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**Working Paper**

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CESifo Working Paper, No. 3946

**Provided in Cooperation with:**

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*Suggested Citation:* Goerke, Laszlo; Herzberg, Frederik; Upmann, Thorsten (2012) : Failure of ad valorem and specific tax equivalence under uncertainty, CESifo Working Paper, No. 3946, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

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Failure of Ad Valorem and Specific Tax  
Equivalence under Uncertainty

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CESIFO WORKING PAPER NO. 3946  
CATEGORY 1: PUBLIC FINANCE  
SEPTEMBER 2012

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# Failure of Ad Valorem and Specific Tax Equivalence under Uncertainty

## Abstract

Applying a framework of perfect competition under uncertainty, we contribute to the discussion of whether or not ad valorem taxes and specific taxes are equivalent. While this equivalence holds without price uncertainty, we show that ad valorem taxes and specific taxes are “almost never” equivalent in the presence of uncertainty if we demand equivalence to hold pathwise. Since we obtain this result under perfect competition, our analysis also provides a further rationale for why the equivalence must fail under imperfect competition.

JEL-Code: H200, H210, H610.

Keywords: ad valorem taxes and specific taxes, revenue neutrality, price uncertainty, concept of pathwise neutrality.

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September 26, 2012

We thank the participants of Session C10 of the 68th Annual Congress of the International Institute of Public Finance in Dresden, Germany, 2012 for valuable comments.

## 1. Introduction

There is a long-standing debate surrounding the question of whether or not specific (unit) taxes and ad valorem taxes are equivalent. Most contributions have focused on imperfectly competitive markets and have analysed the output and welfare effects of (mostly) equal yield substitutions. Cournot (1838, Chap. VI) was the first to recognise the differential effects of ad valorem taxes and specific taxes in a monopoly. Wicksell (1896, p. 20) then clarified that a monopolist's incentives to curtail output are less pronounced in the presence of an ad valorem tax than for a specific tax of equal yield. These insights basically lay dormant until the topic was revived by Suits and Musgrave (1953) and have in recent years been refined in various ways.<sup>1</sup> As a by-product, Suits and Musgrave formally proved the insight widely accepted nowadays that ad valorem taxes and unit taxes are equivalent in a competitive world.

In this paper we challenge this view and show that even in the case of risk neutrality such an equivalence only holds for the special case when there is no uncertainty about the output price (of a commodity of a given quality), and not when the output price is stochastic. We derive this result by applying the concept of *pathwise equivalence*; this contrasts with the usually applied concept of equivalence in expected terms, which merely requires that a tax reform affects neither the *expected* tax proceeds nor the *expected* output level (see, for example, Fraser, 1985, Goerke, 2011 and Kotsogiannis and Serfes, 2012). However, it seems to us that this is an unsatisfactory concept for establishing the equivalence of ad valorem and specific taxation in a world of uncertainty, as it implies that the tax proceeds and/or the output level are generically not constant, and may, in the limiting case, even fluctuate around their respective expected values with probability one. Consequently, a tax reform which keeps the tax yield (and the output level) only constant in expected terms may leave the government with a substantial deficit or surplus,

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<sup>1</sup>For example, Skeath and Trandel (1994) show that the ad valorem tax Pareto-dominates a specific tax of equal yield in a monopoly. Cheung (1998), Schröder (2004) and Dröge and Schröder (2009) assume monopolistic competition; Delipalla and Keen (1992), Denicolò and Matteuzzi (2000), Anderson et al. (2001), Hamilton (2009), and Lapan and Hennessy (2011) inter alia focus on oligopolies, Kind et al. (2009) on two-sided markets and Hamilton (1999) on a monopsony, while Grazzini (2006) and Blackorby and Murty (2007) investigate general equilibrium settings. Within a framework of tax competition, Lockwood (2004) and Akai et al. (2011) compare ad valorem and unit taxes, while Hoffmann and Runkel (2012) compare profit and unit taxes. Finally, Keen (1998) provides a comprehensive survey of contributions.

depending on the path realised and can, therefore, not be viewed as establishing the equivalence of ad valorem taxes and specific taxes. Rather, one may arguably be interested in a tax reform which collects the same public revenue as does the original tax system for *any* possible path, such that the public budget is the same as before with probability one — albeit not necessarily constant (see Keen, 1998). Such pathwise neutrality is also the most relevant concept from a fiscal policy perspective, as restrictions on budget deficits are often defined on a yearly basis and only allow for an (excessive) inter-temporal equalisation of revenue and expenditure variations in exceptional circumstances. For example, the so-called Maastricht criteria of the European Union and the debt brakes in Germany and Switzerland define admissible annual budget deficits as a fraction of GDP. Using this pathwise equivalence concept we show that even under competitive behaviour, ad valorem taxes and specific taxes are not equivalent: that is, they result in different levels of output and/or revenue, except for the degenerate case when the output price is (almost surely) constant.

We believe that this finding provides a new and important answer to a question which has been discussed extensively in the literature. Indeed, the non-equivalence of ad valorem taxes and specific taxes for profit maximising firms in perfectly competitive settings with deterministic prices has thus far been derived in a world with endogenous quality choices, (see Liu, 2003, and Delipalla and Keen, 2006). Intuitively, we would expect this to be the case in such a world because quality adjustments which alter the consumers' willingness to pay and, hence, the equilibrium price, directly affect the revenue collected from ad valorem tax, but have no such impact on the receipts of a specific tax. Furthermore, our contribution is related to three analyses of ad valorem and specific taxation under uncertainty, requiring constant expected revenue. In the tradition of Sandmo (1971), Fraser (1985) assumes risk-averse, price-taking firms facing a convex cost function that have to choose the output level before the uncertainty regarding the price dissolves. Since an ad valorem tax mitigates after-tax price variability, it will have less detrimental output effects for a strictly risk-averse firm than a specific tax. However, in the presence of risk neutrality, both taxes are equivalent.<sup>2</sup> Goerke (2011) shows that even risk-neutral firms that can react to the realisation of the price will increase expected output in response to a substitution of an ad valorem for a specific tax. In addition,

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<sup>2</sup>Fraser (1985) also considers the possibility that prices and output are uncertain, where output uncertainty implies that a given amount of inputs causes an output level that deviates from the expected level.

