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Rational students and resit exams

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Rational Students and Resit Exams

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Rational Students and Resit Exams

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ABSTRACT

Rational Students and Resit Exams*

Resit exams – extra opportunities to do an exam in the same academic year – are widely prevalent in European higher education, but uncommon in the US. I present a simple theoretical model to compare rational student behavior in the case of only one exam opportunity versus the case of two exam opportunities. Numerical examples for a wide range of plausible parameter values show that a second exam opportunity increases the ultimate passing probability only slightly, but strongly reduces average total student effort.

JEL Classification: D01, I21

Keywords: frequency of examinations, education production function, rationality

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1 Introduction

Resit exams – extra opportunities for all students to do an exam in the same academic year – are widely prevalent in European higher education, but uncommon in the US. This note presents a simple theoretical model to compare student behavior in the case of only one exam opportunity versus the case of two exam opportunities.

Numerical examples with plausible parameter values show that a second exam opportunity increases the ultimate passing probability only slightly, but dramatically reduces expected total student effort. In some cases students who are lucky to pass at the first attempt spend less than one fifth of the time they would have spent with only one exam opportunity.

While the present paper is related to the educational literature on optimal testing procedures and setting educational standards – see e.g. Costrell (1994) and Lazaer (2006) – the economics of resit exams does not seem to have received any attention in the literature.[1]

2 The model

Consider the simplest possible model that captures the key issues of interest. A risk neutral rational student needs to decide how much time to spend on preparing for an exam with a binary outcome, pass or fail. The student wishes to maximize the probability of passing, but faces a constant opportunity cost w per unit of time. The student's utility function might be specified as $U = Y + RU_P$. Here $Y = w(Z - t)$ is income, with Z denoting total time available, and t time spent preparing for the exam. $R = 1$ if the student ultimately passes and 0 otherwise. U_P is the utility of passing. U_P can be normalized to 1 without loss of generality through redefining the

[1] In an empirical study, De Paola and Scoppa (2011) compare the case of two exams that each cover different parts of the course material versus one exam covering the whole course material. In the present note the course material is the same for both exam opportunities.

time unit in which w is expressed.

2.1 One exam opportunity

First consider the case with only one exam opportunity. The relationship between the probability of passing, p, and the amount of time spent on exam preparation (or simply "effort"), t, is specified as

$$p(t) = \max\Big(0, \frac{\alpha + \beta t}{1 + \alpha + \beta t}\Big) \tag{1}$$

with $\beta > 0$. Specification (1) captures a number of key features and will allow for analytic solutions of the optimization problem. The passing probability is increasing in effort, with decreasing returns. The parameter α can be interpreted as an indicator of exam difficulty. If $\alpha > 0$, it is possible to pass without exerting any effort, as in the case of multiple choice exams. If $\alpha < 0$, it is impossible to pass without spending at least a minimum amount of time $t_{\min} = -\alpha/\beta$; see figure 1 (dotted line: $\alpha > 0$; solid line: $\alpha < 0$). The case $\alpha < 0$ can also be interpreted as a case with fixed costs (monetary or mental) of doing an exam. The parameter β reflects the ability of the student, with β being larger for a bright student than for a less talented student.[2]

The optimal effort follows from maximizing $p(t) - wt$. The solution is given by

$$\tilde{t} = \beta^{-\frac{1}{2}} w^{-\frac{1}{2}} - \alpha\beta^{-1} - \beta^{-1} \tag{2}$$

with implied passing probability

$$\tilde{p} = p(\tilde{t}) = 1 - w^{\frac{1}{2}}\beta^{-\frac{1}{2}} \tag{3}$$

I assume that $w/\beta < 1$ and sufficiently small to ensure positive effort and passing probability. Higher ability leads to a higher passing rate, but the

[2]It could be argued that both α and β reflect both student ability and exam difficulty. For example, if an exam is poorly designed, β can be close to 0 even for a bright student. In this note I will not elaborate on this.

effect of ability on effort is ambiguous. High opportunity costs lead to lower effort and a lower passing probability. Note that in this model an easy exam (high α) lowers student effort to such an extent that the passing probability remains unaffected. Hence, for a group of homogeneous students passing rates are uninformative about exam difficulty.

