

Reither, Franco; Bennöhr, Lars

**Working Paper**

## Stabilizing rational speculation and price level targeting

Diskussionspapier, No. 104

**Provided in Cooperation with:**

Fächergruppe Volkswirtschaftslehre, Helmut-Schmidt-Universität (HSU)

*Suggested Citation:* Reither, Franco; Bennöhr, Lars (2010) : Stabilizing rational speculation and price level targeting, Diskussionspapier, No. 104, Helmut-Schmidt-Universität - Universität der Bundeswehr Hamburg, Fächergruppe Volkswirtschaftslehre, Hamburg, <https://nbn-resolving.de/urn:nbn:de:gbv:705-opus-26293>

This Version is available at:

<https://hdl.handle.net/10419/66041>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Diskussionspapierreihe  
Working Paper Series



HELMUT SCHMIDT  
UNIVERSITÄT  
Universität der Bundeswehr Hamburg

# STABILIZING RATIONAL SPECULATION AND PRICE LEVEL TARGETING

FRANCO REITHER  
LARS BENNÖHR

Nr./ No. 104  
AUGUST 2010

Department of Economics  
Fächergruppe Volkswirtschaftslehre

Autoren / Authors

**Franco Reither**

Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg  
Institut für Theoretische Volkswirtschaftslehre / Institute of Economic Theory  
Holstenhofweg 85  
22043 Hamburg  
Germany  
fer@hsu-hh.de

**Lars Bennöhr**

Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg  
Institut für Theoretische Volkswirtschaftslehre / Institute of Economic Theory  
Holstenhofweg 85  
22043 Hamburg  
Germany  
lars.bennoehr@hsu-hh.de

Redaktion / Editors

Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg  
Fächergruppe Volkswirtschaftslehre / Department of Economics

Eine elektronische Version des Diskussionspapiers ist auf folgender Internetseite zu finden/  
An electronic version of the paper may be downloaded from the homepage:  
<http://fgvwl.hsu-hh.de/wp-vwl>

Koordinator / Coordinator

Kai Hielscher  
wp-vwl@hsu-hh.de

## STABILIZING RATIONAL SPECULATION AND PRICE LEVEL TARGETING

FRANCO REITHER  
LARS BENNÖHR

### Zusammenfassung/ Abstract

We analyze the contribution of speculation to exchange rate volatility using different assumptions regarding speculation strategies and monetary policy rules. We take the DORNBUSCH (1976) model as the starting point and adopt a slight modification of the money demand specification. With a money supply rule, rational speculation dampens the overshooting of the exchange rate following a money supply shock, compared with speculation based on static expectations. Then, we replace the LM condition by a TAYLOR-type price level targeting rule. The resulting “DORNBUSCH-TAYLOR” model generates a unique saddle point solution even under ‘strict’ inflation targeting, if speculation is based on rational expectations. Under ‘flexible’ inflation targeting, exchange rate overshooting induced by a monetary policy shock is less pronounced under rational speculation than under static speculation. FOREX market equilibrium doesn’t exist at all if speculation is static and monetary policy adopts ‘strict’ inflation targeting.

**JEL-Klassifikation / JEL-Classification:** E44, F31, F41

**Schlagworte / Keywords:** fair rational expectations, DORNBUSCH Model, open economy macroeconomics, speculation, FOREX

## 1. Introduction

Research on speculation impact has a long tradition. Notable contributions are e.g. FRIEDMAN (1953), JOHNSON (1976), DE LONG et al. (1990). But a better understanding of speculation impacts on economies still remains one of the key challenges in economics. This paper stresses the role of FOREX speculation for small open economies. Especially we try to investigate the following questions.

What is the contribution of speculation to the volatility of exchange rates when speculative activity is based on rational expectations? Is exchange rate volatility dampened or is it magnified by rational speculation? Moreover, does the contribution of speculation to exchange rate volatility depend on the specific monetary policy strategy?

In the present paper, we try to address these questions within a simple macroeconomic framework. It consists of a convenient combination of the MUNDELL (1963) and DORNBUSCH (1976) models. In both models money supply is the monetary policy instrument. Price adjustment is completely absent in the MUNDELL model, while DORNBUSCH incorporates a simple PHILLIPS curve describing gradual price adjustment over time. In both models, equilibrium at the foreign exchange market requires the validity of the uncovered interest parity; in this condition, the expectational term vanishes in the MUNDELL model reflecting static expectations, while DORNBUSCH builds on rational (or at least semi-rational) expectations.

As an additional – rather curious – modification, DORNBUSCH – contrary to MUNDELL – does not make use of the conventional money demand specification, but replaces actual output by potential output as the transactions variable. As a consequence, actual output and money supply are completely disconnected, with severe implications for the role of speculation in foreign exchange markets.

In order to concentrate on the contribution of rational speculation to exchange rate volatility, in what follows we apply the common macroeconomic MUNDELL framework, augmented by the PHILLIPS curve specification used by DORNBUSCH, and analyze the differences implied by the two expectations hypotheses. In a second step, we replace the *LM* condition by a TAYLOR-type monetary policy rule; we consider the “strict” as well as the “flexible” inflation targeting version of the interest rate rule.

The plan of the paper is as follows. In Section 2, monetary policy is described with a conventional *LM* condition, where the Central Bank uses money supply as its instrument. Within this framework, we compare market reactions to an unexpected monetary policy shock under static and under rational speculation. In Sections 3 and 4, we replace the *LM* condition by a TAYLOR-type interest rate rule with a price level target. Within this framework, the monetary policy shock is represented by a change of the target price level. Existence and uniqueness of a rational expectations equilibrium is shown in Section 3, while the implications of static expectations are analyzed in Section 4. Section 5 gives a brief summary of the results.

