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Working Paper

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CESifo Working Paper, No. 3983

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Suggested Citation: Knapek, Stephan; Wambach, Achim (2012) : Strategic complexities in the combinatorial clock auction, CESifo Working Paper, No. 3983, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/66871>

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Strategic Complexities in the Combinatorial Clock Auction

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CESIFO WORKING PAPER NO. 3983

CATEGORY 11: INDUSTRIAL ORGANISATION

OCTOBER 2012

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Strategic Complexities in the Combinatorial Clock Auction

Abstract

Recently, the combinatorial clock auction has become more and more common in the auctioning of telecommunication licenses. Although the auction design is complex, the promise is that bidding becomes simple - truth-telling is 'close to optimal'. We show that this claim is too strong. The auction entails several strategic complexities which make bidding non-trivial.

JEL-Code: D440, L960.

Keywords: combinatorial clock auction.

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This version: 25. October 2012

We thank Vitali Gretschno and Nicolas Fugger for helpful comments and suggestions.

1 Introduction

In recent years, the package clock auction or combinatorial clock auction (CCA) has become more and more common in the auctioning of telecommunication licenses.¹

The CCA in its basic design works as follows. In a first phase, the clock phase, clock prices increase on the different categories of frequencies. Bidders just state their demand at the prevailing prices. Clock prices increase until on a category demand is smaller than or equal to supply, when the clock for this category stops. The clock phase is over when on all categories the clock has stopped. In a second phase, the supplementary round, bidders are allowed to make additional bids on any possible bundle of frequencies. These bids are constrained by a pricing rule based on revealed preference which is explained in more detail below. Once all final bids are on the table, the auctioneer determines the winning allocation, which is given by the combination of bids which maximizes the sum of bids. The prices the winners have to pay are determined according to a second price rule: For each bidder, the next best winning allocation, i.e. the winning allocation where all bids from this bidder are excluded, is determined. Then the price this bidder has to pay is equal to the minimum bid he could have made on his allocation in order to beat this second best allocation.²

While the design is quite complex, the promise of the CCA is that this auction makes bidding simpler. E.g. Crampton 2009, Dotecon Ireland 2011 and Czech Regulator 2011 argue that truthful bidding is close to optimal or

¹The CCA has been or will be used among other places in the United Kingdom 2008, Austria 2010, Denmark 2010, the Netherlands 2010, Ireland 2012, Switzerland 2012, Czech Republic 2012 (see KB spectrum).

²Further constraints complicate the determination of final prices. Prices have to be larger than the sum of reserve prices and, more complex, the winning prices should be such that no losing bidder or group of bidders have made bids larger than the winners have to pay, in which case the prices will be modified. In the case of the CCA in Switzerland in 2012, the explanation of the exact pricing rule required 7 pages (Bakom 2011). In the following we mostly abstract from this last complexity, i.e. our reasoning does not rely on the fact that in the real CCA payments are not always equal to the opportunity costs of the winners.

that it "allows bidders to use a simple strategy" and "allows the participants to evaluate the spectrum without ... shadow bids". The recommendation of truthful bidding is useful, as in the clockphase bidders just need to quote on the package which at given prices would lead to the largest profit for them.

A second recommendation is that if all lots in the clock phase have been allocated, i.e. on all categories demand exactly equals supply, then doing nothing (or only a minimal increase of the last bid by one unit) in the supplementary round would be optimal (see e.g. Cramton, 2009, p.26, Dotecon Ireland 2011). This is useful, because the alternative of truthful bidding in the supplementary round would require to determine the willingness to pay for all possible bundles of licenses (in Switzerland, up to 1000 bids were allowed in the supplementary round), which can be very demanding. Furthermore, truthful bidding with a second price rule might create further problems, if the public or at least the regulator is informed about the true willingness to pay of the bidders.³

We show below that both claims (truthful bidding during the clock phase, no further bid in the supplementary round if all lots have been allocated), do not hold.⁴ However, if this simple bidding behavior is non optimal, then the complexities of the auction design make optimal bidding a non trivial task, thus strategic complexities arise.

The paper proceeds as follows. In section 2, we outline a simplified version of the auction rules, abstracting from several complexities which arise in real auctions, like reserve prices, different eligibilities for different categories of frequencies, spectrum caps, and so on. In section 3, we concentrate on the bidding behavior during the supplementary round. We show that not bidding further if all lots have been allocated is not optimal. Furthermore we discuss strategies on how a bidder might increase or decrease the prices his competitors have to pay without any risk of paying more himself. In

³In New Zealand, politicians ran into problems following a second price auction, as the winning firm had to pay much lower prices than its bids (see Milgrom 2004).

⁴In particular, we show that these claims do not hold even without recourse to 'close-to-Vickrey core pricing', see section 5.1.

section 4, bidding during the clock phase is analysed. Here it is shown that truthful bidding is not a dominant strategy. In section 5 we discuss further complexities we encountered when consulting bidders.⁵

2 Auction rules and recommended behavior

2.1 Auction rules

We analyse the CCA in a stripped down version, where further constraints like spectrum caps, reserve prices or 'close-to-Vickrey core pricing' (see section 5.1) are not considered.