2.2 Two exam opportunities

If there is a second unconditional exam opportunity, the *ex ante* ultimate passing probability is $P = p_1 + (1 - p_1)p_2$, where p_1 and p_2 are the passing probabilities at the first and second attempt, respectively.

Optimal effort follows from backward induction. Suppose the student fails the first time. Then he is back in the situation described in the previous subsection, with the only difference being that a time investment equal to $(1 - \delta)t_1$ has already been done. δ is a depreciation factor and likely to be larger the longer the time span between the two exam opportunities. Then it follows from (2) that the optimal time spent preparing for the second opportunity, conditional on $\tilde{t}_1$, is given by

$$\tilde{t}_2 = -(1 - \delta)\tilde{t}_1 + \beta^{-\frac{1}{2}} w^{-\frac{1}{2}} - \alpha\beta^{-1} - \beta^{-1}. \tag{4}$$

As in (3), the passing probability is

$$\tilde{p}_2 = 1 - w^{\frac{1}{2}} \beta^{-\frac{1}{2}} \tag{5}$$

Note that if the student fails at the first attempt, the *total* time spent preparing for both exams, $T = t_1 + t_2$, is exactly the same as the total time spent when there is only one exam, in the absence of depreciation (i.e. $\delta = 0$).

The optimal time investment for the first attempt follows from maximizing

$$p_1(t_1) + (1 - p_1(t_1))\tilde{p}_2 - wt_1 - w(T - (1 - \delta)t_1)(1 - p_1(t_1)) \tag{6}$$

with respect to t_1, where $T = \beta^{-\frac{1}{2}} w^{-\frac{1}{2}} - \alpha\beta^{-1} - \beta^{-1}$. The solution is given by the positive root of a quadratic equation in t_1.

For $\delta = 0$ (no depreciation), the expressions for the optimal time investment at the first attempt, the passing probability at the first attempt, and the ultimate passing probability are given by

$$\begin{cases} \tilde{t}_1 & = 2^{\frac{1}{2}} w^{-\frac{1}{4}} \beta^{-\frac{3}{4}} - \alpha\beta^{-1} - \beta^{-1} \\ \tilde{p}_1 & = 1 - 2^{-\frac{1}{2}} w^{\frac{1}{4}} \beta^{-\frac{1}{4}} \\ \tilde{P} & = 1 - 2^{-\frac{1}{2}} w^{\frac{3}{4}} \beta^{-\frac{3}{4}}. \end{cases} \tag{7}$$

For $\delta = 1$ (full depreciation), the expressions are

$$\begin{cases} \tilde{t}_1 & = w^{-\frac{1}{4}} \beta^{-\frac{3}{4}} - \alpha\beta^{-1} - \beta^{-1} \\ \tilde{p}_1 & = 1 - w^{\frac{1}{4}} \beta^{-\frac{1}{4}} \\ \tilde{P} & = 1 - w^{\frac{3}{4}} \beta^{-\frac{3}{4}}. \end{cases} \tag{8}$$

The analysis shows that a second exam increases the ultimate passing probability ($\tilde{P} > \tilde{p}$). Students who pass at the first attempt spend less time preparing as when there is only one exam opportunity.

2.3 A numerical illustration

In the model students are characterized by four parameters, α, β, δ, and w. For each parameter, I consider two different plausible numerical values and calculate the outcomes of the model for all 16 combinations. The results are presented in table 1.[3]

The results show that, for a wide range of parameter values, a second exam opportunity increases the ultimate passing probability only slightly (P/p), but dramatically reduces expected student effort ($\overline{T}/t$, with $\overline{T} = p_1 t_1 + (1 - p_1)T$), in some cases by almost two thirds. Students who are lucky to pass at the first attempt spend only a fraction of the time they would have spent with only one exam opportunity (t_1/t), in some cases less than one fifth.

[3]To interpret numerical values of w note that a student is indifferent between passing and $1/w$ time units (hours) of non-exam activities.