## 2. 'Overshooting' and 'stabilizing speculation' with a money supply instrument

The basic approach for the analysis of exchange rate behaviour under a money supply regime is DORNBUSCH (1976). DORNBUSCH's theoretical framework is, in turn, very similar to MUNDELL (1963). Both papers describe the perspective of a small open economy with fully integrated financial markets and limited price flexibility. The main improvements made by DORNBUSCH are commonly seen to consist of the introduction of a PHILLIPS curve describing sticky price adjustment, and rational exchange rate expectations. MUNDELL, by contrast, assumed an absolutely fixed price level and static exchange rate expectations.

An additional modification adopted by DORNBUSCH and usually not emphasized in the literature (an exception is OBSTFELD and ROGOFF, 2003) concerns the money demand function: DORNBUSCH uses potential output as the transactions variable in the money demand equation instead of actual output.

For the purposes of the present paper, it seems natural to specify a common block of *IS*, *LM* and *AS* conditions as a combination of the MUNDELL and DORNBUSCH models, and to put the specific form of speculative behaviour into the equilibrium condition for the foreign exchange market, that is the uncovered interest parity condition. The common model block consists of the following three equations,

$$y = a + \delta(e + p^* - p) - \sigma i, \quad (0.1)$$

$$m - p = y - \lambda i, \quad (0.2)$$

$$\dot{p} = \varphi(y - \bar{y}). \quad (0.3)$$

All variables except the interest rates are measured in logs. The parameters  $\sigma, \delta, \lambda$  and  $\varphi$  are defined as positive. (1.1) is the *IS* condition: in goods market equilibrium, output  $y$  equals aggregate demand. Aggregate demand is affected by the real exchange rate ( $e + p^* - p$ ) and the interest rate ( $i$ ), and  $a$  represents autonomous demand. (1.2) is the *LM* condition with nominal money supply  $m$  and aggregate price level  $p$ ; the income elasticity of money demand is set to unity. The *AS* condition (1.3) is a simple version of the PHILLIPS curve as used by DORNBUSCH (1976): price level adjustment in time is proportional to the output gap ( $\bar{y}$  denotes potential output).

With perfect capital mobility and substitutability, equilibrium in the FOREX market requires the equalization of expected returns in domestic and foreign bonds. The specific form of the uncovered interest parity condition (*UIP*) depends on the hypothesis regarding exchange rate expectations. With static expectations (MUNDELL version), *UIP* amounts to

$$i = i^*, \quad (1.4a)$$

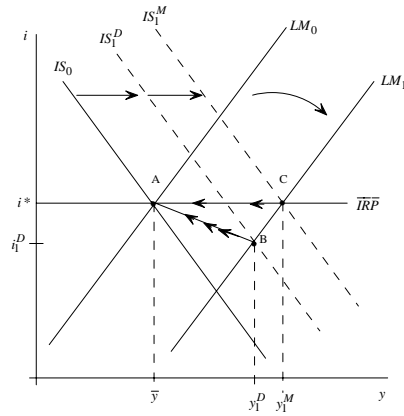
where  $i^*$  is foreign interest rate (exogenous to the small country). The DORNBUSCH version of *UIP* with rational expectations can be written as

$$i = i^* + \alpha(\bar{e} - e), \quad (1.4b)$$

where  $\bar{e}$  is the model-consistent long run equilibrium exchange rate, and  $-\alpha < 0$  is the stable root of the saddle point dynamic equilibrium system. Summarizing so far, equations (1.1) to (1.4a) represent the AS-augmented MUNDELL model (M+), while the combination (1.1) to (1.3) and (1.4b) describe a kind of MUNDELL-DORNBUSCH model (MD).

Admittedly, from a modern theoretical point of view, both models exhibit at least two serious shortcomings. First, they are not able to capture steady state inflation. Second, no distinction is made between real and nominal interest rates. The incorporation of these features is postponed to later research. As an interesting step in this direction, with results similar to ours, see PROAÑO (2009).

Now we are able to analyze the models. We assume a permanent increase of nominal money supply to show how different expectation assumptions and corresponding speculation patterns can alter the model's predictions. The comparative statics are shown in Fig. 1.



**Fig. 1: expansionary monetary shock (MD vs. M+)**

In both models, the monetary expansion leads to an outward shift of the  $LM$  curve because prices adjust slowly and therefore the real money supply increases. The resulting downward pressure to the domestic interest rate induces capital flows towards the foreign currency and domestic currency depreciates. This increases the competitiveness of home products, and the  $IS$  curve shifts to the right.

The main difference between the impact implications of the two models is how far the  $IS$  curve shifts. The M+ model predicts a shift to  $IS_1^M$ , while the DORNBUSCH approach leads to a shift to  $IS_1^D$ . While the market participants in the MD model can anticipate future adjustments of the nominal exchange rate, the static speculators can not. Rational market participants know that future adjustment of prices will ensure economy's return to the old steady state and that this implies downward pressure on the nominal exchange rate. As a result, depreciation in the MD model is less pronounced than in the M+ model, which leads to a smaller  $IS$  shift.

In Fig. 1, point C shows the impact equilibrium with static expectations, while point B represents the rational expectations solution of the MD model. An important implication of these results is that output volatility in the rational expectations framework is lower as well.

In the following, increasing prices lead to a real appreciation which drives down foreign demand for home produced goods. This shifts the  $IS$  curve back to its initial location. The  $LM$  curve is shifted back because higher prices lower real money supply.

The graphical exposition is confirmed by the explicit multiplier analysis. The long run multipliers of the nominal exchange rate and the price level imply that neutrality of money holds in both models. A change of nominal money supply causes an equi-proportional change of prices and the nominal exchange rate.<sup>1</sup> For the impact multipliers, the results are as follows,

$$\frac{de}{dm}(MD) = \frac{1 + \alpha(\lambda + \sigma)}{\delta + \alpha(\lambda + \sigma)} > 1 \quad \text{for } 0 < \delta < 1 \quad \text{and} \quad \frac{de}{dm}(M+) = \frac{1}{\delta} > 1 \quad \text{for } 0 < \delta < 1.$$

It follows that  $\frac{de}{dm}(M+) > \frac{de}{dm}(MD)$ .