Let there be n frequencies for sale, where frequencies are categorized into m categories. We assume that all frequencies in a category are identical to each other.⁶ Let n_j be the number of frequencies (or lots) in category j . It holds that $n = \sum_j n_j$. In latter examples, we mostly use two categories of frequencies ($m = 2$) with two lots each ($n_1 = n_2 = 2$).

The auction consists of two phases. In phase 1, a clock price on each category increases. For given clock prices, bidders indicate their demand, i.e. how many licenses they are willing to take at the current prices. As long as in a category the demand is larger than the number of lots in this category, there is a further round where the clock price increases on that category. If on all categories the demand is equal to or smaller than supply, the clock phase ends.

At the beginning of the auction, bidders obtain a certain eligibility for bidding. The eligibility determines on how many licenses a bidder can bid for in the clock phase. During the course of the clock phase bidders are not allowed to increase their eligibility. I.e. the eligibility for a bid in round t cannot be larger than the eligibility in the previous round $t - 1$. While in

⁵The authors have consulted bidders in recent telecommunication auctions across Europe.

⁶In the real auctions, where frequencies in a given category might differ, the exact allocation of these frequencies is determined in a further allocation stage. We do not consider this stage here.

general, lots in different categories may require different eligibility points, to simplify we assume that lots in every category require the same eligibility. Thus, the eligibility requirement just states that in any new round, the overall demand of a bidder can only stay constant or decrease, but not increase. For simplicity, we assume that eligibility at the beginning of the clock phase encompasses all lots.

In the supplementary round, all bidders can make additional bids on any possible combination (or bundle) of frequencies. However, there is a constraint on bids for bundles which differ from the final package of the clock phase. Suppose bidder i at the end of the clock phase has the leading bid for n_1^i lots of category 1, n_2^i lots of category 2, and so on. We denote this 'final clockround bundle' by $q_f^i = (n_1^i, n_2^i, \dots, n_m^i)$ with eligibility equal to $N_f^i = \sum_j n_j^i$. In what follows we mostly drop the index i . In the supplementary round this bidder can make any bid for his final package, which we denote by $b(q_f)$. If this bidder were to make a bid for a package $q \neq q_f$ in the supplementary round which requires an eligibility of less than or equal to N_f , then this bid is constrained by a revealed preference rule:

$$b(q) \leq b(q_f) + (q - q_f) \times p_f \quad (1)$$

p_f is the vector of prices in the final clockround. Inequality (1) implies that bids of packages smaller or equal in eligibility to q_f are constrained by the bid for the final clockround package plus the difference in package prices at the final round of the clock phase.

If the bidder wants to make a bid on a bundle which requires more eligibility, say $X > N_f$, then the price he is allowed to bid on this bundle has to be smaller than the bid on his final clockround bundle plus some markup, which is defined as follows: During the clock phase, in some round, say t , the bidder reduced his demand from X to Y lots with $Y < X$. The bid the bidder is allowed to make on any bundle which requires the eligibility of X (say q_X) is equal to the largest bid he is allowed to make on the bundle he was quoting for in round t with eligibility Y (q_Y) plus the difference in prices

between the bundles at the clock prices of round t . Formally,

$$b(q_X) \leq b(q_Y) + (q_X - q_Y) \times p_t. \quad (2)$$

The idea behind this rule is that as the bidder was not willing to bid on the larger bundle in round t , when he was still allowed to do so, he 'revealed' that he was at most willing to pay this differences in prices to obtain a bundle of size X compared to the bundle of size Y . Thus his final bids for the larger bundle is constrained by this difference. Note that if $Y > N_f$, then the price the bidder can bid on the bundle of size Y is constrained in a similar manner. Recursively, the maximum bid on q_X is determined by the maximum bid on the final clockround bundle.

Once all final bids are collected, the auctioneer determines the combination of bids which yields the maximum sum of bids. This determines the winning allocation. Note that all bids during the clock phase are valid bids. Hence they are considered in the supplementary round. I.e. if a bidder exited at some prices, then his last bid and also all his other bids in previous rounds still count.

Next, the payments are determined. In the logic of a second price auction, the price a winning bidder has to pay is equal to the lowest bid he would have needed to bid in order to make the winning allocation the winner. I.e. the auctioneer determines the value of a hypothetical winning allocation, where all bids of this particular bidder are excluded. Now this bidder has to pay the difference of the value of this hypothetical allocation minus the value of the bids of all the other winning bidders in the final allocation, as long as this expression is larger than zero. Otherwise he has to pay zero. Thus each winner pays his opportunity costs.⁷

⁷In the real CCA, the pricing formula is more complicated, as e.g. reserve prices have to be met. More critically, there is a second reason why pricing in the real auction is more complex: There is a further requirement, namely that the sum of the final payments should not be smaller than the sum of bids of losing bidders, as otherwise these bidders might complain that although they bid more than the winners have to pay, they did not obtain these frequencies. This further requirement is called 'close-to-Vickrey core pricing' (e.g. Cramton, 2009). We ignore this aspect in the following (but we will comment upon it

2.2 Recommended behavior

As this auction format entails many features of a second price auction, a straightforward recommendation would be to bid truthfully. I.e., during the clock phase, a bidder should bid on a combination of frequencies which maximizes his profit for given clockprices, under the constraint that this bid has to satisfy the eligibility of the bidder in that round.

