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# Revisiting the Great Moderation using the Method of Indirect Inference\*

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## Abstract

We investigate the relative roles of monetary policy and shocks in causing the Great Moderation, using indirect inference where a DSGE model is tested for its ability to mimic a VAR describing the data. A New Keynesian model with a Taylor Rule and one with the Optimal Timeless Rule are both tested. The latter easily dominates, whether calibrated or estimated, implying that the Fed's policy in the 1970s was neither inadequate nor a cause of indeterminacy; it was both optimal and essentially unchanged during the 1980s. By implication it was largely the reduced shocks that caused the Great Moderation—among them monetary policy shocks the Fed injected into inflation.

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## 1 Introduction

John Taylor suggested in Taylor (1993) that an interest rate rule for well-conducted monetary policy fitted the Fed's behaviour since 1987 rather well in a single equation regression. Since then a variety of similar studies have confirmed his finding—most of these have focused on a data sample beginning in the early-to-mid 1980s. For the period from the late 1960s to the early 1980s the results have been more mixed. Thus Clarida, Gali and Gertler (2000) reported that the Taylor Rule fitted but with a coefficient on inflation of less than unity; in a full New Keynesian model this fails under the usual criteria to create determinacy in inflation and they argue that this could be the reason for high inflation and output volatility in this earlier post-war period. They concluded that the reduction in macro volatility between these two periods (the 'Great Moderation') was due to the improvement in monetary policy as captured by this change in the operative Taylor Rule.

This view of the Great Moderation has been widely challenged in econometric studies of the time series. These have attempted to decompose the reduction in macro variance into the effect of parameter changes and the effect of shock variances. Virtually all have found that the shock variances have dominated the change and that the monetary policy rule operating therefore did not change very much.

A further questioning of the Taylor Rule account of the post-war monetary policy has come from Cochrane (2011) and others (Minford, Perugini and Srinivasan, 2002) who argue that the Taylor Rule is not identified as a single equation because a DSGE model with a different monetary policy rule (such as a money supply rule) could equally well

generate an equation of the Taylor Rule form. Therefore much of the work that estimates the Taylor Rule could be spurious.

A way of dealing with this identification problem is to specify the Taylor Rule as one equation in a full DSGE model; in this case the overidentifying restrictions of the model should ensure identification. However, there then remains the question of whether such a model is as good a representation as one that is in general the same but has an alternative monetary policy rule. While some authors have estimated Taylor Rules as part of such a model, none of them to our knowledge has tested such a model against one with a rival rule.

That is precisely our interest in this paper. We wish to investigate whether when identified as part of a DSGE model the Taylor Rule or alternative rules perform best in matching the US data. Having established a valid representation of monetary policy in the post-war US, we would like then to revisit the cause of the Great Moderation.

We look at a particular rival to the Taylor Rule, the Optimal Timeless Rule. This is of interest because in it the Fed is playing a more precisely optimising role than it does in the Taylor Rule which is a simple rule that can be operated with limited current information, namely for output and inflation. The Optimal Timeless Rule assumes that the Fed can solve the DSGE model for all the shocks and so choose in a discriminating way its reaction to each shock. Other than this Optimal Timeless Rule we also look at variants of the Taylor Rule, including one that closely mimics the Optimal Timeless Rule.

To make our testing bounded and tractable we use the monetary rule in conjunction with the most widely-accepted DSGE model representation—where the model is reduced to two equations, a forward-looking ‘IS’ curve and a New Keynesian Phillips curve, plus the monetary rule. We allow each rule/model combination to be calibrated with the best chance of matching the data and then test on that best calibration, using the method of Indirect Inference under which the model’s simulated behaviour is formally tested for congruence with the behaviour of the data. Our efforts here join others that have brought DSGE models to bear on this issue—notably, Ireland (2007), Smets and Wouters

(2007) and the related Le et al. (2011) and Fernandez-Villaverde et al. (2009, 2010). These authors have all used much larger DSGE models, in some cases data that was non-stationary, and in most cases Bayesian estimation methods. Their work is largely complementary to ours and we discuss it, its findings and their relation to ours below. Bayesian estimation is a method for improving on calibrated parameters but our method of Indirect Inference takes matters further and asks if the finally estimated parameters are consistent overall with the data behaviour; if not it searches for some set permissible within the theory that is consistent, getting as close to consistency as possible given the model and the data. This method is the major innovation we introduce for the treatment of the topic here; it is a method based on classical statistical inference which we explain and defend carefully below.

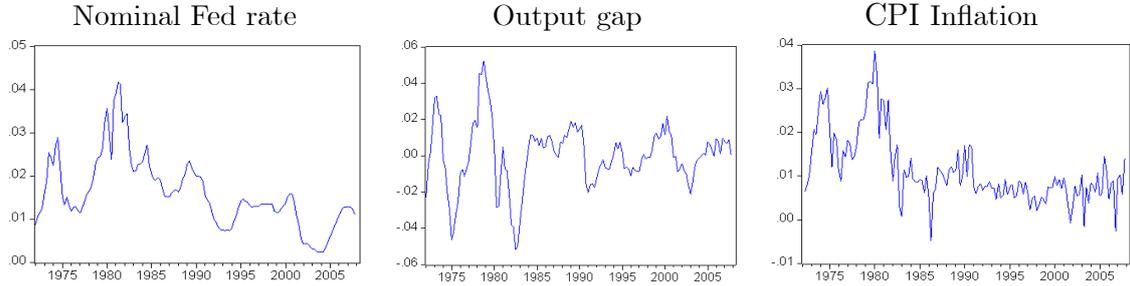
In section 2 we review the work on the Great Moderation; in section 3 that on the Taylor Rule. In section 4 we set out the model and the rules to be tested, and in section 5 our test procedure. In section 6 we show the results; in section 7 we draw out the implications for the Great Moderation; section 8 concludes.

## **2 Causes of the Great Moderation**

The Great Moderation refers to the period during which the volatility of the main economic variables was relatively modest. This began in the US around the early 1980s although there is no consensus on the exact date. Figure 1 below shows the time paths of three main US macro variables from 1972 to 2007: the nominal Fed interest rate, output gap and CPI inflation. It shows the massive fluctuation of the 1970s ceased after the early 1980s, indicating the economy's transition from the Great Acceleration to the Great Moderation.

Changes in the monetary policy regime could have produced the Great Moderation. This is typically illustrated with the three-equation New Keynesian framework, consisting of the IS curve derived from the household's optimization problem, the Phillips curve

Figure 1: Time Paths of Main Macro Variables of the US Economy  
(Quarterly data, 1972-2007)



Data source: the Federal Reserve Bank of St. Louis (<http://research.stlouisfed.org/fred2/>, accessed Nov. 2009).

Fed rate and inflation unfiltered; the output gap is the log deviation of real GDP from its HP trend.

derived from the firm’s optimal price-setting behaviour, and a Taylor Rule approximating the Fed’s monetary policy. Using simulated behaviour from models of this sort, a number of authors suggest that the US economy’s improved stability was largely due to stronger monetary policy responses to inflation (Clarida, Gali and Gertler, 2000; Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006 and Benati and Surico, 2009). The contrast is between the ‘passive’ monetary policy of the 1970s, with low Taylor Rule responses, and the ‘active’ policy of the later period in which the conditions for a unique stable equilibrium (the ‘Taylor Principle’) are met, these normally being that the inflation response in the Taylor Rule be greater than unity. Thus it was argued that the indeterminacy caused by the passive 1970s policy generated sunspots and so the Great Acceleration; with the Fed’s switch this was eliminated, hence the Great Moderation.

By contrast other authors, mainly using structural VAR analysis, have suggested that the Great Moderation was caused not by policy regime change but by a reduction in the variance of shocks. Thus Stock and Watson (2002) claimed that over 70% of the reduction in GDP volatility was due to lower shocks to productivity, commodity prices and forecast errors. Primiceri (2005) argued that the stagflation in the 1970s was mostly due to non-policy shocks. A similar conclusion was drawn by Gambetti, Pappa and Canova (2008), while Sims and Zha (2006) found in much the same vein that an empirical model with variation only in the variance of the structural errors fitted the data best and that alteration in the monetary regime—even if assumed to occur—would not much influence

the observed inflation dynamics.

The logic underlying the structural VAR approach is that, when actual data are modelled with a structural VAR, their dynamics will be determined both by the VAR coefficient matrix that represents the propagation mechanism (including the monetary regime) and by the variance-covariance matrix of prediction errors which takes into account the impact of exogenous disturbances. Hence by analysing the variation of these two matrices across different subsamples it is possible to work out whether it is the change in the propagation mechanism or in the error variability that has caused the change in the data variability. It is the second that these studies have identified as the dominant cause. Hence almost all structural VAR analyses have suggested ‘good shocks’ (or ‘good luck’) as the main cause of the Great Moderation, with the change of policy regime in a negligible role.

Nevertheless, since this structural VAR approach relies critically on supposed model restrictions to decompose the variations in the VAR between its coefficient matrix and the variance-covariance matrix of its prediction errors, there is a pervasive identification problem. As Benati and Surico (2009) have pointed out, the problem that ‘lies at the very heart’ is the difficulty in connecting the structure of a DSGE model to the structure of a VAR. In other words one cannot retrieve from the parameters of an SVAR the underlying structural parameters of the DSGE model generating it, unless one is willing to specify the DSGE model in detail. None of these authors have done this. Hence one cannot know from their studies whether in fact the DSGE model that produced the SVAR for the Great Acceleration period differed or did not differ from the DSGE model producing the SVAR for the Great Moderation period. It is not enough to say that the SVAR parameters ‘changed little’ since we do not know what changes would have been produced by the relevant changes in the structural DSGE models. Different DSGE models with similar shock distributions could have produced these SVARs with similar coefficients and different shock distributions.

Essentially it is this problem that we attempt to solve in the work we present below.

We estimate a VAR for each period and we then ask what candidate DSGE models could have generated each VAR. Having established which model comes closest to doing so, we then examine how the difference between them accounts for the Great Moderation. Since these models embrace the ones put forward by the authors who argue that policy regime change accounts for it, we are also able to evaluate these authors' claims statistically. Thus we bring evaluative statistics to bear on the authors who claim policy regime change, while we bring identification to bear on the authors who use SVARs.

We describe our methods in detail below. But first we discuss the empirical evidence from single equation estimates for the Taylor Rule.

### 3 Taylor Rules, Estimation and Identification

Taylor (1993) suggested that a good rule for monetary policy would set the Federal funds rate according to the following equation:

$$i_t^A = \pi_t^A + 0.5x_t + 0.5(\pi_t^A - \pi^*) + g \quad (1)$$

where  $x_t$  is the percentage deviation of real GDP from trend,  $\pi_t^A$  is the annual rate of inflation averaged over the past four quarters, with inflation target  $\pi^*$  and real GDP growth rate  $g$  both set at 2 percent.