Thus, in both cases the condition  $0 < \delta < 1$  is necessary and sufficient for overshooting. The condition  $\alpha > 0$  is not necessary. This implies that fundamental based speculation is not responsible for overshooting. In contrast, rational behaviour dampens the exchange rate reaction. In this sense, rational speculation stabilizes the economy.<sup>2</sup>

### 3. Rational speculation with a price level targeting

How does the DORNBUSCH model behave when monetary policy follows a TAYLOR-type interest rate rule instead of a money supply target? This is the question addressed in the present section. Formally, we replace the  $LM$  condition (1.2) by an interest rule that does not include the money stock variable.

Additionally, we switch from continuous to discrete time notation. This modification is not only convenient with respect to the numerical calculations we develop below, but also necessary in order to capture the specific dynamic interaction of exchange rate expectations, sticky prices and the interest rate rule. The resulting model consists of the following four equations.

---

<sup>1</sup>  $\frac{d\bar{e}}{dm} = \frac{d\bar{p}}{dm} = 1$

<sup>2</sup> Beside that, the multiplier regarding the impact interest rate level in the MD model, which is responsible for the degree of overshooting is:  $\frac{di}{dm}(MD) = -\frac{\alpha(1-\delta)}{\delta + \alpha(\lambda + \sigma)} \leq 0$  for  $\alpha \geq 0$  and  $0 < \delta < 1$ . A higher multiplier indicates a stronger interest rate reaction and this is equivalent to a smaller exchange rate overshooting.



$$y_t = a + \delta(e_t + p^* - p_t) - \sigma i_t, \quad (2.1)$$

$$p_{t+1} = p_t + \varphi(y_t - \bar{y}) + u_t, \quad (2.2)$$

$$i_t = i^* + E_t e_{t+1} - e_t, \quad (2.3)$$

$$i_t = i^* + \mu_\pi(p_t - \hat{p}) + \mu_y(y_t - \bar{y}). \quad (2.4)$$

Equations (2.1) and (2.3) represent the *IS* and *UIP* conditions in discrete time, respectively. (2.2) is the ‘strong’ version of the *AS* condition with price stickiness: the contemporaneous price level does not respond at all to the output gap.  $u_{t+1}$  is an *iid* stochastic supply shock term ( $u_{t+1} \sim N(0, \sigma^2)$ ).

The novelty in the model is equation (2.4) which represents a specific version of a TAYLOR rule. In order to keep the analogy to the DORNBUSCH original as strong as possible, we postulate that the Central Bank follows a price level target ( $\hat{p}$ ) instead of an inflation target (that is, steady state inflation is zero in this model). In what follows, the monetary policy shock consists of an unexpected change in  $\hat{p}$ .

### 3.1 Derivation of the semi-reduced form

The complete semi-reduced form is a four-equations system that gives the equilibrium values of the endogenous  $y_t, e_t, i_t$  and  $p_{t+1}$  for given values of  $E_t e_{t+1}$ . In order to reduce the dimensionality of the problem, we derive a ‘condensed’ version of the semi-reduced form by eliminating the interest rate variable. Eliminating  $i_t$  from (2.3) and (2.4) gives

$$i^* + E_t e_{t+1} - e_t = i^* + \mu_\pi(p_t - \hat{p}) + \mu_y(y_t - \bar{y}) \quad (2.5)$$

and substituting (2.3) in (2.1) yields

$$y_t = a + \delta(e_t + p^* - p_t) - \sigma(i^* + E_t e_{t+1} - e_t). \quad (2.6)$$

The resulting three-equations system (with  $E_t e_{t+1}$  expressed as  $\hat{e}_{t+1}$ ) is summarized in CRAMER-style in Table 1.

$y_t$	$e_t$	$p_{t+1}$	$\hat{e}_{t+1}$	$p_t$	$u_t$	(eq.)
1	$-(\delta + \sigma)$	0	$-\sigma$	$-\delta$	0	(2.6)
$-\mu_\pi$	0	1	0	1	1	(2.2)
$\mu_y$	1	0	1	$-\mu_\pi$	0	(2.5)

**Table 1**

### 3.2 Model solution

According to the methodology discussed in McCALLUM (1983, 1999), the variables  $p_t$  and  $u_t$  can be identified as the (minimum) state variables, and the RE<sup>3</sup> solution will satisfy the following linear equations,

$$p_{t+1} = \beta_0 + \beta_1 p_t + \beta_2 u_t, \quad (3.1)$$

$$e_t = \lambda_0 + \lambda_1 p_t + \lambda_2 u_t, \quad (3.2)$$

$$y_t = \eta_0 + \eta_1 p_t + \eta_2 u_t, \quad (3.3)$$

with  $\beta_j, \lambda_j$  and  $\eta_j$  as unknown coefficients.<sup>4</sup>

A few points are worth to be emphasized at this stage. First, the parameter  $\beta_1$  plays a pivotal role: for a unique RE equilibrium to exist, the solution has to give exactly one value  $0 < \beta_1 < 1$ ; second, the impact reaction of the exchange rate following a monetary policy shock ( $d\bar{p} = d\hat{p} \neq 0$ ) can be written as:

$$\frac{de_t}{d\bar{p}} = \frac{d\bar{e}}{d\bar{p}} - \lambda_1,$$

thus, ‘overshooting’ requires  $\lambda_1 < 0$ . The implied equilibrium trajectory (see Fig. A1 in the Appendix C) in  $(i, y)$ -space can be derived from (2.4) and (3.3). The expression

$$\frac{di_t}{dy_t} = \frac{\mu_\pi + \eta_1 \mu_y}{\eta_1} =: \tau$$

is the trajectory slope; a unique RE equilibrium requires  $\tau < 0$ .<sup>5</sup> At this point, we report only basic starting steps of the identification procedure. The complete algebra can be found in the Appendix. From (3.2) it follows that

$$E_t e_{t+1} = \lambda_0 + \lambda_1 E_t p_{t+1}, \quad (3.5)$$

and from (3.1) we have

$$E_t p_{t+1} = \beta_0 + \beta_1 p_t \quad (3.6)$$

for  $u_t$  known in  $t$ . Combining (3.5) and (3.6) yields

---

<sup>3</sup> RE means rational expectations

<sup>4</sup>  $\lambda_j$  are not to confuse with the interest rate semielasticity of money demand ( $\lambda$ ).