During the supplementary round, truthful bidding would also be a sensible recommendation, and the constraints on the bids are made such that truthful bidding is possible given that the bidder made truthful bids during the clock phase. I.e., the increments on bundles which require larger eligibility are just equal to the prevailing price at the clock phase at which the bidder reduced his eligibility, thus revealing his willingness to pay for this additional lot. However, in this supplementary round truthful behavior requires bidding on all possible combinations of frequencies, which might be very demanding. Furthermore, by bidding truthfully, bidders will reveal their true willingness to pay for the frequencies. This in turn allows to determine the profits they make in case they obtain an allocation. As the bidders operate in a regulated market, they might not wish to reveal to the regulator how much profit they expect to make in order not to obtain a weaker bargaining position in later negotiations.⁸

Fortunately, the recommendation by the auction designer is such that bidders do not have to reveal their true valuation in this supplementary round (See e.g. Propositions 1 and 2 in Cramton, 2009). In particular, the recommendation is the following: If at the end of the clock phase, all frequencies have been allocated, i.e., in all categories demand exactly equals supply, then no additional bid in the supplementary round is required. If,

in section 5), as the strategic complexities we derive are not a consequence of this 'close-to Vickrey'-rule.

⁸If final bids are not revealed, e.g. if an IT system is used which guarantees that no one has access to the bids, then the worry that the information about the true valuation will be misused is less of a problem. However, it is not clear whether this is legally possible, as the government sells the frequencies and public scrutiny (e.g. by losing bidders) is desirable.

however, at the end of the clock phase in a category the demand is lower than supply, i.e., there is excess supply because the clock stopped when some bidder reduced his demand by more than a single unit, then the bidders who are bidding on this category have to increase their bid by the value of the unsold lots at the final clock prices.⁹

To summarize, the recommendations are that bidders should bid truthfully during the clock phase, and then do nothing in the supplementary round as long as all lots are allocated at the end of the clock phase.

3 Strategic complexities I: Bidding in the supplementary round

In this section we outline strategic complexities in the CCA during the supplementary round. To illustrate our results, we always use the same working vehicle. There are at most two different categories of frequencies, with at most two lots at each category. Categories are numbered by Greek letters (α, β). There are either two or three bidders, numbered by roman letters (A, B , and C).

3.1 No further bid in the supplementary round is not optimal

In this subsection we analyse the advice that if at the end of the clock phase all lots are assigned, additional bidding in the supplementary round is unnecessary. As mentioned above, if optimal, this would be extremely useful, as the other 'simple' alternative of truthful bidding at this stage would imply that the company reveals to the regulator its expected future profits on the assigned frequencies, and indeed on any other bundle of frequencies.

⁹If there are several bidders in a given category with excess supply, then there is a coordination problem among these bidders on who will increase the bid. Here we will abstract from this problem.

The following example shows that following this advice is in general not optimal. There are three bidders (A, B, C), two categories (α, β), two lots in each category. Prices and demand by the three bidders during the clock phase are given in Table 1. The clock phase ends in round two, where all lots have been allocated.

| Round | Price of α | Price of β | Bid of bidder A | Bid of bidder B | Bid of bidder C |
|-------|-------------------|------------------|-----------------|-----------------|-----------------|
| 1 | 0 | 0 | (2,1) | (0,1) | (0,2) |
| 2 | 0 | p | (2,1) | (0,1) | (0,0) |

Table 1: *Prices and demand (bids) during the clock phase.*

In the supplementary phase, bidder A and B follow the advice and make no additional bid, i.e. the final bid of A is p for the package (2, 1). B bids p for the package (0, 1). Bidder C can hand in a bid of $2p$ for the bundle (0, 2), as the last round when he was able to bid on this package is the final round, and $2p$ is the price for this package at final clockround prices. Consider the price he can hand in for the bundle (2, 1). The first round, when he was bidding on a bundle with eligibility smaller than three was in round one. So the bid he can make for this larger bundle is equal to $2p$ (the bid for the bundle (0,2) of round one) plus the difference in the value of the bundles (2,1) and (0,2) at the prices of round one, which is however equal to zero. Thus he can hand in a bid of $2p$ for this bundle. In sum, bidder C and bidder B can outbid the allocation at the end of the clockround, thus A loses.