Table 1: **Rational time inputs and passing probabilities**

				One opportunity		Two opportunities						Comparison		
				t	p	t_1	p_1	t_2	p_2	$\overline{T}$	P	t_1/t	$\overline{T}/t$	P/p
$\alpha = 0,$	$\beta = 0.1,$	$w = 0.001,$	$\delta = 0$	90	.90	34.7	.78	55.3	.90	47.1	.98	.39	.52	1.09
$\alpha = -2,$	$\beta = 0.1,$	$w = 0.001,$	$\delta = 0$	110	.90	54.7	.78	55.3	.90	67.1	.98	.50	.61	1.09
$\alpha = 0,$	$\beta = 0.2,$	$w = 0.001,$	$\delta = 0$	65.7	.93	21.6	.81	44.1	.93	29.9	.99	.33	.45	1.06
$\alpha = -2,$	$\beta = 0.2,$	$w = 0.001,$	$\delta = 0$	75.7	.93	31.6	.81	44.1	.93	39.9	.99	.42	.53	1.06
$\alpha = 0,$	$\beta = 0.1,$	$w = 0.0005,$	$\delta = 0$	131.4	.93	43.2	.81	88.2	.93	59.8	.99	.33	.45	1.06
$\alpha = -2,$	$\beta = 0.1,$	$w = 0.0005,$	$\delta = 0$	151.4	.93	63.2	.81	88.2	.93	79.8	.99	.42	.53	1.06
$\alpha = 0,$	$\beta = 0.2,$	$w = 0.0005,$	$\delta = 0$	95	.95	26.6	.84	68.4	.95	37.4	.99	.28	.39	1.04
$\alpha = -2,$	$\beta = 0.2,$	$w = 0.0005,$	$\delta = 0$	105	.95	36.6	.84	68.4	.95	47.4	.99	.35	.45	1.04
$\alpha = 0,$	$\beta = 0.1,$	$w = 0.001,$	$\delta = 1$	90	.90	21.6	.68	90	.90	43.2	.97	.24	.48	1.08
$\alpha = -2,$	$\beta = 0.1,$	$w = 0.001,$	$\delta = 1$	110	.90	41.6	.68	110	.90	63.2	.97	.38	.57	1.08
$\alpha = 0,$	$\beta = 0.2,$	$w = 0.001,$	$\delta = 1$	65.7	.93	13.8	.73	65.7	.93	27.6	.98	.21	.42	1.06
$\alpha = -2,$	$\beta = 0.2,$	$w = 0.001,$	$\delta = 1$	75.7	.93	23.8	.73	75.7	.93	37.6	.98	.31	.50	1.06
$\alpha = 0,$	$\beta = 0.1,$	$w = 0.0005,$	$\delta = 1$	131.4	.93	27.6	.73	131.4	.93	55.2	.98	.21	.42	1.06
$\alpha = -2,$	$\beta = 0.1,$	$w = 0.0005,$	$\delta = 1$	151.4	.93	47.6	.73	151.4	.93	75.2	.98	.31	.50	1.06
$\alpha = 0,$	$\beta = 0.2,$	$w = 0.0005,$	$\delta = 1$	95	.95	17.4	.78	95	.95	34.7	.99	.18	.37	1.04
$\alpha = -2,$	$\beta = 0.2,$	$w = 0.0005,$	$\delta = 1$	105	.95	27.4	.78	105	.95	44.7	.99	.26	.50	1.04

3 Discussion

The results presented in this note suggest that unconditionally offering students more than one exam opportunity makes little sense. The main effect is that it provides them with a potential windfall gain of passing at the first attempt with minimal effort. Thus the number of resits can strongly affect student effort and thereby the quality of a degree.

Of course, a number of qualifications should be kept in mind when interpreting the results. For example, the model assumes that students have perfect knowledge about the shape of the relationship between effort and passing probability. In practice, students sometimes use the first exam opportunity to learn about its nature and difficulty. Also, the model assumes that students do not derive any intrinsic utility from exam preparation. Highly motivated and conscientious students might act as if the second exam opportunity does not exist and use it only as a last resort. The numerical results also depend on the degree of randomness in the relationship between effort and passing. A well-designed exam exhibits a steep increase in the passing probability beyond a target effort level. Note however that for parameter values closer to that case ($\alpha = -2$; $\beta = 0.2$) the adverse effects of two exam opportunities still emerge.