Known as the original 'Taylor Rule', equation (1) was found to have predicted the movement of actual Fed rates well for much of the period from 1987 until the early 1990s. This success convinced many economists that the Fed's policy at the time could be conveniently described by this equation. A number of variants have also been proposed; for example, one with policy inertia as in Clarida, Gali and Gertler (1999):

$$i_t^A = (1 - \rho)[\alpha + \gamma_\pi(\pi_t^A - \pi^*) + \gamma_x x_t] + \rho i_{t-1}^A \quad (2)$$

with  $\rho$  showing the degree of 'interest rate smoothing'. Others have introduced backward-

or forward-looking behaviour, with inflation and output gap treated as lagged or expected future variables—such as Rotemberg and Woodford (1997, 1998) and Clarida, Gali and Gertler (2000). In most cases the focus is on the period between the early 1980s and some time before the banking crisis of the late 2000s.

Rules of these types are generally found to fit the data well, either as a stand-alone equation in regression analysis, or as part of a full model in DSGE analysis. Giannoni and Woodford (2005) is a recent example of the former, whereas Smets and Wouters (2007) and Ireland (2007) are examples of the latter. However, besides the usual difficulties encountered in applied work (e.g., Castelnuovo, 2003 and Carare and Tchaidze, 2005), these estimates face an identification problem pointed out in Minford, Perugini and Srinivasan (2002) and Cochrane (2011)—see also Minford (2008) which we use in what follows.

Lack of identification occurs when an equation could be confused with a linear combination of other equations in the model. In the case of the Taylor Rule, DSGE models give rise to the same correlations between interest rate and inflation as the Taylor Rule, even if the Fed is doing something quite different, such as targeting the money supply. For example, Minford et al. show this in a DSGE model with Fischer wage contracts.

In effect, unless econometricians know from other sources of information that the central bank is pursuing a Taylor Rule, they cannot be sure they are estimating a Taylor Rule when they specify a Taylor-type equation because under other possible monetary policy rules a similar relationship to the Taylor Rule is implied<sup>1</sup>.

The point can be illustrated using a popular DSGE model with a money supply rule instead of a Taylor Rule as follows:

$$\text{(IS curve): } y_t = \gamma E_{t-1} y_{t+1} - \phi r_t + v_t$$

$$\text{(Phillips curve): } \pi_t = \zeta(y_t - y^*) + \nu E_{t-1} \pi_{t+1} + (1 - \nu)\pi_{t-1} + u_t$$

$$\text{(Money supply target): } \Delta m_t = m + \mu_t$$

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<sup>1</sup>While one may argue that various announcements, proposals and reports published by the central bank directly reveal to econometricians the bank's reaction function. However, what the Fed actually does is not necessarily the same thing as what its officials and governors say it does. So these documents, while illuminating, can complement but cannot substitute for econometric evidence.

(Money demand):  $m_t - p_t = \psi_1 E_{t-1} y_{t+1} - \psi_2 R_t + \varepsilon_t$

(Fisher identity):  $R_t = r_t + E_{t-1} \pi_{t+1}$

The model above implies a Taylor-type relation that looks like:  $R_t = r^* + \pi^* + \gamma \chi^{-1}(\pi_t - \pi^*) + \psi_1 \chi^{-1}(y_t - y^*) + w_t$ , where  $\chi = \psi_2 \gamma - \psi_1 \phi$ , and the error term,  $w_t$ , is both correlated with inflation and output and autocorrelated; it contains the current money supply/demand and aggregate demand shocks and also various lagged values (the change in lagged expected future inflation, interest rate, the output gap, the money demand shock, and the aggregate demand shock). This particular Taylor-type relation was created with a combination of equations- the solution of the money demand and supply curves for interest rate, the Fisher identity and the IS curve for expected future output<sup>2</sup>. But other Taylor-type relations could be created with combinations of other equations, including the solution equations, generated by the model. They will all exhibit autocorrelation and contemporaneous correlation with output and inflation, clearly of different sorts depending on the combination used.

All the above applies to identifying a single equation being estimated; thus one cannot distinguish a Taylor Rule equation from the equations implied by the model and alternative rules when one just estimates that equation. One could attempt to apply further restrictions but such restrictions are hard to find. For example, one might restrict the error process of a Taylor Rule in some distinct way, say to being serially uncorrelated. But the error in a Taylor Rule, which represents ‘monetary judgement’ based on factors other than the two gaps, may well be autocorrelated because those factors are persistent.

However, when a ‘monetary rule’ is chosen for inclusion in a complete DSGE model with rational expectations, then the model imposes over-identifying restrictions through

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<sup>2</sup>From the money demand and money supply equations,  $\psi_2 \Delta R_t = \pi_t - m + \psi_1 \Delta E_{t-1} y_{t+1} + \Delta \varepsilon_t - \mu_t$ . Substitute for  $E_{t-1} y_{t+1}$  from the IS curve and then inside that for real interest rate from the Fisher identity giving  $\psi_2 \Delta R_t = \pi_t - m + \psi_1 (\frac{1}{\gamma}) \{ \varphi (\Delta R_t - \Delta E_{t-1} \pi_{t+1}) + \Delta y_t - \Delta v_t \} + \Delta \varepsilon_t - \mu_t$ ; then, rearrange this as  $(\psi_2 - \frac{\psi_1 \varphi}{\gamma}) \Delta (R_t - R^*) = (\pi_t - m) - \frac{\psi_1 \varphi}{\gamma} \Delta E_{t-1} \pi_{t+1} + \frac{\psi_1}{\gamma} \Delta (y_t - y^*) - \frac{\psi_1}{\gamma} \Delta v_t + \Delta \varepsilon_t - \mu_t$ , where the constants  $R^*$  and  $y^*$  have been subtracted from  $R_t$  and  $y_t$  respectively, exploiting the fact that when differenced they disappear. Finally,  $R_t = r^* + \pi^* + \gamma \chi^{-1}(\pi_t - \pi^*) + \psi_1 \chi^{-1}(y_t - y^*) + \{ (R_{t-1} - R^*) - \psi_1 \varphi \chi^{-1} \Delta E_{t-1} \pi_{t+1} - \psi_1 \chi^{-1} (y_{t-1} - y^*) - \psi_1 \chi^{-1} \Delta v_t + \gamma \chi^{-1} \Delta \varepsilon_t - \gamma \chi^{-1} \mu_t \}$ , where we have used the steady state property that  $R^* = r^* + \pi^*$  and  $m = \pi^*$ .

the expectations terms which involve in principle all the model's parameters. Thus a model with a particular rule is in general over-identified so that estimation by full information methods of that particular model as specified (as in Rotemberg and Woodford, 1997, 1998, Smets and Wouters, 2007, Onatski and Williams, 2004 and Ireland, 2007) is possible. One way of putting this is that there are more reduced-form parameters than structural parameters. Another is to note that the reduced form will change if the structural description of monetary policy changes; this was a point first made by Lucas (1976) in his critique of conventional policy optimization at that time. So if econometricians posit a Taylor Rule then they will retrieve its coefficients and those of the rest of the model under the assumption that it is the true structural monetary rule. They could then compare the coefficients for a model where they assume some other rule. They can distinguish between the two models via their different reduced forms and hence their different fits to the data. Thus it is possible to identify the different rules of monetary policy behavior via full information estimation.

However, the identification problem does not go away, even when a model is over-identified in this way. The problem is that the decision to include the Taylor Rule in such a model has been justified by the fact that it fits the data in single equation estimation; but as we have seen such a choice could be the victim of identification failure as the rule could be mimicking the joint behaviour of the rest of the model and some other (true) monetary rule. If so, including it in the model will produce a mis-specified model whose behaviour will not fit the data as well as the properly-specified model with the true monetary policy equation. To detect this and also to find the true model we need not only to test this model but also to test possible well-specified alternatives. Thus we need to check whether there is a better model which with its over-identifying restrictions may fit the data more precisely.

This points the way to a possible way forward. One may specify a complete DSGE model with alternative monetary rules and use the over-identifying restrictions to determine their differing behaviours. One may then test which of these is more acceptable

from the data’s viewpoint and hence comes closest to the true model. This is precisely the approach taken here.

## 4 A Simple New Keynesian Model for Interest Rate, Output Gap and Inflation Determination

We follow a common practice among New Keynesian authors of setting up a full DSGE model with representative agents and reducing it to a three-equation framework consisting of an IS curve, a Phillips curve and a monetary policy rule.

Under rational expectations the IS curve derived from the household’s problem and the Phillips curve derived from the firm’s problem under Calvo (1983) contracts can be shown to be:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\tilde{i}_t - E_t \pi_{t+1}) + v_t \quad (3)$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w \quad (4)$$

where  $x_t$  is the output gap,  $\tilde{i}_t$  is the deviation of interest rate from its steady-state value,  $\pi_t$  is the price inflation, and  $v_t$  and  $u_t^w$  are the ‘demand shock’ and ‘supply shock’, respectively<sup>3</sup>.

We consider three monetary regime versions widely suggested for the US economy. These are the Optimal Timeless Rule when the Fed commits itself to minimizing a typical quadratic social welfare loss function; the original Taylor Rule (1); and its interest-rate-smoothed version (2).

In particular, the Optimal Timeless Rule is derived following Woodford (1999)’s idea of ignoring the initial conditions confronting the Fed at the regime’s inception. It implies that, if the Fed was a strict, consistent optimizer, it would have kept inflation always

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<sup>3</sup>Note  $\gamma$  and  $\kappa$  are functions of other structural parameters and some steady-state relations (See table 2 for calibrations in what follows). Full derivation of equations (3) and (4) can be found in our Supporting Annex.

equal to a fixed fraction of the first difference of the output gap, ensuring

$$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) \quad (5)$$

where  $\alpha$  is the relative weight it puts on the loss from output variations against inflation variations and  $\gamma$  is the Phillips curve constraint (regarding stickiness) it faces<sup>4</sup>.

Unlike Taylor Rules that specify a systematic policy instrument response to economic changes, this Timeless Rule sets an optimal trade-off between economic outcomes—here, it punishes excess inflation with a fall in the output growth rate. It then chooses the policy instrument setting to achieve these outcomes; thus the policy response is implicit. Svensson and Woodford (2004) categorized such a rule as ‘high-level monetary policy’; they argued that by connecting the central bank’s monetary actions to its ultimate policy objectives this rule has the advantage of being more transparent and robust<sup>5</sup>.