Kotsogiannis and Serfes (2012) show that welfare rankings of the two types of taxes obtained in a Cournot oligopoly with deterministic costs may be reversed if cost uncertainty is allowed for. In his survey, Keen (1998) briefly considers a different approach, namely a government that wants to completely isolate tax revenue from output price variations. The appropriate tax then depends on the price elasticity of demand. Finally, Dickie and Trandel (1996) assume uncertainty about the demand price and a negative production externality. The authors derive conditions under which specific or ad valorem taxes (or quotas) are best suited to internalise this effect for a competitive market and a monopoly.

In this paper we use a discrete time framework to analyse the behaviour of a single risk neutral firm which is aware of tax rates, the marginal cost curve, and the stochastic pre-tax price process, but does not know the realised price when deciding on the output quantity. The government sets the tax rates prior to the revelation of the price level. While tax revenues are, therefore, uncertain, we require that they are unaffected by the structure of taxation (for each possible realisation of the price process). This notion of pathwise revenue neutrality ensures that the substitution of taxes does not implicitly shift the price risk from the firm to the government or vice versa. In such a setting, characterised by uncertainty about the output price, we prove a strikingly simple result: equal-yield ad valorem and specific taxes cannot be equivalent in a perfectly competitive world (unless prices are deterministic). In other words, a tax reform which leaves tax revenue unaffected will induce the competitive firm under consideration to alter its output choice. Since this impact of uncertainty about demand conditions can also arise in the presence of market imperfections, our findings provide a further rationale for why the equivalence of ad valorem taxes and specific taxes can generically not hold under imperfect competition.

This non-equivalence under competitive behaviour is not only a strong theoretical result, but also provides a new perspective on the relative merits of ad valorem taxes and specific taxes from a policy angle. In 2006, consumption taxes generated about 30% of all tax revenues in the OECD (see OECD, 2008), of which about 19% resulted from general consumption taxes (that is, mostly the value-added tax) and the remaining 11% were due to taxes on specific goods and services. Moreover, we have seen a pronounced shift towards value-added, i. e. ad valorem taxation in recent decades. In addition, our results have a bearing on a wider set of issues than simply first-best commodity taxation. They imply that in competitive settings, equal-yield substitutions of ad valorem taxes for specific taxes aimed at correcting externalities, of wage-related for non-wage-related social security contributions, or

of valorem for specific tariffs do have real consequences in the presence of uncertainty.<sup>3</sup> Moreover, tax competition in specific and ad valorem taxes will not only result in different levels of public good provision if domestic tax rates alter the post-tax return to capital (see Lockwood, 2004), but also in a small open economy setting with uncertainty about the future price of capital.

In the remainder of the paper, we begin with describing our analytical framework in Section 2. Section 3 then shows for two concepts of pathwise neutrality — a stricter and a weaker one — that it is (almost surely) impossible to substitute one tax for another tax without affecting tax revenue or output. Section 4 briefly concludes and places our contribution in a wider perspective.

## 2. The Model

We consider an analytical framework in discrete time in which a competitive firm decides about output before the uncertainty about the random output price  $P_t$  is resolved at time  $t \in \mathcal{T} := [0, T] \cap \mathbb{N}_0$ . More precisely, the firm faces a positive stochastic process  $P : \mathcal{T} \times \Omega \rightarrow \mathbb{R}_+$  defined on the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . For informational consistency, we assume that the process  $(P_t)_{t \in \mathcal{T}}$  is adapted to its natural filtration  $\mathbb{F} := (\mathcal{F}_t)_{t \in \mathcal{T}}$ . We assume that  $E_{\mathbb{P}} [P_t | \mathcal{F}_{t-1}] > 0$  almost surely with respect to the probability measure  $\mathbb{P}$ .

Before the firm can sell its output on the market at time  $t$ , it must decide on the quantity to be produced and thus to be ready for supply; normalising the time to market to unity, the firm's market supply  $Q_t$  must be produced at  $t - 1$ . Accordingly, the firm can only determine the quantity sold at time  $t$  on the basis of the information available up to and including time  $t - 1$ . Therefore, the firm must condition its production, and hence its supply decisions on the expected price at time  $t - 1$ :  $\mu_t := E_{\mathbb{P}} [P_t | \mathcal{F}_{t-1}]$ . Formally,  $Q_t$  is  $\mathcal{F}_{t-1}$  adapted for all  $t$ ; that is, the supply at time  $t$  is known at time  $t - 1$ , i. e., the process  $(Q_t)_{t \in \mathcal{T}}$  is predictable with respect to the filtration  $\mathbb{F}$ .

At any time, the government levies two types of taxes on the firm's activity: a specific output tax of level  $s$  ( $s \in \mathbb{R}$ ) and an ad valorem tax of level  $\tau$  ( $\tau \in (-\infty, 1)$ ), where tax rates can be negative and, hence, constitute subsidies. We may think of tax rates that may or may not be adjusted at each instant of time. It is crucial,

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<sup>3</sup>This evaluation is supported by Dickie and Trandel (1996) who show for a very specific setting with linear demand and supply curves that externality-correcting ad valorem taxes and specific taxes are not equivalent in a competitive world.

however, that any change in tax rates does not become effective instantaneously. More precisely, we assume that the tax rate process  $((s_t, \tau_t))_{t \in \mathcal{T}}$  is predictable; that is,  $(s_t, \tau_t)$  is  $\mathcal{F}_{t-1}$ -measurable. This assumption ensures that at time  $t - 1$  the firm knows the tax rates applicable to sales at time  $t$ , i. e., the taxes levied on output  $Q_t$ . This seems to be a natural assumption for products with a relatively short production interval, where governments are likely to need more time to alter tax rates than firms need to change output quantities.

Since the tax system applicable to the revenue of current production decisions is a given constant for the firm, we drop the time index in order to save notational effort, and henceforth write  $(s, \tau)$ , although the tax system need not be constant over time.

The cost structure of the firm is represented by variable cost  $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  plus a certain fixed cost  $f \in \mathbb{R}_+$ , which may best interpreted as the set-up cost. We assume that  $c(0) = 0$ ,  $c' > 0$  and  $c'' > 0$ . Putting the pieces together, the firm's instantaneous (or periodic) profit at time  $t$  (or in period  $t$ ), is then given by

$$(P_t(1 - \tau) - s)Q_t - c(Q_t) - f,$$

if  $Q_t$  is positive, and zero otherwise.

