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Fueling growth when oil peaks: Directed technological change and the limits to efficiency

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Fueling Growth when Oil Peaks: Directed Technological Change and the Limits to Efficiency

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Abstract

While fossil energy dependency has declined and energy supply has grown in the postwar world economy, future resource scarcity could cast its shadow on world economic growth soon if energy markets are forward looking. We develop an endogenous growth model that reconciles the current aggregate trends in energy use and productivity growth with the intertemporal dynamics of forward looking resource markets. Combining scarcity-rent driven energy supply (in the spirit of Hotelling) with profit-driven Directed Technical Change (in the spirit of Romer/Acemoglu), we generate transitional dynamics that can be qualitatively calibrated to current trends. The long-run properties of the model are studied to examine whether current trends are sustainable. We highlight the role of extraction costs in mining.

JEL-Code: O410, Q320, Q430.

Keywords: non-renewable resources, energy, economic growth, innovation, directed technical change.

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1. Introduction

All developed countries have been heavily dependent on fossil fuels over the past decades, which helps explaining why terrorism and oil security are so high on national policy agendas, why "peak oil" stories (e.g. Deffeyes, 2001) engender emotional discussion, and why national governments postpone signing international climate change treaties. Since fossil fuels are nonrenewable resources, their use cannot grow forever and oil dependence might seem to only become a bigger threat. Energy suppliers can be expected to anticipate future scarcity and manage their extraction-production strategies accordingly. Therefore, one has naturally wondered when and to what degree this dependence puts limits to economic growth.

While in the short run oil shocks can have a big impact on the overall economy, there is little evidence that in the second half of the 20th century oil dependence has caused direct problems for economic growth. According to the trends observed during those years, the supply of energy (both in absolute and in per capita terms) has increased steadily and the share of energy in total cost has declined secularly. Although alternative fuels have become cheaper over time, the use of conventional energy has become more attractive at an even faster pace. Technological change has mainly benefited the productivity of conventional fuels, reducing the need to exploit alternative energy sources. An important indicator of these technological developments is the productivity of energy, as measured by value added per unit of energy, which has steadily increased.

Noticing that the path for the supply of conventional (non-renewable) energy has been increasing but knowing that it will inevitable decline in the future, we can naturally predict a peak in oil supply in the future. The marked slowdown in energy use in the first decade of the 21st century might be the beginning of a change in this direction, but the phenomenon is too recent to know if a structural trend change has occurred already.[1] As long as oil productivity improves fast enough to offset scarcity, the economic consequences can be small. A key question is therefore how technological developments respond to oil supply changes. Profit-seeking entrepreneurs will start looking for energy-saving technologies in the face of energy scarcity. Similarly, forward-looking energy suppliers will adjust their production plans to future events like growing demand because of aggregate economic growth or falling demand because of energy saving. Future resource scarcity could cast its shadow on world economic growth soon when energy and technology markets are forward looking.

[1] Per capita energy consumption in USA grew on average at an annual rate 1% over the period 1949-1999, but over the period 200-2010 it declined at a rate of 1% (EIA, 2012).

Our aim is to understand how the trends observed during the second half of the 20th century in aggregate energy use and productivity growth can be linked to future scarcity. We build a model that generate both phases, *viz.* a phase of growth fueled by expanding energy supply followed by a phase of declining oil production, as part of the same fundamental dynamics that arise from profit-maximizing technology development and energy supply.

The Hotelling (1931) model has been the workhorse model to think about how profit-maximizing forward-looking suppliers of nonrenewable resources -like oil- would allocate production over time. It assumes a finite (cumulative) physical supply and predicts rising prices for extracted resources in the long run. Given that these results carry over to partial and general equilibrium models that assume finite resource supply and forward-looking profit maximizing agents, it seems hard to ignore Hotelling's insights when analyzing growth and resource markets. However, because the model is notoriously difficult to reconcile with stylized facts, the practical application of the insights has been questioned. First, as Gaudet (2007) documents, the empirical pattern of prices is difficult to reconcile with the predictions from the simple partial equilibrium models as long as these models ignore uncertainty and cannot endogenize technology responses to energy scarcity. Second, at the aggregate level, there is a difficulty to reconcile growing energy use with the Hotelling framework. For example, merging the standard neoclassical growth model with the Hotelling model, one typically finds that resource use declines monotonically over time (Dasgupta and Heal, 1974, page 17; see also Benchecroun and Withagen, 2011, who examine the transitional dynamics of the model analytically). In the literature there seems to be a gap between, on the one hand, analytical approaches that apply first principles and the Hotelling model but generate extraction patterns that are at odds with facts (e.g. Chakravorty et al., 2008; Van de Ploeg and Withagen, 2011), and, on the other hand, applied and calibrated models that leave out Hotelling for the sake of calibration. For example the DICE model (Nordhaus 1994), the model most often applied to evaluate the consequences of greenhouse emissions associated fossil fuel use, assumes away finiteness of energy supply despite its long time horizon.[2]

In this paper we try to fill this gap by developing an analytically tractable model that both can be calibrated to stylized facts and assumes finite resource supply with profit-

[2] There are two alternative ways to avoid introducing the Hotelling assumption and results. First, there might be a "backstop technology" that makes the scarce resource input superfluous. Second, the resource supply may be physically unlimited, but subject to increasing extraction costs (Heal, 1976). Both approaches basically assume away the economic relevance of physical scarcity and cannot therefore address the economic consequences of physical scarcity. This is not to say that such approaches are not generating useful insights. Rather, we claim that our approach complements these approaches by showing that even with our more "conservative" assumption (i.e. physical finiteness), current trends can be explained.

maximizing intertemporal behavior. By analyzing the key mechanisms of the model, rather than calibrating the model as a black box, we show how the structure of the model allows stylized facts to be replicated. In this procedure, which we dub "qualitative calibration", we analytically derive bounds for parameters we have to pick in each calibration to a model outcome that satisfies a list of well documented stylized facts. This approach allows us to point out that changes in the cost of extraction of the resource play a major role in the calibration. In order to show that this result holds very generally, we deliberately choose to take developments in extraction costs over time as exogenous.

Another contribution of our analytical approach is that we derive the dynamics of our model in terms of a variable that directly captures oil dependence, namely the energy share in GDP. The model predicts a clear-cut relationship between the energy share in GDP, the direction of technological change, and total factor productivity growth. Moreover, given technological change in oil extraction, the dynamics of the energy share can be determined. As a result the model explains how oil dependence – as measured by the energy share in GDP – evolves over time and how it affects the dynamics of technological change and production growth.

When qualitatively calibrating our model, we start from the following stylized facts observed during the last decades of the 20th century: a growing per capita energy supply, declining cost share of energy in GDP, declining energy cost relative to labor cost, and declining energy use per unit of GDP (or, equivalently, increasing energy efficiency), as well as increasing per capita output. We also take into account the evidence of poor substitution between energy and other inputs, and the absence of strong evidence of a predominantly resource-using or resource-saving nature of technological change. The equilibrium dynamics of the model predict that the energy share in GDP first declines and then rises to higher than initial levels. This undershooting phenomenon goes together with fluctuations in total factor productivity (TFP) growth. In the long run, the peak of oil production causes growth rates to fall, partly cushioned by higher rates of TFP growth.

The first building block of the model is the supply of energy, thought of as a non-renewable resource with a fixed initial endowment. From partial equilibrium resource theory, it is well known that the price of an exhaustible resource can be decreasing over a time interval if technological change lowers marginal extraction cost rapidly enough (e.g. Krautkraemer, 1998, p. 2068; Berck and Roberts, 1996). The challenge is to include the dynamics of extraction costs in a general equilibrium framework. Tahvonen and Salo (2001) provided such a framework and obtain an inverted U-shaped time pattern for non-renewable

resource extraction in a model in which energy can be provided by both a renewable and a non-renewable resource; they consider learning-by-doing but not R&D. We complement these analyses by focusing on the macroeconomic level: we consider substitution between resource/energy inputs and labor/capital inputs, while technological change not only affects the resources sector, but also the rest of the economy. In our model, the direction of technological change determines whether oil suppliers speed up or slow down depletion over time.

The second building block of our model is the supply of technology. Three types of technological change are distinguished: labor-saving technological change, energy-saving technological change, and declines in extraction costs. While the latter are assumed to be exogenous for the reason mentioned above, the first two types of technological change are driven by profit incentives in line with Acemoglu's (1998, 2002) theory of endogenous directed technological change. If energy supply grows faster than labor supply, relative energy costs for firms fall (since both inputs are gross complements) and it becomes less profitable to supply resource-saving technologies. However, when the supply of oil has peaked, this pattern reverses. The shifts in the direction of R&D, together with the changes in the scale of the economy as a result from changes in energy supply affect the bias and overall rate of technological change. Existing theoretical papers on growth and non-renewable resources have so far neglected the dynamic interaction between extraction costs and directed technological change. For example, Barbier (1999) and Grimaud and Rougé (2003) assume a unitary elasticity of substitution between inputs so that only one type of technological change can be relevant. In Groth and Schou (2002, 2005), growth is driven by capital accumulation rather than knowledge accumulation. Di Maria and Valente (2008) allow for directed technological change in a Hotelling framework but ignore extraction costs and focus on the steady state.

In section 2, we present the final goods production technology and discuss the behavior of firms, consumers, and resource-owners. In section 3 we investigate how the model can replicate the stylized facts described above and what is the implied path of technology. Declining extraction costs turn out to be essential to explain the combination of observed stylized facts in the short run. With an increasingly smaller share of extraction costs in total energy cost, declines in extraction costs become increasingly less powerful and we conclude that some of the observed patterns on energy use are unlikely to hold in the long run. In section 4 we study the size and direction of technological change by modeling how firms

choose their innovation projects, and we characterize the dynamics of the model. Section 5 concludes.

2. Producers, consumers and resource-owners

A closed economy produces a homogeneous consumption good, using labor and energy services. In turn, labor (energy) services are produced using labor (energy) and a set of specific intermediates. The total supply of labor employed in the final-goods sector, L, is a subset of total population, L^S. The supply of energy results from the endogenous extraction of a non-renewable resource stock. To fix ideas, we will always refer to this input as energy, but a broader interpretation (e.g. materials for production) is possible.