Thus, in order to implement the Optimal Timeless Rule the Fed must fully understand the model (including the shocks hitting the economy) and set its policy instrument (here the Fed rate) to whatever supports the Rule. Nevertheless, the Fed may make errors of implementation that cause the rule not to be met exactly—‘trembling hand’ errors,  $\xi_t$ . Here, since (5) is a strict optimality condition, we think of such policy mistakes as due either to an imperfect understanding of the model or to an inability to identify and react to the demand and supply shocks correctly. This differs from the error in typical Taylor Rules, (1) and (2), which consist of the Fed’s discretionary departures from the rule .

Thus the three model economies with differing monetary policy settings are readily comparable. These are summarised in table 1<sup>6</sup>.

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<sup>4</sup>See also Clarida, Gali and Gertler (1999) and McCallum and Nelson (2004). This is based on defining social welfare loss as ‘the loss in units of consumption as a percentage of steady-state output’ as in Rotemberg and Woodford (1998)—also Nistico (2007); it is conditional on assuming a particular utility function and zero-inflation steady state—more details can be found in our Supporting Annex.

<sup>5</sup>Svensson and Woodford (2004) also comment that such a rule may produce indeterminacy; however this does not occur in the model here.

<sup>6</sup>Note all equation errors are allowed to follow an AR(1) process when the models are tested against the data so omitted variables are allowed for. We also transform the Taylor Rules to quarterly versions so the frequency of interest rate and inflation is consistent with other variables in the model. All constant terms are dropped as demeaned, detrended data will be used, as the ‘data’ section explains below.

Table 1: Competing Rival Models

Baseline framework	
IS curve	$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\tilde{i}_t - E_t \pi_{t+1}) + v_t$
Phillips curve	$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w$
Monetary policy versions	
Optimal Timeless Rule (model one)	$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) + \xi_t$
original Taylor Rule (model two) & transformed equation	$i_t^A = \pi_t^A + 0.5x_t + 0.5(\pi_t^A - 0.02) + 0.02 + \xi_t$ $\tilde{i}_t = 1.5\pi_t + 0.125x_t + \xi_t'$
'IRS' Taylor Rule (2) (model three) & transformed equation	$i_t^A = (1 - \rho)[\alpha + \gamma_\pi(\pi - \pi^*) + \gamma_x x_t] + \rho i_{t-1}^A + \xi_t$ $\tilde{i}_t = (1 - \rho)[\gamma_\pi \pi_t + \gamma'_x x_t] + \rho \tilde{i}_{t-1} + \xi_t'$

Since these models differ only in the monetary policies being implemented, by comparing their capacity to fit the data one should be able to tell which rule, when included in a simple New Keynesian model, provides the best explanation of the facts and therefore the most appropriate description of the underlying policy. We go on to investigate this in what follows.

## 5 The Method of Indirect Inference

We evaluate the models' capacity in fitting the data using the method of Indirect Inference originally proposed in Minford, Theodoridis and Meenagh (2009) and subsequently with a number of refinements by Le et al. (2011) who evaluate the method using Monte Carlo experiments. The approach employs an auxiliary model that is completely independent of the theoretical one to produce a description of the data against which the performance of the theory is evaluated indirectly. Such a description can be summarised either by the

estimated parameters of the auxiliary model or by functions of these; we will call these the descriptors of the data. While these are treated as the ‘reality’, the theoretical model being evaluated is simulated to find its implied values for them.

Indirect inference has been widely used in the estimation of structural models (e.g., Smith, 1993, Gregory and Smith, 1991, 1993, Gouriéroux et al., 1993, Gouriéroux and Monfort, 1996 and Canova, 2005). Here we make a further use of indirect inference, to evaluate an already estimated or calibrated structural model. The common element is the use of an auxiliary time series model. In estimation the parameters of the structural model are chosen such that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the actual data. The optimal choices of parameters for the structural model are those that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. Common choices of this function are the actual coefficients, the scores or the impulse response functions. In model evaluation the parameters of the structural model are taken as given. The aim is to compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of a structural model—which is taken as a true model of the economy, the null hypothesis—with the performance of the auxiliary model when estimated from the actual data. If the structural model is correct then its predictions about the impulse responses, moments and time series properties of the data should statistically match those based on the actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates.

The testing procedure thus involves first constructing the errors implied by the previously estimated/calibrated structural model and the data. These are called the structural errors and are backed out directly from the equations and the data<sup>7</sup>. These errors are then bootstrapped and used to generate for each bootstrap new data based on the structural

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<sup>7</sup>Some equations may involve calculation of expectations. The method we use here is the robust instrumental variables estimation suggested by McCallum (1976) and Wickens (1982): we set the lagged endogenous data as instruments and calculate the fitted values from a VAR(1)—this also being the auxiliary model chosen in what follows.

model. An auxiliary time series model is then fitted to each set of data and the sampling distribution of the coefficients of the auxiliary time series model is obtained from these estimates of the auxiliary model. A Wald statistic is computed to determine whether functions of the parameters of the time series model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

Following Minford, Theodoridis and Meenagh (2009) we take a VAR(1) for the three macro variables (interest rate, output gap and inflation) as the appropriate auxiliary model and treat as the descriptors of the data the VAR coefficients and the variances of these variables. The Wald statistic is computed from these<sup>8</sup>. Thus effectively we are testing whether the observed dynamics and volatility of the chosen variables are explained by the simulated joint distribution of these at a given confidence level. The Wald statistic is given by:

$$(\Phi - \bar{\Phi})' \sum_{(\Phi\Phi)}^{-1} (\Phi - \bar{\Phi}) \quad (6)$$

the squared ‘Mahalanobis distance’, where  $\Phi$  is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with  $\bar{\Phi}$  and  $\sum_{(\Phi\Phi)}$  representing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively<sup>9</sup>.

Figure 2 illustrates the whole testing procedure. While panel A of the figure summarises the main steps just described, the ‘mountain-shaped’ diagram in the second panel gives an example of how the ‘reality’ is compared to model predictions using the Wald

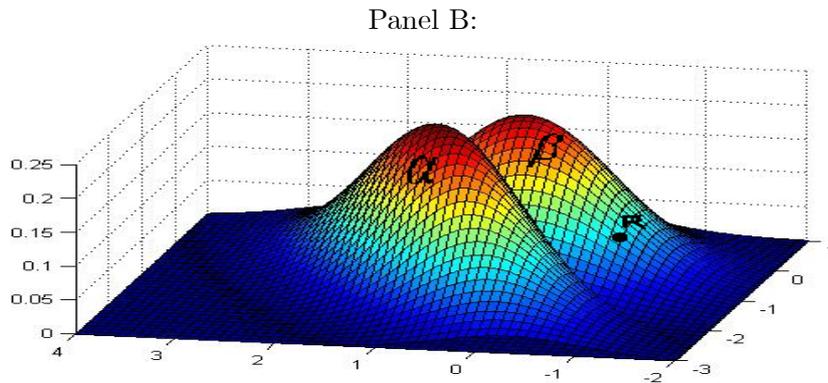
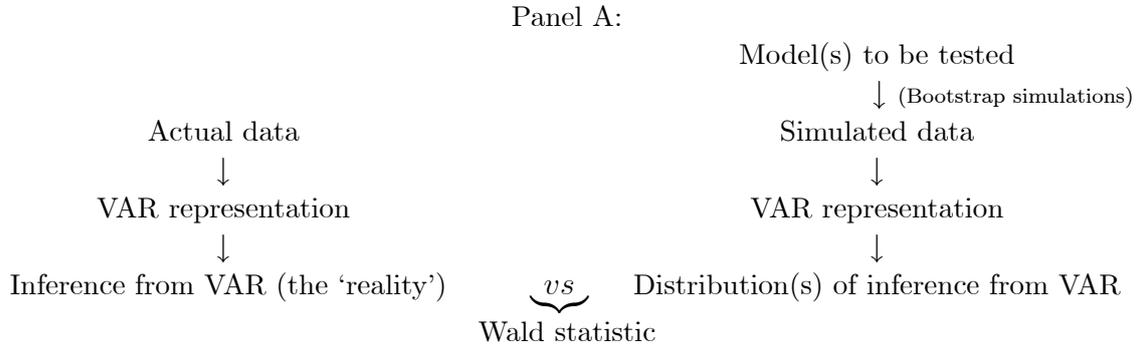
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<sup>8</sup>Note that the VAR impulse response functions, the co-variances, as well as the auto/cross correlations of the left-hand-side variables will all be implicitly examined when the VAR coefficient matrix is considered, since the former are functions of the latter.

<sup>9</sup>Smith (1993), for his demonstration of model estimation, originally used VAR(2) as the auxiliary model. His VAR included the logged output and the logged investment and he tried to maximize the model’s capacity in fitting the dynamic relation between these. To this end he included the ten VAR coefficients (including two constants) in his vector of data descriptors. Here, since a VAR(1) is chosen to provide a parsimonious description of the data and the models are tested against their capacity in fitting the data’s dynamic relations and size, the vector of chosen data descriptors includes nine VAR(1) coefficients and three data variances. No constant is included since the data are demeaned and detrended.

In the Supporting Annex we show our results that follow are robust to the choice of VAR: it turns out that using a VAR of higher orders, though strengthening the test’s rejection power, will not cause change in the ranking between the models.

Figure 2: The Principle of Testing using Indirect Inference



test when two parameters of the auxiliary model are considered. Suppose the real-data estimates of these are given at  $R$  and there are two models to be tested; each implies a joint distribution of these parameters shown by the mountains ( $\alpha$  and  $\beta$ ). Since  $R$  lies outside the 95% contour of  $\alpha$ , it would reject this model at 95% confidence level; the other model that generated  $\beta$  would not be rejected, however, since  $R$  lies inside. In practice there are usually more than two parameters to be considered; to deal with the extra dimensions the test is therefore carried out with the Wald statistic (6).

The joint distribution described above is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is therefore an estimate of the small sample distribution<sup>10</sup>. Such a distribution is generally more accurate for small samples than the asymptotic distribution; it is also shown to be consistent by Le et al. (2011)

<sup>10</sup>The bootstraps in our tests are all drawn as time vectors so contemporaneous correlations between the innovations are preserved.

given that the Wald statistic is asymptotically pivotal. They also showed it had quite good accuracy in small sample Montecarlo experiments<sup>11</sup>.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones ( $H_0$ , the null hypothesis); they can be derived from calibration, estimation, or both. However derived, the test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? Of course only one true model with one set of coefficients is possible. Nevertheless we may have chosen coefficients that are not exactly right numerically, so that the same model with other coefficient values could be correct. Only when we have examined the model with all coefficient values that are feasible within the model theory will we have properly tested it. For this reason we later extend our procedure by a further search algorithm, in which we seek other coefficient sets that could do better in the test.