That under some conditions the supplementary round can alter the clock assignment can also be seen formally. The bid on the larger package X ((2,1) in the example), is constrained by the condition of inequality (1):

$$b(q_X) \leq b(q_Y) + (q_X - q_Y) \times p_t \quad (3)$$

where Y is the smaller package, and t is the round where the bidder was bidding on Y while he could have made a bid on X . Similarly, the bid for the smaller package Y ((0,2) in the example) is constrained by the next

smaller package or the final clockround bundle $((0,0)$ in the example). Here we consider this latter case:

$$b(q_Y) \leq b(q_f) + (q_Y - q_f) \times p_f \quad (4)$$

Now, adding inequalities (3) and (4) leads to:

$$b(q_X) - b(q_f) \leq (q_X - q_Y) \times p_t + (q_Y - q_f) \times p_f \quad (5)$$

So, if

$$(q_X - q_Y) \times p_t \leq (q_X - q_Y) \times p_f, \quad (6)$$

i.e.

$$0 \leq (q_X - q_Y) \times (p_f - p_t), \quad (7)$$

we would have

$$b(q_X) - b(q_f) \leq (q_X - q_f) \times p_f.$$

I.e., the marginal value of awarding q_X to the bidder instead of q_f , would be smaller than the value of the lots at final prices $(q_X - q_f) \times p_f$, such that a change in the final assignment cannot increase value. In that case, further bidding in the supplementary phase will not be necessary.

However, the formal condition (7) can be violated if for some category j it holds that $q_{X,j} < q_{Y,j}$ and at the same time $p_{f,j} > p_{t,j}$, where the index j indicates the j -th element of the vectors. Although the bidder reduces his eligibility overall by going from X to Y , he increases his demand in (at least) one category and there is overdemand in that category. One specific example would be where the bidder, by reducing his eligibility, increases the overall price of the package, i.e. when $(q_X - q_Y) \times p_t$ is negative. Then, e.g. if $p_f = \lambda p_t$ with $\lambda > 1$, we get that $(q_X - q_Y) \times (p_f - p_t) < 0$. In our example, we have $(q_X - q_Y) \times p_t = 0$, as p_t would refer to the prices in round one. However, $(q_X - q_Y) \times p_f = -p$, as q_Y includes two lots in category β , while q_X includes only one lot. This explains the price difference in the final offer

by $C(2p)$ compared to the final clockround offer by $A(p)$.

Our analysis has shown that further bidding in the supplementary round would be necessary to secure the allocation from the end of the clock phase. Bidders can secure this allocation by increasing their bid by an appropriate amount that in principle can be determined via a recursive algorithm based on (5). However, application of (5) would require knowledge on the eligibility of all the other bidders at the start of the auction and a close monitoring of the behaviour of all the other bidders during the clock phase. Both is usually not possible due to limited or even completely missing information. Hence the occurrence of (7) can not be clearly detected.

3.2 Costless punishments of the competitors

While as a first approximation it is useful to assume that bidders primarily care about their own profit, there are good reasons to believe that this might not be the case in reality. In the telecommunication industry typically only a small number of firms compete for a relatively long time. Furthermore, after the auction of telecommunication frequencies, entry into the market is usually limited. In such a market, firms might either be on very competitive terms, or they might arrange themselves. In this and the following subsection, we consider two bidding strategies in the supplementary round, where without incurring any risk, one strategy aims at hurting the competitor, i.e., the competitor has to pay more, while the other strategy is designed in order to support the competitor, i.e., the competitor has to pay less. Paying more or less refers to the comparison with a situation where bidders stick to the recommended rules discussed above.¹⁰

¹⁰Note that very often management of companies as well as operative project teams involved in the preparation of the bidding are very much interested in not paying more for comparable frequencies than its competitors. One of the reasons is that management tries to limit the available budget of its competitors for necessary infrastructure investments following the auction. Another reason is that involved management and project members carry a significant personal risk for their careers, if the own company seemingly overpaid in the auction. Consider for example the CCA in Switzerland in 2012 where bidders had to pay widely differing prices for the acquired frequencies.

To see this, suppose that at the end of the clock stage the final allocation is such that all bidders obtain some frequencies. Final clock prices are given by p_f , the final clockround bundles are given by q_f^i , $i = 1, \dots, k$, where k is the number of bidders. In the supplementary round, bidders can make additional offers such that the final bids on the final clockround packages are given by $b^i(q_f^i) \geq q_f^i \times p_f$. To calculate the price bidder i has to pay, the next best allocation given all final offers has to be determined. We call this allocation q_{-i} , where bidder $j \neq i$ obtains the bundle q_{-i}^j with an offer of b_{-i}^j . Then, if prices are determined by the opportunity costs rule, bidder i has to pay

$$P_i = \sum_{j \neq i} (b_{-i}^j - b^j(q_f^j)) \quad (9)$$

as long as this expression is larger than zero. There are three ways on how a competitor j can influence the price in equation (9): (i) via his offer for the final clockround package ($b^j(q_f^j)$), which will be discussed in subsection (3.3). (ii) via the size of his offer for the next best alternative (b_{-i}^j). (iii) via the determination of the next best alternative (q_{-i}^j). (ii) and (iii) are analysed next.