Following Acemoglu (1998), we assume that the productivity of the primary inputs (labor and energy) mainly depends on the quantity and quality of factor-complementary intermediate goods. The quality is a state variable that increases as a result of R&D effort performed by monopolist firms.

Final goods production

The final consumption good (Y) is produced using labor services (Y_L) and energy services (Y_R) according to the following CES function with elasticity of substitution equal to σ:

$$Y = A \cdot \left(Y_L^{(\sigma-1)/\sigma} + Y_R^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} . \tag{1}$$

Labor (energy) services are produced according to the following Cobb-Douglas/Romer (1990) production functions:

$$Y_L = L^{\beta} \int_0^1 q_{Lk} m_{Lk}^{1-\beta} dk , \tag{2}$$

$$Y_R = R^{\beta} \int_0^1 q_{Rk} m_{Rk}^{1-\beta} dk , \tag{3}$$

where L and R are the primary inputs (labor and energy respectively) employed in the final goods sector, m_{ik} is the use of intermediates of variant k in the production of type i services (i

= R, L), and q_{ik} is the associated quality level. The mass of different intermediates in each sector is normalized to unity.

Final-goods producers take prices as given. Their factor demand (for labor, energy and intermediates) is given by the first-order conditions of the profit maximization problem:

$$p_Y \frac{\partial Y}{\partial Y_L}\frac{\partial Y_L}{\partial L} = w_L \quad \Leftrightarrow \quad \frac{\beta\theta_L p_Y Y}{L} = w_L\,, \tag{4}$$

$$p_Y \frac{\partial Y}{\partial Y_R}\frac{\partial Y_R}{\partial R} = w_R \quad \Leftrightarrow \quad \frac{\beta\theta_R p_Y Y}{R} = w_R\,, \tag{5}$$

$$p_Y \frac{\partial Y}{\partial Y_i}\frac{\partial Y_i}{\partial m_{ik}} = p_{mik} \quad \Leftrightarrow \quad \frac{\theta_i p_Y Y(1-\beta) q_{ik} m_{ik}^{-\beta}}{\int_0^1 q_{ik} m_{ik}^{1-\beta} dk} = p_{mik}\,, \qquad i = R,\, L, \tag{6}$$

where p_Y denotes the price of final output, w_i is the factor price for primary input i, p_{mik} is the price of intermediate good m_{ik}, and $\theta_i \equiv (\partial Y / \partial Y_i) Y_i / Y$ is the production elasticity of sector i's services in final gross output Y. From (4) and (5) we see that the share of gross revenue $p_Y Y$ that is devoted to remunerate primary inputs equals $(\theta_L + \theta_R)\beta$, which equals β because of constant returns to scale in production; the remaining share $(1 - \beta)$ is spent on intermediates, as can be derived from (6). Hence total factor payments equals $\beta p_Y Y$ and θ_i is the share of factor i in factor income. In the sequel we will refer to θ_R as the energy share.

Intermediate goods production and price setting

The market for intermediates is characterized by monopolistic competition (as in Dixit and Stiglitz 1977). Each producer supplies a unique variety and sets a monopoly price. The cost of producing one unit of m_{ik} at quality q_{ik} is q_{ik} units of the final good. Equation (6) reveals that the elasticity of demand for each intermediate good equals $1/\beta$. This fact implies that monopoly prices for intermediates are set as a mark-up over unit costs ($q_{ik} p_Y$). As usual, the mark-up is negatively related to the elasticity of demand $1/\beta$:

$$p_{mik} = q_{ik} p_Y / (1-\beta)\,; \qquad i = R,\, L\,. \tag{7}$$

Substituting this price in the demand function (6), we find that all intermediate goods producers within the same sector i produce the same level of output m_i:

$$m_{ik} = m_i = \theta_i Y(1-\beta)^2 / Q_i \,; \qquad i = R,\, L\,, \tag{8}$$

where Q_L and Q_R represent the state of technology as measured by the average quality of labor-related and energy-related inputs, defined as

$$Q_i \equiv \int_0^1 q_{ik}\, dk \qquad\qquad i = R,\, L\,. \tag{9}$$

Static goods market equilibrium

The goods market equilibrium can be characterized in terms of primary inputs (R and L) and the state of technology (Q_L and Q_R). Substituting equilibrium quantities of intermediate inputs from (8) into the production and demand functions, we first solve for relative factor shares and relative input prices:[3]

$$\frac{\theta_R}{\theta_L} = \left(\frac{Q_R R}{Q_L L}\right)^{-(1-\nu)/\nu} \quad , \tag{10}$$

$$\frac{w_R}{w_L} = \left(\frac{Q_R}{Q_L}\right)^{-(1-\nu)/\nu} \left(\frac{R}{L}\right)^{-1/\nu} \quad , \tag{11}$$

where $\nu = 1 - \beta + \beta\sigma$. Since $\theta_L + \theta_R = 1$, we can solve (10) for each of the factor shares:

$$\theta_R = 1 - \theta_L = \left[\left(\frac{Q_R R}{Q_L L}\right)^{(1-\nu)/\nu} + 1\right]^{-1}. \tag{12}$$

From (11) and (12) we can see that poor substitution ($\sigma < 1$ or, equivalently, $\nu < 1$) means that an increase in the ratio Q_R/Q_L implies a fall in relative energy prices and in the energy share. Therefore, an increase in the ratio Q_R/Q_L, which we call the bias of technology, has the interpretation of energy-saving (equivalently, labor-using or labor-biased) technical change.[4]

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[3] Differentiating (1) and using the definition $\theta_i = (\partial Y/\partial Y_i)Y_i/Y$, we find $\theta_R/\theta_L = (Y_R/Y_L)^{1-1/\sigma}$; using (2)-(3) and (8), we get (10). From (4)-(5) and (10) we get (11).

[4] The opposite holds if $\sigma > 1$, but this is, as will be explained below, not a relevant case.

We next solve for aggregate variables. We express aggregate output as a function of technology and factor inputs by taking into account that the equilibrium level of intermediates use (m_i) depends on output, technology and factor inputs according to (8). In particular, we combine (1), (2), (3), (8), and (12) to arrive at the following expression:[5]

$$Y = \left((Q_L L)^{(\nu-1)/\nu} + (Q_R R)^{(\nu-1)/\nu} \right)^{\nu/(\nu-1)}. \tag{13}$$

This equation shows that, in equilibrium, the average quality levels of intermediates use, Q_i, act as factor augmentation levels. From (13) it is also immediate to interpret ν as the elasticity of substitution between factors (measured in effective terms), to be distinguished from substitution between intermediate services (σ).

From (13), we can directly write output per worker and per capita income growth as

$$Y / L = Q_L (1-\theta_R)^{\nu/(1-\nu)}, \tag{14}$$

$$\hat{Y} - \hat{L}^S = \hat{Q}_L + \theta_R \left(\hat{R} - \hat{L} + \hat{Q}_R - \hat{Q}_L \right) - \left(\hat{L}^S - \hat{L} \right), \tag{15}$$

where hats denote growth rates. Equation (15) shows four sources of growth: changes in the level of (labor-related) technology, growing per capita energy inputs, changes in the bias of technical change and changes in the participation rate. Note also that the second and the third elements are perfect substitutes (regarding growth) and their joint contribution to growth depends on the energy share in output (or production elasticity of energy).

The production of Y serves as intermediates (m) or as final consumption goods (C_Y). Therefore, goods market equilibrium requires $Y = C_Y + \int_0^1 q_{Lk} m_{Lk} dk + \int_0^1 q_{Rk} m_{Rk} dk$. After substituting (8) and (9), we find that net output C_Y is a constant fraction of gross output Y:[6]

$$C_Y = [1-(1-\beta)^2] Y. \tag{16}$$

[5] We have chosen units of Y such that $A = (1-\beta)^{-2(1-\beta)}$, so that the scale constant in (13) becomes unity.

[6] Note that $p_Y C_Y = [1-(1-\beta)^2] p_Y Y = \beta(2-\beta) p_Y Y$ is the value of net output, while $w_R R + w_L L = \beta p_Y Y$ is net factor income (net of intermediates). The excess of net production over net factor payments, $(1-\beta)\beta p_Y Y$, is the monopoly rent accruing to intermediate goods suppliers.

Consumer behavior

The representative consumer maximizes intertemporal utility, specified as $\int_0^\infty \ln c_Y \exp(-\rho t)dt$, subject to his or her budget constraint, where ρ is the utility discount rate, and c_Y is per capita consumption of the Y-good. The logarithmic form of the intertemporal utility function implies that the consumer chooses a consumption path along which total spending grows with the difference between the nominal interest rate r and the utility discount rate ρ, i.e. $\hat{c}_Y + \hat{p}_Y = r - \rho$. Using (16) and assuming that population size, L^S, grows at a constant rate $\hat{L}^S$ so that $\hat{c}_Y = \hat{C}_Y - \hat{L}^S = \hat{Y} - \hat{L}^S$, we may write the Ramsey rule as

$$\hat{p}_Y + \hat{Y} - \hat{L}^S = r - \rho . \tag{17}$$

Natural resource extraction and energy use

The energy input R is obtained from a non-renewable resource stock E. We model extraction costs by assuming that $1+\mu$ units of the resource stock have to be extracted to produce one unit of energy. Hence, the resource stock changes over time according to the following differential equation:

$$\dot{E} = -(1+\mu)R , \tag{18}$$

where a dot over a variable denotes derivation with respect to time. We can interpret μ as the unit resource cost to produce energy. Alternatively, it is a unit extraction or mining cost. Of each unit extracted from the resource stock, a fraction $\mu/(1+\mu)$ is lost in the mining process and only $1/(1+\mu)$ arrives at the market. This specification is similar to the so-called *iceberg costs* in trade literature (introduced by Samuelson, 1954).[7]

We allow unit extraction costs μ to decrease over time. This may reflect improvements in extraction and energy production technology, to the degree that it offsets the increases in extraction costs due to the need to exploit deeper deposits. Evidence for declining extraction costs is documented by, for example, Chermak and Patrick (1995) and Fagan (1997).