Since the firm is only interested in the after-tax price, we define the net price by  $P_t^n := P_t(1 - \tau) - s$ , and write profit in terms of  $P_t^n$  rather than of  $P_t$ . Accordingly, stochastic profit reads as

$$\Pi_t := \pi(P_t^n, Q_t) := (P_t^n Q_t - c(Q_t) - f) \mathbf{1}_{\{Q_t > 0\}}.$$

We assume that the firm (or the management of the firm) is risk neutral and thus maximises at each instant of time the expected profit generated by selling today's production at a random net price tomorrow:

$$\max_{Q_t} E_{\mathbb{P}} [\pi(P_t^n, Q_t) | \mathcal{F}_{t-1}] \quad \forall t \in \mathcal{T}$$

Evidently, the tax rates affect production decisions only through their effect on the expected value of the after-tax price. Thus, the price process  $P = (P_t)_{t \in \mathcal{T}}$  together with the definition of the net price  $P_t^n$  determines the net-price process  $P^n = (P_t^n)_{t \in \mathcal{T}}$  and its associated expected value process  $\mu^n = (\mu_t^n)_{t \in \mathcal{T}}$ . Accordingly, we define the expected *net* price of time  $t$  as  $\mu_t^n := \mu_t^n(s, \tau) := E_{\mathbb{P}} [P_t^n | \mathcal{F}_{t-1}] = (1 - \tau)\mu_t - s$ . Note that

$$E_{\mathbb{P}} [\pi(P_t^n, Q_t) | \mathcal{F}_{t-1}] = (E_{\mathbb{P}} [P_t^n | \mathcal{F}_{t-1}] Q_t - c(Q_t) - f) \mathbf{1}_{\{Q_t > 0\}} \quad (1)$$



due to the  $\mathcal{F}_{t-1}$ -linearity of the conditional expectation operator  $E_{\mathbb{P}}[\cdot | \mathcal{F}_{t-1}]$ .

In the event that the optimal output quantity is positive,  $Q_t$  must be a local maximum of the profit function. Since  $c$  is strictly convex, the function  $Q \mapsto E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}](\omega)Q - c(Q) - f$  is strictly concave for all  $\omega \in \Omega$  and thus has a unique local extremum which is simply the global maximum. Therefore, whenever the optimal  $Q_t$  is positive, it can be uniquely characterised by the first-order condition

$$E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}] - c'(Q_t) = 0,$$

and given by  $Q_t = (c')^{-1}(E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}])$ . Now, due to Equation (1), a positive  $Q_t$  will be optimal if and only if  $E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}]Q_t - c(Q_t) - f > 0$ . Hence, from the first-order condition, we have

$$Q_t > 0 \Leftrightarrow E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}] (c')^{-1}(E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}]) - c((c')^{-1}(E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}])) - f > 0$$

Let  $\Omega_0$  be the event when a positive output level  $Q_t$  is optimal:

$$\begin{aligned} \Omega_0 &= \{Q_t > 0\} \\ &= \{E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}] (c')^{-1}(E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}]) - c((c')^{-1}(E_{\mathbb{P}}[P_t^n | \mathcal{F}_{t-1}])) - f > 0\} \end{aligned}$$

Henceforth, all our analysis will be understood to be restricted to the event  $\Omega_0$ . In particular, for the case of uncertainty, we will argue pathwise throughout and these arguments shall be taken as referring to paths in  $\Omega_0$ , and correspondingly, ‘‘a.s.’’ or ‘‘almost surely’’ will mean *almost surely on  $\Omega_0$* .

*Public revenue* generated by a tax system  $(s, \tau)$  constitutes a stochastic process, denoted by  $(R_t(s, \tau))_t$ , and for all  $t \in \mathcal{T}$ :

$$R_t(s, \tau) := (\tau P_t + s) Q_t(s, \tau).$$

Apparently, the amount of overall taxes paid by the firm per unit sold is an affine function of the price.

We define the (random) set

$$\mathbb{S} \equiv \{(s, \tau) \in \mathbb{R}^2 : s/\mu_t < 1 - \tau\},$$

and for all  $s, \tau \in \mathbb{R}$  the (random) sets

$$\mathbb{S}^s \equiv \{\tau \in \mathbb{R} : (s, \tau) \in \mathbb{S}\} = (-\infty, 1 - s/\mu_t), \quad (2)$$

$$\mathbb{S}_\tau \equiv \{s \in \mathbb{R} : (s, \tau) \in \mathbb{S}\} = (-\infty, (1 - \tau)\mu_t). \quad (3)$$

*Remark 1.*  $Q_t(s, \tau)$  and hence  $R_t(s, \tau)$  are well defined for all  $(s, \tau) \in \mathbb{S}$ .

*Proof.* The assumptions  $c' > 0, c'' > 0$  imply that  $c'$  is strictly monotonic with range  $\mathbb{R}_{>0}$ . Therefore,  $c'$  has an inverse with domain  $\mathbb{R}_{>0}$ . Since  $\mu_t^n(s, \tau) = (1-\tau)\mu_t - s > 0$  if, and only if,  $(s, \tau) \in \mathbb{S}$ , we find that  $Q_t(s, \tau) = (c')^{-1}(\mu_t^n(s, \tau))$  is well defined whenever  $(s, \tau) \in \mathbb{S}$ .  $\square$

Although our model is in discrete time, the following example illustrates the idea of (non-)equivalence within a continuous time framework, applying the popular and well-known process of a geometric Brownian motion.

*Example.* In order to illustrate Remark 1, suppose that the price process  $(P_t)_{t \in \mathcal{T}}$  is given by a geometric Brownian motion (i. e. by an Itô diffusion with proportional drift coefficient function  $\mu(P) = \alpha P$  and proportional diffusion coefficient function  $\sigma(P) = \sigma P$ , so that  $P_t = P_0 \exp((\alpha - \sigma^2/2)t + \sigma W_t)$  by Itô's formula applied to  $P$ ), observed at integer dates, so that the net price process  $P^n$ , defined by  $P_t^n := P_t(1 - \tau) - s$ , is also observed at integer dates. Then

$$\begin{aligned} dP_t^n &= (1 - \tau) dP_t \\ &= \alpha(1 - \tau)P_t dt + \sigma(1 - \tau)P_t dW_t \\ &= \alpha(P_t^n + s) dt + \sigma(P_t^n + s) dW_t. \end{aligned}$$

Since  $P$  is an Itô process, so is  $P^n$ . Observe, however, that if  $P$  follows a geometric Brownian motion,  $P^n$  does *not* follow a geometric Brownian motion under  $\mathbb{P}$ . (Only if  $s = 0$  would this be true.) Since  $P^n$  does not belong to the same class of processes as  $P$ , we may suspect that specific and ad valorem taxes are generically not equivalent. However, this conjecture will be scrutinised more thoroughly below.

