

Schröder, Andreas; Schade, Christian

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# **Integrating a Behavioral Preference Calculus into a Simultaneous Market Entry Game: Analyses of Equilibria for Selected Cases of Prior Gain and Loss Experiences**

Andreas Schröder and Christian Schade

School of Business and Economics

Humboldt-Universität zu Berlin

Spandauer Str. 1

10178 Berlin

email:

[aschroed@wiwi.hu-berlin.de](mailto:aschroed@wiwi.hu-berlin.de)

[schade@wiwi.hu-berlin.de](mailto:schade@wiwi.hu-berlin.de)

**Preliminary Version - Comments Welcome**

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# 1 Introduction

Real decision makers have been argued to be influenced by prior loss or gain experiences, e.g. in stock investment decisions (Shefrin and Statman 1985) and in entrepreneurial decisions (Bowman 1982; Fiegenbaum and Thomas 1988; Fiegenbaum 1990; Wiemann and Mellewigt 1998). These studies deal with the fact that people may be risk taking after losses and risk averse after gains and relate these findings to prospect theory's value function (Kahneman and Tversky 1979; Tversky and Kahneman 1992) that is concave above and convex below certain reference points.

Although the exact influence of prior gains and losses may not always be consistent with the shape of prospect theory's value function (Thaler and Johnson 1990; Weber and Zuchel 2001; Schade, Steul, and Schröder 2002), it may at least be perceived as a more valid description of real decisions makers' behavior in a variety of decision situations than standard (von Neumann and Morgenstern 1947) or subjective expected utility theory (Savage 1957).

It is an open question, yet, how such behavior may influence the way people play games, however. In this paper, we provide first steps of a formal analysis of such behavior. Specifically, we are investigating the effects of all players obeying to prospect theory's value function on mixed equilibria for selected conditions in a symmetric simultaneous market entry game. Whereas simultaneous market entry games only have one equilibrium in pure strategies if the game is asymmetric with individual entry costs (Selten and Güth 1982), they have numerous asymmetric

equilibria in pure strategies if they are symmetric and one symmetric equilibrium in mixed strategies (see Rapoport 1995; Rapoport et al. 1998).

As we are going to show in this paper, there is a unique symmetric equilibrium in mixed strategies also if all players transform payoffs according to prospect theory's value function and a preference calculus proposed by Schade, Steul, and Schröder (2002) formalizing the ideas of earlier studies (see e.g. Shefrin and Statman 1985; Bowman 1982; Fiegenbaum and Thomas 1988; Wiemann and Mellewigt 1998) if they have identical starting points (i.e. identical gain or loss experience). We furthermore analyze the game for the case of three players and calculate the equilibrium points for the cases of symmetric as well as asymmetric starting points (i.e. for one player having a starting point differing from the others).

The paper proceeds as follows. In the next section the assumptions underlying our analysis will be specified, and we motivate the general structure of our model. In the third section, we define the transformed market entry game, i. e. the game integrating our behavioral assumptions. The subsequent section then represents a general, n-player proof of the existence, symmetry, and uniqueness of a mixed strategy equilibrium for the case of identical starting points. We then deal with the analysis of the three-player case. In the final two sections we will briefly discuss our findings and outline the implications for further research.

## 2 Assumptions and Motivation of the Model

We assume that all players obey to a preference calculus based on prospect theory's value function  $v_{PT} : \mathbb{R} \rightarrow \mathbb{R}$  in the parameterized version of Tversky and Kahneman (1992). The functional form for a median decision maker is based on a vast number of experiments on  $n = 1$  games:

$$(1) \quad v_{PT}(x) := \begin{cases} x^\alpha & \text{if } x \geq 0; \\ -\lambda(-x)^\beta & \text{if } x < 0, \end{cases}$$

where the parameters are  $\alpha \approx \beta \approx 0.88$  and  $\lambda \approx 2.25$ .

If players recently experienced a gain or a loss, they are in addition assumed to behave according to the following preference calculus proposed by Schade, Steul, and Schröder (2002):

$$(2) \quad \sigma_{x_0}(x) = v(x_0 + x) - v(x_0),$$

for a starting point  $x_0$ , a reference point of zero, potential future outcomes  $x$ , and  $v$  being prospect theory's value function.

Figure 1 demonstrates the effect of a starting point on the subsequent evaluation of payments. It is easy to see that the valuation of a subsequent outcome is dependent of prior gains or losses.

Note that we do not include probability weighting - also implied by (cumulative) prospect theory (Tversky and Kahneman 1992) - in our preference calculus (for consideration of non-additive beliefs in  $n$ -person games see f.e. Eichberger and Kelsey 2000). The analysis would become more complicated or even untractable. Expected payoff maximization with mixed strategies is defined on the basis of

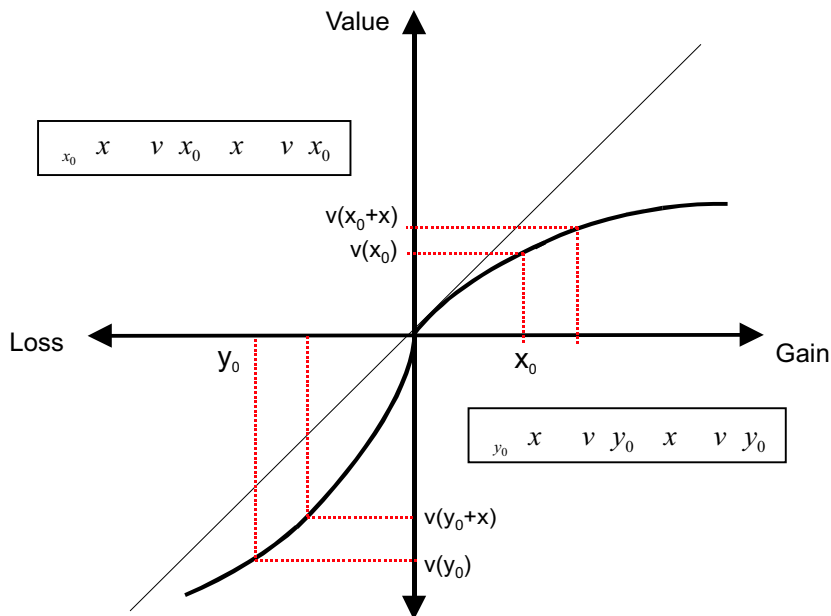


Figure 1: Valuation of an outcome  $x$  with prior loss and gain experiences on the basis of prospect theory's value function and the starting points' calculus

"regular", non-transformed probabilities in game theory.

More important, when integrating these behavioral concepts into a simultaneous market entry game, we will assume that all players obey to the functional form and the parameters valid for a median decision maker according to Tversky and Kahneman (1992), and that this is common knowledge; i. e. all players know that all others evaluate according to prospect theory and the above-specified preference calculus. That assumption is less restrictive than it may appear at first sight. It means nothing but an intuitive understanding of basic behavioral concepts by all players. Players assume that the others are playing in the same way they do themselves, that others are also loss averse, that they are tending to gamble on losses, and that gains make individuals more risk averse etc. The basic idea is the same when Rapoport (1995) assumes risk aversion - and common

knowledge about that - of all players in additional analyses of the simultaneous market entry game. Moreover, many contributions to auction theory operate this way when dealing with risk averse players.