Thus we calculated the minimum-value full Wald statistic for each period using a powerful algorithm based on Simulated Annealing (SA) in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the space<sup>12</sup>. In effect this is Indirect Inference estimation of the model; however here this estimation is being done to find whether the model can be rejected *in itself* and not for the sake of finding the most satisfactory estimates of the model parameters. Nevertheless of course the method does this latter task as a byproduct so that we can use the resulting unrejected model as representing the best available estimated

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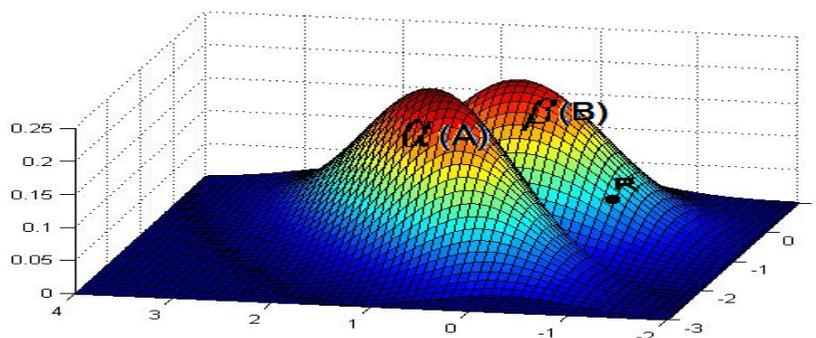
<sup>11</sup>Specifically, they found that the bias due to bootstrapping was just over 2% at the 95% confidence level and 0.6% at the 99% level. They suggested possible further refinements in the bootstrapping procedure which could increase the accuracy further; however, we do not feel it necessary to pursue these here.

<sup>12</sup>We use a Simulated Annealing algorithm due to Ingber (1996). This mimics the behaviour of the steel cooling process in which steel is cooled, with a degree of reheating at randomly chosen moments in the cooling process—this ensuring that the defects are minimised globally. Similarly the algorithm searches in the chosen range and as points that improve the objective are found it also accepts points that do not improve the objective. This helps to stop the algorithm being caught in local minima. We find this algorithm improves substantially here on a standard optimisation algorithm. Our method used our standard testing method: we take a set of model parameters (excluding error processes), extract the resulting residuals from the data using the LIML method, find their implied autoregressive coefficients ( $AR(1)$  here) and then bootstrap the implied innovations with this full set of parameters to find the implied Wald value. This is then minimised by the SA algorithm.

version. The merit of this extended procedure is that we are comparing the best possible versions of each model type when finally doing our comparison of model compatibility with the data.

The principle of estimation using indirect inference is illustrated in figure 3: suppose, as in the case of testing, that we have chosen two parameters of the auxiliary model to describe the reality and the real-data estimates of these are given at  $R$ . Suppose for now the structural model under estimation has two potential sets of parameter values (vectors  $A$  and  $B$ ), each accordingly implies a joint distribution of the descriptive parameters of the auxiliary model shown by the mountains ( $\alpha$  and  $\beta$ ). The contours of these distributions show that the mean of  $\beta$ , compared to that of  $\alpha$ , is closer to  $R$ ,  $B$  is therefore the more preferred parameter set compared to  $A$  from the structural model's viewpoint. Again, in practice one would normally consider for description of the reality more than two parameters of the auxiliary model so that the Wald statistic (6) is used in practice. The SA algorithm is then applied to search for the structural parameters that minimize the Wald value.

Figure 3: The Principle of Estimation using Indirect Inference



One may get several possible outcomes when two models are being compared with the same auxiliary model estimated on a data sample:

- a) one model is rejected, the other is not rejected. In this case only one model is compatible with the behaviour in the data, and the other can be disregarded.
- b) both models are rejected; but the Wald statistic of one is lower than the other's.

c) neither model is rejected but the Wald statistic of one is lower than the other's.

In cases b) and c) we can convert the Wald into a p-value, which can loosely be described as the probability of the model being true given the data. The models can be ranked by these p-values in terms of their probability or 'closeness' to the data behaviour. In case b) this ranking is merely information about possible misspecifications. In case c) one can regard the model with the lower p-value as an approximation to the 'true' model with the higher p-value; thus both are 'true' in this method (i.e. not rejected), but one is a poorer approximation to the true causal structure.

## 6 Data and Results

We evaluate the models against the US experience since the breakdown of the Bretton Woods system using quarterly data published by the Federal Reserve Bank of St. Louis from 1972 to 2007<sup>13</sup>. This covers both the Great Acceleration and the Great Moderation episodes of the US history.

The time series involved for the given baseline model include  $\tilde{v}_t$ , measured as the deviation of the current Fed rates from its steady-state value, the output gap  $x_t$ , approximated by the percentage deviation of real GDP from its HP trend, and the quarterly rate of inflation  $\pi_t$ , defined as the quarterly log difference of the CPI<sup>14</sup>.

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<sup>13</sup><http://research.stlouisfed.org/fred2/>.

<sup>14</sup>Note by defining the output gap as the HP-filtered log output we have effectively assumed that the HP trend approximates the flexible-price output in line with the bulk of other empirical work. To estimate the flexible-price output from the full DSGE model that underlies our three-equation representation, we would need to specify that model in detail, estimate the structural shocks within it and fit the model to the unfiltered data, in order to estimate the output that would have resulted from these shocks under flexible prices. This is a substantial undertaking well beyond the scope of this paper, though something worth pursuing in future work.

Le et al. (2011) test the Smets and Wouters (2007) US model by the same methods as we use here. This has a Taylor Rule that responds to flexible-price output. It is also close to the timeless optimum since, besides inflation, it responds mainly not to the level of the output gap but to its rate of change and also has strong persistence so that these responses cumulate strongly. Le et al. find that the best empirical representation of the output gap treats the output trend as a linear or HP trend instead of the flexible-price output—this Taylor Rule is used in the best-fitting 'weighted' models for both the full sample and the sample from 1984. Thus while in principle the output trend should be the flexible-price output solution, it may be that in practice these models capture this rather badly so that it performs less well than the linear or HP trends.

We have also purposely adjusted the annual Fed rates from the Fred<sup>®</sup> to quarterly rates so the

We should find a break in the VAR process reflecting the start of the Great Moderation. Accordingly we split the time series into two subsamples and estimate the VAR representation before and after the break; the baseline model is then evaluated against the VAR of each subsample separately. We set the break at 1982Q3. Most discussions of the Fed’s behaviour (especially those based on Taylor Rules) are concerned with periods that begin sometime around the mid-1980s but we chose 1982 as the break point here because many (including Bernanke and Mihov, 1998, and Clarida, Gali and Gertler, 2000) have argued that it was around then that the Fed switched from using non-borrowed reserves to setting the Fed Funds rate as the instrument of monetary policy. Such a choice is consistent with the Qu and Perron (2007) test which gives a 95% confidence interval between 1980Q1 and 1984Q4<sup>15</sup>.

For simplicity, the data we use are demeaned so that a VAR(1) representation of them contains no constants but only nine autoregressive parameters in the coefficient matrix; a linear trend is also taken out of the interest rate series for the post-1982 sample to ensure stationarity (See plots and unit root test results in appendix).

The model is calibrated by choosing the parameters commonly accepted for the US economy in the literature. These are summarised in table 2.

The table sets the quarterly discount factor at 0.99, implying a 1% quarterly (or equivalently 4% annual) rate of interest in the steady state.  $\sigma$  and  $\eta$  are set to as high as 2 and 3 respectively as in Carlstrom and Fuerst (2008), who emphasized the values’ consistency with the inelasticity evident in the US data for both intertemporal consumption and labour supply. The Calvo price stickiness ( $\omega$ ) of 0.53 and the price elasticity ( $\theta$ )

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frequencies of all time series kept consistent on quarterly basis. The quarterly interest rate in steady state is given by  $i_{ss} = \frac{1}{\beta} - 1$ .

<sup>15</sup>The Qu-Perron test suggests 1984Q3 as the most likely within the range. We show in the Supporting Annex that our tests are robust to this later choice of switch date.

<sup>16</sup>We have assumed  $Y = C + G$  and used the steady-state  $G/Y$  ratio to calculate the steady-state  $Y/C$  ratio.

<sup>17</sup>Nistico (2007) found that the relative weight  $\alpha$  could be shown as the ratio of the slope of the Phillips curve to the price elasticity of demand, and so  $\alpha = \gamma/\theta$ .

<sup>18</sup>We discuss below (section 6.1.2) the problems with testing the pre-break case and the parameters we therefore use to do so.

Table 2: Calibration of Parameters

Parameters	Definitions	Calibrated Values	
$\beta$	time discount factor	0.99	
$\sigma$	inverse of intertemporal consumption elasticity	2	
$\eta$	inverse of labour elasticity	3	
$\omega$	Calvo contract price non-adjusting probability	0.53	
$\frac{G}{Y}$	steady-state gov. expenditure to output ratio	0.23	
$\frac{Y}{C}$	steady-state output to consumption ratio <sup>16</sup>	$\frac{1}{0.77}$	(implied value)
$\kappa$	$\kappa = \frac{\omega}{(1-\omega)(1-\omega\beta)}$	0.42	(implied value)
$\gamma$	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	2.36	(implied value)
$\theta$	price elasticity of demand	6	
$\frac{\alpha}{\gamma} \equiv \frac{1}{\theta}$	optimal trade-off rate on the Timeless Rule <sup>17</sup>	$\frac{1}{6}$	(implied value)
<i>Parameters on post-break interest-rate-smoothed Taylor Rule<sup>18</sup></i>			
$\rho$	interest rate smoothness	0.76	
$\gamma_\pi$	inflation response	1.44	
$\gamma'_x$	output gap response	0.14	
$\rho_v$	demand shock persistence	pre-break	0.88 (sample estimate)
		post-break	0.93 (sample estimate)
$\rho_{uw}$	supply shock persistence	pre-break	0.91 (sample estimate)
		post-break	0.80 (sample estimate)
$\rho_\xi$	policy shock persistence	pre-break	0.59 (sample estimate)
		post-break	0.38 (sample estimate)
		post-break <sup>18</sup>	0.39 (sample estimate)
		post-break <sup>18</sup>	0.39 (sample estimate)

of demand of 6 are both taken from Kuester, Muller and Stolting (2009); these values imply an average contract length of more than three quarters<sup>19</sup>, while the constant price mark-up over marginal cost is 1.2. The implied steady-state output-consumption ratio of  $1/0.77$  is calculated based on the steady-state government-expenditure-to-output ratio of 0.23 calibrated by Foley and Taylor (2004). The second half of table 2 reports the autoregressive coefficients of the model errors extracted from the data given the calibrated parameters; it shows that in both the Great Acceleration and the Great Moderation the

<sup>19</sup> $2(1-\omega)^{-1} - 1 \approx 3.26$ , to be more precise.

demand and supply shocks are highly persistent compared to the policy shock.

