shows that the allocation of researchers between universities A and B and the non-academic sector is of an overlapping interval form. It generalizes Lemma 1 of Damiano et al. (forthcoming) to include the outside sector.

Figure 1: Overlapping intervals of researchers



Proposition 1. *The equilibrium allocation of researchers is of an overlapping interval form. If the academic sector is non-empty, the interval allocated to academia has $\bar{\theta}$ as its right endpoint. If the outside sector is non-empty, the interval allocated to the outside sector has $\underline{\theta}$ as its left endpoint.*

Proof: I first prove that if $T_A \neq \emptyset$, $T_B \neq \emptyset$, and $m_A \geq m_B$ then $\sup T_A \geq \sup T_B$ and $\inf T_A \geq \inf T_B$. Suppose to the contrary that $\sup T_A < \sup T_B$. Then there exists $\hat{\theta} \in T_B$ such that $r_B(\hat{\theta}) < 1$ and $r_A(\hat{\theta}) = 1$. Since $m_A \geq m_B$, agents of type $\hat{\theta}$ strictly prefer A . This is a contradiction since type $\hat{\theta}$ is higher than the lowest type working for A and thus has the option of moving to A . Suppose now, to reach another contradiction, that $\inf T_A < \inf T_B$. Then there exists $\hat{\theta} \in T_B$ such that $r_B(\hat{\theta}) = 0$ and $r_A(\hat{\theta}) > 0$. Since $m_A \geq m_B$, agents of type $\hat{\theta}$ strictly prefer A . This is again a contradiction since type $\hat{\theta}$ is higher than the lowest type working for A and thus has the option of moving to A .

I now show that if $T_B \neq \emptyset$ and $T_C \neq \emptyset$, then $\sup T_B \geq \sup T_C$ and $\inf T_B \geq \inf T_C$. To see this, note that $T_B \neq \emptyset$ implies that $m_B \in [\inf T_B, \sup T_B]$. Also, note that for $\theta = \inf T_B$, I have that $r_B(\inf T_B) = 0$, so $V_B(\inf T_B) = m_B + w_k$.

Suppose now, in order to reach a contradiction, that $\inf T_C > \inf T_B$. Then for $\theta = \inf T_C$, I have that $r_B(\inf T_C) > 0$, since T_i is defined as the closure of the set of types at which H_i is strictly increasing, so $\inf T_B$ must be a limit point of some neighborhood that belongs to T_B , and this neighborhood must necessarily be to the right of the infimum. Hence,

$$V_B(\inf T_C) > m_B + w_k = V_B(\inf T_B). \quad (2)$$

Since researchers always have the possibility of switching to the outside sector, it must hold that

$$V_B(\inf T_B) \geq V_C(\inf T_B) = c_k, \quad (3)$$

otherwise $\theta \in T_B$ close to $\inf T_B$ would move to the outside sector. Inequalities (2) and (3) imply that $V_B(\inf T_C) > c_k = V_C(\inf T_C)$, so a researcher in T_C close to $\inf T_C$ would be better off switching to B . This is a contradiction since this researcher's type is higher than the lowest type working for B and thus he has the option of moving to B .

Suppose instead, to reach another contradiction, that $\sup T_C > \sup T_B$. Then it must hold that $V_C(\sup T_C) = c_k > V_B(\sup T_C)$, since researchers of type $\theta = \sup T_C$ have the

option of moving to B , but are choosing C over B . However, $r_B(\sup T_C)$ would be 1, so $c_k > V_B(\sup T_C)$ implies that $V_C(\theta) > V_B(\theta)$ for all $\theta \in T_B$. Since researchers can always switch to the outside option, this would imply that $T_B = \emptyset$, a contradiction. A similar argument works to show that if $T_A \neq \emptyset$ and $T_C \neq \emptyset$, then $\sup T_A \geq \sup T_C$ and $\inf T_A \geq \inf T_C$. Since all researchers are allocated to either A , B , or C , $\inf T_C \leq \inf T_B$ and $\inf T_C \leq \inf T_A$ imply that $\inf T_C = \underline{\theta}$, while $\sup T_C \leq \sup T_B$ and $\sup T_B \leq \sup T_A$ imply that $\sup T_A = \bar{\theta}$.

I finally show that T_A , T_B , and T_C are intervals. So see this, suppose that T_A is not an interval. Then there exist types $\hat{\theta}$ and $\tilde{\theta}$, with $\hat{\theta} < \tilde{\theta}$ such that all types on $(\hat{\theta}, \tilde{\theta})$ choose B or C , while types in small neighborhoods below $\hat{\theta}$ and above $\tilde{\theta}$ are in the support set of A . Then it must be that $H_A(\hat{\theta}) = H_A(\tilde{\theta})$ and either $H_B(\hat{\theta}) < H_B(\tilde{\theta})$ and/or $H_C(\hat{\theta}) < H_C(\tilde{\theta})$.

If $H_B(\hat{\theta}) < H_B(\tilde{\theta})$ then, since types $\hat{\theta}$ and $\tilde{\theta}$ have the option of switching between A and B , both types must be indifferent between A and B :

$$\alpha r_A(\hat{\theta}) + m_A = \alpha r_B(\hat{\theta}) + m_B \quad (4)$$

and

$$\alpha r_A(\tilde{\theta}) + m_A = \alpha r_B(\tilde{\theta}) + m_B. \quad (5)$$

Since $r_A(\hat{\theta}) = r_A(\tilde{\theta})$, equations (4) and (5) imply that $r_B(\hat{\theta}) = r_B(\tilde{\theta})$, which contradicts that $H_B(\hat{\theta}) < H_B(\tilde{\theta})$.