Consider an example with two categories α and β , two lots in each category and three bidders (A, B, C). The clock phase has ended at prices of p and 0, when bidder A reduced his demand on α from N to $N - 1$, (see Table 2).

| Round | Price of α | Price of β | Bid of bidder A | Bid of bidder B | Bid of bidder C |
|-------|-------------------|------------------|-----------------|-----------------|-----------------|
| 1 | 0 | 0 | (N,0) | (1,0) | (0,N) |
| 2 | 1 | 0 | (N,0) | (1,0) | (0,N) |
| ... | ... | 0 | (N,0) | (1,0) | (0,N) |
| p | p-1 | 0 | (N,0) | (1,0) | (0,N) |
| p+1 | p | 0 | (N-1,0) | (1,0) | (0,N) |

Table 2: Prices and demand (bids) during the clock phase.

If bidders do not hand in additional bids in the supplementary round,

then bidder A obtains $N - 1$ lots of α . The price bidder A has to pay is zero, as the next best allocation (q_{-A}) is such that bidder B and C obtain identical lots as they obtain in the final allocation, i.e. one lot of α for bidder B and N lots of β for bidder C . Thus P_A as determined by (9) is equal to zero, because $q_{-A}^j = q_f^j$ and thus $b_{-A}^j = b^j(q_f^j)$. With a similar reasoning it is easy to see that bidder B obtains one lot of α for a price of $p - 1$ and C obtains all N lots of β for a price of zero.

In this situation bidder C can increase the price bidder A has to pay without any risk for himself: Bidder C could hand in a bid of up to Np for $(N, 0)$ in the supplementary round. By doing this, he would change for bidder A the next best alternative q_{-i}^j (case (iii)). The best allocation without bidder A would then be that bidder C obtains all lots in α . In case bidder C bids Np , bidder A has to pay a price of $(N - 1)p$, i.e. A has to pay his bid. (A needs to "outbid" the additional bid of C on $N - 1$ lots in α .) More generally, the price bidder A has to pay, is completely determined by bidder C 's bid in the supplementary round on $(N, 0)$. Depending on C 's supplementary bid, A has to pay some price between zero and his last bid of $(N - 1)p$. The bid on α is without risk for C , as he will not win his bid. Hence, C can bid even if he is not interested in obtaining α .

The possibility to increase the prices during the supplementary round has implications for the bidding during the clock phase. The example provided above displays a situation where A tried to outbid B on the category α during the clock phase, but finally decided to give up. Anticipating that C might hurt A with his bid in the supplementary round, it might have been better for A to end the clock phase earlier by reducing his demand on α . This would give bidder C less manoeuvre to increase the prices A has to pay.

The case just discussed is an example for category (iii), i.e., changing the next best alternative in order to influence the final prices. A simple modification allows to see an example for case (ii), i.e., changing the size of the bid for the next best alternative in order to influence the final prices. Suppose C is bidding on $(N, 0)$ for some rounds (say until the price on α is \tilde{p}

with $N\tilde{p} > p$), before switching to $(0, N)$. Then if there are no further bids in the supplementary round, A will have to pay $N\tilde{p} - p$, as the next best alternative is that bidder C obtains N lots of α . Now, by increasing his bid on $(N, 0)$ to Np , bidder C by modifying his offer for the next best alternative (b_{-i}^j) , raises the price bidder A will have to pay.

3.3 Costless collusion

In this subsection we show that a bidder can decrease the prices the others have to pay without any consequence for himself. Suppose bidding has reached the final round of the clock stage and there is a winning allocation. As an example consider Table 3. There is only a single category α with two lots. A was bidding on two lots for α and at the price of 10 he reduced his demand to 1. B was bidding on a single lot.

| Round | Price of α | Bid of bidder A | Bid of bidder B |
|-------|-------------------|-----------------|-----------------|
| 1 | 0 | 2 | 1 |
| 2 | 1 | 2 | 1 |
| ... | ... | 2 | 1 |
| 10 | 9 | 2 | 1 |
| 11 | 10 | 1 | 1 |

Table 3: *Prices and demand (bids) during the clock phase.*

If there is no further bid in the supplementary round, the winning allocation is that A and B get a single lot each. A pays zero, as the next best allocation is that B receives a single lot, which he also receives in the final allocation. B has to pay 8, as the next best allocation is that A obtains two lots at the price of 18. As A is bidding 10 for a single lot in the final clockround, B has to pay $18-10=8$.

Suppose that A raises his bid on his lot in the supplementary round to 18 (or 19, depending on the tie breaking rule). In that case, the winning allocation is still the same, however bidder B has to pay zero as well. The next best allocation is that A obtains a single lot at the price of 18. As that

is the same as his final allocation, the price B has to pay, as determined by (9), is equal to zero.