<sup>5</sup> A derivation can be found in Appendix B.

$$E_t e_{t+1} = \lambda_0 + \lambda_1 \beta_0 + \lambda_1 \beta_1 p_t + \lambda_1 \beta_2 u_t. \quad (3.7)$$

The latter expression is useful to eliminate  $E_t e_{t+1}$ , where needed. For  $\lambda_1$  and  $\eta_1$ , conditional on  $\beta_1$ , we find the solutions

$$\lambda_1 = \frac{\mu_\pi - \delta \mu_y}{(1 + \sigma \mu_y) \beta_1 - (1 + \mu_y (\delta + \sigma))} \quad (3.8)$$

$$\eta_1 = - \frac{\mu_\pi (\delta + \sigma) + \delta (1 - \lambda_1 \beta_1)}{1 + \mu_y (\delta + \sigma)} \quad (3.9)$$

In Appendix C surfaces of  $\lambda_1$  and  $\eta_1$  for different central bank preferences are computed numerically. For the pivotal coefficient  $\beta_1$ , we find a characteristic equation of the form  $\beta_1^2 + a\beta_1 + b = 0$ ,

$$\text{with } a = \frac{\Delta^2 + \varphi \delta (\mu_\pi - \delta \mu_y) - [\varphi (\delta + \mu_\pi (\delta + \sigma)) + \Delta] (1 + \sigma \mu_y)}{\Delta (1 + \sigma \mu_y)}, b = - \frac{\Delta + \varphi (\delta + \mu_\pi (\delta + \sigma))}{1 + \sigma \mu_y}$$

and the Jacobian:  $\Delta = - (1 + \mu_y (\delta + \sigma))$ .

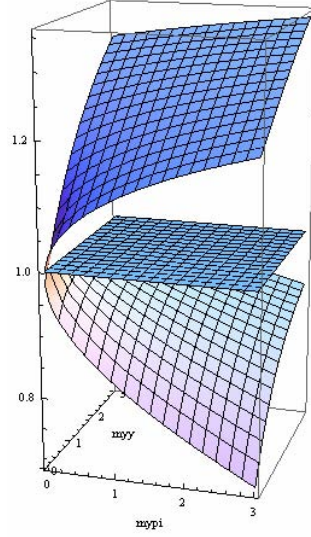
We discuss this quadratic equation numerically in the next section. For a unique RE equilibrium to exist, the two solving roots have to satisfy  $|\beta_1^A| < 1$ , and  $|\beta_1^B| > 1$ ; additionally, a monotonic saddle path requires  $\beta_1^A > 0$ .

### 3.3 Numerical parameter analysis

While we have obtained the identifying restrictions regarding the dynamics of prices, nominal exchange rate and output, it is still questionable if the system is stable for alternative central bank preferences and if adjustment processes are monotonic. We apply numerical calculations to investigate these problems, when central bank behaviour is altered. We assume the following parameter values, which should be inline with common macro-economic sense<sup>6</sup>. These are  $\sigma = 0.3$ ,  $\delta = 0.2$  and  $\varphi = 0.1$ .

---

<sup>6</sup> Further calculations, which are not reported here, have shown that the model is robust to marginal changes in these parameters. To give an example from the open economy literature: McCallum and Nelson use  $\varphi = 0.086$ ,  $\delta = 0.356$  and  $\sigma = 0.2$  (McCallum/Nelson 2001).



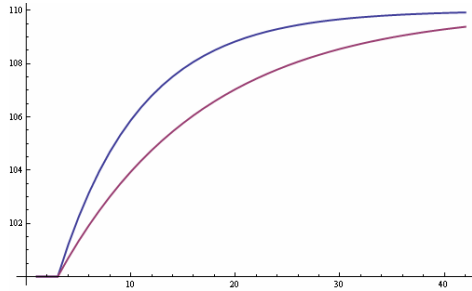
**Fig. 2:  $\beta_1$  for different central banking strategies**

Fig. 2 shows the values of  $\beta_1$  for different values of  $\mu_\pi$  and  $\mu_y$  between zero and three. We obtain two different real solutions for each combination of  $\mu_\pi$  and  $\mu_y$ . Each pair of solution satisfies  $|\beta_1^A| > 1$  and  $-1 < \beta_1^B < 1$ . These conditions are needed to ensure a unique solution and monotonic adjustment.

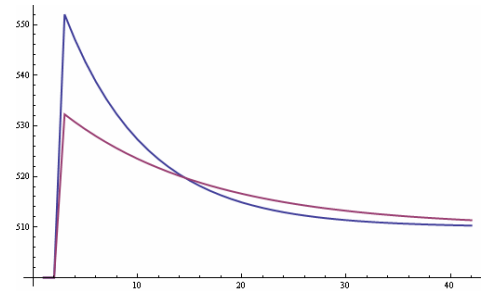
Surfaces for  $\lambda_1, \eta_1$  and  $\tau$  are documented in the Appendix in Figures A1-A3. The values of  $\eta_1$  and  $\tau$  are below zero for all choices of  $\mu_\pi$  and  $\mu_y$  between zero and three. There is an area when  $\mu_\pi$  is smaller than 0.5 and  $\mu_y$  becomes very large where  $\lambda_1$  is not negative, but the policy relevance of an aggressively output stabilizing central bank which doesn't care about price stability seems questionable. For selected combinations of central bank preferences the values of  $\beta_1, \lambda_1, \eta_1$  and  $\tau$  are documented in Tables A1-A3 in the Appendix.