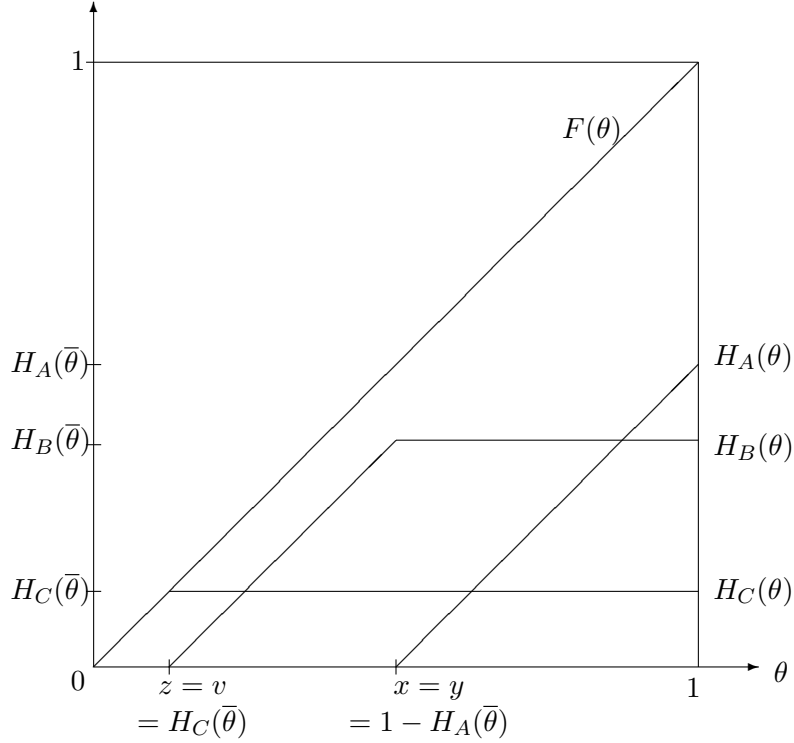
If instead $H_C(\hat{\theta}) < H_C(\tilde{\theta})$ then, since type $\hat{\theta}$ has the option of switching between A and C , I have that $V_A(\hat{\theta}) = c_k$. However, types $\tilde{\theta}$ in a small neighborhood below $\hat{\theta}$ are in T_A , and for these $V_A(\tilde{\theta}) < c_k$ since $r_A(\tilde{\theta}) < r_A(\hat{\theta})$. Therefore, these types would prefer to switch to the outside sector, which contradicts that they belong to the support set of A . A similar argument proves that T_B is an interval.

Suppose finally that T_C is not an interval. Then there exist types $\hat{\theta}$ and $\tilde{\theta}$, with $\hat{\theta} < \tilde{\theta}$ such that all types on $(\hat{\theta}, \tilde{\theta})$ choose A or B , while types in small neighborhoods below $\hat{\theta}$ and above $\tilde{\theta}$ are in the support set of C . Suppose, without loss of generality, that $H_B(\hat{\theta}) < H_B(\tilde{\theta})$. Then, since types $\hat{\theta}$ and $\tilde{\theta}$ have the option of switching between C and B , both types must be indifferent between C and B . Thus, I must have $V_B(\hat{\theta}) = c_k$ and $V_B(\tilde{\theta}) = c_k$. However, this implies that $V_B(\hat{\theta}) = V_B(\tilde{\theta})$, contradicting that $H_B(\hat{\theta}) < H_B(\tilde{\theta})$. ■

Having established that the allocation of researchers will be of an overlapping interval form, and before I proceed to characterize the different possible equilibria, I make the following assumption about the type-distribution of researchers:

Assumption 1. *The distribution of θ is identically and independently uniform on $[\underline{\theta}, \bar{\theta}] = [0, 1]$, i.e. $F(\theta) = \theta$ on $[0, 1]$.*

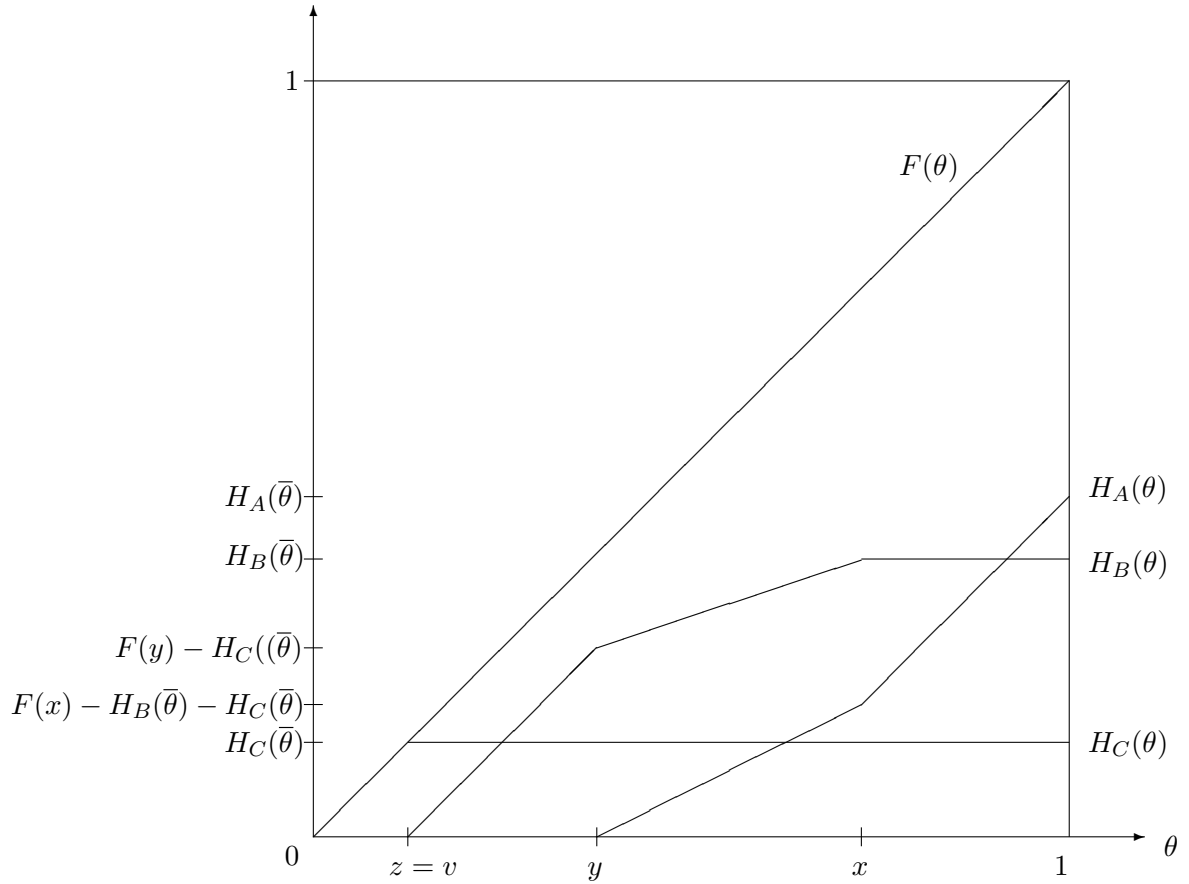
Figure 2: Distribution of researchers across universities when there is full segregation



Note that according to Proposition 1 the researchers sort themselves such that the left side of the type distribution enters the outside sector and the right side enters academia. The equilibria differ in whether they have no, partial, or full overlap between the universities, and whether or not the academic sector is constrained by the outside sector. Here and henceforth I use the term “no overlap” when the overlap consists of only a threshold type, which has measure zero. There are 6 possible scenarios in which the academic sector is active, as well as one scenario with an empty academic sector. As it will be shown in Proposition 2, there is no overlap between the outside and academic sectors in any scenario.