[7] In the partial equilibrium literature, it is common to assume that extraction involves a monetary cost (to pay for the inputs in the mining process) rather than a loss of marketable resources and to assume that extraction costs decrease with the remaining stock of resource (stock effects), and decrease with some technology indicator (technological change). For the sake of analytical tractability, we model extraction costs in terms of resources energy rather than in terms of output. Essentially, this assumption separates the extraction process from the goods market clearing conditions (if extraction required good Y as an input, the non-arbitrage equation (30) would become more complicated).

Krautkraemer (2005) summarizes the literature and concludes that inputs per unit extracted have declined steadily for some decades.

To characterize technology in the resource extraction sector, we define Q_μ as the amount of energy that can be used in production (R) per unit of energy extracted, $(1+\mu)R$, i.e. $Q_\mu \equiv 1/(1+\mu)$ represents efficiency in extraction. Accordingly, $\hat{Q}_\mu$ is the rate of efficiency improvement in extraction. We will refer to $\hat{Q}_\mu$ as improvements in mining efficiency. There is a physical limit to these efficiency improvements, since μ cannot become negative by definition. In the long-run, the most favorable situation would be one with negligible extraction costs, $\lim_{t\to\infty} \mu(t) = 0$, and with all further improvements exhausted, $\lim_{t\to\infty} \hat{Q}_\mu(t) = 0$. In a less favorable situation, extraction costs rise in the long run, possibly due to increasingly difficult geological access to resources. We therefore assume the following:

$$\lim_{t\to\infty} \hat{Q}_\mu(t) \equiv \lim_{t\to\infty} \left(\frac{-\dot{\mu}(t)}{1+\mu(t)} \right) \leq 0 . \tag{19}$$

The resource owners[8] decide on the extraction path in order to maximize their discounted profit, $\int_0^\infty R(t)\, w_R(t) \exp\left[-\int_0^t r(s)ds \right] dt$, subject to (18). The solution to this problem results in the following *modified Hotelling rule* and transversality condition:

$$r = \hat{w}_R + \hat{Q}_\mu , \tag{20}$$

$$\lim_{t\to\infty} E(t) = 0 . \tag{21}$$

Equation (20) is the arbitrage rule for which resource owners are indifferent between, on the one hand, extracting the resource today, selling it, and putting the net revenue in the bank at interest rate r, and, on the other hand, extracting and selling it later, thus benefiting from higher prices and mining efficiency in the future but forgoing the interest payments. The faster mining efficiency improvements ($\hat{Q}_\mu$), the more profitable it is to wait extracting, so the slower prices w_R have to rise or the higher the interest rate r has to be to make the resource owner indifferent. If technological change in the mining sector is fast enough, such that $0 < r < \hat{Q}_\mu$, the energy price w_R decreases because of the cost reduction effect. However, this situation cannot continue forever: whenever μ is non-increasing, $\dot{\mu}$ approaches zero in the

[8] We separately discuss consumption and extraction. Since households own the resource stock, the consumption and extraction decision can be integrated. However, this yields the same results, cf. Groth and Schou (2005).

long run (since μ is non-negative). Then, as time goes to infinity, equation (20) collapses to the standard Hotelling rule, which implies that, in the long run, the energy price increases at a rate equal to the interest rate. The transversality condition (21) implies that the complete resource stock is depleted asymptotically.

3. Stylized facts and the forces behind them

3.1. Stylized facts

We now discuss the main stylized facts regarding growth and energy we want to explain in our model. We first list a set of stylized facts derived from U.S. national accounts during the second half of the 20th century (in parentheses we summarize the fact in the notation of the model):

Stylized facts

1. ($\hat{R} > \hat{L}^S$) per capita energy use increased at an average annual rate of about 1 percent.
2. ($\hat{\theta}_R < 0$) the share of energy cost in GDP declined at an average annual rate of about 1 percent.
3. ($\hat{w}_R < \hat{w}_L$) energy prices per unit of labor cost declined at an average annual rate of almost 1 percent.[9]
4. ($\hat{Y} > \hat{R}$) energy productivity (GDP per unit of energy input) increased at an annual rate of 1.4 per cent on average. Alternatively, energy intensity declined by 1.4 per cent.
5. ($\hat{Y} > \hat{L}^S$) per capita income grew at a faster rate than per capita energy use.

The first four stylized facts are reported in Jones (2002, chapter 9) for the US over the period 1950-1998 (based on data from the Energy Information Administration), and confirmed by Sue Wing and Eckaus (2007). Smulders and De Nooij (2003) present qualitatively similar facts for European countries. The fifth stylized fact is the basic feature of postwar growth in

[9] Energy cost relative to wage cost has declined over the period 1948-1998 at an annual average rate of about 0.9 (Jones, 2002, Figure 9.2). However, as is well known there was a big spike between 1973 and 1985.

the industrialized world (Caselli 2005). Note that this stylized fact is implied by the first and the fourth one.

Fluctuations of energy prices have been large and make it more difficult to establish trends. While we focus on the long-run trend and thus want to average out the energy price spikes around 1973 and 1980, we need to pay attention to the recent years. In Figure 1 we update the figures of Jones (2002) on the energy share. While the trend is declining until 2000, one could interpret the trend as increasing thereafter. We therefore formulate an alternative for fact 2:

Stylized fact 2bis: (U-shaped $\theta_R(t)$) the share of energy cost in GDP has declined for a long period but has started to rise recently.

Two other stylized facts are derived from econometric and other studies that are more sensitive to the approach taken. The first one refers to the direction of technological change. Since technology cannot be observed directly, stylized facts cannot be easily identified for technology trends. Econometric work has provided estimates of the rate and direction of technological change (e.g. by Jorgenson and Fraumendi, 1981; Jorgenson, 1984; Popp, 2002; Sue Wing, 2008; Van der Werf, 2008). Most studies estimate at a sectoral level and find indications of energy-using technological change in at least some industries. All studies provide evidence of significant effects of disembodied technological change on energy efficiency improvements in a substantial part of the economy.[10] However, the evidence is mixed with respect to the direction of technical change: in the aggregate, technological change is not strongly biased in either the energy-saving or energy-using direction. To capture this evidence as a *stylized* fact, we will argue that technical change is close enough to being neutral (or, equivalently, unbiased) over the relevant period and that, by appropriate choice of an initial point in time (denoted by $t = 0$), technical change is unbiased initially.[11]

The final fact we deal with refers to substitution elasticities. Studies based on CES production function estimation typically find elasticities of substitution smaller than 1, but significance and range vary with the exact specification (e.g. Prywes 1986; Kuper and Van

[10] Sue Wing (2008) aggregates sectoral estimates into a measure of the effect of technical change on aggregate energy intensity. This however yields an estimate of total factor productivity improvements rather than of the bias of technological change. The latter would require aggregating up to the effect of technology on the demand for energy relative to other inputs (R/L) rather than on the energy intensity (R/Y).

[11] As shown below, for our basic results it is not crucial to assume that technological change is exactly neutral, only that it is sufficiently close to neutral. Nevertheless, neutral technical change as a starting point simplifies the calibration exercise considerably.

Soest, 2002; Van der Werf, 2008; see Neumayer 2003 for a brief survey). In the CGE literature on energy and climate change, the consensus is to assume substitution elasticities not exceeding one (see Van der Werf 2008, Table 1, for an overview).

Summing up, we consider the following two additional stylized facts:

Stylized facts (continued)

6. ($\hat{Q}_R(0) = \hat{Q}_L(0)$) initially, neutral factor-augmenting technological change.

7. ($\nu < 1$) poor substitution between energy and other inputs.

3.2. Qualitative calibration

We want to check if our analytical model has the right structure to be qualitatively calibrated to the observed stylized facts. By this we mean that, instead of collecting representative data on variables of interest and setting the parameters of the model so as to generate specific values for the endogenous variables that match collected data, we can identify the empirically relevant *bounds* on the variables of interest and find the corresponding *range* of parameter values that keep the endogenous variables within the empirically relevant bounds. While the former quantitative calibration procedure is the traditional proven method for employing CGE models in policy evaluation, our qualitative approach is more suited to assess the empirical relevance of analytical models.

The stylized facts in the previous subsection provide the bounds on the variables of interest. We proceed by deriving the predictions of our model for these variables of interest, after which we determine the bounds on the parameters. In this section we focus on the role of directed technical change and mining efficiency, by identifying the bounds on these variables in particular.

The following lemma summarizes how the key variables of the model depend on the three types of technological change, the discount rate and the evolution of the participation rate, L/L^S.

Lemma 1. Per capita energy supply, the energy cost share, energy prices relative to wages, labor productivity, and energy productivity evolve over time according to the following equations:

$$\hat{R} - \hat{L}^s = n(\hat{Q}_\mu - \rho) - (1-n)\left[(\hat{Q}_R - \hat{Q}_L) - (\hat{L} - \hat{L}^s)\right], \tag{22}$$

$$\hat{\theta}_R = -(1-n)\left[(\hat{Q}_R - \hat{Q}_L) + (\hat{Q}_\mu - \rho) - (\hat{L} - \hat{L}^s)\right], \tag{23}$$

$$\hat{w}_R - \hat{w}_L = -\left(\frac{\theta_R(1-n)}{1-\theta_R}\right)(\hat{Q}_R - \hat{Q}_L) - \left(\frac{1-n\theta_R}{1-\theta_R}\right)\left[(\hat{Q}_\mu - \rho) - (\hat{L} - \hat{L}^s)\right], \tag{24}$$

$$\hat{Y} - \hat{R} = \hat{Q}_R - n(1-\theta_R)\left[(\hat{Q}_R - \hat{Q}_L) + (\hat{Q}_\mu - \rho) - (\hat{L} - \hat{L}^s)\right], \tag{25}$$

$$\hat{Y} - \hat{L}^s = \hat{Q}_L + n\theta_R\left[(\hat{Q}_R - \hat{Q}_L) + (\hat{Q}_\mu - \rho)\right] + (1-n\theta_R)(\hat{L} - \hat{L}^s), \tag{26}$$

where

$$n \equiv \frac{\nu}{\nu + (1-\nu)(1-\theta_R)} \in (\nu, 1). \qquad \blacksquare$$

In (22)-(26) n can be interpreted as the effective elasticity of substitution between energy and other inputs (notice that for small θ_R, as in the data, we have $n \approx \nu$).