### 3 Transforming a Simple Simultaneous Market Entry Game

Our analysis is based on a simplified version of a game introduced by Rapoport (1995) and Rapoport et al. (1998). In this  $n$ -person noncooperative game, each player  $i$  has the pure strategy set:

$$(3) \quad S_i = \{s_{i1}, s_{i2}\} \equiv \{1, 0\},$$

where 1 represents the decision to enter and 0 represents the decision not to enter the market. All players face the payoff function

$$(4) \quad M_i(s) := \begin{cases} K(c - m(s)) & \text{if } s_i = 1; \\ 0 & \text{if } s_i = 0. \end{cases}$$

For any strategy vector  $s = (s_1, \dots, s_n)$ ,  $s_i \in S_i$ ,  $m(s) := \sum_{i=1}^n s_i$  is the number of players entering the market ( $s_i = 1$ ). The value  $c$  symbolizes a certain limited market capacity:  $1 < c < n$ , and the parameter  $K \geq 1$  is a constant.

This game has the following equilibria in pure strategies: If  $c$  is an integer, then there are two possible types of equilibria. Either exactly  $c$  players enter the market (the number of equilibrium points in this case is  $\binom{n}{c}$ ). Or the number

of entering players is  $c - 1$ , with  $\binom{n}{c-1}$  equilibrium points. If  $c$  is not an integer, then the number of equilibria is  $\binom{n}{|c|}$ , with  $|c|$  entries, where  $|c|$  is the integral part of  $c$ .

The game has the following symmetric equilibrium  $\tilde{p}$  in mixed strategies:

$$\tilde{p} = \frac{c-1}{n-1},$$

with  $n \cdot \tilde{p}$  expected entries.

Assuming the preference calculus introduced in section 2 (formula (1) and (2)), we get the following game including the subjective payoff functions:

$$(5) \quad T_i(s) := \begin{cases} v_{PT}[x_i + K(c - m(s))] - v_{PT}[x_i] & \text{if } s_i = 1; \\ 0 & \text{if } s_i = 0. \end{cases}$$

A market entry game with payoff functions  $T_i$  ( $i = 1, \dots, n$ ) will be called a transformed game. Note that a game will also be called transformed in the following if subjective payoffs are - thus generalizing prospect theory's prediction - monotonically increasing functions of objective payoffs.

## 4 Equilibria of the Transformed Game

In this section, we analyze equilibria for selected cases of gain and loss experiences. We first of all provide a proof of the uniqueness of a symmetric mixed strategy equilibrium for the n-player case when gain or loss experiences are identical, i. e. when all players have made the same gain or loss experience. For the n-player



game, equilibria have to be determined numerically. We will introduce a method how to do so. We will then concentrate on a three-player game and also analyze the case of different gain and loss experiences. In the three-player case, equilibria can be determined analytically. On the basis of an analytical solution, we will demonstrate that resulting equilibria in general differ from the equilibria predicted by normative game theory. But they converge to the normative equilibrium if gain or loss experiences approach infinity.

In the following analyses we use the regular definition of a Nash equilibrium: Let  $T_i(q_i, p_{-i})$  be the payoff of player  $i$  from playing the mixed strategy  $q_i$  while each of the other players plays his components of his mixed strategy vector  $p := (p_1, \dots, p_n)$ . Then the vector of mixed strategies  $\tilde{p}$  is a Nash equilibrium, if for  $i = 1, \dots, n$  and all strategies  $q_i \in P_i$ , the payoff to  $i$  of playing  $q_i$ ,  $T_i(q_i, \tilde{p}_{-i})$ , does not exceed the return from playing  $\tilde{p}$ ,  $T_i(\tilde{p})$ :

$$(6) \quad T_i(\tilde{p}) \geq T_i(q_i, \tilde{p}_{-i}),$$

for all  $q_i \in P_i$ .

In the case of strategy sets with two elements, the equilibrium probabilities  $\tilde{p}_i$  are given by:

$$(7) \quad \begin{aligned} i) & \quad \text{if } \tilde{p}_i = p(s_{i1}) \Rightarrow q_i = 1 - p(s_{i1}), \\ ii) & \quad \text{if } \tilde{p}_i = 1 - p(s_{i1}) \Rightarrow q_i = p(s_{i1}). \end{aligned}$$

We characterize a mixed strategy equilibrium as follows: In a *mixed equilibrium point*  $\tilde{p}$ ,

$$(8) \quad \frac{\partial M_i(\tilde{p})}{\partial \tilde{p}_i} = 0,$$

for all  $i = 1, \dots, n$ , where  $\frac{\partial M_i(p)}{\partial p_i}$  is the partial derivative of the expected payoff function with respect to  $i$ 's mixed strategy.

Instead of directly investigating prospect theory's parameterized value function, we analyze the general case of a transformation of payoffs with monotonically increasing functions where  $v_{PT}$  is a special case. Obviously all analyses therefore also apply to  $v_{PT}$ . A map  $v : \mathbb{R} \rightarrow \mathbb{R}$  is called *strictly monotonic increasing*, if

$$(9) \quad v(a) < v(b) \Leftrightarrow a < b, \text{ for all } a, b \in \mathbb{R}.$$

If  $v$  is differentiable, then

$$(10) \quad v \text{ is strictly monotonic increasing} \Leftrightarrow v'(a) > 0, \text{ for all } a \in \mathbb{R};$$

$$v \text{ is strictly monotonic decreasing} \Leftrightarrow v'(a) < 0, \text{ for all } a \in \mathbb{R}.$$

## 4.1 Analyses for n-Player Game

### 4.1.1 Pure Strategies Equilibria

**Proposition 1** *The transformed market entry game has multiple pure strategy equilibria.*

**Proof:** We distinguish between two situations, depending on whether the capacity  $c$  is an integer or not.

First we study the situation, where  $c$  is an integer: Let  $\tilde{p}$  be a pure strategy equilibrium.

Then, every strategy vector  $\tilde{p} \in S$  is an equilibrium, if

$$(11) \quad m(\tilde{p}) = \sum_{i=1}^n \tilde{p}_i = c$$

or

$$(12) \quad m(\tilde{p}) = \sum_{i=1}^n \tilde{p}_i = c - 1.$$

With respect to equation (11), we have to distinguish between two cases:

*Case 1:* Let  $\tilde{p}_i = 0$ . Then player  $i$  has the payoff:

$$(13) \quad T_i(\tilde{p}) = 0.$$

The only pure strategy alternative is  $q_i = 1$  (see (7)). Here, the payoff would be

$$(14) \quad T_i(q_i, \tilde{p}_{-i}) = v[x_i + K(c - (c + 1))] - v[x_i] = v[x_i - K] - v[x_i] < 0,$$

because  $K \geq 1$  and  $v$  is a strictly increasing map. Condition (6) is satisfied

because

$$(15) \quad T_i(\tilde{p}) > T_i(q_i, \tilde{p}_{-i}).$$

*Case 2:* Let  $\tilde{p}_i = 1$ . Then player  $i$  has the payoff:

$$(16) \quad T_i(\tilde{p}) = v[x_i + K(c - c)] - v[x_i] = 0.$$

The only pure strategy alternative here is  $q_i = 0$  (see (7)). Playing this strategy implies

$$(17) \quad T_i(q_i, \tilde{p}_{-i}) = 0.$$

Condition (6) is again satisfied since

$$(18) \quad T_i(\tilde{p}) = T_i(q_i, \tilde{p}_{-i}).$$

With respect to equation (12), we again have to distinguish between two cases:

*Case 1:* Let  $\tilde{p}_i = 0$ . Then player  $i$  has the payoff:

$$(19) \quad T_i(\tilde{p}) = 0.$$

The only pure strategy alternative is  $q_i = 1$  (see (7)). This leads to the following payoff:

$$(20) \quad T_i(q_i, \tilde{p}_{-i}) = v[x_i + K(c - c)] - v[x_i] = 0.$$

Condition (6) is satisfied since

$$(21) \quad T_i(\tilde{p}) > T_i(q_i, \tilde{p}_{-i}).$$

*Case 2:* Let  $\tilde{p}_i = 1$ . Then player  $i$  has the payoff:

$$(22) \quad T_i(\tilde{p}) = v[x_i + K(c - (c - 1))] - v[x_i] > 0.$$

The only pure alternative is playing  $q_i = 0$  (see (7)). For the payoff we get

$$(23) \quad T_i(q_i, \tilde{p}_{-i}) = 0.$$

Condition (6) is again met since

$$(24) \quad T_i(\tilde{p}) > T_i(q_i, \tilde{p}_{-i}).$$

Now we look at all vectors  $q$  with  $m(q) > c$ . Then an integer  $d \geq 1$  exists so that  $m(q) = c + d$ . Thus there is a number  $d$  of players who have an incentive to stay out of the market, because for all players  $i$  with  $q_i = 1$

$$(25) \quad T_i(q_i, q) = v[x_i + K(c - (c + d))] - v[x_i] < 0 = T_i(p_i, q);$$

see also (9).

Now let  $m(q) < c - 1$ . Then for an integer  $e \geq 1$  we get  $m(q) = c - 1 - e$ .

Here,  $e$  players have an incentive to enter the market, because with  $q_i = 0$

$$(26) \quad T_i(q_i, q) = 0 < v[x_i + K(c - (c - 1 - e))] - v[x_i] = T_i(p_i, q).$$

Summarizing, strategy vectors' with  $m(q) > c$  and  $m(q) < c - 1$  cannot be equilibria in pure strategies. Altogether we therefore have  $\binom{n}{c}$  pure strategy vectors fulfilling (11) and  $\binom{n}{c-1}$  pure strategy vectors fulfilling (12).

Second, if  $c$  is not an integer, then  $|c|$  players enter the market, where  $|c|$  is the integral part of  $c$ . Here, we have  $\binom{n}{|c|}$  pure strategy equilibria. The proof is analogous to that for (12).

□

#### 4.1.2 Mixed Strategy Equilibria for Identical Starting Points

##### Existence and characteristics of a mixed strategy equilibrium

We study the situation where every player has the same prior gain or loss experience (in the following called "starting point")  $x := x_i \in \mathbb{R}$  ( $i = 1, \dots, n$ ).

Thus for the payoff functions we get:  $T_1 = T_2 = \dots = T_n := \varphi$  with  $\varphi = v(x + K(c - k)) - v(x)$  for  $k = 1, 2, \dots, n$ . Every  $n$ -person game with finite strategy sets has a mixed strategy equilibrium according to the theorem by John Nash (1950, pg. 288). If a game is symmetric, at least one symmetric mixed strategy

equilibrium must exist (Nash 1950, pg. 289).

In the following we are investigating such equilibria. Here, we use the description "mixed strategy equilibrium" if players actually randomize. First we analyze a three-player game. In a mixed strategy equilibrium holds (see also (29)):

$$q_2 q_3 \varphi(1) + (p_2 q_3 + q_2 p_3) \varphi(2) + p_2 p_3 \varphi(3) = 0,$$

$$q_1 q_3 \varphi(1) + (p_1 q_3 + q_1 p_3) \varphi(2) + p_1 p_3 \varphi(3) = 0,$$

$$q_1 q_2 \varphi(1) + (p_1 q_2 + q_1 p_2) \varphi(2) + p_1 p_2 \varphi(3) = 0,$$

with  $\varphi(k) = v(x + K(c - k)) - v(x)$  and  $q_i = 1 - p_i$  ( $i = 1, 2, 3$ ).

Now we compare the first with the second equation. Solving the first equation for  $p_2$ , we get

$$p_2 = \frac{-p_3 \varphi(2) - q_3 \varphi(1)}{p_3 \varphi(3) + q_3 \varphi(2) - q_3 \varphi(1) - p_3 \varphi(2)}$$

Solving the second equation for  $p_1$  we get

$$p_1 = \frac{-p_3 \varphi(2) - q_3 \varphi(1)}{p_3 \varphi(3) + q_3 \varphi(2) - q_3 \varphi(1) - p_3 \varphi(2)}$$

Thus

$$p_1 = p_2.$$

Now we compare the second with the third equation. Solving the second equation for  $p_3$  we get

$$p_3 = \frac{-q_1 \varphi(1) - p_1 \varphi(2)}{q_1 \varphi(2) + p_1 \varphi(3) - p_1 \varphi(2) - q_1 \varphi(1)}$$

Solving the third equation for  $p_2$  we get

$$p_2 = \frac{-q_1 \varphi(1) - p_1 \varphi(2)}{q_1 \varphi(2) + p_1 \varphi(3) - p_1 \varphi(2) - q_1 \varphi(1)}$$

Thus

$$p_2 = p_3.$$

Therefore

$$p_1 = p_2 = p_3.$$

In a four-player game

$$q_2 q_3 q_4 \varphi(1) + (p_2 q_3 q_4 + q_2 p_3 q_4 + q_2 q_3 p_4) \varphi(2) + (p_2 p_3 q_4 + q_2 p_3 p_4 + p_2 q_3 p_4) \varphi(3) + p_2 p_3 p_4 \varphi(4) = 0,$$

$$q_1 q_3 q_4 \varphi(1) + (p_1 q_3 q_4 + q_1 p_3 q_4 + q_1 q_3 p_4) \varphi(2) + (p_1 p_3 q_4 + q_1 p_3 p_4 + p_1 q_3 p_4) \varphi(3) + p_1 p_3 p_4 \varphi(4) = 0,$$

$$q_1 q_2 q_4 \varphi(1) + (p_1 q_2 q_4 + q_1 p_2 q_4 + q_1 q_2 p_4) \varphi(2) + (p_1 p_2 q_4 + q_1 p_2 p_4 + p_1 q_2 p_4) \varphi(3) + p_1 p_2 p_4 \varphi(4) = 0,$$

$$q_1 q_2 q_3 \varphi(1) + (p_1 q_2 q_3 + q_1 p_2 q_3 + q_1 q_2 p_3) \varphi(2) + (p_1 p_2 q_3 + q_1 p_2 p_3 + p_1 q_2 p_3) \varphi(3) + p_1 p_2 p_3 \varphi(4) = 0.$$

Again, from the first and second equation it follows that  $p_2 = p_3$ , from the second and third:  $p_3 = p_2$ , and from the third and fourth:  $p_4 = p_3$ . Thus

$$p_1 = p_2 = p_3 = p_4.$$

In an  $n$ -player game: Analyzing the structure of the terms containing the combinations of entry probabilities, we find that the  $t$ 's and  $(t + 1)$ 's equations only differ in the  $(t + 1)$ 's and  $t$ 's entry probabilities, respectively. Thus from the  $t$ 's and  $(t + 1)$ 's equations we conclude that

$$p_{t+1} = p_t,$$

for all  $t \in \{1, \dots, n\}$ , and therefore

$$p_1 = p_2 = \dots = p_n.$$

**Corollary 1** *Let  $\tilde{p} = (\tilde{p}_1, \dots, \tilde{p}_n)$  represent a mixed strategy equilibrium in a symmetric transformed market entry game. Then it must be symmetric, i. e.*

$$\tilde{p}_1 = \tilde{p}_2 = \dots = \tilde{p}_n.$$

### Uniqueness of a symmetric mixed strategy equilibrium

**Proposition 2** *The symmetric transformed market entry game has a unique symmetric mixed strategy equilibrium  $\tilde{p}$ .*

**Proof:** For pure strategies we demonstrated the existence of multiple asymmetric equilibria. (Alternatively one could show that all players entering the market is no equilibrium, and the same holds for none entering the market.) According to Corollary 1, we are thus allowed to conclude that a mixed strategy equilibrium must be symmetric.