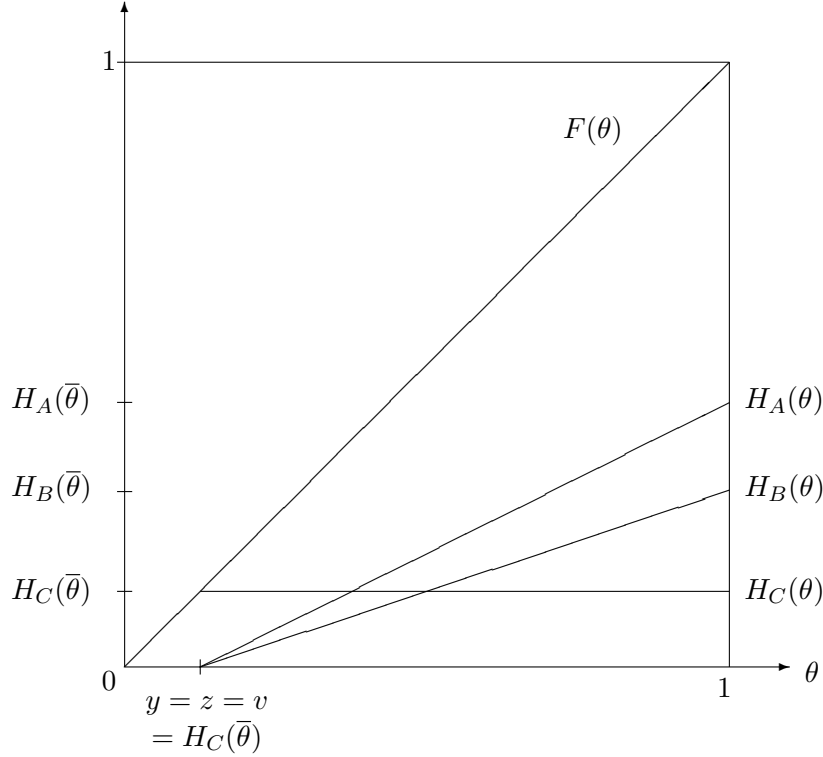
Unconstrained Full Segregation (UFS). In UFS, there is no overlap between the universities and the wage difference between the outside and academic sectors is sufficiently low that the outside option does not affect the academic sector. Both universities are able to satisfy their demands. The better university succeeds in hiring an interval at the top of the ability distribution and the less good university succeeds in hiring an interval just below that of the better university. Figure 2 illustrates a situation with full segregation.

Figure 3: Distribution of researchers across universities when there is partial overlap



Unconstrained Partial Overlap (UPO). In UPO, there is partial overlap of positive measure between the universities. Also, the outside option is low enough that it does not affect the academic sector, and hence the universities are still able to satisfy their demands for researchers. The better university hires from the top of the ability distribution, but there are types where some of the researchers of this type work for the better university and others work for the less good university. Figure 3 illustrates a situation with partial overlap.

Figure 4: Distribution of researchers across universities when there is full overlap



Unconstrained Full Overlap (UFO). In UFO, there is full overlap between the universities. All types who work in academia have a certain fraction of researchers working for A and the remainder for B. As a consequence, in UFO the institutions are of the same average quality. Similarly to UFS and UPO, the outside option is low enough that it does not constrain the academic sector, and thus the universities' demands for researchers are satisfied. Figure 4 illustrates a situation with full overlap.

Constrained Equilibria. When the outside option is sufficiently attractive it affects the academic sector because the universities are then unable to fully satisfy their demands for researchers. For each type of unconstrained equilibrium, there is a corresponding constrained equilibrium. I denote these **Constrained Full Segregation (CFS)**, **Constrained Partial Overlap (CPO)**, and **Constrained Full Overlap (CFO)**. In each of these the size of the academic sector is constrained by the outside option. Figures 2, 3, and 4 also illustrate CFS, CPO, and CFO equilibria, respectively.

Empty Academic sector (EAS). The final scenario is one of an empty academic sector. This will occur if the outside option is sufficiently high that all researchers want to work only in the outside sector.

When there is excess demand for researchers, the model is underdetermined. Let $\tilde{H}_i(\bar{\theta})$ denote university i 's demand for researchers and let $H_i(\bar{\theta})$ denote the measure of researchers university i actually succeeds in hiring. I close the model with the following assumption about the allocation of researchers under excess demand:

Assumption 2. *When there is excess demand, each university succeeds in hiring a fraction ξ of its demand, where ξ is determined endogenously by the wage difference between the outside and academic sectors as well as by the universities' demands and the preference parameter α . That is, $H_i(\bar{\theta}) = \xi \tilde{H}_i(\bar{\theta})$ for $i = A, B$, where $\xi = \xi(c - w, \tilde{H}_A(\bar{\theta}), \tilde{H}_B(\bar{\theta}), \alpha)$.*

Given the universities' demands $\tilde{H}_A(\bar{\theta})$ and $\tilde{H}_B(\bar{\theta})$, which of the above-described equilibria prevails is determined by the wage difference $c - w$ between the outside and academic sectors as well as by the weight α the researchers give to ranking compared with peer-effect in their utility function.

Proposition 2 presents the stable sorting equilibria.

Proposition 2. *Under Assumptions 1 and 2 there exists a unique stable sorting equilibrium. Which category the equilibrium belongs to varies across $(c - w, \alpha)$ -space as depicted in Figure 5.*

Proof: To prove Proposition 2, I solve for the cut-offs v , z , y , and x , as well as for the slopes of the non-normalized type distributions H_A , H_B , and H_C across $(c - w, \alpha)$ -space. I can then solve for the regions of $(c - w, \alpha)$ -space in which the different equilibria exist and for the size of the academic sector in the constrained equilibria. I finally show that the equilibria depicted in Figure 5 are stable and that there is a unique stable equilibrium across $(c - w, \alpha)$ -space. Note that equation (1) gives that