### 3. Neutrality of the Tax System

**3.1. Neutrality Under Certainty.** In a perfectly competitive world without risk, where the output price is a given constant, specific taxes and ad valorem taxes are equivalent (see, inter alia, Suits and Musgrave, 1953; Musgrave, 1959, p. 305; Bishop, 1968; Kay and Keen, 1983; Dickie and Trandel, 1996; and Liu, 2003 for formal derivations). In the present setting this result can be verified easily as follows. The firm chooses output such that the after-tax price equals marginal cost

$$P(1 - \tau) - s = c'(Q(s, \tau)).$$

Differentiating this equation with respect to  $\tau$  and  $s$ , we find that

$$-P = c''(Q(s, \tau)) \frac{\partial Q}{\partial \tau}, \quad -1 = c''(Q(s, \tau)) \frac{\partial Q}{\partial s}.$$

for all  $(s, \tau)$ , and thus conclude that  $\frac{\partial Q}{\partial \tau} = P \frac{\partial Q}{\partial s}$ . It follows that a substitution of the specific tax for the ad valorem tax such that output is held constant requires<sup>4</sup>

$$-\frac{ds}{d\tau}\Big|_{Q \text{ fixed}} = P.$$

That is, along any iso- $Q$  curve, the marginal rate of substitution between the two tax rates equals the gross price.

On the other hand, if we differentiate public revenue, amounting to  $R(s, \tau) = (\tau P + s)Q(s, \tau)$ , with respect to  $\tau$  and  $s$ , yielding

$$\begin{aligned}\frac{\partial R(s, \tau)}{\partial s} &= Q(s, \tau) + (\tau P + s) \frac{\partial Q(s, \tau)}{\partial s}, \\ \frac{\partial R(s, \tau)}{\partial \tau} &= PQ(s, \tau) + (\tau P + s) \frac{\partial Q(s, \tau)}{\partial \tau},\end{aligned}$$

for all  $(s, \tau)$ , and then exploit the fact that  $\frac{\partial Q}{\partial \tau} = P \frac{\partial Q}{\partial s}$ , we obtain  $\frac{\partial R}{\partial \tau} = P \frac{\partial R}{\partial s}$ . As before, we thus find that<sup>5</sup>

$$-\frac{ds}{d\tau}\Big|_{R \text{ fixed}} = P.$$

Hence, the iso-output curves and the iso-revenue curves have the same slope. Therefore, as soon as an iso-output and an iso-revenue curve have at least one point in common, they must already coincide.

We thus conclude that for any two tax systems  $(s^1, \tau^1)$  and  $(s^2, \tau^2)$ , we have

$$R(s^1, \tau^1) = R(s^2, \tau^2) \quad \Leftrightarrow \quad Q(s^1, \tau^1) = Q(s^2, \tau^2).$$

This establishes the equivalence between specific taxes and ad valorem taxes under certainty in a very broad sense, generalising the proofs found for specific settings in the contributions mentioned above.

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<sup>4</sup>Formally, any function  $s_{\bar{Q}}$ , such that  $\hat{Q}(\tau) := Q(s_{\bar{Q}}(\tau), \tau) = \bar{Q}$  is constant in  $\tau$ , satisfies

$$0 = \frac{d}{d\tau} \hat{Q}(\tau) = s'_{\bar{Q}}(\tau) \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial s} + \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial \tau} = (s'_{\bar{Q}}(\tau) + \mu) \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial s},$$

whence  $\frac{ds}{d\tau} \equiv s'(\tau) = -P$  for all  $\tau$ .

<sup>5</sup>Formally, any function  $s_{\bar{R}}$ , such that  $\hat{R}(\tau) := R(s_{\bar{R}}(\tau), \tau) = \bar{R}$  is constant in  $\tau$ , satisfies

$$0 = \frac{d}{d\tau} \hat{R}(\tau) = s'_{\bar{R}}(\tau) \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial s} + \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial \tau} = (s'_{\bar{R}}(\tau) + \mu) \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial s},$$

whence  $\frac{ds_{\bar{R}}}{d\tau} \equiv s'_{\bar{R}}(\tau) = -P$  for all  $\tau$ .

**3.2. Neutrality Under Uncertainty.** For simplicity, we assume that there are only two dates in  $\mathcal{T}$ , viz.  $\mathcal{T} = \{0, 1\}$ , called “today” and “tomorrow”, where today’s information set is trivial:  $\mathcal{F}_0 = \{\emptyset, \Omega\}$ . (At the cost of considerable notational effort, one could generalize the analysis below to more general discrete  $\mathcal{T}$ .) We shall suppress the subscript 1 and thus write  $P, \mu, Q, R, s, \tau$  instead of  $P_1, \mu_1, Q_1, R_1, s_1, \tau_1$ , respectively. Then, based on our general assumption that  $\mu, Q, s, \tau$  are previsible and the simplifying assumption in this subsection that the initial information set is trivial, we know that  $\mu, Q, s, \tau$  are (deterministic) constants, whereas  $P, R$  are random variables. The following analytical steps are all pathwise on  $\Omega_0$ , that is, they hold for almost every  $\omega \in \Omega_0$ , and the argument  $\omega$  shall be consistently suppressed.

On the basis of these simplifying hypotheses, tax levels  $s, \tau$  and output quantity  $Q(s, \tau)$  are simply constants, whereas tomorrow’s output price is a random variable  $P$  with mean  $\mu$ . Hence, a tax system is a pair  $(s, \tau) \in \mathbb{S}$ , and the tax revenue resulting from the tax system  $(s, \tau)$  equals

$$R(s, \tau) := (\tau P + s) Q(s, \tau),$$

wherein

$$Q(s, \tau) = (c')^{-1}(\mu^n) = (c')^{-1}((1 - \tau)\mu - s).$$

(Recall that  $Q$  is well defined since  $c'' > 0$ , whence  $c'$  is strictly increasing and thus injective.)