We denote with  $P_i\{m_{-i}(s) = k\}$  the probability that  $k$  other players (not including the  $i$ 's player) are in the market.

For the expected payoff function of player  $i$  we get:

$$\begin{aligned} T_i(p) &= \sum_{k=0}^{n-1} p_i \cdot P_i\{m_{-i}(s) = k\} \cdot (v[x + K(c - (k + 1))] - v[x]) \\ (27) \quad &= p_i \cdot \sum_{k=0}^{n-1} P_i\{m_{-i}(s) = k\} \cdot (v[x + K(c - (k + 1))] - v[x]), \end{aligned}$$

for all  $i = 1, \dots, n$ . Furthermore, we define

$$(28) \quad \varphi(k) := v[x + K(c - k)] - v[x],$$

for  $k \in \{0, \dots, n\}$ . Then for the derivative of the  $i$ 's player's expected payoff



function and an equilibrium  $\tilde{p}$  the following must hold (see (8)):

$$(29) \quad \frac{\partial T_i(\tilde{p})}{\partial \tilde{p}_i} = \sum_{k=0}^{n-1} \tilde{P}_i\{m_{-i}(s) = k\} \cdot \varphi(k+1) = 0,$$

for all  $i = 1, \dots, n$ . The expression  $\tilde{P}_i\{m_{-i}(s) = k\}$  symbolizes the equilibrium probability that  $k$  players (not including the  $i$ 's player) enter the market. If  $\tilde{p}$  is a symmetric mixed strategy equilibrium point (see corollary 1), the following must hold:

$$(30) \quad \tilde{p}_1 = \tilde{p}_2 = \dots = \tilde{p}_n := \tilde{\pi}.$$

Thus the system implied by (29) is reduced to

$$(31) \quad \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k \cdot \varphi(k+1) = 0.$$

For further analyses we define a map  $\wp : \mathbb{R} \rightarrow \mathbb{R}$  with

$$(32) \quad \wp(\pi) = \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \pi)^{n-k-1} \pi^k \cdot \varphi(k+1).$$

Obviously a *zero*  $\tilde{\pi}$  of  $\wp$  in  $[0, 1]$  is a *Nash equilibrium*  $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n)$  with  $\tilde{p}_i = \tilde{\pi}$  for all  $i = 1, \dots, n$ . Thus we only have to show that the *polynomial*  $\wp$  has a *unique zero* in  $[0, 1]$ :

It is

$$(33) \quad \wp(0) = \varphi(1) = v[x + K(c-1)] - v[x] > 0,$$

because of (9) and for  $c > 1$ .

Also,

$$(34) \quad \wp(1) = \varphi(n) = v[x + K(c-n)] - v[x] < 0,$$

because of (9) and for  $c < n$ .

Obviously  $\varphi$  is *continuous* for arguments in  $[0, 1]$ . Thus we have to show that  $\varphi$  is *strictly monotonic decreasing*. Or in other words, we proof that the *criterion*  $\varphi'(\pi) < 0$  is fulfilled for all  $\pi \in [0, 1]$  (see also (10)), whereby  $\varphi'$  is the *derivative* of  $\varphi$ .

We get

$$(35) \quad \varphi'(\pi) = \sum_{k=0}^{n-1} \binom{n-1}{k} [k\pi^{k-1}(1-\pi)^{n-k-1} - \pi^k(n-k-1)(1-\pi)^{n-k-2}] \cdot \varphi(k+1).$$

In addition, the following equalities hold:

$$(36) \quad \binom{n-1}{k+1} (k+1) = \frac{(n-1)!}{(k+1)!(n-k-2)!} (k+1) = \frac{(n-1)!}{k!(n-k-2)!}$$

and

$$(37) \quad \binom{n-1}{k} (n-k-1) = \frac{(n-1)!}{k!(n-k-1)!} (n-k-1) = \frac{(n-1)!}{k!(n-k-2)!}.$$

From (35), (36), and (37) we get:

$$(38) \quad \varphi'(\pi) = \sum_{k=1}^{n-1} \binom{n-1}{k} k\pi^{k-1}(1-\pi)^{n-k-1} \cdot [\varphi(k+1) - \varphi(k)].$$

Because  $v$  is strictly monotonic increasing we get

$$(39) \quad \varphi(k+1) - \varphi(k) = v[x + K(c - (k+1))] - v[x + K(c - k)] < 0,$$

for all  $k = 1, \dots, n-1$  and  $1 < c < n$ . Thus  $\varphi'(\pi) < 0$  for all  $\pi \in [0, 1]$ . It follows that  $\varphi$  has a *unique zero* in  $(0, 1)$ .

□

## Remark

If we want to investigate monotonically decreasing transformations of the payoff function of the Rapoport Game, then the proofs for existence and uniqueness of a symmetric mixed strategy equilibrium can be carried out analogously: Here,  $\varphi(0) < 0$  and  $\varphi(1) > 0$ . Consequently, one would have to demonstrate that  $\varphi'$  is monotonically increasing, here.

### 4.1.3 Numerical Analysis

If we are interested in computing a symmetric Nash equilibrium, then we must find a  $\tilde{\pi}$  so that the following equation is fulfilled:

$$(40) \quad \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k \cdot \varphi(k+1) = 0,$$

with

$$(41) \quad \varphi(k+1) := v[x + K(c - (k+1))] - v[x].$$

In the special case  $v(y) := y$ , we have the situation of Rapoport's game in (4).

Here, an analytical solution of (40) is possible and validates Rapoport et al's findings:

$$(42) \quad \begin{aligned} & \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k \cdot K(c - (k+1)) \\ &= K(c-1) \cdot \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k - K \sum_{k=0}^{n-1} k \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k. \end{aligned}$$

Using the two equalities

$$(43) \quad \sum_{k=0}^{n-1} \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k = 1 \text{ (binomial formula)}$$

and

$$(44) \quad \sum_{k=0}^{n-1} k \binom{n-1}{k} (1 - \tilde{\pi})^{n-k-1} \tilde{\pi}^k = (n-1)\tilde{\pi} \text{ (binomial expected value),}$$

we get for (42):

$$(45) \quad K(c-1) - K(n-1)\tilde{\pi} = 0 \Rightarrow \tilde{\pi} = \frac{c-1}{n-1}.$$

For the transformation of payoffs via prospect theory's value function  $v_{PT}$  defined in (1) and according to the preference calculus introduced in (2), a Nash equilibrium may only be determined numerically. For this means, we have implemented a simple secant method (Judd 1998, pg. 158-159) in a java routine.