$$r_i(\theta) = \frac{H_i(\theta)}{H_i(\bar{\theta})}.$$

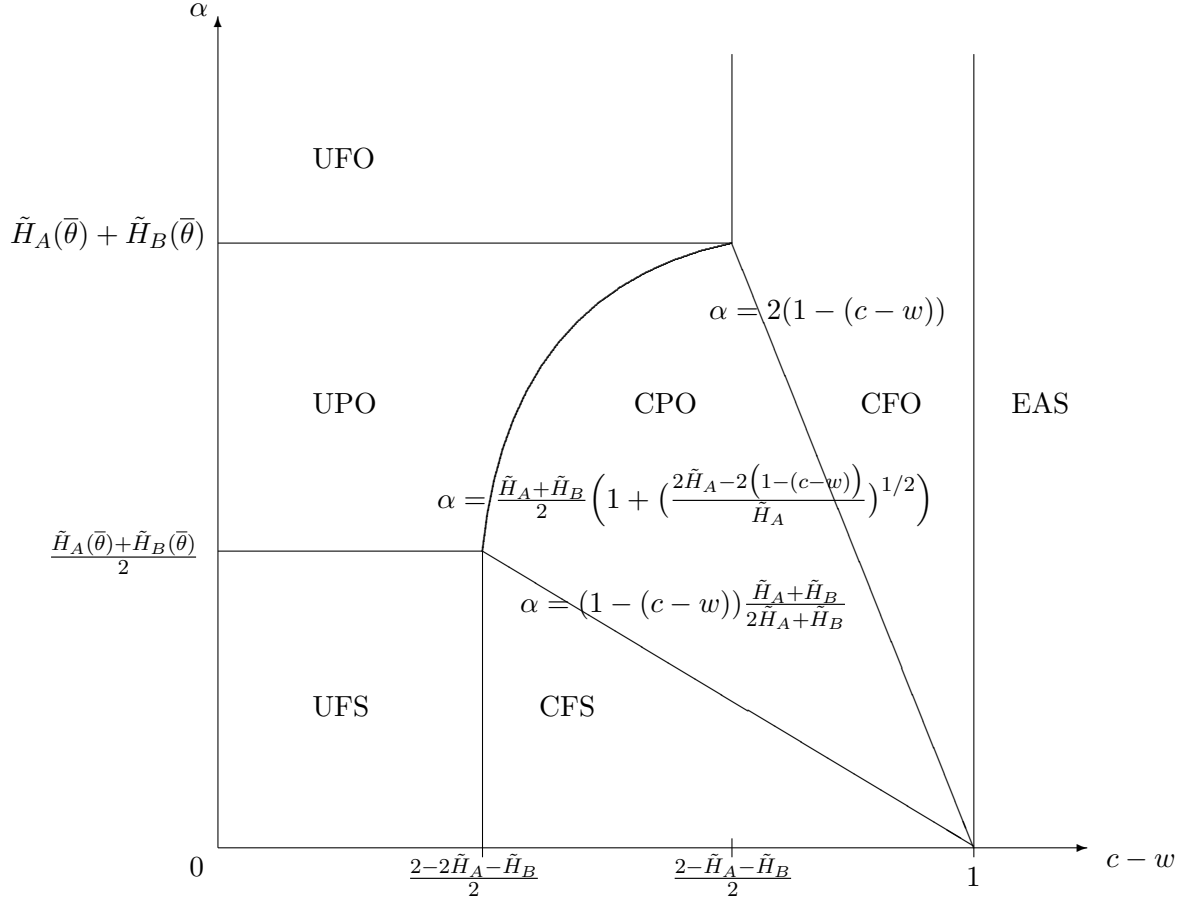
A researcher of type v is the lowest ability researcher who works for B and therefore has rank $r_B(v) = 0$. In a constrained equilibrium, a researcher of type v is indifferent between B and the outside option. Therefore, I have that

$$V_B(v) = V_C(v) \Leftrightarrow \alpha \frac{H_B(v)}{H_B(\bar{\theta})} + m_B + w = c \Leftrightarrow m_B + w = c. \quad (6)$$

Researchers with $\theta \in (v, z)$ are also indifferent between B and the outside option. I thus have that for $\theta \in (v, z)$,

$$V_B(\theta) = V_C(\theta) \Leftrightarrow \alpha \frac{H_B(\theta)}{H_B(\bar{\theta})} + m_B + w = c \Leftrightarrow H_B(\theta) = 0,$$

Figure 5: Sorting Equilibria



using the result in (6). This implies that no researchers in (v, z) work for B . Thus, $v = z$.

In an unconstrained equilibrium, the lowest ranked researcher at B is of type $v = 1 - \tilde{H}_A - \tilde{H}_B$, and has higher utility at B than he would have with the outside option. Therefore, I have that

$$V_B(v) > V_C(v) \Leftrightarrow \alpha \frac{H_B(v)}{H_B(\bar{\theta})} + m_B + w > c \Leftrightarrow m_B + w > c \Leftrightarrow c - w - m_B < 0. \quad (7)$$

For researchers with $\theta \in (v, z)$ to be indifferent between B and the outside option I must

have that for $\theta \in (v, z)$,

$$V_B(\theta) = V_C(\theta) \Leftrightarrow \alpha \frac{H_B(\theta)}{H_B(\bar{\theta})} = c - w - m_B,$$

which gives a contradiction with $H_B(\theta) \geq 0$, using the result in (7). This again implies that no researchers in (v, z) work for B . I have therefore established that $v = z$.

A researcher of type z is the highest ability researcher who works in the outside sector. Hence, $H_C(\theta)$ is constant and equal to $H_C(\bar{\theta})$ on $[z, 1]$. With $z = v$, $H_B(z) = H_A(z) = 0$, and it follows that $H_C(z) = F(z) - H_A(z) - H_B(z) = F(z)$. Since $F(z) = z$, it follows that $z = H_C(\bar{\theta}) = H_C(1)$. On $[0, z)$ everyone prefers the outside sector. I conclude that the non-normalized distribution of types in the outside sector, is

$$H_C(\theta) = \begin{cases} F(\theta) & \text{if } \theta \in [0, z) \\ H_C(\bar{\theta}) & \text{if } \theta \in [z, 1]. \end{cases}$$

Consider first full segregation. With full segregation, $x = y$. The non-normalized distributions of types, H_A , H_B , and H_C , under full segregation are illustrated in Figure 2. By Proposition 1, it is the top end of the ability distribution that will choose to work in academia. With full segregation, all of the highest ability researchers choose to work for the better university A , while the best of the remaining researchers choose to work for B . Only a researcher with $\theta = x$ is indifferent between A and B . I easily find, in addition to the values for $H_C(\theta)$ found above, that under full segregation

$$H_A(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, x) \\ F(\theta) - H_B(\bar{\theta}) - H_C(\bar{\theta}) & \text{if } \theta \in [x, 1] \end{cases}$$

and

$$H_B(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, z) \\ F(\theta) - H_C(\bar{\theta}) & \text{if } \theta \in (z, x) \\ H_B(\bar{\theta}) & \text{if } \theta \in [x, 1]. \end{cases}$$

In particular, I have that $z = v = H_C(\bar{\theta})$ and $x = y = 1 - H_A(\bar{\theta}) = H_B(\bar{\theta}) + H_C(\bar{\theta})$.