Then, for any fixed level of output  $\bar{Q}$ , the equation  $Q(s, \tau) = \bar{Q}$  implicitly defines the specific tax as a function of the ad valorem tax (and  $\bar{Q}$ , with the implicit parameter  $\mu$ ):

$$s = \phi(\tau, \bar{Q}).$$

Because  $c'' > 0$ ,  $c'$  is strictly increasing, whence so is  $(c')^{-1}$ , and hence  $Q(\cdot, \tau)$  is strictly decreasing (in  $s$ ) and thus invertible for any fixed  $\tau$ . Therefore,  $\phi(\tau, \cdot)$  is well defined and strictly decreasing (in  $s$ ) for all  $\tau$ .

Note that  $R(s, \tau)$  is a random variable for any pair of tax rates  $(s, \tau)$ . If  $R(\cdot, \tau)$  is  $\mathbb{P}$ -almost surely invertible for all  $\tau$  (which a priori does not need to be the case), then for any fixed level of tax proceeds  $\bar{R}$ , the equation  $R(s, \tau) = \bar{R}$  implicitly defines, say, the specific tax as a function of the ad valorem tax (and  $\bar{R}$  with the implicit parameter  $\mu$ ):

$$s = \psi(\tau, \bar{R}).$$

In this situation, any given tax system  $(s^\circ, \tau^\circ)$ , yields a well defined revenue  $R^\circ := R(s^\circ, \tau^\circ)$  and output  $Q^\circ := Q(s^\circ, \tau^\circ)$ . That is,  $(s^\circ, \tau^\circ)$  results in  $R^\circ$  and  $Q^\circ$ .

We may now define the neutrality of the tax system as follows.

**Definition 1.** *We say that the price-affine taxation structure satisfies tax neutrality (given the model  $(c, P, \mathbb{P})$ ) if, and only if, for all tax systems  $(s^1, \tau^1)$  and  $(s^2, \tau^2)$ , one has  $R(s^1, \tau^1) = R(s^2, \tau^2)$  almost surely whenever  $Q(s^1, \tau^1) = Q(s^2, \tau^2)$ .<sup>6</sup>*

In light of Definition 1, the result found in Section 3.1 may be formulated as follows: If  $P$  is deterministic and constant, the model  $(c, P, \mathbb{P})$  satisfies the requirement of *tax neutrality*.

One may relax the notion of neutrality by simply requiring that Definition 1 need not hold for all pairs  $(s, \tau) \in \mathbb{S}$ , but only for some suitable subset  $D \subseteq \mathbb{S}$ , that is for all  $(s, \tau) \in D \subseteq \mathbb{S}$  (which we may call *neutrality on  $D$* ). A natural specification of  $D$  is to consider a *complete* substitution of the specific tax by an ad valorem tax (and vice versa), such that both tax systems lead to the same  $(R, Q)$ -pair.

**Definition 2.** *The price-affine taxation structure is neutral under complete substitution (given the model  $(c, P, \mathbb{P})$ ) if, and only if, the equivalence*

$$R(0, \tau) = R(s, 0) \text{ a. s.} \quad \Leftrightarrow \quad Q(0, \tau) = Q(s, 0)$$

*holds for all  $s \in \mathbb{S}_0$  and  $\tau \in \mathbb{S}^0$  (with the sets  $\mathbb{S}^0$  and  $\mathbb{S}_0$  defined as in (2) and (3), respectively).*

This is, of course, a weaker concept of neutrality than that as defined in Definition 1.

Now, we may reformulate the question: given some pair  $(R^\circ, Q^\circ)$ , is there any tax system  $(s^\circ, \tau^\circ)$  such that  $R^\circ := R(s^\circ, \tau^\circ)$  and  $Q^\circ := Q(s^\circ, \tau^\circ)$ ? If so, we will call  $R^\circ$  and  $Q^\circ$  *compatible*.

**Definition 3.** *A pair  $(R^\circ, Q^\circ)$  is said to be compatible if, and only if, there is some tax system  $(s^\circ, \tau^\circ) \in \mathbb{S}$  such that  $R^\circ = R(s^\circ, \tau^\circ)$  almost surely and  $Q^\circ = Q(s^\circ, \tau^\circ)$ .*

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<sup>6</sup>More formally (taking into account the fact that we have restricted our attention to sample paths  $\omega$  with positive production, i. e.  $\omega \in \Omega_0$ ), the price-affine taxation structure satisfies tax neutrality (given the model  $(c, P, \mathbb{P})$ ) if, and only if,

$$\mathbb{P}(\Omega_0 \cap \{R(s^1, \tau^1) = R(s^2, \tau^2)\}) = \mathbb{P}(\Omega_0) \quad \Leftrightarrow \quad Q(s^1, \tau^1) = Q(s^2, \tau^2)$$

for all  $(s^1, \tau^1)$  and  $(s^2, \tau^2)$ .

*Remark 2.* The price-affine taxation structure satisfies tax neutrality (given the model  $(c, P, \mathbb{P})$ ) if, and only if, for all compatible pairs  $(R^\circ, Q^\circ)$ , the tax rate  $\psi(\tau, R^\circ)$  is well defined  $\mathbb{P}$ -almost surely and for all  $\tau \in \mathbb{R}$ , we have

$$\psi(\tau, R^\circ) = \phi(\tau, Q^\circ)$$

$\mathbb{P}$ -almost surely.

*Proof.* Based on Definition 1, tax neutrality means that for all  $(s^1, \tau^1)$  and  $(s^2, \tau^2)$ ,

$$R(s^1, \tau^1) = R(s^2, \tau^2) \text{ a. s.} \quad \Leftrightarrow \quad Q(s^1, \tau^1) = Q(s^2, \tau^2).$$

In other words, tax neutrality means that for all tax systems  $(s^1, \tau^1) \in \mathbb{S}$ ,

$$\{(s^2, \tau^2) : R(s^1, \tau^1) = R(s^2, \tau^2) \text{ a. s.}\} = \{(s^2, \tau^2) : Q(s^1, \tau^1) = Q(s^2, \tau^2)\}.$$

Note, however, that the set of compatible pairs of random variables is simply

$$\{(R(s^1, \tau^1), Q(s^1, \tau^1)) : (s^1, \tau^1) \in \mathbb{S}\}.$$

Therefore, tax neutrality actually means that for all compatible pairs  $(R^\circ, Q^\circ)$ , we have

$$\{(s^2, \tau^2) \in \mathbb{S} : R^\circ = R(s^2, \tau^2) \text{ a. s.}\} = \{(s^2, \tau^2) \in \mathbb{S} : Q^\circ = Q(s^2, \tau^2)\}.$$