The calculations show that equilibrium probabilities are dependent of the parameter  $K$ , the capacity  $c$ , and also of the number of players  $n$ . For starting points  $x < 0$ , the entry probability is always larger, and for starting points  $x > 0$  always smaller than the normative solution  $\frac{c-1}{n-1}$ . A numerical sensitivity analysis also seems to allow for the tentative conclusion that for  $x \rightarrow -\infty$  and  $x \rightarrow +\infty$ , entry probabilities converge strictly monotonic to the normative equilibrium.

For the three-player game, this tentative conclusion can be analytically validated (see the next section).

## 4.2 Analytical Solutions for the Three-Player Game

The further analyses cannot be carried out for monotonically increasing functions in general, anymore. Instead, we have to be more restrictive and analyze convex and concave maps. Herewith, we are however still more general than prospect theory's value function.

A function  $v : \mathbb{R} \rightarrow \mathbb{R}$  will be called *strictly concave*, if for all  $a, b \in \mathbb{R}$  and for all  $\lambda \in (0, 1)$ ,

$$(46) \quad v((1 - \lambda)a + \lambda b) > (1 - \lambda)v(a) + \lambda v(b).$$

If for all  $a, b \in \mathbb{R}$  and for all  $\lambda \in (0, 1)$ ,

$$(47) \quad v((1 - \lambda)a + \lambda b) < (1 - \lambda)v(a) + \lambda v(b),$$

then we call  $v$  *strictly convex*.

If  $v : \mathbb{R} \rightarrow \mathbb{R}$  is *twice differentiable*, then we can characterize (46) and (47) as follows:

$$(48) \quad \text{Let } v''(x) < 0, \text{ then for all } x \in \mathbb{R} \Rightarrow v \text{ is strictly concave for } x \in \mathbb{R},$$

$$(49) \quad \text{Let } v''(x) > 0, \text{ for all } x \in \mathbb{R} \Rightarrow v \text{ is strictly convex of } x \in \mathbb{R}.$$

### 4.2.1 Basic Considerations

**Lemma 1** *Let  $v : \mathbb{R} \rightarrow \mathbb{R}$  be strictly monotonic increasing and  $\varepsilon > 0$ . If  $v$  is also strictly concave, then, for all  $x \in \mathbb{R}$*

$$(50) \quad \frac{v(x) - v(x - \varepsilon)}{v(x + \varepsilon) - v(x)} > 1 \text{ resp. } \frac{v(x - \varepsilon) - v(x)}{v(x + \varepsilon) - v(x)} < -1.$$

If  $v$  is also strictly convex, then, for all  $x \in \mathbb{R}$

$$(51) \quad \frac{v(x) - v(x - \varepsilon)}{v(x + \varepsilon) - v(x)} < 1 \text{ resp. } \frac{v(x - \varepsilon) - v(x)}{v(x + \varepsilon) - v(x)} > -1.$$

**Proof:** For  $a := x - \varepsilon$ ,  $b := x + \varepsilon$ , and  $\lambda := \frac{1}{2}$  we get:

$$(1 - \lambda)a + \lambda b = x.$$

Because of (46),

$$v(x) > \frac{1}{2}v(x - \varepsilon) + \frac{1}{2}v(x + \varepsilon)$$

or

$$(52) \quad 2v(x) > v(x - \varepsilon) + v(x + \varepsilon).$$

Subtracting  $v(x)$  and  $v(x - \varepsilon)$ , and dividing by the right-hand side of the inequation (possible since  $v$  is strictly monotonic increasing), results in (50).

The correctness of (51) can be shown accordingly.

□

**Lemma 2** Let  $v : \mathbb{R} \rightarrow \mathbb{R}$  be differentiable strictly monotonic increasing and  $\varepsilon > 0$ . Also let  $w : \mathbb{R} \rightarrow \mathbb{R}_+$  with

$$(53) \quad w(x) := \frac{v(x) - v(x - \varepsilon)}{v(x + \varepsilon) - v(x)}.$$

If  $v$  is differentiable and  $v'$  is strictly monotonic decreasing then  $w$  is strictly monotonic decreasing if and only if

$$(54) \quad \frac{v'(x) - v'(x - \varepsilon)}{v'(x + \varepsilon) - v'(x)} > w(x).$$

If  $v$  is differentiable and  $v'$  is strictly monotonic increasing then  $w$  is strictly monotonic decreasing if and only if

$$(55) \quad \frac{v'(x) - v'(x - \varepsilon)}{v'(x + \varepsilon) - v'(x)} < w(x).$$

**Proof:** We show, that the criterion  $w'(x) < 0$  (see (10)) is fulfilled:

$$(56) \quad w'(x) = \frac{[v'(x) - v'(x - \varepsilon)][v(x + \varepsilon) - v(x)] - [v(x) - v(x - \varepsilon)][v'(x + \varepsilon) - v'(x)]}{[v(x + \varepsilon) - v(x)]^2}.$$

Also:

$$(57) \quad w'(x) = \frac{v'(x) - v'(x - \varepsilon)}{v(x + \varepsilon) - v(x)} - w(x) \cdot \frac{v'(x + \varepsilon) - v'(x)}{v(x + \varepsilon) - v(x)}.$$

*Case 1:* If  $v'$  is strictly monotonic decreasing, then  $v'(x) - v'(x - \varepsilon) < 0$  and  $v'(x + \varepsilon) - v'(x) < 0$  and we have:

$$(58) \quad \begin{aligned} w'(x) &< 0 \\ &\Leftrightarrow \\ v'(x) - v'(x - \varepsilon) &< w(x) \cdot (v'(x + \varepsilon) - v'(x)) \\ &\Leftrightarrow \\ \frac{v'(x) - v'(x - \varepsilon)}{v'(x + \varepsilon) - v'(x)} &> w(x). \end{aligned}$$

*Case 2:* If  $v'$  is strictly monotonic increasing, then  $v'(x) - v'(x - \varepsilon) > 0$  and  $v'(x + \varepsilon) - v'(x) > 0$  and we have:

$$(59) \quad \begin{aligned} w'(x) &< 0 \\ &\Leftrightarrow \\ v'(x) - v'(x - \varepsilon) &< w(x) \cdot (v'(x + \varepsilon) - v'(x)) \\ &\Leftrightarrow \\ \frac{v'(x) - v'(x - \varepsilon)}{v'(x + \varepsilon) - v'(x)} &< w(x). \end{aligned}$$

□

**Lemma 3** *Let  $w : \mathbb{R} \rightarrow \mathbb{R}_+$  be strictly monotonic decreasing and  $w$  and  $\sqrt{w}$  differentiable. Then the map  $u : \mathbb{R} \rightarrow \mathbb{R}$  with*

$$(60) \quad u(x) := \frac{1 - \sqrt{w(x)}}{1 - w(x)}$$

*is strictly increasing.*

**Proof:** We show, that  $u'(x) > 0$  is satisfied (see (10)):

$$(61) \quad \begin{aligned} u'(x) &= \frac{[-\frac{1}{2}w(x)^{-\frac{1}{2}} \cdot w'(x)][1 - w(x)] - [1 - \sqrt{w(x)}][-w'(x)]}{[1 - w(x)]^2} \\ &= \frac{-w'(x)}{2 \cdot \sqrt{w(x)}[1 - w(x)]^2} \cdot [\sqrt{w(x)} - 1]^2 > 0, \end{aligned}$$

because  $w'(x) < 0$ .