## 6.1 Results for calibrated models

The test results for the models considered are presented in what follows; these are based on the nine autoregressive coefficients of a VAR(1) representation and three variances of the model variables, the chosen descriptors of the dynamics and volatility of the data as discussed above. Our evaluation is based on the Wald test, and we calculate two kinds of Wald statistic, namely, a ‘directed Wald’ that accounts either only for dynamics (the VAR coefficients) or only for the volatility (the variances) of the data, and a ‘full Wald’ where these features are jointly evaluated. In both cases we report the Wald statistic as a percentile, i.e. the percentage point where the data value comes in the bootstrap distribution. The models’ performance in each subsample follows.

### 6.1.1 Model performance in the Great Moderation:

We start with the post-1982 period, the Great Moderation subsample, as this has been the main focus of econometric work to date. Table 3 summarises the performance of model one with the Optimal Timeless Rule<sup>20</sup>.

Panel A shows while two out of the nine VAR(1) coefficients (i.e., the interest rate’s response to the lagged output gap and the output gap’s response to its own lagged value) estimated with the actual data are found to lie beyond their respective model-implied

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<sup>20</sup>The VAR(1) notation is as follows: 
$$\begin{bmatrix} \tilde{i}_t \\ x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} \tilde{i}_{t-1} \\ x_{t-1} \\ \pi_{t-1} \end{bmatrix} + \Sigma_t,$$
 with the variable order being interest rate (1), output gap (2) and inflation (3).

Although the Wald statistics provide us with our tests, we also report for this first case only the calculated 95% bounds for each individual estimate of our descriptors. These show where the data estimate for each descriptor lies within the model distribution for that descriptor alone. These may give clues about sources of model misspecification. These comparisons are similar to the widespread comparison of moments (including cross-moments) in the data with those simulated from the model. However, these comparisons do not take account of these moments’ joint distribution which is relevant to whether the data is compatible with the model on all these features simultaneously. Unfortunately the individual data moment comparisons taken as a group are not a reliable guide to whether the data moments will lie inside the model’s joint distribution for them—see Le, Minford and Wickens (2010). For this the Wald must be used.

Table 3: Performance of the Optimal Timeless Rule Model in the Great Moderation

Panel A: Directed Wald- data dynamics				
VAR(1) coefficients	95% lower bound	95% upper bound	Values estimated with real data	In/Out
$\beta_{11}$	0.7408	0.9689	0.8950	In
$\beta_{12}$	-0.0316	0.0329	0.0395	Out
$\beta_{13}$	-0.0709	0.0896	0.0315	In
$\beta_{21}$	-0.2618	0.8132	-4.28e-05	In
$\beta_{22}$	0.4102	0.7617	0.8243	Out
$\beta_{23}$	-0.3954	0.3056	-0.0657	In
$\beta_{31}$	-0.3197	0.2122	0.0105	In
$\beta_{32}$	0.0050	0.1735	0.0979	In
$\beta_{33}$	0.1090	0.5052	0.2353	In
Directed Wald percentile for dynamics			86.4	

Panel B: Directed Wald- data volatilities				
Volatilities of endogenous variables	95% lower bound	95% upper bound	Values calculated with real data	In/Out
$Var(\tilde{z})$	0.0042	0.0264	0.0156	In
$Var(x)$	0.0686	0.1627	0.1620	In
$Var(\pi)$	0.0095	0.0204	0.0149	In
Directed Wald percentile for volatilities			89.6	

Note: Estimates reported in panel B are magnified by 1000 times as their original values.

Panel C: Full Wald statistic	
Chosen data features	Full Wald percentile
Dynamics+Volatilities	77.1

95% upper bound, the test returns a directed Wald percentile of 86.4. This means at 95% (or even at 90%) confidence level the real-data-based estimates are easily explained by their joint distribution generated from model simulations, indicating that the model has in general captured the dynamic features of the data pretty precisely.

Panel B then examines the model's capacity to explain the data's volatility. It shows the observed data variances not only lie individually within the 95% bounds but are also jointly explained by the model at the 95% level (indeed, also marginally at 90%), since the directed Wald is 89.6. Thus compared to the data the Timeless Rule model is also

correctly sized.

The model’s overall fit to data is then evaluated in panel C, where the full Wald jointly considers the two aspects just assessed. The full Wald percentile reported is 77.1. Such a low Wald percentile indicates that what we observe in reality is fairly close to the model’s implication on average; thus even at the 90% confidence level the data fail to reject the model jointly on both dynamics and size. The conclusion is that the US facts do not reject the Timeless Rule model as the data-generating process post-1982.

This is not the case, however, when Taylor Rules of the standard sort are substituted for it. Table 4 suggests when the original Taylor Rule (1) or the interest-rate-smoothed Taylor Rule (2) is combined with the same IS-Phillips curve framework on these commonly accepted calibrations, from all perspectives the post-1982 data strongly reject the model at 99%.

Table 4: Wald Statistics for Typical Taylor Rule Models in the Great Moderation

Tests for chosen data features	Baseline model with	
	original Taylor Rule (model two)	‘IRS’ Taylor Rule (model three)
Directed Wald for dynamics	100	99.8
Directed Wald for volatilities	99.2	99
Full Wald for dyn. & vol.	100	99.7

### 6.1.2 Model performance in the Great Acceleration:

We now proceed to evaluate how the models behave before 1982, the Great Acceleration period. Table 5 reveals the performance of the Optimal Timeless Rule model.

We can see that although the model does not behave as well here as it did in the Moderation subsample in explaining the data dynamics, with a directed Wald of 98.2 the directed Wald for data volatilities at 89.6 lies within the 90% confidence bound. Overall, the full Wald percentile of 97.3 falls between the 95% and the 99% confidence bounds. So

Table 5: Performance of the Optimal Timeless Rule Model in the Great Acceleration

Tests for chosen data features	Wald percentiles
Directed Wald for dynamics	98.2
Directed Wald for volatilities	89.6
Full Wald for dyn. & vol.	97.3

while the model fits the facts less well than in the case of the Great Moderation, it just about fits those of the turbulent Great Acceleration episode if we are willing to reject at a higher threshold. As we will see next, it also fits them better than its rival Taylor Rule models.

Unfortunately we are unable to test the DSGE model with the generally proposed pre-1982 Taylor Rules because the solution is indeterminate, the model not satisfying the Taylor Principle. Such models have a sunspot solution and therefore any outcome is possible and also consistent formally with the theory. The assertion of those supporting such models is that the solutions, being sunspots, accounted for the volatility of inflation. Unfortunately there is no way of testing such an assertion. Since a sunspot can be anything, any solution for inflation that occurred implies such a sunspot—equally of course it might not be due to a sunspot, rather it could be due to some other unspecified model. There is no way of telling. To put the matter technically in terms of indirect inference testing using the bootstrap, we can solve the model for the sunspots that must have occurred to generate the outcomes; however, the sunspots that occurred cannot be meaningfully bootstrapped because by definition the sunspot variance is infinite. Values drawn from an infinite-variance distribution cannot give a valid estimate of the distribution, as they will represent it with a finite-variance distribution. To draw representative random values we would have to impose an infinite variance; by implication all possible outcomes would be embraced by the simulations of the model and hence the model cannot be falsified. Thus the pre-1982 Taylor Rule DSGE model proposed is not a testable theory of this

period<sup>21</sup>.

However, it is open to us to test the model with a pre-1982 Taylor Rule that gives a determinate solution; we do this by making the Taylor Rule as unresponsive to inflation as is consistent with determinacy, implying a long-run inflation response of just above unity. Such a rule shows considerably more monetary ‘weakness’ than the rule typically used for the post-1982 period, as calibrated here with a long-run response of interest rate to inflation of 1.5 in (1).

We implement this weak Taylor Rule across a spectrum of combinations of smoothing coefficient and short-run response to inflation, with in all cases the long-run coefficient equalling 1.001. The Wald test results are shown in table 6.

What we see here is that with a low smoothing coefficient the model encompasses the variance of the data well, in other words picking up the Great Acceleration. However, when it does so, the data dynamics reject the model very strongly. If one increases the smoothing coefficient, the model is rejected less strongly by the data dynamics and also overall but it is then increasingly at odds with the data variance. In all cases the model is rejected strongly overall by the data, though least badly with the highest smoothing coefficient. Thus the testable model that gets nearest to the position that the shift in US post-war behaviour was due to the shift in monetary regime (reflected on Taylor Rule coefficients) is rejected most conclusively.

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<sup>21</sup>We could use the approach suggested in Minford and Srinivasan (2011) in which the monetary authority embraces a terminal condition designed to eliminate imploding (as well as exploding) sunspots. In this case the model is forced to a determinate solution even when the Taylor Principle does not hold. However in our sample here we find that the model only fails to be rejected with inflation response parameters well in excess of unity—see below—while as we see from table 6 being consistently rejected for parameters that get close to unity. So parameter values below unity, where the Taylor Principle does not apply, seem unlikely to fit the facts and we have not therefore pursued them here using this terminal condition approach.

<sup>22</sup>T-value normalization of the Wald percentiles is calculated based on Wilson and Hilferty (1931)’s method of transforming chi-squared distribution into the standard normal distribution. The formula used here is:  $Z = \{[(2M^{squ})^{1/2} - (2n)^{1/2}]/[(2M^{squ^{95th}})^{1/2} - (2n)^{1/2}]\} \times 1.645$ , where  $M^{squ}$  is the square of the Mahalanobis distance calculated from the Wald statistic equation (6) with the real data,  $M^{squ^{95th}}$  is its corresponding 95% critical value on the simulated (chi-squared) distribution,  $n$  is the degrees of freedom of the variate, and  $Z$  is the normalized t value; it can be derived by employing a square root and assuming  $n$  tends to infinity when the Wilson and Hilferty (1931)’s transformation is performed.