Hence, tax neutrality says that for all compatible pairs  $(R^\circ, Q^\circ)$  and for all  $\tau^2$ , one has

$$\begin{aligned} \{s^2 \in \mathbb{S}_{\tau^2} : R^\circ = R(s^2, \tau^2) \text{ a. s.}\} &= \{s^2 \in \mathbb{S}_{\tau^2} : Q^\circ = Q(s^2, \tau^2)\} \\ &= \{\phi(\tau^2, Q^\circ)\}. \end{aligned}$$

Thus, tax neutrality is satisfied if, and only if, for all compatible pairs  $(R^\circ, Q^\circ)$  and all  $\tau^2$ , there is a unique  $s^2 \in \mathbb{S}_{\tau^2}$  such that  $R^\circ = R(s^2, \tau^2)$  a. s. and this  $s^2$  equals  $\phi(\tau^2, Q^\circ)$ .  $\square$

According to this remark, the iso- $Q$  tax paths and the iso- $R$  tax paths coincide, provided that the levels  $Q^\circ$  and  $R^\circ$  are compatible with each other, meaning that there is some tax system  $(s^\circ, \tau^\circ)$  such that  $R^\circ = R(s^\circ, \tau^\circ)$  and  $Q^\circ = Q(s^\circ, \tau^\circ)$ .

The task is now to investigate whether or not the price-affine taxation structure satisfies neutrality in the sense of either of the definitions given above. — As shown, the price-affine taxation structure is neutral for a constant price process, so one might expect this neutrality to hold for some (non-empty) class of non-trivial random processes as well. However, our example of an Itô process (see page

7) should make us sceptical in this regard — and in fact, we show now that the conjecture of some broader neutrality is misguided.

To complete the proof of non-equivalence, note that the firm chooses output such that the expected after-tax price equals marginal cost

$$\mu(1 - \tau) - s = c'(Q(s, \tau)).$$

Differentiating this equation with respect to  $\tau$  and  $s$ , we find that

$$-\mu = c''(Q(s, \tau)) \frac{\partial Q}{\partial \tau}, \quad -1 = c''(Q(s, \tau)) \frac{\partial Q}{\partial s}$$

for all  $\tau$ , and thus conclude that  $\frac{\partial Q}{\partial \tau} = \mu \frac{\partial Q}{\partial s}$ . It follows that a substitution of the specific tax for the ad valorem tax such that output is held constant requires that<sup>7</sup>

$$-\frac{ds}{d\tau} \Big|_{Q \text{ fixed}} = \mu.$$

That is, along any iso- $Q$  curve, the marginal rate of substitution between the two tax rates equals the expected gross price.

On the other hand, if we differentiate public revenue, amounting to  $R(s, \tau) = (\tau P + s)Q(s, \tau)$ , with respect to  $\tau$  and  $s$ , yielding

$$\begin{aligned} \frac{\partial R}{\partial s} &= Q(s, \tau) + (\tau P + s) \frac{\partial Q}{\partial s} \\ \frac{\partial R}{\partial \tau} &= PQ(s, \tau) + (\tau P + s) \frac{\partial Q}{\partial \tau} \end{aligned}$$

for all  $(s, \tau)$ , and then exploit the fact that  $\frac{\partial Q}{\partial \tau} = \mu \frac{\partial Q}{\partial s}$ , we find that  $\frac{\partial R}{\partial \tau} = (P - \mu)Q(s, \tau) + \mu \frac{\partial R}{\partial s}$ . This implies that<sup>8</sup>

$$-\frac{ds}{d\tau} \Big|_{R \text{ fixed}} = \mu + \frac{(P - \mu)Q(s, \tau)}{\frac{\partial R(s, \tau)}{\partial s}}.$$

<sup>7</sup>Formally, any function  $s_{\bar{Q}}$ , such that  $\hat{Q}(\tau) := Q(s_{\bar{Q}}(\tau), \tau) = \bar{Q}$  is constant in  $\tau$ , satisfies

$$0 = \frac{d}{d\tau} \hat{Q}(\tau) = s'_{\bar{Q}}(\tau) \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial s} + \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial \tau} = (s'_{\bar{Q}}(\tau) + \mu) \frac{\partial Q(s_{\bar{Q}}(\tau), \tau)}{\partial s},$$

whence  $\frac{ds_{\bar{Q}}}{d\tau} = s'_{\bar{Q}}(\tau) = -\mu$  for all  $\tau$ .

<sup>8</sup>Formally, any function  $s_{\bar{R}}$ , such that  $\hat{R}(\tau) := R(s_{\bar{R}}(\tau), \tau) = \bar{R}$  is constant in  $\tau$ , satisfies

$$\begin{aligned} 0 &= \frac{d}{d\tau} \hat{R}(\tau) = s'_{\bar{R}}(\tau) \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial s} + \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial \tau} \\ &= (s'_{\bar{R}}(\tau) + \mu) \frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial s} + (P - \mu)Q(s_{\bar{R}}(\tau), \tau), \end{aligned}$$

whence

$$\frac{ds_{\bar{R}}}{d\tau} \equiv s'_{\bar{R}}(\tau) = -\mu + \frac{(\mu - P)Q(s_{\bar{R}}(\tau), \tau)}{\frac{\partial R(s_{\bar{R}}(\tau), \tau)}{\partial s}}$$

for all  $\tau$ .

This shows that a given iso-output curve  $s_{\bar{Q}}$  (in the situation described in Remark 2:  $\phi(\cdot, \bar{Q})$ ) and a given iso-revenue curve  $s_{\bar{R}}$  (in the situation described in Remark 2:  $\psi(\cdot, \bar{R})$ ) have almost surely the same slope if, and only if, either  $P = \mu$  almost surely or  $Q(s, \tau) = 0$  for all  $(s, \tau)$  on the iso-revenue curve. Using the implicitly defined functions  $\phi, \psi$ , this means that for any  $\bar{Q}, \bar{R}$  the equation  $\frac{\phi(\cdot, \bar{Q})}{\partial s} = \frac{\psi(\cdot, \bar{R})}{\partial s}$  (let alone  $\psi(\cdot, \bar{R}) = \phi(\cdot, \bar{Q})$ ) is equivalent to the statement that either  $P = \mu$  or  $\bar{Q} = Q(\psi(\cdot, \bar{Q}), \cdot) = 0$ . Hence, in light of Remark 2, the price-affine taxation structure satisfies tax neutrality given the model  $(c, P, \mathbb{P})$  if, and only if,  $P = \mu$  almost surely or  $Q^\circ = 0$  for all compatible  $(R^\circ, Q^\circ)$ .<sup>9</sup> The latter case, however, is absurd; hence, equivalence requires  $P = \mu$  almost surely.