The peer effect in university A is then given by

$$m_A = \frac{2 - H_A}{2}$$

while the peer effect in university B is then given by

$$m_B = \frac{2 - 2H_A - H_B}{2}.$$

The full segregation equilibrium is unconstrained as long as $\frac{2 - 2\bar{H}_A - \bar{H}_B}{2} \geq c - w$, that is, as long as the peer effect in B when both universities' demands are satisfied is at least

as high as the wage difference between the outside and academic sectors. As long as this is satisfied, the lowest ranked researcher in B has higher utility from working at B than from working in the outside sector. Therefore, the border between UFS and CFS is given by

$$\frac{2 - 2\tilde{H}_A - \tilde{H}_B}{2} = c - w.$$

Full segregation also requires that the difference in peer effect is large enough that no researcher wishes to move from B to A . Under UFS, this condition is given by

$$m_A - m_B = \frac{\tilde{H}_A + \tilde{H}_B}{2} \geq \alpha. \quad (8)$$

As long as (8) is satisfied, the lowest ranked researcher at A has at least as high utility from staying at A as from moving to B . Thus, the north border of UFS is given by

$$\alpha = \frac{\tilde{H}_A + \tilde{H}_B}{2}.$$

When instead $\frac{2 - 2\tilde{H}_A - \tilde{H}_B}{2} < c - w$, the full segregation equilibrium is constrained and the size of the academic sector is determined by the equation

$$\frac{2 - 2H_A - H_B}{2} = c - w. \quad (9)$$

That is, the size of the academic sector is pinned down by $m_B = c - w$ such that the lowest ranked researcher in B gets exactly the same utility from working at B as he would from working in the outside sector. By assumption 2, $H_i(\bar{\theta}) = \xi \tilde{H}_i(\bar{\theta})$ for $i = A, B$ when the universities are constrained. Thus, equation (9) implies that under CFS the proportion of the universities' demand that will be satisfied is given by

$$\xi = \frac{2(1 - (c - w))}{2\tilde{H}_A + \tilde{H}_B}. \quad (10)$$

Under CFS, with $m_B = c - w$ the difference in peer effect between A and B is given by $m_A - m_B = \frac{2 - H_A}{2} - (c - w)$. Hence, the condition for the lowest ranked researcher at A having at least as high utility from staying at A as from moving to B is

$$\frac{2 - H_A}{2} - (c - w) \geq \alpha.$$

Plugging in that $H_i(\bar{\theta}) = \xi \tilde{H}_i(\bar{\theta})$ for $i = A, B$ and $\xi = \frac{2(1 - (c - w))}{2\tilde{H}_A + \tilde{H}_B}$ as found in (10), I find the north-east border for CFS to be given by

$$\alpha = (1 - (c - w)) \frac{\tilde{H}_A + \tilde{H}_B}{2\tilde{H}_A + \tilde{H}_B}. \quad (11)$$

Consider now partial overlap. With partial overlap, $y < x < 1$. The non-normalized distributions of types, H_A , H_B , and H_C , under partial overlap are illustrated in Figure 3. Again, by Proposition 1, it is the top end of the ability distribution that will choose to work in academia. When there is overlap, all researchers with ability θ in the interval $[y, x]$ are indifferent between the two universities, thus both $H_A(\theta)$ and $H_B(\theta)$ are increasing on $[y, x]$ as illustrated in Figure 3.

To find the non-normalized type distributions H_A and H_B under partial overlap, I first find their values at the critical points z , y , and x . With $z = v$, I see that $H_B(y) = F(y) - H_C(\bar{\theta})$, since researchers in (z, y) are allocated exclusively to B . A researcher of type x is the highest ability researcher who works for B , hence $H_B(x) = H_B(\bar{\theta})$ and $H_B(\theta)$ is constant thereafter. Since y is the lowest type who works for A , $H_A(y) = 0$. Finally, researchers in $(x, 1]$ are allocated exclusively to A , hence $H_A(x) = F(x) - H_B(\bar{\theta}) - H_C(\bar{\theta})$.

Since a researcher of type y is indifferent between A and B I must have that

$$\begin{aligned} V_B(y) = V_A(y) &\Leftrightarrow \alpha \frac{F(y) - H_C(\bar{\theta})}{H_B(\bar{\theta})} + m_B + w_k = \alpha \frac{H_A(y)}{H_A(\bar{\theta})} + m_A + w_k \\ &\Leftrightarrow \alpha \frac{y - H_C(\bar{\theta})}{H_B(\bar{\theta})} + m_B = m_A. \end{aligned} \quad (12)$$

Since a researcher of type x is also indifferent between A and B I have that

$$\begin{aligned} V_B(x) = V_A(x) &\Leftrightarrow \alpha \frac{H_B(\bar{\theta})}{H_B(\bar{\theta})} + m_B + w_k = \alpha \frac{F(x) - H_B(\bar{\theta}) - H_C(\bar{\theta})}{H_A(\bar{\theta})} + m_A + w_k \\ &\Leftrightarrow m_B = \alpha \frac{x-1}{H_A(\bar{\theta})} + m_A. \end{aligned} \quad (13)$$

Equations (12) and (13) imply that

$$\frac{y - H_C(\bar{\theta})}{H_B(\bar{\theta})} = \frac{1 - x}{H_A(\bar{\theta})}. \quad (14)$$

Furthermore, researchers with $\theta \in (y, x)$ are also indifferent between A and B . Hence, for $\theta \in (y, x)$,

$$\begin{aligned} V_B(\theta) = V_A(\theta) &\Leftrightarrow \alpha \frac{H_B(\theta)}{H_B(\bar{\theta})} + m_B + w_k = \alpha \frac{H_A(\theta)}{H_A(\bar{\theta})} + m_A + w_k \\ &\Leftrightarrow H_B(\theta) = \frac{H_B(\bar{\theta})}{H_A(\bar{\theta})} H_A(\theta) + \frac{H_B(\bar{\theta})}{\alpha} (m_A - m_B). \end{aligned} \quad (15)$$

It follows from (15) that $H_A(\theta)$ and $H_B(\theta)$ must be linear on (y, x) , and if $H_A(\bar{\theta}) > H_B(\bar{\theta})$, then $H_B(\cdot)$ is flatter than $H_A(\cdot)$.