According to (22), improvements in mining efficiency favor future extraction over current extraction. Intuitively, resource owners bring the oil to the market when it can be mined relatively more efficiently. A high discount rate ρ has the opposite effect and makes the economy extract now rather than later. Together, $\hat{Q}_\mu - \rho$ can be called the "effective postponement stimulus" or "effective patience factor" (cf. Sinclair 1992). The more patient the market effectively is, the more extraction is postponed, which makes it more likely that energy supply will grow rather than decline. Growth in the participation rate, $\hat{L} - \hat{L}^S$, also provides an incentive to postpone extraction, since in future more labor input is available to productively combine with energy. On the other hand, resource-saving technological change in production, as measured by $\hat{Q}_R - \hat{Q}_L$, reduces energy use over time.

Technological change and discount rates also affect the dynamics of the energy share and the relative energy price. According to (23) and (24), energy-saving technological change as well as mining extraction efficiency improvements makes energy effectively less scarce and thus reduces the energy share and relative energy price, while discounting tends to result in increasing energy share and relative energy price. Again, improving mining efficiency and low discounting make resource owners postpone extraction, which makes energy more

abundant and cheaper in the future and reduces the relative price and the share of energy cost in production. The opposite happens with the participation rate: an increasing labor force makes energy scarcer (and thus more expensive) as compared to labor.

Finally, note that mining efficiency improvements and energy technological change play separate roles: with respect to *relative* energy use and energy prices, mining efficiency improvements and energy-augmented technological change work in opposite directions (cf. (22) and (24)); but for *aggregate* per capita growth, energy efficiency and energy share, these two types of technological change appear as perfect substitutes (cf. (23), (25) and (26)).

The next step in our qualitative calibration is to find the bounds for the rate of directed technical change that make the model consistent with the three first stylized facts. The following lemma presents these bounds:

Lemma 2 (Energy intensity, energy share and relative energy prices vs. technology bias)
Stylized facts 1-3 hold if and only if the bias of technical change is bounded in the following way:

$$\max\left\{\hat{L}-\hat{L}^S-\hat{Q}_\mu+\rho,\frac{\hat{L}-\hat{L}^S-\hat{Q}_\mu+\rho}{(1-\nu)\theta_R}\right\}<\hat{Q}_R-\hat{Q}_L<\frac{\nu}{(1-\nu)\theta_L}(\hat{Q}_\mu-\rho)+\hat{L}-\hat{L}^S. \qquad \blacksquare$$

Lemma 1 states that technological change must be not too biased in either energy-using or energy-saving direction to replicate the first three stylized facts. The reason is that per capita energy growth and energy share growth both decrease with energy-saving technological change, while the stylized facts require that energy supply and energy share move in opposite directions. The bounds on the rate of directed technical change in Lemma 2 are consistent with the empirical studies and also with our representation of them in stylized fact number 6 (i.e., technical change is initially neutral). As we show in the following proposition (Proposition 1), if we assume neutral technical change (fact 6), then it is immediate to identify necessary and sufficient conditions for all the other stylized facts to hold.

Proposition 1 (short-run matching with stylized facts)
If fact 6 holds, the following pair of conditions is necessary and sufficient for replicating stylized facts 1- 5:

(i) $\hat{Q}_\mu > \rho+\max\left\{\left(\hat{L}-\hat{L}^S\right),\frac{-(1-\nu)\theta_L}{\nu}\left(\hat{L}-\hat{L}^S\right)\right\}$,

(ii) $\hat{Q}_R > n\theta_L \left[(\hat{Q}_\mu - \rho) - (\hat{L} - \hat{L}^S) \right]$. ■

Note that, under poor substitution (stylized fact 7), condition (i) implies $\hat{Q}_\mu > \rho$, since the second term at the RHS is clearly nonnegative. Therefore, Proposition 1 shows that the model can replicate the stylized facts provided extraction costs fall sufficiently fast, so that there is a positive and large enough "postponement stimulus" ($\hat{Q}_\mu - \rho > 0$). Condition (ii) states that energy-augmenting technical change must be large enough. This is needed to ensure that energy efficiency is increasing even though it is getting more intensively used and its share in total income is declining.

The proposition highlights the role of extraction costs. In particular, the trends of declining energy dependence (fact 2) and growing energy supply (fact 1) are to be understood in the model as the result of the physical constraints in extraction of resources becoming less severe (i.e. $\hat{Q}_\mu$ positive and sufficiently large). This insight follows directly from the Ramsey rule (17), the Hotelling rule (20) and the demand for energy (equation (5) differentiated with respect to time), which jointly imply the following relationship:

$$\hat{R} - \hat{L}^S = \hat{\theta}_R + \hat{Q}_\mu - \rho .$$

To have $\hat{R} - \hat{L}^S > 0$, fact 1, and $\hat{\theta}_R < 0$, fact 2, we must have $\hat{Q}_\mu - \rho > 0$.

3.3. "Peaking oil" and the driving forces in the long run

Since the total stock of energy resources is non-renewable, the flow of energy supply cannot keep growing and must peak at some instant. The question then is to what extent the current trends in energy supply, energy share, energy price, energy productivity, and economic growth can be sustained when oil has peaked and per capita energy growth turn negative. To examine this, we now use the *steady state* of the model (more specifically, the long run version of equations (22)-(26)). For the moment, we take rates of technological change as given and asymptotically constant. Moreover, we assume that, in the long run, the participation rate in the final-goods sector is constant.

Equation (23) is then a differential equation in the single variable θ_R, and reveals that depending on relative magnitudes of the different types of technical change, the energy share either becomes zero or one, as we state in the following proposition.

Lemma 3 (Solow versus Cake-eating model) Assume $\sigma < 1$, constant rates of technological change and constant participation rate L/L^S in the long run. Then, in the long run,

(i) if $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L > \rho$, only labor contributes to per capita output growth, $\theta_R = 0$ and $\hat{Y} - \hat{L}^S = \hat{Q}_L$.

(ii) if $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L < \rho$, only energy contributes to per capita output growth, $\theta_R = 1$ and $\hat{Y} - \hat{L}^S = \hat{Q}_\mu + \hat{Q}_R - \rho$.

(iii) if $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L = \rho$, both energy and labor contribute to growth and per capita growth rate equals the rate of labor-augmenting technological change, $0 < \theta_R < 1$ and $\hat{Y} - \hat{L}^S = \hat{Q}_\mu + \hat{Q}_R - \rho = \hat{Q}_L$. ■

This lemma shows that, in cases (i) and (ii), the model behaves in the long run as the Solow Neoclassical model and the Dasgupta/Heal (1974, 1979) cake-eating model, respectively. Poor substitution implies that the relatively scarce factor dominates. If the rates of technological change which affect directly energy inputs are relatively large as compared to labor-saving technical change ($\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L > \rho$), energy becomes less and less scarce in effective terms, and the economy is no longer dependent on oil in the long run (θ_R tends to 0). Then the economy's per capita growth rate equals the rate of labor–augmenting technological change (as in the Solow-model).

On the other hand, with low improvements in energy or mining efficiency, energy becomes the constraining factor, the economy becomes increasingly dependent on oil and the role of labor becomes negligible (θ_R tends to 1). Per capita growth is then determined by resource-related technical change and the discount rate (as in a cake-eating model, Dasgupta and Heal, 1979)[12]. Only if the effects of energy-related technological change on relative energy demand exactly offset those of labor-related technological change, the energy share stays constant over time and both energy and labor keep contributing to output.

We can now find expressions for the rate of change of other key variables in the long run, which allow us to check whether the stylized facts that we listed above can be sustained in the long run. We conclude the following:

[12] Note, however, that the standard cake-eating model does not distinguish between energy efficiency improvements and mining efficiency improvements, as we do.

Proposition 2 (Long Run) Assume $\sigma < 1$. If rates of technological change are constant in the long run with $\lim_{t\to\infty} \hat{Q}_\mu(t) \le 0$, $\lim_{t\to\infty} \hat{Q}_R(t) \ge 0$ and $\lim_{t\to\infty} \hat{Q}_L(t) \ge 0$, then, in the long run,

(i) Stylized facts 1 ($\hat{R} > \hat{L}^S$) and 3 ($\hat{w}_R < \hat{w}_L$) cannot be sustained;

(ii) Stylized fact 2 ($\hat{\theta}_R < 0$) is sustained if and only if $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L > \rho$;

(iii) Stylized fact 4 ($\hat{Y} > \hat{R}$) is sustained unless $\hat{Q}_R = 0$;

(iv) Stylized fact 5 ($\hat{Y} > \hat{L}^S$) is sustained unless $\hat{Q}_R + \hat{Q}_\mu \le \rho$. ■

This proposition describes the long-run situation, allowing for either constant or increasing extraction costs in the long run. The first part reports that per capita energy supply must peak and relative energy prices must rise in the long run (therefore, stylized facts 1 and 3 will get reversed in the future). Growing scarcity of energy inputs tends to fuel increases in energy productivity, because of diminishing returns. Positive energy- and labor-augmenting technological progress adds to this tendency, which supports continuation of stylized fact 4. However, if energy-related technological change is low, energy inputs become scarcer over time not only in physical terms, but also in effective terms, and the energy share must increase in contrast to what characterizes current short-run dynamics (stylized fact 2). Then energy becomes asymptotically the sole driver of GDP growth, (i.e. $\lim_{t\to\infty} \theta_R = 1$, see Lemma 3) and only energy-augmenting technical progress can drive energy productivity improvements. If energy-augmenting technological progress is absent, energy productivity becomes asymptotically stagnant (as stated in the third part of the proposition).

For per capita income to keep increasing, we need positive technological progress related to the factor that becomes effectively scarcer over time: if the role of energy vanishes over time (when $\hat{Q}_\mu + \hat{Q}_R - \rho > \hat{Q}_L$), labor is the binding factor and positive labor-augmenting technical progress drives per capita growth as in the Solow world; if energy is the binding factor in the long run, energy-augmenting technical progress needs to be large enough to offset the drag on growth because of depletion (as driven by discount rate ρ) and increases in extraction costs (in particular, we need $\hat{Q}_R > \rho - \hat{Q}_\mu$).