We have thus proved:

**Theorem 1.** *The price-affine taxation structure satisfies tax neutrality given the model  $(c, P, \mathbb{P})$  if, and only if,  $P$  is almost surely constant (on  $\Omega_0$ ), i. e. if there is essentially no uncertainty.*

The intuition for the finding summarised in Theorem 1 can best be obtained from inspection of the tax revenue along an iso-output curve:  $\tilde{R}(\tau) \equiv R(s_{\bar{Q}}(\tau), \tau)$ . Differentiating  $\tilde{R}$  (for any given level  $\bar{Q}$ ) yields

$$\begin{aligned} \frac{d\tilde{R}(\tau)}{d\tau} &= \frac{\partial R(s_{\bar{Q}}(\tau), \tau)}{\partial s} s'_{\bar{Q}}(\tau) + \frac{\partial R(s_{\bar{Q}}(\tau), \tau)}{\partial \tau} \\ &= (P - \mu)\bar{Q}. \end{aligned}$$

Apparently, the firm will not be affected by this (partial) substitution of the ad valorem tax ( $\tau$ ) for the specific tax ( $s$ ), as this tax substitution allows expected profits to be kept constant, which does not then affect the firm's production decision, for it is assumed to be risk neutral. Public revenue, however, rises (falls) as a result of this substitution if the output price exceeds (falls short of) its expected level. To keep both, output and public revenue constant in the course of this substitution, the output price must be equal to its expected level. However, this can only be true for all paths (i. e. for all  $\omega \in \Omega$ ) if  $P$  is almost surely constant.

In analyses of equal-yield tax substitution, a weaker concept of revenue neutrality has occasionally been employed. Following Suits and Musgrave (1953), later contributions such as, for example, Delipalla (2009) and Hamilton (2009) investigate

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<sup>9</sup>The reason for this is that, given any  $Q^\circ \in \mathbb{R}_+$ , one can always choose  $s^\circ, \tau^\circ$  such that  $(1 - \tau^\circ)\mu - s^\circ = c'(Q^\circ) > 0$  (whence  $(s, \tau) \in \mathbb{S}$  readily), so that  $Q(s^\circ, \tau^\circ) = Q^\circ$ . If we then simply put  $R^\circ = R(s^\circ, \tau^\circ) = (\tau^\circ P + s^\circ)Q^\circ$ , the pair  $(R^\circ, Q^\circ)$  is compatible.



what Delipalla and Keen (1992) define as a  $P$ -shift. This shift implies constant tax revenue at the initial price and, hence, ignores second-order effects. In our model this entails that  $\frac{ds}{d\tau}\Big|_{R \text{ fixed}} = P$ . Comparison of  $\frac{ds}{d\tau}\Big|_{R \text{ fixed}}$  and  $\frac{ds}{d\tau}\Big|_{Q \text{ fixed}}$  clarifies that our non-neutrality result will also apply if this alternative budgetary restriction is imposed.

Having established a negative answer to the question of whether a partial substitution of an ad valorem tax for a specific tax in a model of price uncertainty guarantees tax neutrality, it is natural to ask whether at least neutrality under *complete substitution* can be achieved in the presence of uncertainty. Even to this question we shall have to respond negatively.

Suppose neutrality under complete substitution is satisfied given the model  $(c, P, \mathbb{P})$ . Since  $R(s, 0) = sQ(s, 0)$  and  $R(0, \tau) = \tau PQ(0, \tau)$  for all  $s \in \mathbb{S}_0$  and  $\tau \in \mathbb{S}^0$ ,<sup>10</sup> an equivalent formulation of tax neutrality under complete substitution would be that

$$\forall \tau \in \mathbb{S}^0 \quad \forall s \in \mathbb{S}_0 \quad (Q(s, 0) = Q(0, \tau) \Leftrightarrow sQ(s, 0) = \tau PQ(0, \tau) \text{ a. s.}) \quad (4)$$

On the other hand, the arguments for tax neutrality under certainty (see Subsection 3.1) yield the equivalence

$$\forall \tau \in \mathbb{S}^0 \quad \forall s \in \mathbb{S}_0 \quad (Q(s, 0) = Q(0, \tau) \Leftrightarrow sQ(s, 0) = \tau \mu Q(0, \tau)) \quad (5)$$

(because by definition  $R(s, 0) = sQ(s, 0)$  and in a model without uncertainty we also obtain  $R(0, \tau) = \tau \mu Q(0, \tau)$ ).

Now fix an arbitrary  $\tau \in \mathbb{S}^0 = (-\infty, 1)$  and put  $s^\tau = \tau \mu \in \mathbb{S}_0$ . Then  $Q(s^\tau, 0) = Q(0, \tau)$ ,<sup>11</sup> and hence by equivalence (5),

$$s^\tau Q(s^\tau, 0) = \tau \mu Q(0, \tau).$$

However, assuming that neutrality under complete substitution is satisfied, equivalence (4) yields also  $s^\tau Q(s^\tau, 0) = \tau PQ(0, \tau)$  almost surely, whence

$$\tau \mu Q(0, \tau) = \tau PQ(0, \tau) \text{ a. s.}$$

<sup>10</sup>This follows directly from the definition  $R(s, \tau) = (\tau P + s)Q(s, \tau)$  (for all  $(s, \tau) \in \mathbb{S}$ ).

<sup>11</sup>Since  $c'$  is strictly monotonic (as  $c'' > 0$ ) and thus invertible, we obtain

$$Q(s^\tau, 0) = Q(0, \tau) \Leftrightarrow \underbrace{c'(Q(s^\tau, 0))}_{\mu - s^\tau} = \underbrace{c'(Q(0, \tau))}_{=(1-\tau)\mu} \Leftrightarrow s^\tau = \tau \mu.$$

Hence, if neutrality under complete substitution is satisfied, then for all  $\tau \in \mathbb{S}^0$ , either  $\tau = 0$  or  $Q(0, \tau) = 0$  or  $P = \mu$  almost surely. Thus, if neutrality under complete substitution were satisfied but not  $P = \mu$  almost surely, then for all  $\tau \in \mathbb{S}^0$  either  $\tau = 0$  or  $Q(0, \tau) = 0$ , so  $\mathbb{S}^0 \subseteq \left\{0, 1 - \frac{c'(0)}{\mu}\right\}$ ,<sup>12</sup> which leads to a contradiction since  $\mathbb{S}^0 = (-\infty, 1)$ .