Table 6: Wald Percentiles for ‘Weak’ Taylor Rule Models in the Great Acceleration (with ‘weak’ rule defined as having a long-run interest rate response to inflation equalling 1.001)

Taylor Rule: $\tilde{u}_t = \rho\tilde{u}_{t-1} + \gamma_\pi\pi_t + \xi_t$		Wald percentiles for chosen features (Normalized t-values in parenthesis <sup>22</sup> )		
Parameter versions	Error dynamics	Directed Wald for dynamics	Directed Wald for volatilities	Full Wald for dyn. & vol.
$\rho = 0, \gamma_\pi = 1.001$	$\xi_t \sim \text{AR}(1)$	100 (39.81)	78.9 (0.22)	100 (40.24)
$\rho = 0.3, \gamma_\pi = 0.7007$	$\xi_t \sim \text{AR}(1)$	100 (30.26)	92 (1.08)	100 (28.01)
$\rho = 0.5, \gamma_\pi = 0.5005$	$\xi_t \sim \text{AR}(1)$	100 (22.69)	95.9 (1.77)	100 (21.98)
$\rho = 0.7, \gamma_\pi = 0.3003$	$\xi_t \sim \text{iid}$	100 (19.26)	98.2 (2.73)	100 (18.24)
$\rho = 0.9, \gamma_\pi = 0.1001$	$\xi_t \sim \text{iid}$	100 (9.09)	99 (3.56)	100 (9.03)

### 6.1.3 Ireland’s alternative Taylor Rule representation of Fed policy:

In a recent paper Ireland (2007), unlike the other New Keynesian authors we have cited above, estimates a model in which there is a non-standard Taylor Rule that is held constant across both post-war episodes. His policy rule always satisfies the Taylor Principle because unusually it is the change in the interest rate that is set in response to inflation and the output gap so that the long-run response to inflation is infinite. He distinguishes the policy actions of the Fed between the two subperiods not by any change in the rule’s coefficients but by a time-varying inflation target which he treats under the assumptions of ‘opportunism’ largely as a function of the shocks to the economy. Ireland’s model implies that the cause of the Great Moderation is the fall in shock variances. However, since these also cause a fall in the variance of the inflation target, which in turn lowers the variance of inflation, part of this fall in shock variance can be attributed to monetary policy.

Table 7: Performance of the Model with the Unrestricted Ireland Rule

Ireland's rule in unrestricted form: & equivalent transformation <sup>23</sup> :	$\tilde{i}_t = \tilde{i}_{t-1} + \gamma_\pi \pi_t + \gamma_g (g_t - g) + \xi_t$	$\tilde{i}_t = \tilde{i}_{t-1} + \gamma_\pi \pi_t + \gamma_g (x_t - x_{t-1}) + \xi_t$
Tests for chosen data features	pre-1982 sample	post-1982 sample
Directed Wald for dynamics	98.9	79
Directed Wald for volatilities	78.8	89.4
Full Wald for dynamics & volatilities	98.1	71.1

Note: 1. Ireland (2007)'s ML estimates suggest  $\gamma_\pi = 0.91$ ,  $\gamma_g = 0.23$ .

2. All equation errors follow AR(1) process according to the data and model.

It turns out that Ireland's model is hardly distinguishable from our Optimal Timeless Rule model. His Taylor Rule changes interest rate until the Optimal Timeless Rule is satisfied, in effect forcing it on the economy. Alternatively we can write his rule as a rule for inflation determination (i.e. with inflation on the left hand side), which reveals that it is identical to the Timeless Rule's setting of inflation apart from the term in the change in interest rate and some slight difference in the coefficient on output gap change<sup>24</sup>. Since the Ireland rule is so similar to the Optimal Timeless Rule, it is not surprising that its empirical performance is also similar. We embedded his rule in the same model and obtained Wald percentiles for it that are hardly different: 71.1 in the Great Moderation (against 77.1 for the Timeless Rule model) and 98.1 in the Great Acceleration (against 97.3).

<sup>23</sup>While Ireland originally specified  $\tilde{i}_t = \tilde{i}_{t-1} + \gamma_\pi \pi_t + \gamma_g (g_t - g) - \gamma_\pi \pi_t^* - \Delta \pi_t^* + \Omega_t$ , the exercise here tests its unrestricted form:  $\tilde{i}_t = \tilde{i}_{t-1} + \gamma_\pi \pi_t + \gamma_g (g_t - g) + \xi_t$ , where  $\xi_t = -\gamma_\pi \pi_t^* - \Delta \pi_t^* + \Omega_t$ . In particular, this unrestricted Ireland rule is rewritten as  $\tilde{i}_t = \tilde{i}_{t-1} + \gamma_\pi \pi_t + \gamma_g (x_t - x_{t-1}) + \xi_t$  so it can be evaluated within our baseline framework; such an equivalent transformation is derived by writing:  $g_t - g = \ln y_t - \ln y_{t-1} - (\ln y_t^{hptr} - \ln y_{t-1}^{hptr}) = \ln y_t - \ln y_t^{hptr} - (\ln y_{t-1} - \ln y_{t-1}^{hptr}) = x_t - x_{t-1}$ .

<sup>24</sup>Note the transformed Ireland rule can be rewritten as  $\pi_t = \frac{1}{\gamma_\pi} (\tilde{i}_t - \tilde{i}_{t-1}) - \frac{\gamma_g}{\gamma_\pi} (x_t - x_{t-1}) - \frac{1}{\gamma_\pi} \xi_t$  to resemble the Optimal Timeless Rule; its coefficient on the output gap change according to Ireland's estimation is 0.25, close to that of 0.17 on the Timeless Rule we used above.

Ireland's Taylor Rule can in principle be distinguished from the Optimal Timeless Rule via his restriction on the rule's error. As noted earlier we cannot apply this restriction within our framework here so that Ireland's Taylor Rule in its unrestricted form here only differs materially from the Optimal Timeless Rule in the interpretation of the error. But from a welfare viewpoint it makes little difference whether the cause of the policy error is excessive target variation or excessively variable mistakes in policy setting; the former can be seen as a type of policy mistake. Thus both versions of the rule imply that what changed in it between the two subperiods was the policy error.

It might be argued that the success of Ireland's rule reveals that a type of Taylor Rule does after all fit the data well. This would be true. But in the context of the debate over the cause of the Great Moderation it is to be firmly distinguished from what we call the 'standard Taylor Rule' under which shifts in the rule's parameters are regarded as the cause. In Ireland's rule there are no such shifts and as we have seen the behaviour under it is essentially identical to that under the Optimal Timeless Rule setting.

## **6.2 Simulated Annealing and model tests with final parameter selection**

The above results based on calibration thus suggest that the Optimal Timeless Rule, when embedded in our IS-Phillips curves model, outperforms testable Taylor Rules of the standard sort in representing the Fed's monetary behaviour since 1972. In both the Great Acceleration and the Great Moderation the only model version that fails to be strongly rejected is the one in which the optimal timeless policy was effectively operating.

However, fixing model parameters in such a way is an excessively strong assumption in terms of testing and comparing DSGE models. This is because the numerical values of a model's parameters could in principle be calibrated anywhere within a range permitted by the model's theoretical structure, so that a model rejected with one set of assumed parameters may not be rejected with another. Going back to what we have just tested,

this could mean that the Taylor Rule models were rejected not because the policy specified was incorrect but because the calibrated IS and Phillips curves had failed to reflect the true structure of the economy. Thus, to compare the Timeless Rule model and Taylor Rule models thoroughly one cannot assume the models' parameters are fixed always at particular values; rather one is compelled to search over the full range of potential values the models can take and test if these models, with the best set of parameters from their viewpoints, can be accepted by the data.

Accordingly we now allow the model parameters to be altered to achieve for each model the lowest Wald possible, subject to the theoretical ranges permitted by the model theory<sup>25</sup>. This estimation method is that of Indirect Inference; we use the Simulated Annealing (SA) algorithm for the parameter search. In this process we allow each model to be estimated with different parameters for each episode. Thus we are permitting changes between the episodes in both structural parameters and the parameters of monetary policy; in so doing we are investigating whether either structural or policy rule changes were occurring and so contributing to the Great Moderation<sup>26</sup>.

### 6.2.1 The estimated Optimal Timeless rule model:

The SA estimates of the timeless rule model in both the post-war subperiods are reported in table 8. We can see that this estimated model is not very different from its calibrated version in the Great Moderation. However for the Great Acceleration period the estimation now suggests substantially lower elasticities of intertemporal consumption (the inverse of  $\sigma$ ) and labour supply (the inverse of  $\eta$ ), and a much higher Calvo contract non-adjusting probability ( $\omega$ ); with lower  $\gamma$  the latter implies a much flatter Phillips curve.

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<sup>25</sup>We fix the time discount factor  $\beta$  and the steady-state consumption-output ratio  $\frac{C}{Y}$  as calibrated in table 2; other parameters are allowed to vary within  $\pm 50\%$  of the calibrated values—which are set as initial values here—unless stated otherwise.

<sup>26</sup>It could be argued that deep parameters such as the elasticity of intertemporal substitution and Calvo price-change probabilities should remain fixed across the two periods. However, with such radically different environments these parameters could have differed; for example Le et al (2011) find evidence that the degree of nominal rigidity varied across periods and interpret this as a response to changing variability. Here therefore we allow the data to determine the extent of change.

The estimation also suggests the Fed had a low relative weight on output variations ( $\alpha$ ) pre-1982 but that high nominal rigidity forced it to reduce inflation more strongly in response to output growth (due to higher  $\alpha/\gamma$ ). The shocks' persistence is not much altered in either period from that in the calibrated model.

Table 8: SA Estimates of the Optimal Timeless Rule Model

Parameters	Definitions	Calibrations	SA estimates		
			Pre-1982	Post-1982	
$\beta$	time discount factor	0.99	fixed	fixed	
$\sigma$	inverse of intertemporal consumption elasticity	2	1.01	1.46	
$\eta$	inverse of labour elasticity	3	2.04	3.23	
$\omega$	Calvo contract price non-adjusting probability	0.53	0.79	0.54	
$\frac{G}{Y}$	steady-state gov. expenditure to output ratio	0.23	fixed	fixed	
$\frac{Y}{C}$	steady-state output to consumption ratio	$\frac{1}{0.77}$	fixed	fixed	
$\kappa$	$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$	0.42	0.06	0.40	
$\gamma$	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	2.36	0.19	2.06	
$\alpha$	relative weight of loss assigned to output variations against inflation	0.39	0.20	0.58	
$\frac{\alpha}{\gamma} \equiv \frac{1}{\theta}$	optimal trade-off rate on the Timeless Rule	$\frac{1}{6}$	$\frac{1}{0.95}$	$\frac{1}{3.6}$	
$\theta$	price elasticity of demand	6	0.95	3.6	
$\rho_v$	demand shock persistence	pre-break	0.88	0.92	—
$\rho_{uw}$	supply shock persistence	pre-break	0.91	0.86	—
$\rho_\xi$	policy shock persistence	pre-break	0.59	0.14	—
$\rho_v$	demand shock persistence	post-break	0.93	—	0.94
$\rho_{uw}$	supply shock persistence	post-break	0.80	—	0.79
$\rho_\xi$	policy shock persistence	post-break	0.38	—	0.42

Table 9 shows that estimation brings the model substantially closer to the data. This is particularly so for the pre-1982 period where the calibrated model was rejected at 95% confidence; here the necessary parameter changes were substantial to get the model to fit, as we have just seen. The Full Wald percentile in both episodes is now around 70%, so that the model easily fails to be rejected at 95%.