Empirical research, preferably based on field experiments, is required to obtain more insight into the benefits and costs of resit exams (including the costs of designing and grading exams for faculty) and the optimal institutional design of evaluating students.

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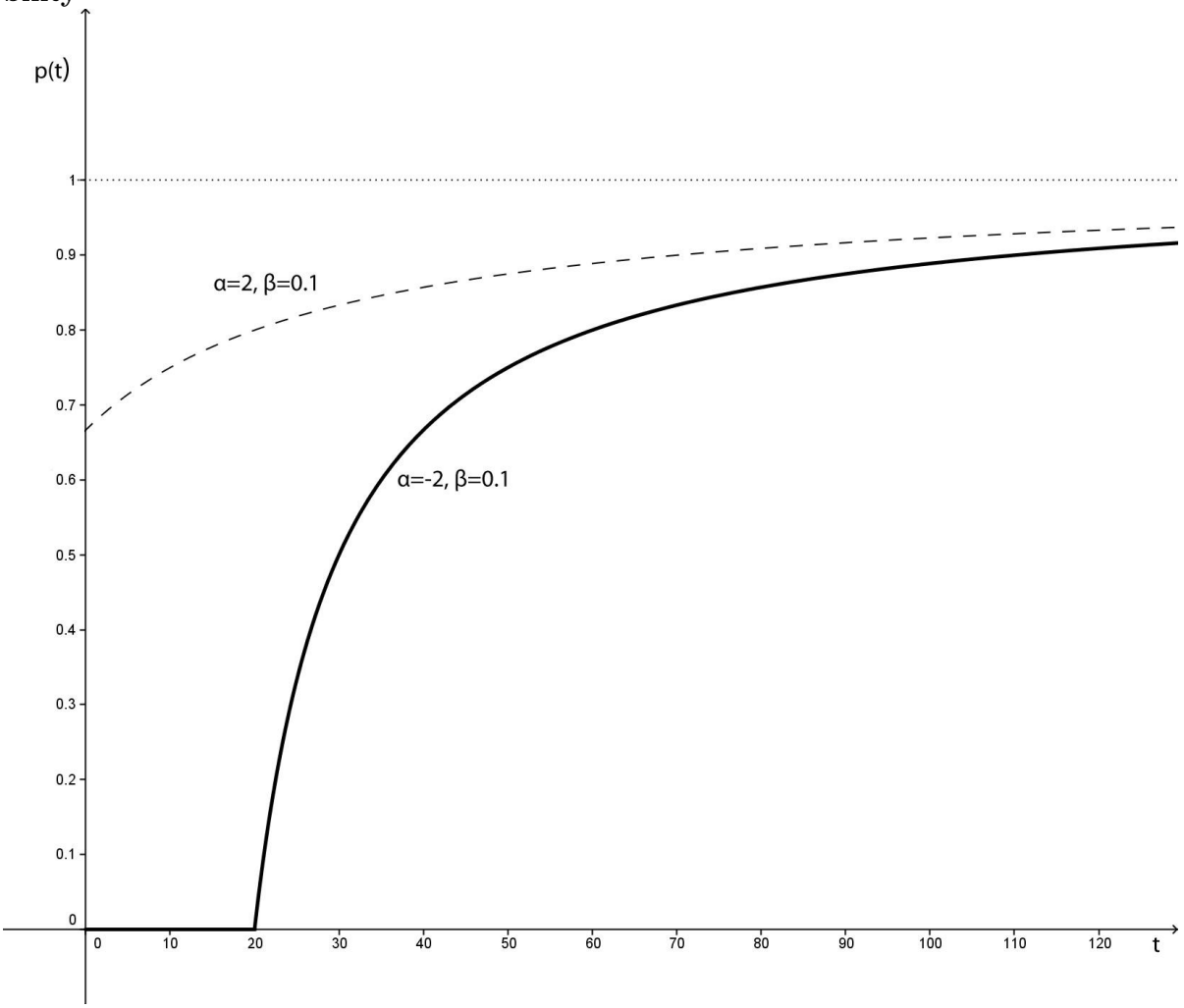
Figure 1: **The relationship between time spent and passing probability**