This observation actually holds more generally. Suppose that at the end of the clock stage the final allocation is such that all bidders obtain some frequencies. Final clock prices are given by p_f , the final clockround bundles are given by q_f^i , $i = 1, \dots, k$, where k is the number of bidders. Assume that all bidders raise their bids on their final clockround bundle such that the new bid is equal to the maximum bid they made during the clock phase. That is they increase their bid from $q_f^i \times p_f$ to b_{max}^i , where b_{max}^i is the largest package bid of bidder i during the clock phase. In that case every bidder has to pay zero. This is due to the fact that the second best allocation, i.e., the next best allocation where bidder i is excluded is given by the final bids of all the other bidders, and this leads to a joint bid of $\sum_{j \neq i} b_{max}^j$. Thus the payment of bidder i , which is given by the next best allocation minus the bids of the others in the final allocation, is equal to zero.

4 Strategic complexities II: Bidding in the clock phase

This section deals with the bidding behavior during the clock phase. First, we show that truthful bidding is not a dominant strategy. Second, we argue how a bidder can with his bidding during the clock phase increase the price another bidder has to pay without increasing his own prices.

4.1 Truthful bidding during the clock phase is not a dominant strategy

Consider the following example.¹¹ One category (α), two bidders (A and B) and two lots. The preferences of A are such that he would be willing to pay up to 14 for two lots, and nothing for a single lot. The willingness to pay of B for a single lot is 11 and for two lots is 15.

Thus initially both bidders have a demand of 2, such that the overall demand is equal to 4. If both bidders bid truthfully, bidder B reduces his demand to 1 at the price of 5, and A reduces his demand to zero when the clock price reaches 8, see Table 4. If A exits at 8, B raises his bid to 11 for a single lot and 15 for two lots in the supplementary round. B wins both lots.

| Round | Price of α | Bid of bidder A | Bid of bidder B |
|-------|-------------------|-----------------|-----------------|
| 1 | 0 | 2 | 2 |
| 2 | 1 | 2 | 2 |
| ... | ... | 2 | 2 |
| 5 | 4 | 2 | 2 |
| 6 | 5 | 2 | 1 |
| 7 | 6 | 2 | 1 |
| 8 | 7 | 2 | 1 |
| 9 | 8 | 0 | 1 |

Table 4: *Prices and demand (bids) during the clock phase.*

Now suppose that instead of exiting at 7, A stays in the auction at the clockprice of 8 and then exits. The first observation is that if B bids truthfully in the supplementary round, then the behavior of A would not be optimal. In that case, B raises his bid in the supplementary round to 15 for two lots (i.e., bids truthfully), implying that A wins the auction, as he has entered a bid of 16 for two lots. However, he has to pay 15 and thus makes a loss. Therefore,

¹¹The argument provided in this section that truthful bidding is not a dominant strategy even during the clock phase, does not rely on the fact that prices are 'close-to-Vickrey core prices'. As mentioned above, we assume that prices are determined according to the opportunity costs rule.

if B bids truthfully during the supplementary round, then truthful bidding for A is optimal. However, this does not imply that it is a dominant strategy for A to bid truthfully, i.e. to exit at 8, and we discuss two possible (and realistic) scenarios where exiting later in the clock phase might be optimal.

B only bids in the supplementary round if it is possibly profitable for him. Suppose A enters a bid of 16 for two lots and exits the auction at the clock-price of 9. So B is sure to lose the auction with a bid of 15. So why should B enter a bid in the supplementary round? The general recommendation for the bidding strategy in this situation (where demand at the final clock price is smaller than supply) states that the person with the winning bid has to increase the price for the winning package by the clock price times the number of lots for which there is no demand. This implies that B needs to hand in a bid of at least 16 which would be larger than his valuation. Thus if B enters a bid in the supplementary round only if this is profitable for him, then staying into the auction at the price of 8 would be preferable for A . While A loses the contest if he exits at 8, he wins the contest for a price of 9 if he exits at 9. Hence, bidding truthfully is not a dominant strategy.

B updates his valuations.

In the preceding analysis we assumed that B has a fixed valuation for the frequency (11 for a single lot and 15 for two lots). However, if valuations were ex ante fully known, then an ascending auction would not be necessary. Bidders could just hand in their valuations in a generalized Vickrey auction. One of the reasons why an ascending auction is used is that bidders obtain information during the auction which they can use to redetermine their valuations. And as after the clock phase there is additional time until the supplementary bid has to be made, it is well conceivable that valuations are updated in between. Consider the following example: Suppose B has a preliminary valuation of 9 for a single lot and 13 for two lots. In case A exits at 8, the management of B might reason that by just increasing the bid slightly above the preliminary valuation to 14, B will win two lots. As this increase in valuations leads to a 'sure' win, the management might quite

likely be willing to increase its valuation. But, if A remains in the auction until prices reach 9, and then exits, B might find it too expensive to outbid A in the supplementary round. Thus A wins.