### 3.4 Impulse response functions

Impulse responses for two different central bank preference settings have been computed. The first set of graphs illustrates pure inflation targeting and the second assumes a central bank which incorporates output stabilization. The chosen parameter values are  $\mu_y = 0.5$  or zero and  $\mu_\pi = 0.5$  in both cases. The other parameters are chosen as in the former section. Figures 3-6 illustrate the adjustment of prices, nominal exchange rate output and nominal interest rate after an exogenous ten percent raise of  $\hat{p}$ . The blue lines denote strict inflation targeting and the pink lines represent model's behaviour when output targeting is a central bank's goal.

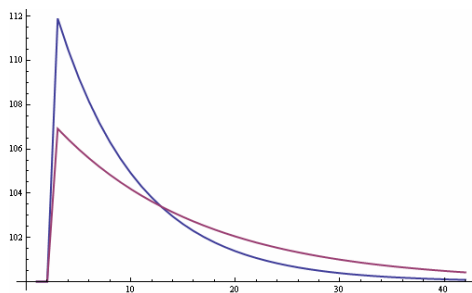


**Fig. 3: Prices**

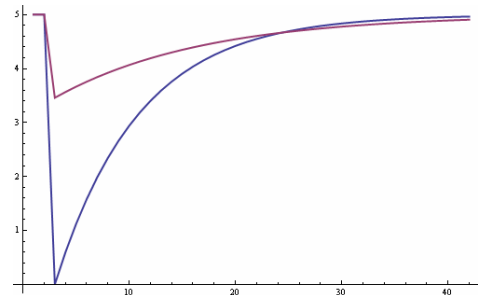


**Fig. 4: Exchange rate**

The price adjustment under strict inflation targeting runs faster than under the mixed approach. There is an exchange rate overshooting in both cases. Strict inflation targeting leads to depreciation of 10 percent at its peak while in the other setting depreciation reaches only a little more than six percent. Both experiments yield a persistent change of the exchange rate.



**Fig. 5: Output**



**Fig. 6: Nominal Interest rate**

The shock yields an impact boost on output of twelve respectively seven percent. Output targeting leads to slower adjustment. Because of the Phillips curve restriction there is - in both cases - no room for a persistent change of the economy's output. Not surprisingly, output deviation from steady state is smaller when central bank looks for output stabilization.

Fig. 6 shows the reaction of the nominal interest rate. The shock is followed by a sharp drop of it. Central bank's interest rate response to the target change is smaller when output stabilization is taken into account. The interest rate drop under strict inflation targeting is three times higher. The stronger interest rate drop is responsible for the stronger output reaction and the stronger devaluation of the exchange rate, as seen in Fig. 4 and 5. The main results from these experiments are first, that disregarding output stabilization leads to higher volatility of the exchange rate after the shock occurs. The second interesting finding is that under strict inflation targeting the adjustment process runs faster. There seems to be a trade-off between macroeconomic volatility and adjustment speed.

#### 4. Static expectations with price level targeting

In discrete time-stochastic notation, static expectations amount to the hypothesis that the exchange rate follows a random walk,

$$e_t = e_{t-1} + z_t, \quad (4.1)$$

with  $z_t \sim N(0, \sigma_z^2)$ , and, accordingly

$$E_t e_{t+1} = e_t. \quad (4.2)$$

This means that FOREX equilibrium requires  $i_t = i^*$ , and the interest rate rule degenerates to the constraint on  $p_t$  and  $y_t$

$$\mu_y (y_t - \bar{y}) = -\mu_\pi (p_t - \hat{p}). \quad (4.3)$$

The resulting temporary equilibrium system is given in Table 2.

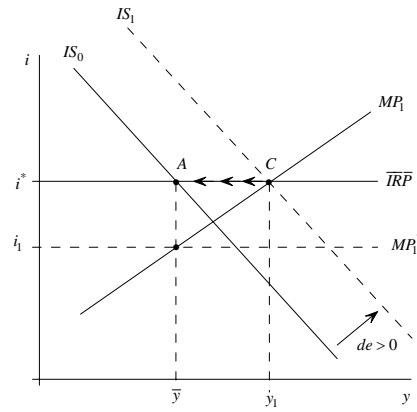
$y_t$	$e_t$	$p_{t+1}$	$p_t$	$\hat{p}$
1	$-\delta$	0	$-\delta$	0
$-\mu_\pi$	0	1	1	1
$\mu_y$	0	0	$-\mu_\pi$	$\mu_\pi$

**Table 2**

The Jacobian to this system is  $\mathcal{A}_{SE} = -\delta\mu_y$ , indicating that no determinate equilibrium exists for ‘strict’ inflation targeting ( $\mu_y = 0$ ). This feature is also revealed by the exchange rate multiplier for the monetary policy shock,

$$\frac{de}{d\hat{p}} = \frac{\mu_\pi}{\delta\mu_y} > 0, \text{ which implies } \lim_{\mu_y \rightarrow 0} \frac{de}{d\hat{p}} = +\infty.$$

The economic reasoning behind these results runs as follows. The decision of the Central Bank to raise the target price level ( $d\hat{p} > 0$ ) leads to a lower interest rate and a FOREX disequilibrium in favour of the foreign currency. The resulting depreciation of the domestic currency has an expansionary effect on output demand. If  $\mu_y > 0$ , this effect exerts an upward pressure to the domestic interest rate, via the interest rate rule. The price level cannot contribute to the correction in period  $t$ , since it is not able to adjust before  $t+1$ . With ‘strict’ inflation targeting ( $\mu_y = 0$ ), the interest rate does not respond to output expansion, and there is no limiting force to the depreciation of domestic currency.