□

#### 4.2.2 Equilibria for Identical Starting Points

**Proposition 3** *Let  $c = 2$ ,  $K = 1$ , and prospect theory's value function represent the subjective transformation of payoffs. Then the transformed three-player's market entry has the following symmetric mixed equilibrium:*

$$(62) \quad \tilde{\pi} = \frac{1 - \sqrt{-\alpha}}{1 + \alpha},$$

*with*

$$(63) \quad \alpha = \frac{v_{PT}(x - 1) - v_{PT}(x)}{v_{PT}(x + 1) - v_{PT}(x)}.$$



**Proof:** Because of proposition 2 and the respective proof, the three-players market entry game equilibrium  $\tilde{\pi}$  is the (unique) solution in  $(0, 1)$  of the polynome

$$(64) \quad (1 - \pi)^2(v_{PT}(x + 1) - v_{PT}(x)) + \pi^2(v_{PT}(x - 1) - v_{PT}(x)) = 0.$$

The candidates for such a zero are:

$$(65) \quad \tilde{\pi}_1 = \frac{1 + \sqrt{-\alpha}}{1 + \alpha} \text{ and } \tilde{\pi}_2 = \frac{1 - \sqrt{-\alpha}}{1 + \alpha},$$

with

$$(66) \quad \alpha := \frac{v_{PT}(x - 1) - v_{PT}(x)}{v_{PT}(x + 1) - v_{PT}(x)}.$$

The expression  $\alpha$  and thus the zero is dependent on whether the convex or concave "part" of  $v_{PT}$  is active. For the sake of simplicity, we only analyze situations where either the convex or the concave part of  $v_{PT}$  is "active", or where  $x \equiv 0$ ; this covers starting points  $x \in (-\infty, -1] \cup [1, +\infty) \cup \{0\}$ .

*Case 1:* Let  $v_{PT}$  be strictly concave. Then from (50) in Lemma 1 we get  $\alpha < -1$  or  $1 + \alpha < 0$ . Moreover  $1 + \sqrt{-\alpha} > 0$  and  $1 - \sqrt{-\alpha} < 0$ . Thus  $\tilde{\pi}_2$  is the zero.

*Case 2:* Let  $v_{PT}$  be strictly convex. Then from (50) in Lemma 1 follows  $-1 < \alpha < 0$  or  $0 < 1 + \alpha < 1$ . Moreover  $1 + \sqrt{-\alpha} > 1$  and  $\tilde{\pi}_1 > 1$ . Thus  $\tilde{\pi}_2$  is the zero also in this case.

*Case 3:*  $x \equiv 0$ . Then  $\alpha = -2.25$ ; this implies that the third case is a special case of the second.

□

### Remark

Note that the equilibrium reported in (65) and (66) differs from the equilibrium reported in section 3 (see (45)). Thus our analysis leads to equilibrium entry probabilities of our players that differ from the normative game. Consider e.g. the case where  $x$  is equal to zero: The entry probability  $\tilde{p}_i$  in the normative game that can be derived from (45) will always be 0.5 independent of  $x$ . But in the transformed game ((65), (66)),  $\tilde{\pi}_i = 0.4$  ( $i = 1, 2, 3$ ). The expected number of entries in the normative case is therefore 1.5 - already only 75% of market capacity -, but even reduces to 1.2 or 60% in the transformed game.

Interpretation: If players use prospect theory's value function instead of a linear utility model, entry probabilities are reduced even if the starting point is equal to reference point.

**Proposition 4** *Let  $c = 2$  and  $K = 1$ . Then for the Nash equilibrium  $\tilde{\pi}$  of a three-players market entry game we have: For*

$$(67) \quad \tilde{\pi} = \frac{1 - \sqrt{w(x)}}{1 - w(x)},$$

with

$$(68) \quad w(x) := \frac{v_{PT}(x) - v_{PT}(x-1)}{v_{PT}(x+1) - v_{PT}(x)},$$

we have

$$(69) \quad \lim_{x \rightarrow +\infty} u(x) = \frac{1}{2} = \lim_{x \rightarrow -\infty} u(x).$$

Also is

$$(70) \quad u(x) := \frac{1 - \sqrt{w(x)}}{1 - w(x)},$$

strictly monotonic increasing for  $x \in (-\infty, -1) \cup (+1, +\infty)$ .

**Proof:** First we show the identities in (69). Therefore we analyze the limits

$\lim_{x \rightarrow +\infty} w(x)$  and  $\lim_{x \rightarrow -\infty} w(x)$  (for  $v_{PT}$  and  $\beta \in (0, 1)$ ):

$$(71) \quad \lim_{x \rightarrow +\infty} w(x) = \lim_{x \rightarrow +\infty} \frac{x^\beta - (x-1)^\beta}{(x+1)^\beta - x^\beta} = \lim_{x \rightarrow +\infty} \frac{1 - (\frac{x-1}{x})^\beta}{1 - (\frac{x}{x+1})^\beta}.$$

Because

$$(72) \quad \lim_{x \rightarrow +\infty} 1 - \left(\frac{x-1}{x}\right)^\beta = \lim_{x \rightarrow +\infty} 1 - \left(\frac{x}{x+1}\right)^\beta = 0,$$

we can utilize L'Hospital's theorem. This implies that

$$(73) \quad \lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow +\infty} \frac{f'(x)}{g'(x)}$$

with

$$f(x) := 1 - \left(\frac{x-1}{x}\right)^\beta$$

and

$$g(x) := 1 - \left(\frac{x}{x+1}\right)^\beta.$$

Thus

$$(74) \quad \lim_{x \rightarrow +\infty} w(x) = \lim_{x \rightarrow +\infty} \frac{-\beta \left(\frac{x-1}{x}\right)^{\beta-1} (-(x+1)^2)(x-1)}{-\beta \left(\frac{x}{x+1}\right)^{\beta-1} (-x^2)x} = 1.$$

We also have

$$(75) \quad \lim_{x \rightarrow -\infty} w(x) = \lim_{x \rightarrow -\infty} \frac{-\lambda [(-x)^\beta - (-(x-1))^\beta]}{-\lambda [(-(x+1))^\beta - (-x)^\beta]} = \lim_{x \rightarrow +\infty} \frac{1 - \left(\frac{x+1}{x}\right)^\beta}{1 - \left(\frac{x}{x-1}\right)^\beta}.$$

Since

$$(76) \quad \lim_{x \rightarrow +\infty} 1 - \left(\frac{x+1}{x}\right)^\beta = \lim_{x \rightarrow +\infty} 1 - \left(\frac{x}{x-1}\right)^\beta = 0,$$

we can use L'Hospital's theorem:

$$(77) \quad \lim_{x \rightarrow -\infty} w(x) = \lim_{x \rightarrow +\infty} w(-x) = \lim_{x \rightarrow +\infty} \frac{-\beta\left(\frac{x+1}{x}\right)^{\beta-1}(-(x-1)^2)(x+1)}{-\beta\left(\frac{x}{x-1}\right)^{\beta-1}(-x^2)x} = 1.$$

Now we calculate  $\lim_{x \rightarrow +\infty} u(x)$  and  $\lim_{x \rightarrow -\infty} u(x)$ . As in (74), we have

$$(78) \quad \lim_{x \rightarrow +/\infty} 1 - w(x) = \lim_{x \rightarrow +/\infty} 1 - \sqrt{w(x)}.$$

Therefore the suppositions of L'Hospital's theorem are satisfied. We get:

$$(79) \quad \lim_{x \rightarrow +/\infty} u(x) = \lim_{x \rightarrow +/\infty} \frac{1}{2} \frac{w'(x)}{\sqrt{w(x)}w'(x)} = \frac{1}{2}.$$

Thus the statement in (69) is proofed.