Therefore, tax neutrality under complete substitution implies that  $P = \mu$  almost surely, in which case even tax neutrality in the sense of Definition 1 is satisfied. Hence, the price-affine taxation structure satisfies tax neutrality under complete substitution if, and only if,  $P = \mu$  almost surely.

**Theorem 2.** *The price-affine taxation structure satisfies tax neutrality under complete substitution given the model  $(c, P, \mathbb{P})$  if, and only if,  $P = \mu$  almost surely (on  $\Omega_0$ ), i. e. if there is essentially no uncertainty.*

According to Theorems 1 and 2, in a world of stochastic output prices a substitution of ad valorem taxes for specific taxes (or vice versa) such that a competitive firm's output decision is not affected (output neutrality) requires that the government has to bear more of the volatility of the output price: public revenue is subject to the realisation of the price process and the substitution thus has a stochastic effect on public revenue. As a consequence, a neutrality result does (almost surely) not hold.

#### 4. Conclusion

In this paper we have scrutinised the (non-)equivalence of ad valorem taxes and specific taxes on the output of a competitive firm. The starting point of our contribution is the well-known result that in a world without uncertainty, a competitive firm will not alter its output level (of a commodity of a given quality) in response to a balanced-budget substitution of an ad valorem tax for a specific tax (see Suits and Musgrave, 1953; Keen, 1998). While the overwhelming majority of contributions analysing the relative merits of ad valorem taxes and specific taxes is based on deterministic settings, Fraser (1985), Goerke (2011) and Kotsogiannis and Serfes (2012) have extended the framework to allow for price and cost uncertainty, respectively. Because uncertainty introduces volatility of public tax revenue, an appropriate concept of neutrality has to be specified. In this regard, the common approach is to apply the concept of a revenue neutrality in *expected terms*. Using

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<sup>12</sup>Since  $c'$  is invertible, the definition of  $Q$  yields  $Q(0, \tau) = 0 \Leftrightarrow (1 - \tau)\mu = c'(0)$ .

this, it is easy to show that for a competitive, risk-neutral firm that must decide on output volume before the realisation of the output price, a substitution of a specific tax for an ad valorem tax safeguards neutrality in the sense that it keeps expected tax revenue constant whenever expected output is kept constant (and vice versa).

In our analysis we employ exactly this framework of a competitive, risk neutral firm and maintain all assumptions underlying the standard neutrality prediction, except that we apply an alternative equivalence concept that is, from our point of view, more plausible: When a public budget must be met in any, or at least in any foreseeable, state of the world, holding *expected* levels of public revenue and output constant seems to be too weak a concept, as realised levels, tax proceeds and output, generically differ from their respective targeted levels (given the state of the world). This implies that neutrality in expected terms does not typically bring about neutrality with respect to realised (or actual) variables. Therefore, we employ a stricter neutrality concept requiring that for each possible state of the world, the proposed tax reform safeguards neutrality; that is, both tax systems bring about the same tax revenue *and* the same output decision in each state of the world. Assuming such *pathwise neutrality*, we show that a substitution of an ad valorem tax for a specific tax (or vice versa) will alter the path of a competitive firm's output decisions and of public tax proceeds such that the tax reform does not satisfy neutrality, except for the virtual absence of uncertainty — that is, if the price process is deterministic. We also demonstrate that even if we only allow for a complete substitution of either tax for the other, neutrality still cannot be attained. En route to this result we have also proved in a very general manner the well-known result of the irrelevance of the structure of commodity taxation in a world of (gross) price certainty.

The intuition behind the non-neutrality result, or the relevance of the tax structure in a setting with uncertainty, is that ad valorem taxes affect the volatility of the after-tax price, whereas specific taxes do not. As a consequence, any given tax structure will specifically divide the quantity effects of gross price uncertainty between the firm and the government. Any change in the structure of commodity taxation will alter this division, and in consequence, it is almost surely impossible to construct a tax reform which affects neither output choices nor tax revenue (for any possible state of the world).

Conceptually, our analysis has focused on the behaviour of a single competitive firm. In order to literally establish the non-equivalence of ad valorem and specific

commodity taxes for a competitive market, we have to acknowledge the behaviour of both the supply and the demand side along the equilibrium path for any proposed tax reform. However, since equivalence cannot even be obtained for a single competitive firm, we see no scope for attaining equivalence in this broader setting of a competitive market (except possibly for purely pathological cases). Also, since our results are basically determined by the requirement of pathwise equivalence, along with strict convexity of the profit function, and neither of these two assumptions is dependent on the (additional) hypothesis of the firm being a price-taker, we are quite confident that our findings generically apply to any non-competitive firm. In fact, this seems to be quite a straightforward result, given the multitude of non-neutrality results for imperfectly competitive settings under deterministic prices (see Keen, 1998). Finally, note that in order to derive our non-equivalence result, we do not need risk aversion; it fully suffices to assume price uncertainty along with our equivalence concept of pathwise neutrality. Thus, also in this regard our findings have wider implications, for there is apparently little scope for arriving at tax neutrality (in the sense of pathwise neutrality) if the firm is either risk averse or risk loving.

We also believe that the results derived here for a competitive firm, can be applied to other frameworks as well; for example, if the tax base of an ad valorem tax includes at least a fraction of the costs of inputs, such as it will be the case for a VAT, the findings for price uncertainty will basically apply to cost uncertainty, as well. Moreover, our results carry over to the context of income taxation: A stronger emphasis on wage-related components of income taxation and a reduced impact of hours-related aspects of taxation would be equivalent to a shift from specific to ad valorem taxation. Our findings indicate that a change in the composition of a given level of income taxation will alter labour supply behaviour and tax revenue differently if there is uncertainty about the gross wage. Furthermore, regulatory interventions are often discussed in the context of “prices versus quantities” (Weitzman, 1974). In the presence of uncertainty, price-related regulatory instruments are likely to have effects different from those of quantity-related tools. Dickie and Trandel (1996) have established such differential effect for the case of a negative production externality. In addition, Earle et al. (2007) clarify that price caps imposed on risk-neutral Cournot oligopolists in a setting with demand uncertainty are likely to result in equilibrium effects which are at variance with those in a deterministic world. — Based on these findings and our results, we surmise that

the non-equivalence of regulatory instruments due to uncertainty applies to a much wider range of settings.

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