**Fig. 7: Static Expectations**

The situation with static speculation is illustrated in Fig. 7. The upward sloping line  $MP_1$  represents the ‘flexible’ inflation targeting rule ( $\mu_y > 0$ ) after the expansionary monetary policy shock; the depreciation of domestic currency induces an outward shift of the  $IS$  line (from  $IS_0$  to  $IS_1$ ), and the impact equilibrium is found in point  $C$ . By contrast, the flat line  $MP_1'$  represents ‘strict’ inflation targeting, where the outward shift of the  $IS$  line does not induce a higher domestic interest rate. In this regime, no determinate equilibrium exists. As it regards exchange rate volatility, point  $C$  in Fig. 7 clearly indicates that, with static expectations, overshooting is more pronounced than with rational expectations.

## 5. Summary

With monetary policy following a money supply strategy, both static and rational speculation generate a well defined equilibrium path. Under both expectational behavior patterns, a monetary policy shock induces exchange rate overshooting. With static expectations, overshooting is more pronounced than with rational speculation. In this sense, rational expectations reduce exchange rate volatility. The necessary and sufficient condition for overshooting is a sufficiently weak sensitivity of output demand vis-à-vis the exchange rate ( $0 < \delta < 1$ ). These results apply for a conventional money demand function with actual output as the transactions variable.

The stabilizing property of rational speculation is even confirmed if monetary policy follows a Taylor-type interest rate rule. With ‘flexible’ inflation targeting ( $\mu_y > 0$ ), equilibrium is well defined under both expectational schemes. With ‘strict’ inflation targeting ( $\mu_y = 0$ ), however, equilibrium remains well defined under rational speculation, while under static expectations no stable equilibrium exists at all. The stabilizing property of rational speculation is stronger when monetary policy adopts ‘flexible’ inflation targeting than with ‘strict’ inflation targeting.

**References**

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990), Positive-Feedback Investment Strategies and Destabilizing Rational Speculation, *Journal of Finance* Vol. 45(2), 374-397.

Friedman, Milton (1953), The Case for Flexible Exchange Rates. in: *Essays in Positive Economics*, University Of Chicago Press.

Johnson, Harry G. (1976) , Destabilizing Speculation: A General Equilibrium Approach ,*The Journal of Political Economy*, Vol. 84(1), 101-108

McCallum, Bennett T. (1983), On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective. *Journal of Monetary Economics* Vol. 11(2), 139–168.

McCallum, Bennett T. (1999), Role of the Minimal State Variable Criterion in Rational Expectations Models, *International Tax and Public Finance*, Vol. 6(4), 621-639.

McCallum, Bennett T. and Edward Nelson (2000), Monetary Policy for an Open Economy: an Alternative Framework with Optimizing Agents and Sticky Prices, *Oxford Review of Economic Policy*, Vol. 16(4), 74-91.

Mundell, Robert (1963), Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates, *Canadian Journal of Economics and Political Science* Vol. 29, 475-485.

Obstfeld Maurice and Kenneth Rogoff (2003), *Foundations of International Macroeconomics*, Chap. 9, MIT Press.

Proaño, Christian R. (2009), Heterogeneous Behavioral Expectations, FX Fluctuations and Dynamic Stability in a Stylized Two-Country Macroeconomic Model, *IMK Working Paper 3/2009*.

Taylor, John B. (1993), Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, 195-21.



### Appendix A. Partial multipliers to the semi-reduced form (Table 1)

Output

$$\frac{\partial y_t}{\partial \hat{e}_{t+1}} = \frac{\delta}{1 + \mu_y(\delta + \sigma)} > 0, \quad \frac{\partial y_t}{\partial p_t} = -\frac{\delta + \mu_\pi(\delta + \sigma)}{1 + \mu_y(\delta + \sigma)} < 0, \quad \frac{\partial y_t}{\partial u_t} = 0;$$

Price level

$$\frac{\partial p_{t+1}}{\partial \hat{e}_{t+1}} = \frac{\varphi\delta}{1 + \mu_y(\delta + \sigma)} > 0, \quad \frac{\partial p_{t+1}}{\partial p_t} = \frac{1 + \mu_y(\delta + \sigma) - \varphi(\delta + \mu_\pi(\delta + \sigma))}{1 + \mu_y(\delta + \sigma)}, \quad \frac{\partial p_{t+1}}{\partial u_t} = 1;$$

Exchange rate

$$\frac{\partial e_t}{\partial \hat{e}_{t+1}} = \frac{1 + \sigma\mu_y}{1 + \mu_y(\delta + \sigma)} > 0 \text{ (and } < 1), \quad \frac{\partial e_t}{\partial p_t} = -\frac{\mu_\pi - \delta\mu_y}{1 + \mu_y(\delta + \sigma)} < 0 \text{ for } \mu_y < \mu_\pi / \delta, \quad \frac{\partial e_t}{\partial u_t} = 0.$$

### Appendix B UC-MSV approach

Equation (3.1) implies

$$E_t p_{t+1} = \beta_0 + \beta_1 p_t, \quad (\text{B1.1})$$

For  $u_t$  known in  $t$  and

$$\beta_0 = (1 - \beta_1)\hat{p} \quad (\text{for } \bar{p} = \hat{p}). \quad (\text{B1.2})$$

Subtracting the steady state condition ( $\bar{p} = \beta_0 + \beta_1\bar{p}$ ) from (3.1) and rearranging yields

$$p_{t+1} = (1 - \beta_1)\bar{p} + \beta_1 p_t + \beta_2 u_{t+1},$$

The identifying restriction regarding the steady state exchange rate is

$$\lambda_0 = \bar{e} - \lambda_1 \hat{p}, \quad (\text{for } \bar{p} = \hat{p}). \quad (\text{B1.3})$$