I can find the slopes of H_A and H_B on $[y, x]$ from the critical points above and conclude that under constrained overlap the non-normalized distribution of types in university A is

$$H_A(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, y) \\ \frac{x + H_A(\bar{\theta}) - 1}{x - y} \theta - y \frac{x + H_A(\bar{\theta}) - 1}{x - y} & \text{if } \theta \in [y, x] \\ F(\theta) - H_B(\bar{\theta}) - H_C(\bar{\theta}) & \text{if } \theta \in (x, 1] \end{cases}$$

while the non-normalized distribution of types in university B is

$$H_B(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, z] \\ F(\theta) - H_C(\bar{\theta}) & \text{if } \theta \in (z, y) \\ \frac{1-y-H_A(\bar{\theta})}{x-y}\theta - x\frac{1-y-H_A(\bar{\theta})}{x-y} + H_B(\bar{\theta}) & \text{if } \theta \in [y, x] \\ H_B(\bar{\theta}) & \text{if } \theta \in (x, 1]. \end{cases}$$

The peer effect in university B is then given by

$$m_B = \int_z^y \frac{1}{H_B(\bar{\theta})} \theta d\theta + \int_y^x \frac{1-F(y)-H_A(\bar{\theta})}{H_B(\bar{\theta})(x-y)} \theta d\theta = \frac{1}{2} \frac{y^2 - z^2}{H_B(\bar{\theta})} + \frac{1}{2} \frac{1-y-H_A(\bar{\theta})}{H_B(\bar{\theta})} (x+y), \quad (16)$$

while the peer effect in university A is

$$m_A = \int_y^x \theta \frac{F(x)+H_A(\bar{\theta})-1}{H_A(\bar{\theta})(x-y)} d\theta + \frac{1}{H_A(\bar{\theta})} \int_x^{\bar{\theta}} \theta d\theta = \frac{1}{2} \left(\frac{x-1}{H_A(\bar{\theta})} + 1 \right) (x+y) + \frac{1}{2} \frac{\bar{\theta}^2 - x^2}{H_A(\bar{\theta})}. \quad (17)$$

Using (16) and (17), the difference in peer effect between A and B is given by

$$m_A - m_B = \frac{1}{2} (x+y) \left(1 + \frac{x-1}{H_A(\bar{\theta})} - \frac{1-y-H_A(\bar{\theta})}{H_B(\bar{\theta})} \right) + \frac{1}{2} \frac{1-x^2}{H_A(\bar{\theta})} - \frac{1}{2} \frac{y^2 - z^2}{H_B(\bar{\theta})}.$$

This and equation (14) imply that

$$m_A - m_B = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2H_A(\bar{\theta})^2} (1-x)(2H_A(\bar{\theta}) - 1 + x).$$

This and equation (13) can be used to find that

$$x = 1 - H_A(\bar{\theta}) + H_A(\bar{\theta}) \left(\frac{2\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} - 1 \right) \quad (18)$$

Plugging this into (14) then gives that

$$y = 1 - H_A(\bar{\theta}) - H_B(\bar{\theta}) \left(\frac{2\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} - 1 \right). \quad (19)$$

Using (17), (18), and (19), I now have that

$$m_A = \frac{2 - H_A(\bar{\theta}) - H_B(\bar{\theta})}{2} + \frac{2\alpha H_B(\bar{\theta})}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right) \quad (20)$$

and

$$m_B = \frac{2 - H_A(\bar{\theta}) - H_B(\bar{\theta})}{2} + \frac{2\alpha H_A(\bar{\theta})}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \left(\frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} - 1 \right). \quad (21)$$

From equations (18) and (19) it follows that $x \geq y$ is equivalent to

$$\alpha \geq \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2}, \quad (22)$$

which is consistent with the north border of UFS. From equation (18) it also follows that $x \leq 1$ if and only if

$$\alpha \leq H_A(\bar{\theta}) + H_B(\bar{\theta}). \quad (23)$$

Once $x = 1$, the overlap is full rather than partial.

The partial overlap equilibrium is unconstrained as long as the peer effect in university B , which is given in (21), calculated with the universities actual demands $\tilde{H}_A(\bar{\theta})$ and $\tilde{H}_B(\bar{\theta})$ is at least as high as the wage difference between the outside and academic sectors. That is, the equilibrium is UPO as long as

$$\frac{2 - \tilde{H}_A(\bar{\theta}) - \tilde{H}_B(\bar{\theta})}{2} + \frac{2\alpha\tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} \left(\frac{\alpha}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} - 1 \right) \geq c - w. \quad (24)$$

Equality in (24) defines a curve in $(c - w, \alpha)$ -space, which is the border between UPO and CPO. Solving for α in (24) and noting that the positive root is the relevant one, the border is given by

$$\alpha = \frac{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}{2} \left(1 + \left(\frac{2\tilde{H}_A(\bar{\theta}) - 2(1 - (c - w))}{\tilde{H}_A(\bar{\theta})} \right)^{1/2} \right).$$

It follows from (22) and (23) that the south border of UPO is given by

$$\alpha = \frac{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}{2},$$

while the north border of UPO is given by

$$\alpha = \tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta}).$$

When instead (24) does not hold, the partial overlap equilibrium is constrained and the size of the academic sector is determined by the equation

$$\frac{2 - H_A(\bar{\theta}) - H_B(\bar{\theta})}{2} + \frac{2\alpha H_A(\bar{\theta})}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \left(\frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} - 1 \right) = c - w. \quad (25)$$

That is, the size of the academic sector is pinned down by $m_B = c - w$ such that the lowest ranked researcher in B gets exactly the same utility from working at B as he would from working in the outside sector.

To find the proportion of the universities' demand that will be satisfied in CPO, I insert $H_i(\bar{\theta}) = \xi \tilde{H}_i(\bar{\theta})$ for $i = A, B$ into (25) and solve for ξ . This gives that

$$\xi = \frac{1 - (c - w) - \frac{2\alpha \tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} + \left(\left(1 - (c - w) - \frac{2\alpha \tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} \right)^2 + 4 \frac{\alpha^2 \tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} \right)^{1/2}}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}, \quad (26)$$

using that only the positive root is relevant.

Equation (25) together with (23) with equality give that the border between CPO and CFO is given by

$$\alpha = 2(1 - (c - w)).$$

Plugging this into (26) gives that at the border between CPO and CFO,

$$\xi = \frac{\alpha}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}.$$

The border between CPO and CFS is given by (11).

Consider finally full overlap. With full overlap, $x = 1$ and $y = H_C(\bar{\theta})$. In this case, A and B share all employed types, i.e. all employed types are indifferent between A and B . Then

$$m_A = m_B = \frac{2 - H_A(\bar{\theta}) - H_B(\bar{\theta})}{2}. \quad (27)$$

This scenario is illustrated in Figure 4.

The full overlap equilibrium is unconstrained as long as

$$\frac{2 - \tilde{H}_A(\bar{\theta}) - \tilde{H}_B(\bar{\theta})}{2} \geq c - w. \quad (28)$$

Once (28) is not satisfied, the full overlap equilibrium is constrained. In CFO, the size of the academic sector is determined by (28) with equality. Under CFO, the fraction of the universities' demand that is satisfied is given by

$$\xi = \frac{2(1 - (c - w))}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}. \quad (29)$$

Thus, when $c - w = 1$, $\xi = 0$ and the academic sector is empty. In full overlap equilibria, since all employed types are indifferent between A and B , equation (4) is satisfied for all employed types. Therefore, full overlap equilibria are in principle possible across $(c - w, \alpha)$ -space. However, as I will now show they are only stable in the areas depicted in Figure 5.