### 6.2.2 Taylor Rule model under estimation:

In estimating the Taylor Rule model alternative we substitute the smoothed version (equation (2) in section 3 above) for the Optimal Timeless Rule in the identical IS-Phillips

Table 9: Performance of the Timeless Rule Model under Calibration and Estimation

Tests for chosen features	Pre-1982 under		Post-1982 under	
	calibration	estimation	calibration	estimation
Directed Wald for dynamics	98.2	81.9	86.4	77.7
Directed Wald for volatilities	89.6	32.5	89.6	90.3
Full Wald for dynamics & volatilities	97.3	71.7	77.1	68.6

curves framework. This specification covers all Taylor Rule versions we considered in the earlier evaluation, as when  $\rho$  is zero it reduces to the original Taylor Rule while when  $\gamma_\pi$  is just above unity it turns to be a weak Taylor Rule variant.

As with the Optimal Timeless Rule model the estimation process achieves a substantial improvement in the closeness of the Taylor Rule model to the data in both episodes. Pre-1982 the best weak Taylor Rule version was strongly rejected; after estimation it is still rejected at the 95% level but not at the 99% level. Most importantly, the estimates include a much stronger Taylor Rule response to inflation than the calibrated version for this early episode; hence the evidence supports the view that the Taylor Rule principle was easily satisfied in this period. The response is essentially the same as that found in the later period by this estimation process: the weaker the response, the further the model is from fitting the data. Table 10 shows the details. The elasticity of intertemporal consumption and that of labour are found to be fairly similar to those estimated with the Optimal Timeless Rule, as is the Calvo rigidity parameter which is again higher in the first episode. For the model to get close to the data there needs to be interest rate smoothing in both episodes.

The resulting Wald statistics in table 11 thus show that the Taylor Rule model is now close to passing at 95% pre-1982 and passes comfortably post-1982. However relative to the Timeless Rule it is substantially further from the data, as summarized in table 12

Table 10: SA Estimates of the Taylor rule Model

Parameters	Definitions	Calibrations	SA estimates		
			Pre-1982	Post-1982	
$\beta$	time discount factor	0.99	fixed	fixed	
$\sigma$	inverse of intertemporal consumption elasticity	2	1.15	1.16	
$\eta$	inverse of labour elasticity	3	2.66	3.85	
$\omega$	Calvo contract price non-adjusting probability	0.53	0.79	0.61	
$\frac{G}{Y}$	steady-state gov. expenditure to output ratio	0.23	fixed	fixed	
$\frac{Y}{C}$	steady-state output to consumption ratio	$\frac{1}{0.77}$	fixed	fixed	
$\kappa$	$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$	0.42	0.06	0.25	
$\gamma$	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	2.36	0.23	1.33	
$\gamma_\pi$	interest-rate response to inflation	1.44	2.03	2.06	
$\gamma'_x$	interest-rate response to output gap	0.14	0.001	0.06	
$\rho$	interest rate smoothness	0.76	0.42	0.89	
$\rho_v$	demand shock persistence	pre-break	n/a	0.91	—
$\rho_{uw}$	supply shock persistence	pre-break	n/a	0.87	—
$\rho_\xi$	policy shock persistence	pre-break	n/a	0.58	—
$\rho_v$	demand shock persistence	post-break	0.93	—	0.95
$\rho_{uw}$	supply shock persistence	post-break	0.80	—	0.77
$\rho_\xi$	policy shock persistence	post-break	0.39	—	0.40

where the p-values are also reported. This suggests that, although it is possible to fit the post-1982 period with a Taylor Rule model, policy is better understood in terms of the Timeless Rule model.

### 6.2.3 The identification problem revisited in the light of our results

Having established that the Optimal Timeless Rule model gives the best representation of the key features of the US post-war data, we can now ask whether this model can also account for the single-equation findings for the Taylor Rule.

The above suggests that the widespread success reported in single-equation Taylor Rule regressions on US data could simply represent some sort of statistical relation emerging from the model with the Optimal Timeless Rule. To examine this possibility, we treat the Optimal Timeless Rule model as the true model and ask whether the existence of empirical Taylor Rules would be consistent with that. Technically this is again a process

<sup>27</sup>The results for the best testable weak Taylor Rule version as in table 6.

Table 11: Performance of Taylor Rule Model under Calibration and Estimation

Tests for chosen features	Pre-1982 under		Post-1982 under	
	calibration <sup>27</sup>	estimation	calibration	estimation
Directed Wald for dynamics	100	98	99.8	89.6
Directed Wald for volatilities	99	40.6	99	94.9
Full Wald for dynamics & volatilities	100	96.1	99.7	87.6

Table 12: Summary of Model Performance with Estimated Parameters

Tests for chosen features	Pre-1982 with		Post-1982 with	
	Timeless rule	Taylor Rule	Timeless rule	Taylor Rule
Directed Wald for dynamics (and p-value)	81.9 (0.26)	98 (0.01)	77.7 (0.30)	89.6 (0.12)
Directed Wald for volatilities (and p-value)	32.5 (0.76)	40.6 (0.75)	90.3 (0.13)	94.9 (0.05)
Full Wald for dyn. & vol. (and p-value)	71.7 (0.38)	96.1 (0.03)	68.6 (0.41)	87.6 (0.15)

of model evaluation basing on indirect inference; but instead of a VAR here Taylor Rule regression coefficients are used as the data descriptors for the model to fit.

Table 13 shows the OLS estimates of several popular Taylor Rule variants when these are fitted, respectively, to data for both the post-war episodes. To compare the regression results here with those commonly found in the US Taylor Rule literature where unfiltered interest rate data is normally used we must emphasize that here for the post-82 subsample a linear trend is taken out of the interest rate series so that stationarity is ensured. These Taylor Rules, when estimated on the stationary data we have used here, generally fail to satisfy the Taylor Principle, in much the same way as in pre-1982. Thus econometrically the standard estimates of the long-run Taylor Rule response to inflation post-1982 are

Table 13: ‘Taylor Rules’ in the Data (with OLS): consistency with the estimated Timeless Rule Model

Panel A: ‘Taylor Rules’ in the Great Acceleration					
‘Taylor Rule’ versions	$\gamma_\pi$	$\gamma_x$	$\rho$	Adj.R <sup>2</sup>	Wald percentiles
$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho \tilde{i}_{t-1} + \xi_t$	0.09	0.06	0.90	0.84	31.9
$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \xi_t$ $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t$	0.30	0.07	0.92	0.85	69.1
$\tilde{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \xi_t$	0.60	-0.01	n/a	0.24	36.9
$\tilde{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \rho \tilde{i}_{t-1} + \xi_t$	-0.11	0.06	0.82	0.83	68.5

Panel B: ‘Taylor Rules’ in the Great Moderation					
‘Taylor Rule’ versions	$\gamma_\pi$	$\gamma_x$	$\rho$	Adj.R <sup>2</sup>	Wald percentiles
$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho \tilde{i}_{t-1} + \xi_t$	0.08	0.05	0.89	0.92	11.5
$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \xi_t$ $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t$	0.07	0.06	0.93	0.90	95.6
$\tilde{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \xi_t$	0.26	0.13	n/a	0.24	17.1
$\tilde{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \rho \tilde{i}_{t-1} + \xi_t$	0.03	0.04	0.89	0.91	89.7

biased by the non-stationarity of the interest rate. There is little statistical difference in the estimates across the two periods. The reported Wald percentiles indicate that these empirical ‘Taylor Rules’ are indeed consistent with what the Timeless Rule implies: in both panels the Taylor Rule regressions estimated are all within or on the 95% confidence bounds implied by the estimated Timeless Model.

This illustrates the identification problem with which we began this paper: a Taylor Rule regression having a good fit to the data may well be generated by a model where there is no structural Taylor Rule at all. Here we suggest that the Timeless Rule model we have found gets closest to fitting US data in each episode is also generating these Taylor Rule single-equation relationships.

#### 6.2.4 The ‘interest rate smoothing’ illusion: a further implication

Another issue on which the above sheds light is the phenomenon of ‘interest rate smoothing’. Clarida, Gali and Gertler (1999) noted that the Optimal Timeless Rule required nominal interest rate to be adjusted in a once-and-for-all manner, but that empirical evidence from Taylor Rule regressions usually displayed clear interest rate smoothing. This they argued created a ‘puzzle’: that sluggish interest rate movements could not be justified as optimal.

While various authors have tried to explain such a discrepancy either from an economic (e.g., Rotemberg and Woodford, 1997, 1998; Woodford, 1999, 2003a, b) or from an econometric (e.g., Sack and Wieland, 2000; Rudebusch, 2002) viewpoint, the Taylor Rule regressions above show that ‘smoothing’ is a regression result that is generated by the Optimal Timeless Rule model, in which there is no smoothing present. The source of inertia in the model is the persistence in the shocks themselves.

## 7 What Caused the Great Moderation?

We have found that the Optimal Timeless Rule is the best guide to US monetary policy since the Bretton Woods; we have also obtained estimates of the model under a Taylor Rule, which though fitting the data considerably less well nevertheless fail to be rejected in absolute terms by the data. These models enable us finally to examine the causes of the Great Moderation. We have made a number of empirical findings about changes in the structural parameters, the parameters of the monetary rule trade-off, and the behaviour of the shocks. We now examine the contribution of each of these changes to the Great Moderation.

Table 14 shows that under our preferred model with the Optimal Timeless Rule the Great Moderation is almost entirely the result of reduced volatility in the shocks. There is a small contribution to lowered inflation variance from the policy parameters; but otherwise the contribution from both structural and policy parameters is slightly to increase

macro variance in the later period. If one then examines which shocks' volatility fell, the table (15) following shows that it did so for all three of our shocks, with a fall in standard deviation of 60-70%.

If we look at the Taylor Rule model the story is essentially the same. As we saw above the inflation response of the Taylor Rule hardly changes across the two periods. The main change is a doubling of the smoothing parameter which accordingly contributes about a third of the reduction in interest rate variance. Otherwise structural and policy parameter changes contribute negligibly to the variance reduction. Thus again the reduction in shock variability dominates as the cause of the Great Moderation. Here too all the shocks have large falls in standard deviation; the largest at 86% is the monetary shock (tables 16 and 17).