As a side remark, note that this example displays a further complexity. As long as B exits before the clock price reaches 8, the relevant second best alternative is that B obtains two lots at the price of 4, thus A has to pay 8. This implies that in the clock price region between 5 and 8, the price A has to pay in case of winning does not depend on the clock price. Once the clock price is above 8, the relevant alternative in case B exits the auction is that B obtains one lot at the clock price at which he exited. Thus A has to pay this price in case of winning. But note that A is bidding on two lots (and is assumed to win two lots), while the price he has to pay is equal to the single clock price. Figure 1 shows the bids and payments of A as a function of the clockprice at which B exits.¹²

4.2 Threats and counterthreats

In subsection 3.2 we discussed strategies where hurting the competitor came without risk of retaliation. Here we discuss a strategy where bidders try to increase the price of their competitor during the clock phase with little or no consequences for themselves.

Consider the following scenario, which is displayed in Table 5. There is one category with two lots. A and B both desire a single lot each. However while B bids on a single lot, A is bidding on two lots. As long as A remains in the contest with a demand of two, the price will increase further. If at some clock price p , A reduces his demand to one, the clock phase ends. If

¹²One should caution here that in the CCA a bidder does usually not observe how many competitors he has on a particular category. The standard information provided to each bidder during a round is whether there is more demand than supply on a given category or not for the prevailing clock price. This latter complexity requires that bidder A can identify whether at the price of 5 it was bidder B who reduced his demand from two to one, or whether some other bidder C reduced his demand from one to zero. However, in some situations there might be only two bidders present, or even if more bidders participate in the auction, it might be that only two bidders are expected to bid in a particular category, e.g. if only two bidders have aligned frequencies.

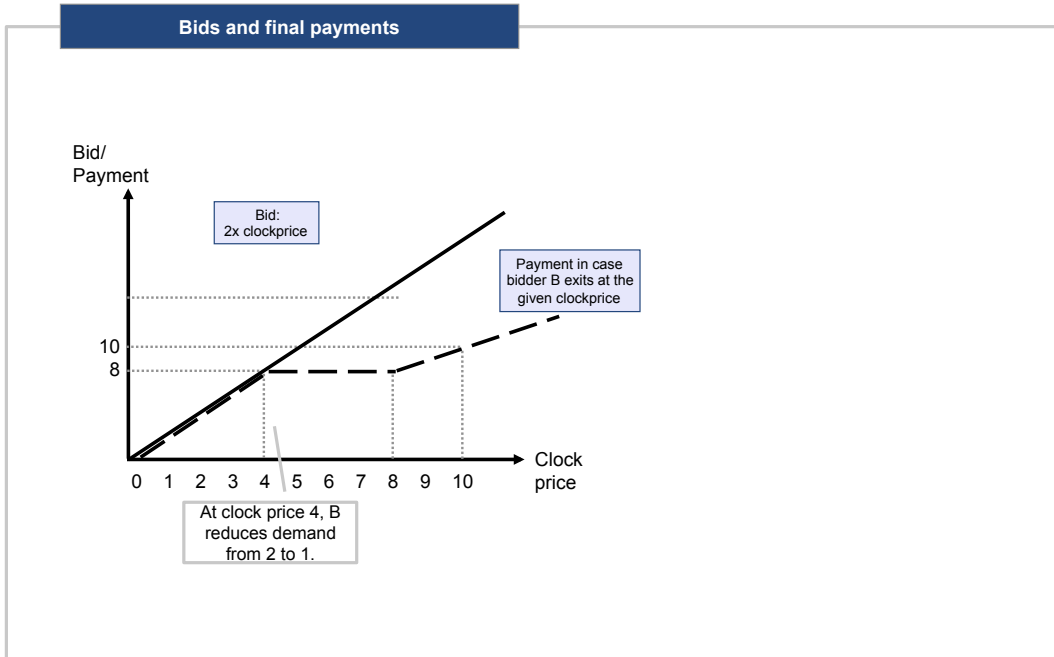


Figure 1: *Prices do not increase strict monotonically*

there are no additional bids in the supplementary round (and even if there are - here they will not change the final allocation), A and B both win one lot, B pays $p - 2$ while A pays zero.

| Round | Price of α | Bid of bidder A | Bid of bidder B |
|-------|-------------------|-----------------|-----------------|
| 1 | 0 | 2 | 1 |
| 2 | 1 | 2 | 1 |
| ... | ... | 2 | 1 |
| p | $p-1$ | 2 | 1 |
| $p+1$ | p | 1 | 1 |

Table 5: *Prices and demand (bids) during the clock phase.*

In the context of telecommunication auctions, it is well conceivable that there are lots which a specific telecommunication company needs to acquire. Thus such a strategy of increasing the price B has to pay might come with little risk. The main risk is that B exits the auction, such that A wins both

lots at the exit price of B .