I show stability by applying Theorem 6.5 in Stokey and Lucas (1989). Let μ denote the mean type in academia. That is,

$$\mu = \frac{2 - H_A(\bar{\theta}) - H_B(\bar{\theta})}{2}.$$

Note that

$$\frac{H_A(\bar{\theta})}{H_A(\bar{\theta}) + H_B(\bar{\theta})} m_A + \frac{H_B(\bar{\theta})}{H_A(\bar{\theta}) + H_B(\bar{\theta})} m_B = \mu. \quad (30)$$

Let t denote the difference in average types between the universities, i.e.

$$t = m_A - m_B.$$

Under full overlap, $t = 0$, while under full segregation, $t = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2}$. Under partial overlap,

$$t \in \left(0, \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2} \right).$$

Define

$$D(t) = m_A(t) - m_B(t).$$

By (30),

$$D(t) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{H_B(\bar{\theta})} (m_A(t) - \mu). \quad (31)$$

A stationary point has $D(t) = t$. By Theorem 6.5 in Stokey and Lucas (1989), a stationary point is stable if the derivative of $D(t)$ is less than one in absolute value.

Taking the derivative of $D(\cdot)$ in (31) gives

$$D'(t) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{H_B(\bar{\theta})} \frac{dm_A}{dt}. \quad (32)$$

Taking the derivative w.r.t. t in (17) yields

$$\frac{dm_A}{dt} = \frac{1}{2} \left(\frac{x-1}{H_A(\bar{\theta})} + 1 \right) \frac{dy}{dt} + \frac{1}{2} \left(\frac{x-1}{H_A(\bar{\theta})} + 1 \right) \frac{dx}{dt} + \frac{1}{2} \frac{x+y}{H_A(\bar{\theta})} \frac{dx}{dt} - \frac{x}{H_A(\bar{\theta})} \frac{dx}{dt}. \quad (33)$$

By (12),

$$t = \alpha \frac{y-1 + H_A(\bar{\theta}) + H_B(\bar{\theta})}{H_B(\bar{\theta})},$$

which gives that

$$\frac{dy}{dt} = \frac{H_B(\bar{\theta})}{\alpha}. \quad (34)$$

Similarly, by (13),

$$t = \alpha \frac{1-x}{H_A(\bar{\theta})},$$

yielding that

$$\frac{dx}{dt} = -\frac{H_A(\bar{\theta})}{\alpha}. \quad (35)$$

Combining (32), (33), (34), and (35), I now have that

$$D'(t) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{\alpha} \left(\frac{x - 1 + H_A(\bar{\theta})}{2H_A(\bar{\theta})} - \frac{y - 1 + H_A(\bar{\theta})}{2H_B(\bar{\theta})} \right). \quad (36)$$

Now, consider first full overlap allocations. For these, $t = 0$ and $m_A = m_B$. Hence, $D(0) = 0$, so full overlap allocations are stationary points. Plugging $x = 1$ and $y = 1 - H_A(\bar{\theta}) - H_B(\bar{\theta})$ into (36) gives that

$$D'(0) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{\alpha}.$$

Thus, $D'(0) \leq 1$ if and only if $\alpha \geq H_A(\bar{\theta}) + H_B(\bar{\theta})$. This implies that UFO allocations are only stable for $\alpha \geq \tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})$. Under CFO on the other hand, equation (29) implies that $H_A(\bar{\theta}) + H_B(\bar{\theta}) = 2(1 - (c - w))$. Therefore, CFO allocations are stable only for

$$\alpha \geq 2(1 - (c - w)). \quad (37)$$

EAS allocations are CFO allocations with $\xi = 0$, and are therefore stable because (37) is satisfied when $c - w \geq 1$.

Consider next full segregation allocations. For these $t = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2}$ and $m_A = \frac{2 - H_A(\bar{\theta})}{2}$. By inserting this and the expression for μ into (31), it follows that $D(t) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2} = t$. Therefore, $t = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{2}$ is a stationary point. Under full segregations $x = y = 1 - H_A(\bar{\theta})$. Plugging this into (36) gives that $D'(t) = 0$ for all t . It follows that the full segregation equilibria are stable.

Finally consider partial overlap allocations. By (20) and (21), $t = 2\alpha \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right)$. Inserting (17) and the expression for μ into (31) it follows that $D(t) = 2\alpha \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right) = t$. Therefore, $t = \alpha \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right)$ is a stationary point. By (12), $y = \frac{tH_B(\bar{\theta})}{\alpha} + 1 - H_A(\bar{\theta}) - H_B(\bar{\theta})$, while by (13), $x = 1 - \frac{tH_A(\bar{\theta})}{\alpha}$. Plugging these expressions into (36) gives that $D'(t) = \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{\alpha} \left(1 - \frac{t}{\alpha} \right)$. It follows that $D' \left(2\alpha \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right) \right) = 2 - \frac{H_A(\bar{\theta}) + H_B(\bar{\theta})}{\alpha}$. Thus, $D' \left(2\alpha \left(1 - \frac{\alpha}{H_A(\bar{\theta}) + H_B(\bar{\theta})} \right) \right) \leq 1$ if and only if $\alpha \leq H_A(\bar{\theta}) + H_B(\bar{\theta})$.

This implies that UPO allocations are stable for $\alpha \leq \tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})$. Using that ξ under CPO is given by (26), CPO allocations are stable if

$$\alpha \leq 1 - (c - w) - \frac{2\alpha\tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} + \left(\left(1 - (c - w) - \frac{2\alpha\tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} \right)^2 + 4 \frac{\alpha^2\tilde{H}_A(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})} \right)^{1/2},$$

which is equivalent to $\alpha \leq 2(1 - (c - w))$. ■

Note that my model allows for an additional dimension compared to Damiano et al. (forthcoming), namely the value of the outside option. Introducing the outside option allows me to analyze what happens as the value of the outside option varies, which may endogenously induce both voluntary and involuntary exit of researchers from academia. Moreover, as seen in Proposition 2 and discussed subsequently, there is an interesting interaction between the preference parameter α and the value of the outside option, which naturally cannot be analyzed in the absence of an outside sector.

For low values of α and $c - w$, the equilibrium is UFS. Since α is low, the researchers care little about their ranking within their institution. Instead they care a lot about the peer-effect. Therefore, the researchers sort themselves into the two institutions in non-overlapping intervals with all types in $[1 - \tilde{H}_A(\bar{\theta}), 1]$ working for A and all types in $[1 - \tilde{H}_A(\bar{\theta}) - \tilde{H}_B(\bar{\theta}), 1 - \tilde{H}_A(\bar{\theta})]$ working for B . The remaining types work in the outside sector.