We conclude from Lemma 3 and Proposition 2 part iv that whether the per capita growth prospects change after oil has peaked depends crucially on which of the two is relatively large: energy-related or labor-related technical change. With such a large role for technology, it becomes crucial to examine the driving forces behind technological change. In the next section we complete the presentation of the model by introducing the mechanism

with drives the rate of energy-saving technological change. This will allow us to study the full dynamics of the model and get further insights about the connections between technical change, energy use and economic growth. We will find that the knife-edge situation from the third part of Lemma 3 arises as the long-run equilibrium of the model when innovation is explicitly considered.

4. The direction of technical change and the dynamics of the model

4.1. Modeling endogenous factor-augmenting technological change

We now model the economic mechanisms driving factor-augmenting technological change by assuming that each intermediate goods producer improves the quality of his or her good by investing in-house in research and development activities. We choose the simplest endogenous growth formulation for the investment technology of a single firm:[13]

$$\dot{q}_{ik} = \frac{Q_i}{\kappa_i} D_{ik}; \qquad i=R,L, \tag{27}$$

where D_{ik} represents the amount of resources (labor) spent on research and development by firm k in sector i. Apart from the cost parameter κ_i, the productivity in development activities depends on spillovers. An individual firm builds on the knowledge accumulated in the past by all firms in the sector (see Popp, 2002, for evidence with respect to energy-related research). This knowledge stock is proxied by the current aggregate quality level Q_i. The firm takes it as given and neglects that its own current development efforts expand the knowledge stock on which future development builds. Thus, intertemporal spillovers arise, which play an important role in preventing the returns to innovation to fall over time. Since production costs rise with the quality level of the product, the return on subsequent innovation tends to fall. However, intertemporal spillovers reduce the cost of innovation, which boosts the rate of

[13] Our formulation implies that we end up with a "first generation" endogenous growth model a la Romer (1990) and Grossman and Helpman (1991). In these models a bigger economy grows faster (as per the so-called scale effect). However, we are interested in the evolution of the energy-related variables rather than the effect of the size of the economy, so that the scale effect is not problematic in our case. Following Howitt (1999), we can easily remove the scale effect by assuming that the mass of firms in each sector equals N (rather than one), that N grows at the same rate as population, and that spillovers are related to the average quantity of knowledge per firm. It turns out that most of the equations in this paper go through after redefining L^S as being the amount of effective labor supplied per person (rather than the total amount of labor) and L and D as employment in production and research, respectively, as a share of the total workforce. None of the conclusions will be qualitatively affected by this change in modeling. Details are available upon request.

return. Under the present specification, both forces exactly offset each other in the long run and rates of return can be sustained.

Each firm k in sector i chooses innovation effort D_{ik} in order to maximize the net present value of its profits. The associated optimal control problem results in the following no-arbitrage equation:[14]

$$\left(\frac{\beta}{1-\beta}\right)\frac{p_Y m_i Q_i}{w_D \kappa_i} + \hat{w}_D - \hat{Q}_i \leq r, \quad \text{with equality if } D_{ik} > 0\,; \qquad i = R,L\,. \tag{28}$$

where w_D is the unit cost of development effort D_{ik} and can be understood as the researcher's wage. Equation (28) states that each firm invests until the marginal net returns from investment equal the cost of capital r and no investment takes place if even at zero investment the cost of capital is higher than the returns. The first term on the left-hand side is the direct return from supplying goods of higher quality. Profits rise with quality in proportion to sales m_i and the average quality Q_i. The term $\hat{w}_D - \hat{Q}_i$ equals the expected rate of change in the cost of quality improvements. Fast quality growth in the sector (captured by $\hat{Q}_i$) implies large spillovers and cheaper development in the future, which provides an incentive to postpone innovation and reduces the returns to investment. A higher future cost of development effort (captured by $\hat{w}_D$) has an opposite effect.

4.2. The dynamics with endogenous technological change

We now derive expressions for, first, sectoral differences in technological change, which will amount to the bias of technological change, and, second, measures of aggregate technological change, which will give insight in total factor productivity change. We assume that there is a constant supply of labor, L^S, which is perfectly mobile across production and research. Hence we have $L + D = L^S$, were D is total R&D effort. We denote total research effort by D, which is allocated over the labor and energy sectors:

$$D = D_L + D_R\,, \tag{29}$$

[14] The firm maximizes the present value of profits, $\int [p_{mik} m_{ik} - q_{ik} m_{ik} p_y - w_D D_{ik}] \exp\left[-\int_0^t r(s)ds\right] dt$, subject to (6) and (27) and using m_{ik} and D_{ik} as control variables, taking as given all prices, aggregate variables and quantities of other firms. See the appendix for the derivation of equation (28).

where $D_i = \int_0^1 D_{ik} dk$ is aggregate research effort in sector i ($i = L, R$). We find sector-level technological change by aggregating (27) over all firms k in sector i and substituting (9), which yields:

$$\hat{Q}_i = D_i / \kappa_i ; \qquad i = R,L. \tag{30}$$

This equation clarifies that κ_i represents the amount of effort needed for each percentage point of growth of average quality Q_i.

We now choose the normalization $\kappa_L + \kappa_R = 1$, which ensures that our cost parameters κ_i reflect *relative* differences in effort requirements only (absolute effort levels will by governed by total labor, cf. equation (38) below).

Substituting (8) to eliminate $m_i Q_i$, we write the no-arbitrage expressions in (28) as

$$r - \hat{w}_D \geq \frac{\beta(1-\beta)}{w} \frac{\theta_i}{\kappa_i} - \hat{Q}_i, \quad \text{with equality if } \hat{Q}_i = D_i / \kappa_i > 0 ; \qquad i = L, R , \tag{31}$$

where $w \equiv w_D / p_Y Y$ represents the wage, scaled by GDP.

For the moment we will focus on interior solutions (i.e., there is positive R&D in both sectors); we defer the discussion of corner solutions until we build our phase diagram. Under this assumption, subtracting the equations for R and L in (31) from each other, using $\theta_L = 1 - \theta_R$ and $\kappa_L + \kappa_R = 1$, we can derive the following expression which governs the evolution of the bias of technological change:

$$\hat{Q}_R - \hat{Q}_L = \frac{\beta(1-\beta)}{w \kappa_L \kappa_R} (\theta_R - \kappa_R) . \tag{32}$$

Equation (32) reveals that the direction of technical change is driven by the energy share, which represents the importance of energy for production. If energy turns out to be very important for production (i.e. when the energy share θ_R is high as compared to the unit cost of energy-complementary R&D, κ_R), then it pays to allocate more resources to the energy sector. The opposite happens when the energy share is low.

To derive implications for aggregate innovation, again for an interior solution, using (29) and (30) we get $D = \kappa_L \hat{Q}_L + \kappa_R \hat{Q}_R$. Adding the equations for R and L in (31) after multiplying by κ_i yields

$$r - \hat{w}_D = \frac{\beta(1-\beta)}{w} - D \,. \tag{33}$$

Noting that $\hat{w} = \hat{w}_D - \hat{Y} - \hat{p}_Y$ and substituting (17), we find

$$\dot{w} = (D + \rho)w - \beta(1-\beta) \,. \tag{34}$$

Since labor is perfectly mobile across production and research, in equilibrium $w_L = w_D$ must hold. Using this condition together with $L + D = L^S$ and (4) we find the following expression for the equilibrium allocation of labor between production and research:

$$L = \beta \frac{1-\theta_R}{w} \,, \tag{35}$$

$$D = L^S - \beta \frac{1-\theta_R}{w} \,. \tag{36}$$

Using (31) under the assumption of interior solution, together with (33) to eliminate $r - \hat{w}_D$ and (36) to eliminate D, we can derive the amount of research in each sector:

$$D_i = \beta(1-\beta)\frac{\theta_i - \kappa_i}{w} + \kappa_i \left[L^S - \beta \frac{1-\theta_R}{w} \right]; \qquad i = L, R \,. \tag{37}$$

Define conventional total factor productivity growth $\hat{Q}_{TFP}$ as the growth of output minus the appropriately weighted growth of inputs which, using (15), can be written as $\hat{Q}_{TFP} \equiv \hat{Y} - \left[\theta_R \hat{R} + (1-\theta_R)\hat{L} \right] = \theta_R \hat{Q}_R + (1-\theta_R)\hat{Q}_L$, where the second equality comes from (13). Using (29), (31) and (33), in an interior solution total factor productivity growth can be written as

$$\hat{Q}_{TFP} = D + \left(\frac{\beta(1-\beta)}{\kappa_L \kappa_R} \right) \frac{(\theta_R - \kappa_R)^2}{w}. \tag{38}$$

The following lemma shows that, in an interior solution, the dynamics of the whole model can be represented by a two-dimensional system in variables θ_R and w.

Lemma 4 (System dynamics).

In an interior solution with research in both sectors, the dynamics of the model evolve according to the following equations:

$$\hat{w} = \left(\rho + L^S \right) - \frac{\beta}{w} \left[2 - \beta - \theta_R \right], \tag{39}$$

$$\hat{\theta}_R = \frac{-\beta(1-\nu)(1-\theta_R)}{w} \left[\left(1 + \frac{1-\beta}{\kappa_L \kappa_R} \right) \theta_R - \left(2 - \beta + \frac{1-\beta}{\kappa_L} \right) + \left(\frac{L^S + \hat{Q}_\mu}{\beta} \right) w \right]. \quad \blacksquare \tag{40}$$

For a constant, non-positive, rate of mining efficiency improvements, $\hat{Q}_\mu \leq 0$, equations (39) and (40) define a phase diagram in the (θ_R,w) plane with negative sloped lines along which w and θ_R, respectively, are constant, and a negatively sloped saddlepath that leads to the interior steady state. However, in the short run, we can have positive mining efficiency improvements. In fact, since $\hat{Q}_\mu$ is time-varying, this is a non-autonomous system. In the (θ_R, w) plane, a change in mining efficiency improvements implies a rotation of the $\dot{\theta}_R = 0$ locus around a point on the horizontal axis. Figure 2 depicts the phase diagram in the (θ_R,w) plane. We have included two $\dot{\theta}_R = 0$ loci, one for $\hat{Q}_\mu > 0$, the other for $\hat{Q}_\mu = 0$. The figure also shows the non-negativity constraints for D, D_L and D_R, which can be derived from (36) and (37). The phase-diagram shows the equilibrium path for the case in which the rate of mining efficiency improvements monotonically falls from a value that exceeds the discount rate to the value zero. This case results in an energy share that, in line with stylized fact 2bis, decreases initially, but eventually increases. In the remainder of this section we generalize the conditions under which the energy share falls and we check the implications for the other key variables from the list of stylized facts.