Table 14: Accountability of Factor Variations for Reduced Data Volatility  
(Timeless Rule model)

Reduced data volatility caused by	Interest rate	Output gap	Inflation
Reduced shocks	115.3%	106.9%	90%
Chg in policy paras	-4.3%	-2.5%	12.7%
Chg in structural para	-11%	-4.5%	-2.7%

Thus what we find is that the Great Moderation is essentially a story of 'good shocks' as proposed in the time-series studies we cited earlier. Also we have found no evidence of the weak monetary regime regarded by an earlier DSGE model literature as responsible for the Great Acceleration and in the same vein no evidence of much change in the monetary regime during the Great Moderation. However, what we do find about monetary policy is that the 'trembling hand' trembled enormously more in the earlier period than in the later; thus monetary error is a large source of the Great Acceleration and its reduction an important reason for the Moderation. For those that embrace a Taylor Rule model in spite of its poorer data fit the story is the same—in this case monetary 'judgement' was

Table 15: Reduced Size of Shocks  
(Timeless Rule model)

Standard deviation of	Pre-1982	Post-1982	Reduction
Demand shock	0.0625 (0.0050)	0.02 (0.0012)	60%
Supply shock	0.4767 (0.0667)	0.1419 (0.0298)	70%
Policy shock	0.0148 (0.0127)	0.0055 (0.0032)	63%

Note: 1. Values in parentheses are sample estimates of standard deviation of innovations.

2. The standard deviation of the shocks is calculated using  $sd(terr.)=sd(innov)/(1-\rho)$ ;

$\rho$  is the sample estimate of shock persistence reported in table 8.

Table 16: Accountability of Factor Variations for Reduced Data Volatility (II)  
(Taylor Rule model)

Reduced data volatility caused by	Interest rate	Output gap	Inflation
Reduced shocks	66.7%	99%	99.9%
Chg in policy paras	34.2%	1.8%	3.9%
Chg in structural para	-0.9%	-0.8%	-3.8%

substantially more erratic in its effect in the earlier period.

## 7.1 A comparison with other recent DSGE models

As we noted earlier, Ireland (2007), Smets and Wouters (2007), Le et al. (2011) and Fernandez-Villaverde et al. (2009, 2010) have also estimated models of these periods and we can compare their results in a general way with ours. Other than Ireland, these models follow the model of Christiano et al. (2005). Smets and Wouters use this model with some small modifications; they estimate it by Bayesian methods. Le et al. add a competitive sector and reestimate the model using Indirect Inference, since they found

Table 17: Reduced Size of Shocks (II)  
(Taylor Rule model)

Standard deviation of	Pre-1982	Post-1982	Reduction
Demand shock	0.0533 (0.0050)	0.0280 (0.0012)	48%
Supply shock	0.5777 (0.0751)	0.1474 (0.0339)	75%
Policy shock	0.0145 (0.0061)	0.0020 (0.0012)	86%

Note: 1. Values in parentheses are sample estimates of standard deviation of innovations.

2. The standard deviation of the shocks is calculated using  $sd(err.)=sd(innov)/(1-\rho)$ ;  
rho is the sample estimate of shock persistence reported in table 10.

the model was rejected quite badly overall by the data with the previously estimated ones. When reestimated in this way they found that the model was accepted, at 99% for the full post-war period and at 95% for the Great Moderation period, for the key subset of variables, output, inflation and interest rate when represented by a VAR(1). Fernandez-Villaverde et al. add moving volatility in the errors and drift in the parameters of the Taylor Rule; like Smets and Wouters they estimate the model by Bayesian methods.

What is striking about all these studies is that none of them find evidence of much difference in monetary regime between the two periods—interestingly, Fernandez-Villaverde et al. find variations of ‘monetary toughness’ *within* both periods, while not finding much difference on average *across* the two. Both Smets and Wouters and Le et al. in their reworking of them find little change in the inflation response coefficient of the Taylor Rule. In this these models echo Ireland, even though their Taylor Rule representations differ from his. Thus these studies agree with ours in finding that it is the shocks that account for the difference in volatility.

Nevertheless, all also agree with us that the scale of the monetary shock has declined between the two periods. Thus a pattern is visible in ours, Ireland’s and these other

studies: while the monetary regime did not apparently change much, the scale of the monetary ‘error’ fell between the two periods. Ireland interprets this, based on his connection of it with other shocks, as ‘opportunism’, where the Fed was allowing the inflation target to drift with events, pushing it downwards when events allowed this to be done with less perceived cost. Other studies, like ours, do not model it other than as a pure error.

An important implication of the lack of regime change is that there is no evidence of indeterminacy in the earlier period according to any of these studies including ours. Thus all these studies that are based on full information system estimates cannot find the evidence that appears to come out of single-equation studies that the earlier period’s Taylor Rule responded weakly to inflation. As we have seen this is consistent with the lack of identification of the Taylor Rule as a single equation; indeed as we have seen the models that fit the data overall could easily have ‘generated’ single-equation Taylor Rules of this ‘weak’ type.

## 8 Conclusion

In this study we have used the method of Indirect Inference to estimate and test a three-equation DSGE model against the data for the Great Acceleration and the Great Moderation. The method has the advantage over alternatives that it tests the model overall in its ability to fit the data’s behaviour. Nevertheless, in spite of differences in method, our results echo those of other recent work where DSGE models of greater complexity than ours have been estimated by a variety of methods. We have found that the monetary regimes being followed in the two periods are rather similar. We have also found that, while these regimes can be represented by Taylor Rules of the usual sort, they more closely fit the facts if represented by an Optimal Timeless Rule, essentially the same as the Taylor Rule form suggested by Ireland, which he also finds fits the facts best.

A corollary of this finding is that there is no evidence of indeterminacy due to the

‘weakness’ of the monetary regime during the Great Acceleration. Previous findings to this effect seem to have arisen from single-equation estimates that suffered from a lack of identification and are quite consistent with the DSGE models estimated here.

By implication we also find, in common with these other studies using full DSGE models, that the Great Moderation was mainly the result of ‘good shocks’—a fall in the variance of the errors in the model. This reinforces the results of a large number of time-series studies using Structural VARs, but it does so through finding structural DSGE parameters that can replicate these VARs and so allows them to be interpreted structurally.

Nevertheless, the falling variance of shocks includes that of monetary shocks. Within this fall lies the remedying of a failure of monetary policy. Whether this is due to ‘opportunistic’ pursuit of varying inflation targets as in Ireland, to sheer inefficiency, or to some other reason, our work cannot say; this remains a fruitful avenue for future work. Clearly and perhaps not surprisingly given the size and novelty of the shocks bombarding the 1970s economy, monetary policy was far from perfect in this early period. But at least we and other recent DSGE modellers are clear that it was not just plain stupid.

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## Appendix

### A Qu-Perron Test for Structural Break

Table A.1: Qu-Perron Test Result

Estimated break date	95% confidence interval		supLR test statistic for fixed number of breaks	5% critical value
	lower	upper		
1984Q3	1980Q1	1984Q4	164.84	31.85

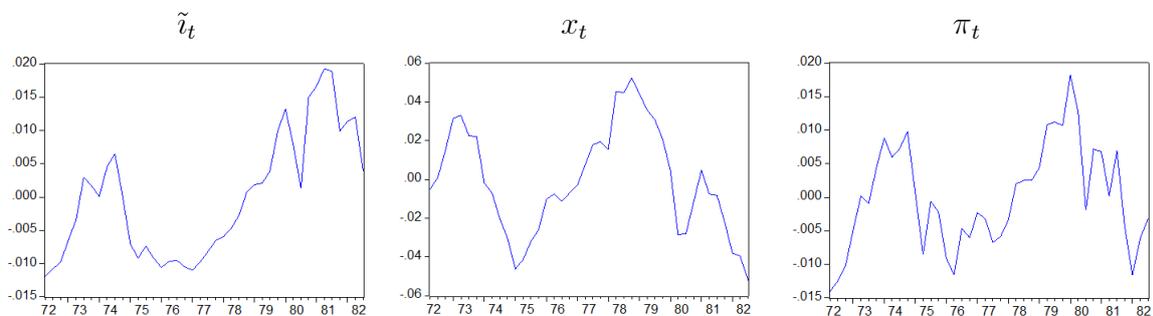
Note: a. Time series model: VAR(1) (without constant). b.  $H_0$ : no structural break;  $H_1$ : one structural break.

c. Observation sample (adjusted): 1972Q2—2007Q4.

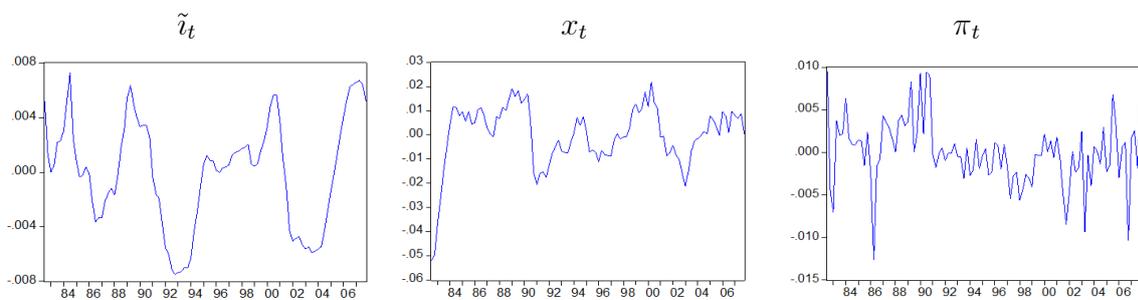
## B Plots of Subepisode Time Series

Figure A.1: Demeaned, Detrended Time Series

Panel A: Pre-1982 sample (1972Q2-1982Q3)



Panel B: Post-1982 sample (1982Q3-2007Q4)



Note:  $\tilde{i}_t \equiv$  deviation of quarterly Fed rate from steady-state value;  $x_t \equiv$  log difference of quarterly real GDP from HP trend;  $\pi_t \equiv$  quarterly CPI inflation

## C Unit Root Test for Stationarity

Table A.2: Unit Root Test Result

Panel A: pre-break sample (1972Q3—1982Q3)

Time series	5% critical value	10% critical value	ADF test statistics	p-values*
$\tilde{I}_t$	-1.95	-1.61	-1.71	0.0818
$x_t$	-1.95	-1.61	-1.67	0.0901
$\pi_t$	-1.95	-1.61	-2.86	0.0053

Panel B: post-break sample (1982Q4—2007Q4)

Time series	5% critical value	10% critical value	ADF test statistics	p-values
$\tilde{I}_t$	-1.95	-1.61	-2.91	0.0040
$x_t$	-1.95	-1.61	-4.42	0.0000
$\pi_t$	-1.95	-1.61	-3.34	0.0010

Note: "\*" denotes the Mackinnon (1996) one-sided p-values.