If α increases, holding $c - w$ constant, the equilibrium is UPO. The researchers now care enough about their ranking that some of the bottom types from university A will find it worthwhile to move to university B . The peer-effect will be smaller at B , but their ranking will be higher. Hence, the equilibrium will be of the overlapping interval form, with $y < x$. All types in $[1 - \tilde{H}_A(\bar{\theta}) - \tilde{H}_B(\bar{\theta}), 1]$ work in the academic sector, while the remaining types work in the outside sector. As α increases, the overlap between the universities grows and the quality difference between them decreases.

If $\alpha \geq \tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})$ (still holding $c - w$ constant) the equilibrium is UFO. Then there is no quality difference between the universities. The researchers care enough about ranking that even the slightest quality difference would warrant the lowest ranked researcher in the better institution to move to the less good institution. Still, all types in $[1 - \tilde{H}_A(\bar{\theta}) - \tilde{H}_B(\bar{\theta}), 1]$ work in the academic sector, while the remaining types work in the outside sector.

In all three unconstrained equilibria, those researchers who leave academia do so involuntarily. Perhaps surprisingly, the involuntary exit occurs exactly because the universities are unconstrained which happens when the outside option is not sufficiently attractive. That is, once the universities' demands for researchers are satisfied, there are no more jobs in the academic sector, but the outside option is sufficiently unattractive that each of the remaining types, who each have measure zero and thus individually would not affect the average quality of the universities, would have liked to work in academia.

The intuition that the equilibrium changes from full segregation to partial overlap to full overlap as α increases through the unconstrained equilibria can also be found in Damiano et al. (forthcoming). However, they have no outside sector, and the universities' demand is always for the full set of researchers. Hence, in Damiano et al. all researchers always work in the academic sector. There is no exit, neither voluntary nor involuntary. Naturally, their model cannot be used to analyze what happens when the outside option becomes more attractive.

Suppose instead that starting from UFS we make the outside option more attractive by increasing $c - w$ while holding α constant. Then the equilibrium becomes CFS. The outside

option is now attractive enough that the outside sector constrains the academic sector. The universities can no longer satisfy their full demands, since more researchers are attracted to the outside sector and leave the academic sector voluntarily. The size of the academic sector is such that the peer effect in university B is exactly equal to the wage difference between the outside and academic sectors. With that size, the lowest ranked researcher in B is indifferent between staying at B and moving to the outside sector. Therefore, in CFS the researchers sort themselves into the two institutions and the outside sector in non-overlapping intervals with all types in $[1 - H_A(\bar{\theta}), 1]$ working for A , all types in $[1 - H_A(\bar{\theta}) - H_B(\bar{\theta}), 1 - H_A(\bar{\theta})]$ working for B , and the remaining types in the outside sector.

As $c - w$ increases through CFS (still holding α constant), the lowest ranked researchers at B find it worthwhile to move to the outside sector. This increases the peer effect at B , which can therefore attract some of the researchers from A . As a consequence, the quality difference between the two universities decreases. In particular, when $c - w$ increases beyond $1 - \frac{2\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}\alpha$ the difference in peer-effect becomes low enough that the utility from ranking starts to dominate for some types. Therefore, the equilibrium becomes CPO. This equilibrium is of the overlapping interval form with $y < x$ and the universities are unable to satisfy their full demands. The outside sector constrains the academic sector, the size of which is again such that the peer effect in university B is exactly equal to the wage difference between the outside and academic sectors. All types in $[1 - H_A(\bar{\theta}) - H_B(\bar{\theta}), 1]$ sort themselves into the academic sector, while the remaining types choose the outside sector.

The quality difference between the universities decreases further as we increase $c - w$ through CPO. When $c - w$ becomes sufficiently high, there is no quality difference between A and B and the equilibrium is CFO. Finally, when $c - w \geq 1$ the academic sector is empty and the equilibrium is EAS.

In each of the constrained equilibria, all exit from academia is voluntary. Also, to summarize the discussion above, in CFS and CPO, when the outside option becomes better it not only increases competition between academia and the outside sector, but also increases competition within the academic sector, since the quality difference between the universities decreases. As $c - w$ increases through the constrained equilibria, the proportion ξ of the universities' demands that is satisfied decreases. These observations are summarized in Corollary 1.

Corollary 1. *In a constrained equilibrium, if the wage difference $c - w$ between the outside and academic sectors increases, the proportion ξ of the universities' demands that is satisfied decreases. In a CPO or CFS equilibrium, an increase in $c - w$ also decreases the quality difference $m_A - m_B$ between the universities.*

Proof: That ξ is decreasing in $c - w$ follows from taking the derivative with respect to $c - w$ in (10), (26), and (29). In CFS, $m_A - m_B = \frac{\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}{2\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta})}(1 - (c - w))$ and it

follows immediately that $\frac{d(m_A - m_B)}{d(c-w)} < 0$. In CPO, $m_A - m_B = 2\alpha \left(1 - \frac{\alpha}{\xi(\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta}))}\right)$. Taking the derivative gives that $\frac{d(m_A - m_B)}{d(c-w)} = \frac{2\alpha^2}{\xi^2(\tilde{H}_A(\bar{\theta}) + \tilde{H}_B(\bar{\theta}))} \frac{d\xi}{d(c-w)}$, which is negative since $\frac{d\xi}{d(c-w)} < 0$. ■

A final thing to note from Figure 5 is that a higher value of α implies that a higher value of $c - w$ is needed before the outside sector starts to constrain the academic sector. The reason is that a higher α is associated with a higher weight on the utility component stemming from working in academia relative to the utility component stemming from salary.

4 Concluding remarks

This paper has developed an equilibrium sorting model with utility maximizing researchers who differ in their ability on one side of the market, and on the other side universities and an outside sector. I have solved for the allocation of researchers' ability across sectors and within the academic sector as well as for how changes in the outside option affects this distribution. The analysis enables comparative statics analysis between fields with different attractiveness of the outside option.

The most important results are the following: The top of the ability distribution is allocated to the academic sector, while the bottom of the ability distribution is allocated to the outside sector. For low values of the outside option, the academic sector is unaffected by it and exit from academia happens involuntarily. For higher values of the outside option, the academic sector will be affected by it and by changes in its value. In particular, the universities will be unable to satisfy their demands for researchers, all exit from academia is voluntary, and when the value of the outside option increases the difference in average quality between the higher and lower ranked universities decreases.

The comparative statics insights suggest that in fields where the outside option is very attractive, one would expect the difference in research quality between top ranked and lower ranked departments to be smaller than the corresponding difference in a field where the outside option is less attractive. It would be interesting in future research to investigate whether the difference in research quality between the highest ranked department and a department ranked e.g. just outside the top-10 percent of departments differs across fields in the manner suggested by Proposition 2 and Corollary 1.

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