While in many auctions, bidding for a category a competitor desires will increase the prices of the competitor, the second pricing rule of the CCA allows to increase the prices without having to pay more for the own lots in that category. With a strategy as displayed above, one competitor (A) raises the price for the other competitor (B), without having to pay this increase in prices himself.¹³

A bidder in a CCA should prepare himself for the possibility that other bidders can increase his prices with little risk for themselves. There are several ways on how B could counteract this threat by A . If B has some bidding rights from other categories, B might enter a bid for two lots in category α . If that occurs at some price \tilde{p} , and B reduces his demand to one, A has to pay $\min\{2\tilde{p} - p, 0\}$ if he reduces his demand to one at the price of p . The risk of having to pay more for his lot might deter A from applying this strategy further. Another way on how B could retaliate is by bidding on a different category β , in which A ideally has a large stake. As is well known from the literature on strategic demand reduction and on signaling in auctions, the (counter-)threat of starting a bidding war in β might be sufficient to deter A from increasing the prices in α .

5 Further complexities

Here we comment upon further complexities, which arise due to particular design features or due to particularities of decision taking inside firms.

5.1 Close-to-Vickrey core pricing

As already mentioned, in the real CCA payments are not determined purely according to the logic of a second price auction. It is required that the

¹³In a dynamic ascending auction as in Germany 2010, such a strategy would not have worked. There, by increasing the prices on one band, all lots in that band become more expensive. Increasing the prices of a competitor without paying more could only work in the dynamic ascending auction if a firm did not want to win any lot in that category.

sum of the final payments should not be smaller than the sum of bids of an alternative set of bidders, potentially leading to an increase in the payments of one or more winning bidders. In this case, as the own bid is an upper limit for the own payment, a bidder can now influence his own payments by his own bids. This in turn destroys the logic behind truthful bidding. This effect is well documented in the literature (A. Erdil and P. Klemperer 2009, J.K. Goeree and Y. Lien 2009), and we only discuss a simple example here to illustrate this phenomenon.

There is one category, two lots and three bidders (A , B , C). The clock phase has ended at the price of 2, when bidder C , who was bidding on two lots, exited, while A and B , both bidding on a single lot, remained in the auction. Suppose both A and B , e.g. by bidding truthfully, raise their bid to 3. Then the second price rule would imply that both have to pay zero, as the second best alternative for both is that the other bidder gets a single lot with his bid of 3.

However, as the final bid of C for the two lots is equal to 2, close-to-Vickrey core pricing implies that both A and B have to pay 1 instead of zero. Now, if A instead of bidding 3 would hand in a bid of 2, the final prices with a strict second price rule would still be zero. Close-to-Vickrey core pricing now implies that A pays $4/5$, while B pays $6/5$. Thus by reducing his bid A reduces his payment as well.

5.2 CFOs want to have control over their expenses

Typically there is a lot at stake for bidders participating in CCAs and the corresponding valuations amount to significant investments for companies if they would have to be spent. Therefore, during the auction, bidders usually have to report to e.g. their supervisory board on the development of the auction and especially on the momentary expenses at that stage of the auction.¹⁴ Additionally, CFOs usually want to keep close control over their

¹⁴Thus the well known fact that larger bidders tend to pay less than smaller bidders per unit (see e.g. Table 3 and the subsequent discussion) creates further complexities when

expenses and the value at stake. However, in the CCA, the second price logic (or even more complex the 'Close-to-Vickrey core prices') makes this requirement hard to fulfill. Any computation of the actual payments would require knowledge on individual bids of competitors, which is not available. Therefore, in most cases only estimates of varying reliability (and hence often very limited usefulness) could be provided.

Despite the limited relevance of auction expenses for optimal bidding behaviour (apart from budget limitations), bidders nevertheless tend to put huge effort in determining actual expenses or even act according to actual expenses and not auction prices.

5.3 Bidding with a budget

In many cases, bidding according to valuations is limited by available resources. On the positive side, these budget constraints limit the problem of computing absolute valuations. For straightforward bidding during the clock phase, foremost relative valuations between different bundles of frequencies are relevant for bidding. Then budgets are required on the marginal value of additional lots. Nevertheless, there might still be the requirement for an absolute budget limit, for example for a minimal set of lots a bidder needs to successfully continue his operations.

From a bid strategy point of view, budget limitations force bidders to change to "cheaper" packages, either because the budget for some marginal value or the absolute budget limit is reached during the auction, although another package would still be more profitable. Additional problems might occur due to a change of a budget limit during the auction, which might lead to ex-ante suboptimal bids. Note however, that none of these problems is specific to the CCA.

A problem that is however specific to the CCA arises due to the fact that there might not be enough budget to secure the end-of-clock-round bid, compare section 3.1. Thus cases are conceivable where during the clock phase

communicating the results to the public.

a bidder optimally chooses a different bundle even if his preferred package (at the given prices) is still 'in the budget'.

6 Conclusion

Bidding in the combinatorial clock auction is not straight forward. In this article we show pitfalls which might arise, both during the clock phase and in the supplementary round. We outline bidding strategies which are designed in order to hurt the competitor or to help the competitor and thus to hurt the regulator. Once truthful bidding is no longer on the table as an optimal strategy, the complexities in the design of the auction make it a non trivial task for the bidders to determine their best bidding behavior.

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