We now show that it is possible to find initial values for the key variables such that the short term dynamics of the model is qualitatively consistent with all the stylized facts reported

above. Since fact 7 refers to the elasticity of substitution, which is modeled as a parameter, we impose $\nu < 1$ as an assumption for this parameter and investigate the possibility to generate facts 1-6 as (short-run) outcomes of the model.

Lemma 5 (calibration: matching the model dynamics to the stylized facts)

If the initial conditions and parameters are chosen to satisfy all of the following:

(i) $\theta_R(0) = \kappa_R$,

(ii) $\rho < \hat{Q}_\mu(0) < L^S(1-\beta)/\kappa_L$,

(iii) $\rho < \hat{Q}_\mu(t)$ for $t \in [0, T)$,

then there is a value $T_1 > 0$ such that, if $T \geq T_1$, the model is consistent with stylized facts 1-7.

■

Condition (i) implies that the energy share that is observed in the data provides the value for the cost parameter κ_R. The model is calibrated to generate the desired (observed) initial value for the energy share by choosing the initial value of the resource stock.[15] Condition (iii) ensures that the economy benefits long enough from efficiency improvements in energy supply so that the energy share can fall over time and energy supply grows.

We can now start from this "qualitative calibration" in Lemma 5 to explore what developments the model predicts.

Proposition 3 (dynamics and steady state of the qualitatively calibrated model)

After calibrating the short-run dynamics of the model (with endogenous factor-augmenting technological progress and exogenous improvements in mining efficiency) to stylized facts 1-6 through making assumptions (i)-(iv) from Lemma 5, we find that over time

(i) the energy share initially falls; its long-run value exceeds its initial value;

(ii) technological change starts neutral but becomes increasingly energy-using initially; in the long run it is energy-saving;

(iii) per capita income growth will stabilize at a lower long-run value as compared to the initial value.

(iv) the relative price of energy (in terms of labor costs) is initially decreasing and it is increasing in the long run.

[15] For any combination of initial energy share and path of mining efficiency, $\{\theta_R(0), \hat{Q}_\mu(t)\}$, the system in (38)-(39) defines $w(0)$, such that $\theta_R(t)$ and $w(t)$ approach constants (their steady state values) in the long run. Along the associated path, cumulative extraction has to equal the initial stock $E(0)$ to satisfy the transversality condition in (21). Since $E(0)$ does not enter any of the other equations, we can choose $E(0)$ to accomplish this.

Proposition 3 shows that, once we account for endogenous technical change, the trend described by stylized fact 2 must be reversed: eventually energy scarcity will cause its share to increase over time. As discussed above, recent observations suggest an increasing trend for the energy share. Thus, we find that calibrating the model for t=0 to facts 1-6, with declining energy share in particular, generates an increasing energy share later on, thus replicating stylized fact 2bis. This reversal cannot be explained with constant exogenous rates of technical change (cf. Proposition 2). Therefore, studying the direction of technical change turns out to be crucial to explain the observed trends in energy use and economic growth. Under endogenous directed technical change, the initial improvements in mining efficiency induce energy-using technical change. Nevertheless, in the long, when the possibilities to increase mining efficiency are exhausted, technical changes becomes necessarily energy-saving, and a higher energy dependence (higher energy share) can no longer be avoided. The higher long-run energy share comes with a lower long-run growth rate.

One desirable feature of both Lemma 5 and Proposition 3 is that they are *general* results in the sense that they allow to calibrate the model and characterize its dynamic evolution without any specification for the path of mining improvements. They only use the assumptions that initial improvements should be (upper and lower) bounded in the short run and that in the long run mining efficiency cannot keep improving, but do not use any additional assumption on the specific time evolution of $Q_\mu(t)$. As a consequence, Proposition 3 characterizes only what happens at time zero and compares this to what happens in the long run. It does not characterize what happens in the "medium run", since this will depend on how mining efficiency improvements proceed between time zero and the long run. We now explore the implications of an additional assumption, regarding the medium run mining efficiency improvements: if we assume these do never accelerate over time, we find the results as summarized in the following proposition.

Proposition 4 (dynamics with non-accelerating mining efficiency improvements)

After calibrating the short-run dynamics of the model (with endogenous factor-augmenting technological progress and exogenous improvements in mining efficiency) to stylized facts 1-6 through making assumptions (i)-(iii) from Lemma 5, and assuming in addition that $\hat{Q}_\mu(t)$ is non-increasing over time, we find that over time

(i) the energy share first falls then rises to a higher than initial level;

(ii) total research effort (D) first falls, then rises;

(iii) technological change becomes more and more energy-using first, but then gradually becomes less energy-using and turns into energy-saving;

(iv) total factor productivity growth either initially declines and then rises or it initially declines, then rises, then declines again and finally rises.

The dynamics of the (qualitatively calibrated) model imply that the energy share continues decreasing for a while as long as mining efficiency improvements are large enough. At this stage, energy is abundant and there are little incentives for energy-saving technological change. However, when these improvements become smaller and smaller, and scarcity of resources increases, the energy share starts increasing. This triggers energy-saving technological change, which slows down the increase in the energy share. Indeed, in the long run, the model predicts a constant energy share, as a result of the interaction between resource extraction and endogenous directed technological change. These results allow us to be more precise about the economic forces driving the energy share, which can not be done if technical change is not modeled. (cf. Proposition 2).

Research effort increases after oil has peaked. This is because with scarcer energy, labor productivity in goods production falls, the wage falls, and R&D becomes relatively cheaper. However, per capita income growth is lower in the steady state than at time zero: the increase in total factor productivity is not enough to offset the decline in per capita energy use and the decline in mining efficiency improvements.

5. Conclusions

We have developed an endogenous growth model that connects the rate and direction of technical change to energy use and optimal resource extraction within a dynamic general equilibrium framework. The novel element is that both technical change and resource use are assumed to be endogenous variables resulting from market arbitrage activities by rational optimizing agents.

This model is shown to be compatible with the key stylized facts about economic growth and energy use observed during the second half of the 20th century. Specifically, we show that when resource extraction costs steadily decline, the model generates steadily increasing per capita energy use and decreasing energy share in GDP.

We also conclude that if we take the dynamics of the model towards its steady state as an indication for possible future trends, some important postwar trends in energy use and prices can be expected to be reversed in the future. Indeed, aggregate energy-related data on the first decade of the 21st century could be interpreted as confirming our prediction, although these recent data could also reflect temporary shocks rather than a change of the long-run trend. First, the model predicts that per capita energy use will start declining at some point in the future, and will keep declining in the long run. Since by definition the stock of nonrenewable resources is finite, increasing energy use is unsustainable. The increased scarcity of resources shifts technical change progressively towards energy-saving technological change. This stimulates total factor productivity growth, which thus partly offsets the negative impact of oil having peaked.

Second, the effect of efficiency gains in resource-extraction technologies is likely to become progressively smaller as compared to the exhaustion effect, which means that the observed decrease in energy prices relative to labor prices is likely to be reversed in the long run as the energy price evolution converges to the standard Hotelling rule. In a similar way, the evolution of energy share is also likely to show a U-shaped pattern. Some of this reversal of the decline in energy dependency appeared in recent years. Our model therefore suggests that the spikes in the energy share around the oil crises and the high current level might be interpreted as the start of a structurally higher energy dependency.

Our model highlights what may drive the dynamics of growth and energy scarcity over time. The exhaustion of essential natural resources, in itself, does not necessarily lead to lower growth in future. Rather, it is the exhaustion of the potential to improve the efficiency with which we exploit these physical resources that might slow down growth.

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Appendix A: Proofs.

Proof of Lemma 1

Using (17), (20) and differentiating (4) we get $\hat{w}_R - \hat{w}_L = \rho - \hat{Q}_\mu + \hat{L} - \hat{L}^S - \hat{\theta}_L$. We differentiate (11) and (12), and use the result to eliminate $\hat{w}_R - \hat{w}_L$ and $\hat{\theta}_L$, respectively. After rearranging we obtain (22). Differentiating (12) and using (22) we get (23). Differentiating (4)-(5) and (12), we get (24). Finally, (25) and (26) follow from (15) and (22). QED.

Proof of Lemma 2

From (22), (23) and (24), we get

$$\hat{R} - \hat{L}^s > 0 \Leftrightarrow \hat{Q}_R - \hat{Q}_L < \hat{L} - \hat{L}^s + \frac{n(\hat{Q}_\mu - \rho)}{1-n},$$

$$\hat{\theta}_R < 0 \Leftrightarrow \hat{Q}_R - \hat{Q}_L > (\hat{L} - \hat{L}^s) - (\hat{Q}_\mu - \rho),$$

$$\hat{w}_R - \hat{w}_L < 0 \Leftrightarrow (\hat{Q}_R - \hat{Q}_L) > \frac{(1 - n\theta_R)}{\theta_R(1-n)}\left[(\hat{L} - \hat{L}^s) - (\hat{Q}_\mu - \rho)\right],$$

which, using the definition of n, can be combined to get the bounds displayed in the lemma. QED.