Subtracting the steady state restriction from equation (3.2) and rearranging yields

$$e_t = \bar{e} - \lambda_1 \bar{p} + \lambda_1 p_t + \lambda_2 u_{t+1}, \quad (\text{B1.4})$$

From (3.3) it follows, that

$$\eta_0 = \bar{y} - \eta_1 \hat{p} \quad (\text{for } \bar{p} = \hat{p}). \quad (\text{B1.5})$$

Subtracting the steady state restriction from equation (3.3) and rearranging yields

$$y_t = \bar{y} - \eta_1 \bar{p} + \eta_1 p_t + \eta_2 u_t. \quad (\text{B1.6})$$

Using the partial derivatives from Table 1, it is possible to identify  $\eta_1$  as

$$\begin{aligned} \frac{dy_t}{dp_t} &= \frac{\partial y_t}{\partial \hat{e}_{t+1}} \frac{d\hat{e}_{t+1}}{dp_t} + \frac{\partial y_t}{\partial p_t} = \eta_1, \\ &= -\frac{\delta}{\Delta} \lambda_1 \beta_1 + \frac{\delta + \mu_\pi (\delta + \sigma)}{\Delta}, \\ &= \frac{\mu_\pi (\delta + \sigma) + \delta (1 - \lambda_1 \beta_1)}{\Delta} = \eta_1. \end{aligned} \quad (\text{B 1.7})$$

Where  $\Delta$  is the Jacobian ( $\Delta = -(1 + \mu_y (\delta + \sigma))$ ). Then the next step is the derivation of the  $(y, i)$  trajectory.

Rearranging equation (B 1.6) yields

$$p_t - \bar{p} = \frac{1}{\eta_1} (y_t - \bar{y}) - \frac{\eta_1}{\eta_2} u_t. \quad (\text{B1.8})$$

Inserting (B 1.8) in (2.4) gives

$$i_t - i_t^* = \left( \frac{\mu_\pi}{\eta_1} + \mu_y \right) (y_t - \bar{y}) - \frac{\mu_\pi \eta_1}{\eta_2} u_t \text{ and}$$

$$\frac{di_t}{dy_t} = \frac{\mu_\pi + \eta_1 \mu_y}{\eta_1} =: \tau \quad (\text{trajectory slope}). \quad (\text{B1.9})$$

The coefficients  $\beta_1$  and  $\lambda_1$  are identified as follows. Substitution of  $\hat{e}_{t+1}$  through (3.7) into the semi-reduced form using the partial multipliers given in Appendix A gives the identifying restrictions

$$\frac{dp_{t+1}}{dp_t} = \frac{\varphi (\delta + \mu_\pi (\delta + \sigma)) + \Delta - \varphi \delta \lambda_1 \beta_1}{\Delta} = \beta_1 \text{ and} \quad (\text{B2.1})$$

$$\frac{de_t}{dp_t} = \frac{\mu_\pi - \delta \mu_y - (1 + \sigma \mu_y) \lambda_1 \beta_1}{\Delta} = \lambda_1. \quad (\text{B2.2})$$

These are two independent equations that can be solved for  $\beta_1$  and  $\lambda_1$ . Rearranging (B 2.1) leads to

$$\lambda_1 = \frac{\varphi(\delta + \mu_\pi(\delta + \sigma)) + \Delta(1 - \beta_1)}{\varphi\delta\beta_1}, \quad (\text{B 2.3})$$

and from (B 2.2) we find

$$\lambda_1 = \frac{\mu_\pi - \delta\mu_y}{\Delta + (1 + \sigma\mu_y)\beta_1}. \quad (\text{B 2.4})$$

Combining (B 2.3) and (B 2.4) leads to a quadratic equation of the form

$$\beta_1^2 + a\beta_1 + b = 0, \text{ where}$$

$$a = \frac{\Delta^2 + \varphi\delta(\mu_\pi - \delta\mu_y) - [\varphi(\delta + \mu_\pi(\delta + \sigma)) + \Delta](1 + \sigma\mu_y)}{\Delta(1 + \sigma\mu_y)}, \quad (\text{B 2.5})$$

$$b = -\frac{\Delta + \varphi(\delta + \mu_\pi(\delta + \sigma))}{1 + \sigma\mu_y}. \quad (\text{B 2.6})$$

### C. Numerical results

$\mu_\pi \backslash \mu_y$	0	0,5	1,0
0,5	0.88098	0.93082	0.95520
1,0	0,83139	0,88808	0,92090
1,5	0,79127	0,85299	0,89138

**Table A1: Different values of  $\beta_1$**

$\mu_\pi \backslash \mu_y$	0	0,5	1,0
0,5	-4,20099	-2,22773	-1,16173
1,0	-5,93070	-3,93521	-2,64171
1,5	-7,18639	-5,20322	-3,81008

**Table A2: Different values of  $\lambda_1$**

$\mu_\pi \backslash \mu_y$	0	0,5	1,0
0,5	-1,19020	-0,69178	-0,44796
1,0	-1,68614	-1,11917	-0,79103
1,5	-2,08728	-1,47012	-1,08616

Table A3: Different values of  $\eta_1$

$\mu_\pi \backslash \mu_y$	0	0,5	1,0
0,5	-0,4200	-,2228	-,1162
1,0	-0,5931	-,3935	-,2642
1,5	-0,7186	-,5203	-,3810