For  $x \geq 0$ ,  $v_{PT}$  is defined as:

$$v_{PT}(x) = x^\beta,$$

with  $\beta \approx 0.88$  (see also (1)). For the derivatives we get:

$$v'_{PT}(x) = \beta x^{\beta-1} > 0, v''_{PT}(x) = \beta(\beta-1)x^{\beta-2} < 0.$$

Therefore  $v$  is strictly monotonic increasing and  $v'$  strictly monotonic decreasing.

Because of Lemma 2 we must show, that:

$$\frac{v'_{PT}(x) - v'_{PT}(x - \varepsilon)}{v'_{PT}(x + \varepsilon) - v'_{PT}(x)} > w(x).$$

Also we have to show, that:

$$\frac{\beta \cdot (x^{\beta-1} - (x-1)^{\beta-1})}{\beta \cdot ((x+1)^{\beta-1} - x^{\beta-1})} > \frac{(x-1)x^\beta - x(x-1)^\beta}{x(x+1)^\beta - (x+1)x^\beta}.$$

This is equivalent to

$$\frac{(x-1)x^\beta - x(x-1)^\beta}{x(x+1)^\beta - (x+1)x^\beta} > \frac{x^{\beta+1}(x-1) - x(x-1)^{\beta+1}}{(x+1)^{\beta+1}x - x^{\beta+1}(x+1)}.$$

Because the monotony condition of  $v'_{PT}$  this inequality is fulfilled if

$$((x-1)x^\beta - x(x-1)^\beta)((x+1)^{\beta+1}x - x^{\beta+1}(x+1)) < (x^{\beta+1}(x-1) - x(x-1)^{\beta+1})(x(x+1)^\beta - (x+1)x^\beta)$$

This condition can be rewritten to

$$(x-1)^{1-\beta} < x^{1-\beta},$$

and the correctness of the inequality is obvious. Thus it follows by Lemma 3 that  $u$  is strictly monotonic increasing.

The case  $v_{PT}(x) = -\lambda \cdot (-x)^\beta$  for  $x < 0$  can be shown accordingly.

□

Figure 2 represents the real valued function

$$(80) \quad \varphi(x) := \frac{1 - \sqrt{-\alpha}}{1 + \alpha},$$

where  $x$  are variable (but for all players identical) starting points and  $\varphi$  describes the variation of equilibrium entry probabilities of all players if all starting points are symmetrically varied.

Interpretation: According to figure 2, equilibrium entry probabilities first of all differ between the situations of negative and positive starting points (of all players). As could be expected, entry probabilities are lower for positive rather than negative starting points since players are risk averse above and risk seeking below their reference point.

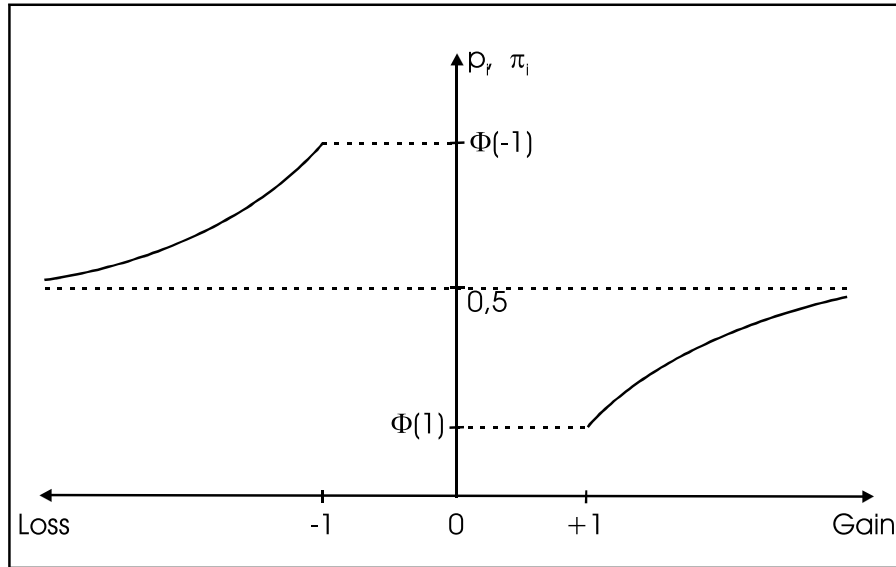


Figure 2: Entry probabilities with varied identical starting points in the transformed game

Moving the starting points of all players upwards leads to an increase in the entry probability that eventually is equal to the normative solution for positive infinity. Moving the starting points of all players downwards leads to a decrease in the entry probability that again comes out to be equal to the normative solution for negative infinity. Both results are intuitive, since players may want to adjust their entry probabilities downwards (upwards) if not only their own starting situation is getting worse (better) but the same applies to all others who in turn also have stronger (smaller) incentives to enter (before considering the incentives of the others).

### 4.2.3 Equilibria for Different Starting Points

In this section we again restrict our analyses to starting points  $x, y \in (-\infty, -1] \cup [1, +\infty) \cup \{0\}$ .

**Proposition 5** *Let  $x_1 = x_2 := y$ ,  $x_3 := x$  and  $c = 2$ . Then the transformed three-player market entry game has a unique mixed strategy equilibrium  $\tilde{\pi}$  with*

$$(83) \quad \tilde{\pi}_1 = \tilde{\pi}_2 = \frac{1 - \sqrt{-\alpha}}{1 + \alpha},$$

for

$$(84) \quad \alpha = \frac{v_{PT}(x-1) - v_{PT}(x)}{v_{PT}(x+1) - v_{PT}(x)},$$

$$(85) \quad \tilde{\pi}_3 = \frac{1}{1 + \left(\frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}\right)\alpha_y},$$

with

$$(86) \quad \alpha_y = \frac{v_{PT}(y) - v_{PT}(y-1)}{v_{PT}(y+1) - v_{PT}(y)}.$$

**Proof:** Let  $\tilde{\pi} = (\tilde{\pi}_1, \tilde{\pi}_2, \tilde{\pi}_3)$  be a mixed equilibrium point. Then for the partial derivatives of expected payoff functions of the  $i$ 's player ( $i = 1, 2, 3$ ) (see also (8) and (29)) the following must be fulfilled:

$$(87) \quad \frac{\partial T_i(\tilde{\pi})}{\partial \tilde{\pi}_i} = 0$$

Therefore

$$(88) \quad (1 - \tilde{\pi}_2)(1 - \tilde{\pi}_3)(v_{PT}(y+1) - v_{PT}(y)) + \tilde{\pi}_2\tilde{\pi}_3(v_{PT}(y-1) - v_{PT}(y)) = 0,$$

$$(89) \quad (1 - \tilde{\pi}_1)(1 - \tilde{\pi}_3)(v_{PT}(y+1) - v_{PT}(y)) + \tilde{\pi}_1\tilde{\pi}_3(v_{PT}(y-1) - v_{PT}(y)) = 0,$$

$$(90) \quad (1 - \tilde{\pi}_1)(1 - \tilde{\pi}_2)(v_{PT}(x+1) - v_{PT}(x)) + \tilde{\pi}_1\tilde{\pi}_2(v_{PT}(x-1) - v_{PT}(x)) = 0.$$