Proof of Proposition 1

Using $\hat{Q}_R = \hat{Q}_L$ in (25), it follows that condition (ii) is both necessary and sufficient for fact 4. Using again $\hat{Q}_R = \hat{Q}_L$ and manipulating equations (22), (23) and (24) we find

$$\begin{aligned} \hat{R} - \hat{L}^s > 0 \quad &\Leftrightarrow \quad \hat{Q}_\mu > \rho - \frac{(1-n)}{n} \cdot (\hat{L} - \hat{L}^s) \\ \hat{\theta}_R < 0 \quad &\Leftrightarrow \quad \hat{Q}_\mu > \rho + \hat{L} - \hat{L}^s \\ \hat{w}_R - \hat{w}_L < 0 \quad &\Leftrightarrow \quad \hat{Q}_\mu > \rho + \hat{L} - \hat{L}^s. \end{aligned}$$

Combining these expressions, we conclude that (i) is necessary and, moreover, it is sufficient for facts 1, 2 and 3. Finally, facts 1 and 4 together imply the fulfillment of fact 5. Therefore, conditions (i) and (ii) are also sufficient for fact 5. QED.

Proof of Lemma 3

From (23), we have that $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L > \rho$ implies $\hat{\theta}_R < 0$ and, therefore, $\lim_{t\to\infty} \theta_R(t) = 0$. If, on other hand, $\hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L < \rho$ then $\hat{\theta}_R > 0$ and $\lim_{t\to\infty} \theta_R(t) = 1$. Expressions for $\hat{Y} - \hat{L}^S$ follow from (26). QED.

Proof of Proposition 2

Define $\hat{Q}_\rho \equiv \hat{Q}_\mu + \hat{Q}_R - \hat{Q}_L - \rho$. From (23) it follows that, if $\hat{Q}_\rho > 0$, $\lim_{t\to\infty} \theta_R(t) = 0$ and, therefore, $\lim_{t\to\infty} n(t) = \nu$. On the other hand, if $\hat{Q}_\rho < 0$, $\lim_{t\to\infty} \theta_R(t) = 0$ and, therefore, $\lim_{t\to\infty} n(t) = 1$. Using these results in (22)-(26) and checking the three relevant cases ($\hat{Q}_\rho \lesseqgtr 0$), we get the following long-run expressions, from which results (i)-(iv) are immediate:

$$\begin{aligned}
&\hat{R} - \hat{L} = -(\rho - \hat{Q}_\mu) - \max\{0, (1-\nu)\hat{Q}_\rho\},\\
&\hat{\theta}_R = \min\{0, -(1-\nu)\hat{Q}_\rho\},\\
&\hat{w}_R - \hat{w}_L = \rho - \hat{Q}_\mu,\\
&\hat{Y} - \hat{R} = \hat{Q}_R - \max\{0, \nu\hat{Q}_\rho\},\\
&\hat{Y} - \hat{L} = \min\{\hat{Q}_\mu + \hat{Q}_R - \rho, \hat{Q}_L\}.
\end{aligned}$$

QED

Proof of Lemma 4

(39) immediately follows after using (36) in (34) to eliminate D. From (35) we get $\hat{L} = -\hat{\theta}_R \theta_R / (1-\theta_R) - \hat{w}$. Using this expression to eliminate $\hat{L}$, (32) to eliminate the growth of technological bias and (39) to eliminate $\hat{w}$, (40) is obtained. QED

Proof of Lemma 5

It follows from (32) that (i) guarantees the fulfillment of stylized fact **6**, $\hat{Q}_R(0) = \hat{Q}_L(0)$.

Differentiating (35), using (23) to eliminate $\hat{\theta}_R$, the definition for n, as well as (32) and (i), we get $\hat{L}(0) = (1-\nu)\kappa_R\left[\hat{Q}_\mu(0) - \rho\right] - (\kappa_L + \nu\kappa_R)\hat{w}(0)$ which, using (39), we can rewrite as

$$\hat{L}(0) = (1-\nu)\kappa_R[\hat{Q}_\mu(0) - \rho] - (\kappa_L + \nu\kappa_R)\left[\rho + L^S - \frac{\beta(\kappa_L + 1 - \beta)}{w(0)}\right]. \qquad \text{(A.1)}$$

We define $\underline{w}$ and $\overline{w}$ as the values of w associated to the intersection between the $\dot{\theta}_R = 0$ locus and the $\dot{w} = 0$ locus for the highest and lowest value of $\hat{Q}_\mu(t)$, respectively. We also define $w^* \equiv \beta(1-\beta+\kappa_L)/[L^S + \hat{Q}_\mu(0)]$ as the value of w associated to the intersection between the $\theta_R = \kappa_R$ line and the $\dot{\theta}_R = 0$ locus for $\hat{Q}_\mu(0)$.

By construction, in a steady state with constant θ_R and w, we have $\overline{w} \geq w \geq \underline{w}$. Moreover, the equilibrium dynamics imply

$$\underline{w} \leq w(t) \leq \overline{w} \quad \text{for all } t \geq 0, \tag{A.2}$$

since, otherwise, (39) implies that the solution would not converge to the steady state.

To get $\dot{\theta}_R(0) < 0$ (stylized fact 2) $w(0)$ has to start above the $\dot{\theta}_R = 0$ locus, that is $w(0) > w^*$. We now prove by contradiction that condition (ii) and a large enough T guarantee this inequality. Suppose $w(0) < w^*$. Then, as long as $\rho < \hat{Q}_\mu$, it follows from (39) and (40) that w would fall and θ_R increase over time. It also follows that if $\rho < \hat{Q}_\mu$ holds for a long enough period, w would keep declining until it is smaller than $\underline{w}$, while θ_R is still below the steady state value. But according to (A.2), $w < \underline{w}$ cannot be an equilibrium. It follows that only $w(0) > w^*$ can be an equilibrium, which implies $\dot{\theta}_R(0) < 0$. From (A.2), $w(0) > w^*$ is only feasible if $w^* < \overline{w}$, which requires $\hat{Q}_\mu(0) > \rho + \kappa_L \kappa_R (1-\beta)^{-1}[\rho - \max\{\hat{Q}_\mu(t)\}]$, which holds under condition (iii). This proves that stylized fact 2 is replicated with (iii) and sufficiently large T. [16]

Substituting (A.1) and (i) (this lemma) into (22), we see that (iii) and $w(0) < \overline{w}$, which was proven above, are sufficient to replicate stylized fact **1**.

Substituting (A.1), (23) and (i) (this lemma) into (24), we see that stylized fact **3** is replicated for $w(0) > w^*$ (which was proven above). Hence if stylized fact 2 is replicated and (i) holds, stylized fact **3** is replicated as well.

It follows from (25), (i) (this lemma), (39), and $w(0) < \overline{w}$ that (ii) is sufficient to replicate stylized fact **4**.

[16] Similarly, for given T, the length of period during which mining efficiency improves at a rate faster than ρ, we can ensure $\dot{\theta}_R(0) < 0$ by increasing the rate of mining efficiency improvements before T.

Since $\hat{Y}-\hat{L}_p=(\hat{Y}-\hat{R})+(\hat{R}-\hat{L}_p)$ and both terms in parentheses are positive if stylized facts 1 and 2 are satisfied, stylized fact **5** needs no further conditions. QED

Proof of Proposition 3

Solving (39) and (40) for the long-run energy share and wage rate from Lemma 3, we find:

$$\theta_R(\infty)=\kappa_R+(\kappa_L+1-\beta)a/(a+b)>\kappa_R=\theta_R(0),$$

$$w(\infty)=\beta(1-\beta)(\kappa_L+1-\beta)/(a+b),$$

where $a\equiv\kappa_L\kappa_R[\rho-\hat{Q}_\mu(\infty)]$ and $b\equiv(L^S+\rho)(1-\beta)$. The calibration in Lemma 5 ensures $\theta_R(0)=\kappa_R<\theta_R(\infty)$ and $\dot{\theta}_R(0)>0$. This proves part **(i)**.

Technological change is energy-using (energy-saving) if $\hat{Q}_R-\hat{Q}_L<(>)0$. The short run part of **(ii)** follows from equation (32) and the initial condition of θ_R. Concerning the long run result, using (i) in (32), we get $\hat{Q}_R(\infty)-\hat{Q}_L(\infty)=\rho-\hat{Q}_\mu(\infty)>0$.

Substituting (30), (32), and (37) into (26), we express per capita growth as a function of w, θ_R and $\hat{L}$. To find long-run per capita growth from the resulting expression, we substitute $\hat{L}=0$ and the long-run values of w and θ_R, which yields:

$$\hat{Y}(\infty)=\hat{Q}_L(\infty)=\left(\frac{1-\beta}{1-\beta+\kappa_L}\right)L^S-\left(\frac{\kappa_L}{1-\beta+\kappa_L}\right)\rho-\left(\frac{1-\beta}{1-\beta+\kappa_L}\right)\kappa_R[\rho-\hat{Q}_\mu(\infty)].$$

Short-run growth can be expressed, using (A.1) and $\theta_R(0)=\kappa_R$, as:

$$\hat{Y}(0)=\kappa_R[L^S+\hat{Q}_\mu(0)]-\rho+(\kappa_L-\beta)\kappa_L\beta/w(0).$$

If $\kappa_L=\beta$, we can compare long-run and short-run growth without knowing $w(0)$:

$$\hat{Y}(0)-\hat{Y}(\infty)=(1-\beta)[\hat{Q}_\mu(0)-\rho]+(1-\beta)\kappa_R[\rho-\hat{Q}_\mu(\infty)]>0.$$

If $\kappa_L<\beta$, we find a lower bound for short-run growth, using $w(0)>w^*$:

$$\hat{Y}(0)>\left(\frac{1-\beta}{1-\beta+\kappa_L}\right)L^S-\left(\frac{\kappa_L}{1-\beta+\kappa_L}\right)\rho+\left(\frac{1-\beta}{1-\beta+\kappa_L}\right)[\hat{Q}_\mu(0)-\rho].$$

If $\kappa_L>\beta$, we find a lower bound for short-run growth, using $w(0)<\overline{w}$ and assuming $\hat{Q}_\mu(0)=\max_{t\geq 0}\hat{Q}_\mu(t)$:

$$\hat{Y}(0)>\left(\frac{1-\beta}{1-\beta+\kappa_L}\right)L^S-\left(\frac{\kappa_L}{1-\beta+\kappa_L}\right)\rho+\left(\frac{1-\beta}{1-\beta+\kappa_L}+\frac{\kappa_L\kappa_R}{1-\beta}\right)\kappa_R[\hat{Q}_\mu(0)-\rho].$$

Since $\rho - \hat{Q}_\mu(0) < 0 < \rho - \hat{Q}_\mu(\infty)$, we find $\hat{Y}(\infty) < \hat{Y}(0)$ for all three cases. This proves part **(iii)**. QED.