Table A4: Different values of  $\tau$

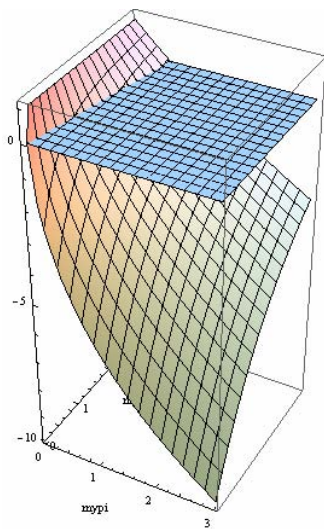


Fig. A1:  $\lambda_1$  for different central banking strategies

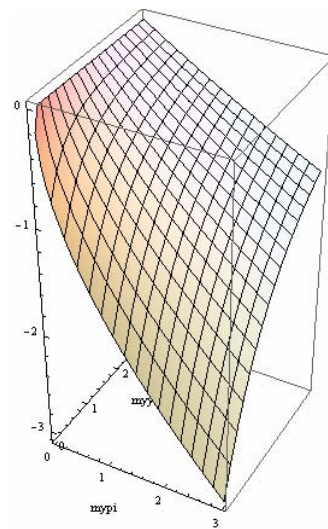


Fig. A2:  $\eta_1$  for different central banking strategies

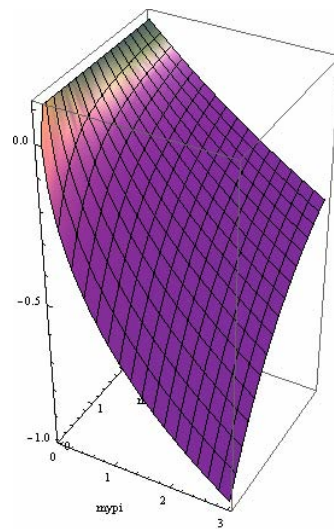


Fig. A3:  $\tau$  for different central banking strategies



## DISKUSSIONSPAPIERE DER FÄCHERGRUPPE VOLKSWIRTSCHAFTSLEHRE

### DISCUSSION PAPERS IN ECONOMICS

Die komplette Liste der Diskussionspapiere ist auf der Internetseite veröffentlicht / for full list of papers see:  
<http://fgvwl.hsu-hh.de/wp-vwl>

#### 2010

- 104 Reither, Franco; Bennöhr, Lars. Stabilizing Rational Speculation and Price Level Targeting, August 2010.
- 103 Christmann, Robin. Warum brauchen wir Richter?, August 2010.
- 102 Hackmann, Johannes; Die einkommensteuerliche Berücksichtigung von Scheidungs- und Kinderunterhalt im Vergleich, Juni 2010.
- 101 Schneider, Andrea; Zimmermann, Klaus W. Fairness und ihr Preis, Juni 2010.
- 100 von Arnould, Andreas; Zimmermann, Klaus W. Regulating Government ('s Share): The Fifty-Percent Rule of the Federal Constitutional Court in Germany, März 2010.

#### 2009

- 99 Kruse, Jörn. Wissen für demokratische Entscheidungen, Dezember 2009.
- 98 Horgos, Daniel; Zimmermann, Klaus W. It Takes Two to Tango: Lobbies and the Political Business Cycle, September 2009.
- 97 Berlemann, Michael; Zimmermann, Klaus W. Gewerkschaften im Bundestag: Gemeinwohlorientiert oder Lobbyisten?, September 2009.
- 96 Kruse, Jörn. Priority and Internet Quality, August 2009.
- 95 Schneider, Andrea. Science and teaching: Two-dimensional signalling in the academic job market, August 2009.
- 94 Kruse, Jörn. Das Governance-Dilemma der demokratischen Wirtschaftspolitik, August 2009.
- 93 Hackmann, Johannes. Ungereimtheiten der traditionell in Deutschland vorherrschenden Rechtfertigungsansätze für das Ehegattensplitting, Mai 2009.
- 92 Schneider, Andrea; Klaus W. Zimmermann. Mehr zu den politischen Segnungen von Föderalismus, April 2009.
- 91 Beckmann, Klaus; Schneider, Andrea. The interaction of publications and appointments - New evidence on academic economists in Germany, März 2009.
- 90 Beckmann, Klaus; Schneider, Andrea. MeinProf.de und die Qualität der Lehre, Februar 2009.
- 89 Berlemann, Michael; Hielscher, Kai. Measuring Effective Monetary Policy Conservatism, February 2009.
- 88 Horgos, Daniel. The Elasticity of Substitution and the Sector Bias of International Outsourcing: Solving the Puzzle, February 2009.
- 87 Rundshagen, Bianca; Zimmermann, Klaus W.. Buchanan-Kooperation und Internationale Öffentliche Güter, Januar 2009.

#### 2008

- 86 Thomas, Tobias. Questionable Luxury Taxes: Results from a Mating Game, September 2008.
- 85 Dluhosch, Barbara; Zimmermann, Klaus W.. Adolph Wagner und sein „Gesetz“: einige späte Anmerkungen, August 2008.
- 84 Zimmermann, Klaus W.; Horgos, Daniel. Interest groups and economic performance: some new evidence, August 2008.
- 83 Beckmann, Klaus; Gerrits, Carsten. Armutsbekämpfung durch Reduktion von Korruption: eine Rolle für Unternehmen?, Juli 2008.
- 82 Beckmann, Klaus; Engelmann, Dennis. Steuerwettbewerb und Finanzverfassung, Juli 2008.
- 81 Thomas, Tobias. Fragwürdige Luxussteuern: Statusstreben und demonstratives Konsumverhalten in der Geschichte ökonomischen Denkens, Mai 2008.
- 80 Kruse, Jörn. Hochschulen und langfristige Politik. Ein ordnungspolitischer Essay zu zwei Reformutopien, Mai 2008.
- 79 Kruse, Jörn. Mobile Termination Carrier Selection, April 2008.
- 78 Dewenter, Ralf; Haucap, Justus. Wettbewerb als Aufgabe und Problem auf Medienmärkten: Fallstudien aus Sicht der „Theorie zweiseitiger Märkte“, April 2008.
- 77 Kruse, Jörn. Parteien-Monopol und Dezentralisierung des demokratischen Staates, März 2008.
- 76 Beckmann, Klaus; Gattke, Susan. Status preferences and optimal corrective taxes: a note, February 2008.