From (88) and (89) we directly get

$$(91) \quad \tilde{\pi}_1 = \tilde{\pi}_2.$$

Using this result in (90) we get

$$(92) \quad \tilde{\pi}_1 = \tilde{\pi}_2 = \frac{1 - \sqrt{-\alpha}}{1 + \alpha},$$

with

$$(93) \quad \alpha = \frac{v_{PT}(x-1) - v_{PT}(x)}{v_{PT}(x+1) - v_{PT}(x)}.$$

(The other zero is not valid, see the proof for Proposition 3.) Using (92) and (93)

for (89) and (87), respectively, we get:

$$(94) \quad \tilde{\pi}_3 = \frac{1}{1 + \left(\frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}\right)\alpha_y},$$

with

$$(95) \quad \alpha_y = \frac{v_{PT}(y) - v_{PT}(y-1)}{v_{PT}(y+1) - v_{PT}(y)}.$$

□

We now take a closer look at the situation implied by Proposition 5. We define

$$(96) \quad \varphi_y(x) = \frac{1}{1 + \left(\frac{\tilde{\pi}_1(x)}{1 - \tilde{\pi}_1(x)}\right)\alpha_y},$$

describing the equilibrium entry probability  $\tilde{\pi}_3$  if  $y$  is fixed and  $x$  is variable, and

$$(97) \quad \varphi_x(y) = \frac{1}{1 + \left(\frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}\right)\alpha_y(y)},$$

describing the entry probability  $\tilde{\pi}_3$  if  $x$  is fixed and  $y$  is variable.



**Proposition 6** *Let  $x_1 = x_2 := y$ ,  $x_3 := x$  and  $c = 2$ . Then the identities*

$$(98) \quad \lim_{x \rightarrow +\infty} \varphi_y(x) = \frac{1}{1 + \alpha_y} = \lim_{x \rightarrow -\infty} \varphi_y(x),$$

where  $\alpha_y$  is given in (86), and

$$(99) \quad \lim_{y \rightarrow +\infty} \varphi_x(y) = \frac{1}{1 + \frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}} = \lim_{y \rightarrow -\infty} \varphi_x(y),$$

where  $\tilde{\pi}_1$  is given in (83), respectively, hold.

**Proof:** The statement in (98) obviously follows from (69) in Proposition 4 and the statement in (99) is the result of the limit analysis in (74) and (77) (see Proof of Proposition 4).

□

Because the monotony properties of  $\varphi_y(x)$  and  $\varphi_x(y)$  induced by the monotony properties of  $u$  and  $w$  in Proposition 4 and because  $\varphi_y(x)$  and  $\varphi_x(y)$  are continuous, we conclude:

**Corollary 2** *The functions defined in (96) and (97) are monotone for  $x, y \in (-\infty, -1) \cup (+1, +\infty)$  and we have:*

$$(100) \quad \varphi_x(y) > \frac{1}{1 + \frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}}, \text{ if } y < -1 \text{ and } \varphi_x(y) < \frac{1}{1 + \frac{\tilde{\pi}_1}{1 - \tilde{\pi}_1}}, \text{ if } y > +1$$

and

$$(101) \quad \varphi_y(x) < \frac{1}{1 + \alpha_y}, \text{ if } x < -1 \text{ and } \varphi_y(x) > \frac{1}{1 + \alpha_y}, \text{ if } x > +1$$

To interpret these results, we analyze a special case where  $x$  is fixed and equal to 0. Here, 0.6 is the limit for the entry probability  $\pi_3 := \tilde{\pi}_3$  if  $y$  is varied up to

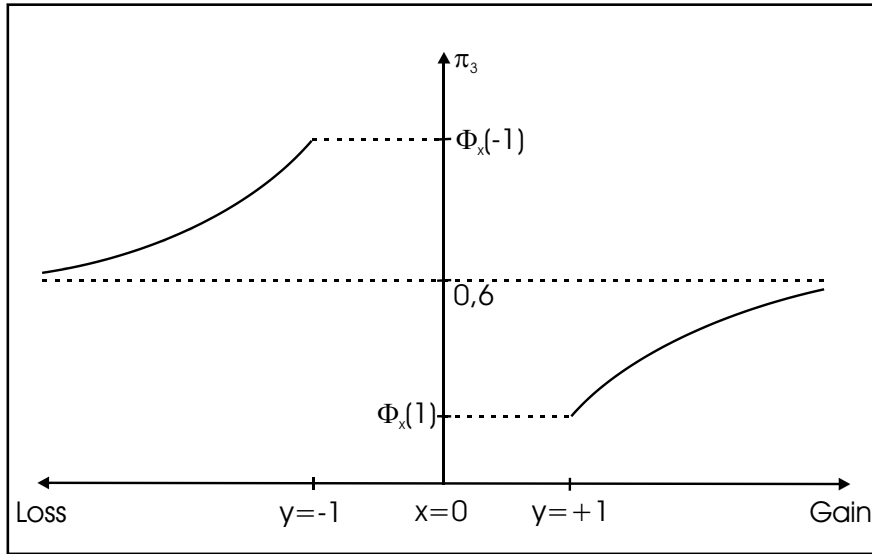


Figure 3: Entry probability  $\pi_3$  with fixed  $x$  and varied starting points  $y$  positive and down to negative infinity. A good intuition for our findings may be gained from figure 3.

Interpretation: We hold the starting point of the third player constant and equal to 0 and vary the starting points of the two others. We look at the entry probability of the third. If the starting point values of the two first players are small and positive, the entry probability of the third player is smaller as compared to the situation when the starting point values of the two first players are large and positive.

If the starting point values of the two first players are small and negative, however, the entry probability is higher as compared to the situation when they are large and negative.

In the situation described in (101), the value of  $y$  is fixed and  $x$  is varied. Within the loss area  $\pi_3$  converges from below and within the gain area from

above, both against the limit value 0.302. Additionally, we found out that within the loss area an increasing  $\pi_3$  corresponds with a decreasing  $\pi_1 (= \pi_2 := \tilde{\pi}_1)$ . Within the gain area the reverse holds.

## 5 Discussion

The integration of behavioral concepts into the simultaneous market entry game leads to unique equilibria in mixed strategies for the cases analyzed, here. Not surprisingly, equilibrium entry probabilities differ between the normative game and the behavioral game as has been demonstrated in the tree-player case. For a potentially more accurate prediction of real players' behavior - given that the equilibrium concept itself holds in reality - it may therefore be essential to take into account behavioral phenomena such as loss aversion and gambling on losses. Interesting phenomena also arise on the aggregated market level. Here, expected total entry is smaller in the behavioral rather than the normative game already without any prior gain or loss experience. The loss aversion feature of prospect theory's weighting function seems to deter individuals from entry.

## 6 Implications and Further Research

Assuming that behavioral assumptions are common knowledge and keeping the general framework of game theory - i. e. equilibrium analysis - seems to open the door for an integration of behavioral concepts into  $n > 1$  games. More behavioral concepts, more normatively irrelevant drivers of real decision makers'

behavior, and more games may be analyzed using this approach. This makes it feasible to test behavioral assumptions in laboratory experiments on  $n > 1$  games. Experimental economics may benefit from this approach since normative and descriptive predictions may be compared. The first step in this research should be a replication of a (slightly modified) Rapoport (1995) simultaneous market entry game with different starting conditions (loss or gain experiences) manipulated. The entry probabilities should be measured according to a study of Anderhub et al (1999).

Further research on the simultaneous market entry game should address the uniqueness of a mixed strategy equilibrium for the  $n$ -player case with asymmetric starting conditions.

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