Proof of Proposition 4

To prove part (i), define $\theta_R^*(t)$ as the value of θ_R associated to the intersection between the $\dot{\theta}_R = 0$ locus and the $\dot{w} = 0$ locus. Since the system is non-autonomous, $\theta_R^*(t)$ is time-varying. Actually, under the assumption that $\hat{Q}_\mu(t)$ is non-increasing over time, $\theta_R^*(t)$ is monotonically increasing. Then, as illustrated in the phase-diagram, the dynamics of the system can be divided in two stages: In the first stage, $\theta_R > \theta_R^*(t)$ and saddle-path stability implies that θ_R is decreasing. In the second stage $\theta_R(t) \le \theta_R^*(t)$ and saddle-path stability implies that $\theta_R(t)$ is increasing. The fact that $\theta_R(0) < \theta_R(\infty)$ ensures that stage 2 will be reached.

To derive (ii), we plot the iso-*D* lines in the (θ_R,w) phase diagram: from (36), these are given by $w = (1-\theta_R)\beta/(L^S - D)$ and their slopes are negative and steeper than the $\dot{w} = 0$ locus and *a fortiori* steeper than the equilibrium path. Combining these isoclines with the equilibrium (θ_R,w) trajectory, we find (ii).

To derive (iii), we plot the iso-$(\hat{Q}_R - \hat{Q}_L)$ lines in the (θ_R,w) phase diagram: from (32), these are rays emanating from point κ_R on the horizontal axis defined by $w = \beta(1-\beta)(\kappa_L\kappa_R)^{-1}(\hat{Q}_R - \hat{Q}_L)^{-1}(\theta_R - \kappa_R)$. Combining these isoclines with the equilibrium (θ_R,w) trajectory, we find (iii).

Substituting (36) in (38), we find $\hat{Q}_{TFP} = L^S + (\beta/w)[(\theta_R - \kappa_R)^2(1-\beta)/\kappa_R\kappa_L + \theta_R - 1]$, from which we derive iso-TFP-growth curves. These curves have a minimum at $\theta_R = \theta_T \equiv [1 - \kappa_L/2(1-\beta)]\kappa_R < \kappa_R$. From Lemma 5 we have $\theta_R(0) = \kappa_R$ and from part (i) of this proposition we have that θ_R first declines and then rises. We first infer from (39)-(40) that the equilibrium path in the θ_R,w plane cannot be steeper than the iso-TFP-growth curves. This must imply that TFP growth first falls. Then consider what happens if θ_R falls over time (initial phase). If θ_R never falls below θ_T, TFP growth keeps falling until θ_R starts rising (second phase). If, on the other hand, θ_R falls below θ_T, TFP growth starts rising before θ_R starts rising. When θ_R starts rising, TFP growth falls until θ_R exceeds θ_T again after which TFP growth increases over time. This proves (iv). QED

Appendix B. The firm's maximization problem: optimality condition (28) and the transversality condition

Intermediate goods firm k in sector i maximizes the present value of its profits, given by $\int e^{-rt}\left(p_{mik}m_{ik} - q_{ik}m_{ik}p_y - w_D D_{ik}\right)dt$, subject to (6) and (27) and using m_{ik} and D_{ik} as controls, taking as given all prices, aggregate variables and quantities of other firms. Collecting terms that are beyond the firm's control in $\overline{P}$, we may write (6) as $p_{mik} = \overline{P}q_{ik}m_{ik}^{1-\beta}$. Then, we may write the current-value Hamiltonian (H_{ik}) and the first order conditions as

$$H_{ik} = \overline{P}q_{ik}m_{ik}^{1-\beta} - q_{ik}m_{ik}p_Y - w_D D_{ik} + p_{qik}\left(Q_i/\kappa_i\right)D_{ik} \tag{B.1}$$

$$(1-\beta)\overline{P}q_{ik}m_{ik}^{-\beta} = q_{ik}p_Y \tag{B.2}$$

$$p_{qik} = w_D\kappa_i / Q_i \tag{B.3}$$

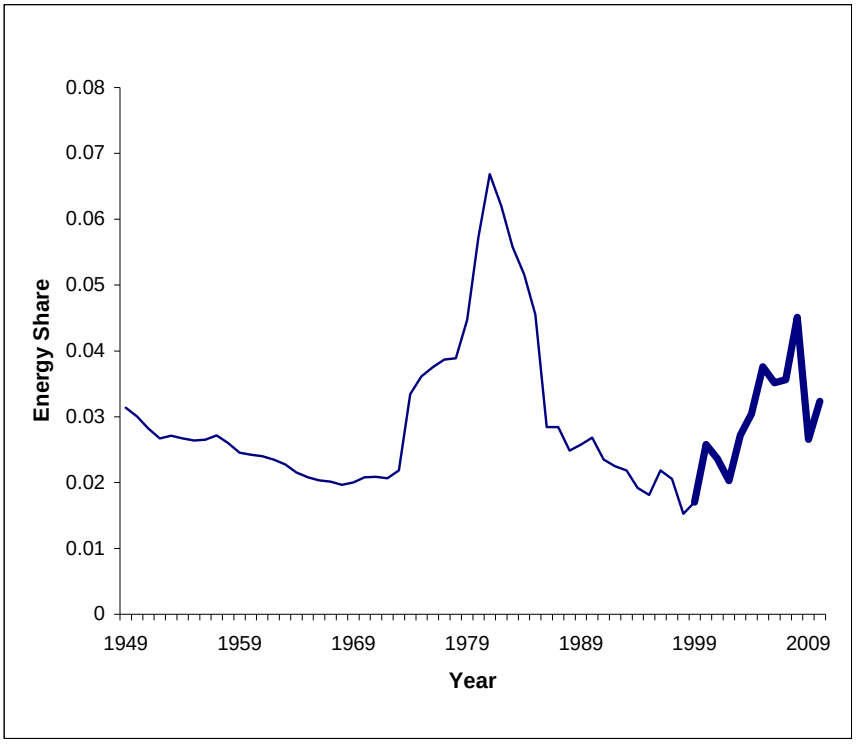
$$\overline{P}m_{ik}^{1-\beta} - m_{ik}p_Y = rp_{qik} - \dot{p}_{qik} \tag{B.4}$$

Eliminating $\overline{P} = p_{mik}/q_{ik}m_{ik}^{1-\beta}$ in (B.2) we find (7). From (B.3) we get $\hat{p}_{qik} = \hat{w}_D - \hat{Q}_i$. Using this equation together with (B.3) and (B.4) to eliminate p_{qik} and $\hat{p}_{qik}$, we get (28). The transversality condition reads $\lim_{t\to\infty} p_{qik}q_{ik}e^{-rt} = 0$. Integrating over all firms in sector i, using (9), and eliminating p_{qik} with (B.3), we find:

$$\lim_{t\to\infty}\int_0^1 p_{qik}q_{ik}e^{-rt}dk = \lim_{t\to\infty}\frac{w_D\kappa_i}{Q_i}e^{-rt}\int_0^1 q_{ik}dk = w_D\kappa_i\lim_{t\to\infty}e^{-rt} = 0$$

Hence, a necessary condition for the transversality condition to hold is $\lim_{t\to\infty} r(t) - \hat{w}_D(t) \geq 0$.

Figure 1 US energy share.



Calculated as total energy expenditure (energy consumption times energy price) over GDP. Data from "Annual Energy Review" from the US Energy Information Administration. The thin line replicates Jones (2002, Table 9.3). The thick line is our update (using the same source as Jones).

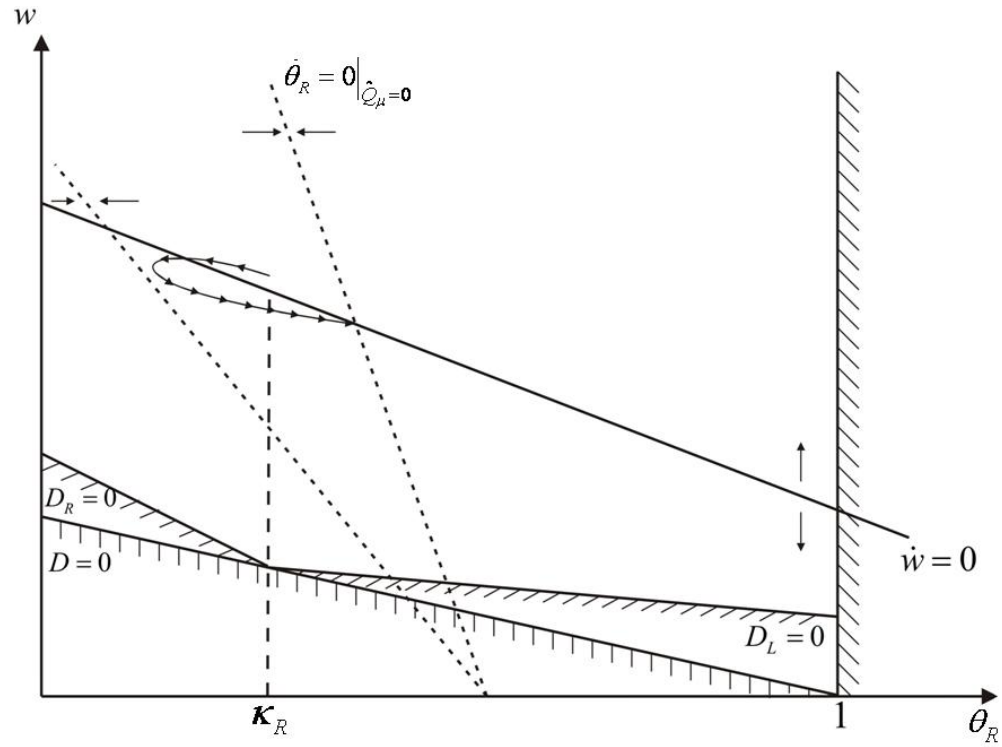



Figure 2 Phase diagram (endogenous technological change)