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Did sunspot cause the Great Depression?

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Did Sunspot Forces Cause the Great Depression?*

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Abstract

We apply a dynamic general equilibrium model to the period of the Great Depression. In particular, we examine a modification of the real business cycle model in which the possibility of indeterminacy of equilibria arises. In other words, agents' self-fulfilling expectations can serve as a primary impulse behind fluctuations. We find that the model, driven only by these measured sunspot shocks, can explain well the entire *Depression era*. That is, the decline from 1929-1932, the subsequent slow recovery, and the recession that occurred in 1937-1938.

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1 Introduction

There has been a recent resurgence in interest among macroeconomists in the Great Depression. Perhaps because of the recent events in the U.S. economy – the record-long boom followed by what may now have become a global recession – curiosity about this unique episode has piqued. Of particular relevance to us is the application of neoclassical modelling techniques, which had previously only been applied to postwar episodes, to this historic period. For example, Cole and Ohanian (1999a) examine the efficacy of the real business cycle model at explaining both the decline from 1929 to 1933 and the subsequent slow recovery throughout the rest of the decade. While their model explains 40% of the decline, they are left with the puzzle of how to explain the weakness of the U.S. recovery. They suggest that some other type of shock must be responsible.

In this paper we identify such a shock. In particular, we examine a modification of the real business cycle model in which the possibility of indeterminacy of equilibria arises. The indeterminacy arises when, in the presence of relatively low increasing returns to scale in production, changes in agents' expectations are self-fulfilling and therefore serve as a primary impulse behind fluctuations. We find that such a model, driven only by these measured sunspot shocks, can explain well the entire *Depression era*. That is, the decline and subsequent slow recovery as well as the recession that occurred in 1937-1938. Because of this, we believe that we have found the “other shock.”

Most popular theories of the Great Depression stem from Friedman and Schwartz' (1963) monumental work blaming inept monetary policy, or reprehend bank failures for deteriorating the effectiveness of financial intermediaries (Bernanke, 1983). These findings are related to the often stressed viewpoint that the United States' adherence to the Gold Standard was a crucial element of the economic decline (Eichengreen, 1992). In more recent work, Bordo, Erceg and Evans (2000) identify a sequence of negative shocks to money growth dating from 1930:I to 1933:I, which coincides with the U.S. administration completely abandoning the Gold Standard in April of 1933. They evaluate these shocks in a model with nominal wage stickiness and find that these money growth innovations help to explain a large share of the decline in output experienced over that period.

However, both Bordo et al. (2000) and Cole and Ohanian (1999a) find that their models predict a swift recovery as well, when in fact output stayed depressed for the complete decade: per capita output still remained more

than twenty percent below trend in 1939. In Bordo et al. (2000), expansionary policies by the Federal Reserve, in which money supply grew at spectacular rates after 1932, induce a quickly rebounding economy. In Cole and Ohanian (1999a), it is total factor productivity that started to return to trend very quickly.

This all suggests that important nonmonetary, domestic forces kept the economy off track. Correspondingly, Bordo et al. (2000) and Cole and Ohanian (2001) shift attention to New Deal labor policy that facilitated inflating real wages. Still, Cole and Ohanian's (2001) technology-driven cartel-model closes the reported gap between the perfect markets real business cycle model and U.S. output by only a half. Perhaps even more important, it appears to miss the 1937-1938 recession – the third largest recession in American history in terms of output loss – altogether.¹

Here, we look towards shocks to confidence as an alternative explanation for the entire Depression era, as we have defined it above. Of course, ours is not the first approach that highlights the effects of changes in confidence during the Depression. For example, Temin (1976) emphasizes a sudden contraction of aggregate demand after 1929. In conventional Keynesian jargon, he classifies this drop as a collapse of autonomous spending. Romer (1990) picks up on his observation and reports an increasing state of uncertainty following the October 1929 stock market crash. Indeed, she finds that this uncertainty led to delaying expenditures on durable goods. Both Temin (1976) and Romer (1993) note that expectations turn from uncertain to pessimistic during 1930. Temin writes:

“Sometime in the fall of 1930, then, businessmen became convinced that prosperity was no longer just around the corner. The timing of this change is not known with precision, but [...] it would appear that businessmen's and probably also consumers' expectations built up during the 1920s about the normal state of business activity were not shattered immediately by the stock-market crash; they only dissolved a year after the crash.” [Temin, 1976, p. 79]

Moreover, Simons (1948) places great emphasis on the state of business confidence in explaining the Great Depression. Such evidence can also be found outside of the academic literature in economics. As exemplified in

¹Cole and Ohanian also do not provide a theory for the years 1929 to 1934 but rather simply calibrate their model to be 24 percent below its steady state in 1934.

the following quote from *Business Week*, the public recognizes depressed expectations by as early as the Spring of 1930:

“Business is now suffering chiefly from a pain in the expectations.” [*Business Week*, May 14, 1930, p. 1]

We take these ideas a step further and propose that changes in nonfundamental confidence were the driving force behind all three events of the Great Depression era. That is, we hypothesize that the deep and prolonged extrinsic pessimism of agents can be explained by factors unrelated to fundamentals. In support of this idea, Hart (1933), a contemporary sociologist concludes:

“[t]hat the depression has been fundamentally a psychological phenomenon has been reiterated at intervals in various magazines.” [Hart, 1933, p. 677]

We propose, as did Temin, that the delayed fall in confidence caused what might have been a typical recession to worsen. In order to test this hypothesis, we apply the notion that *animal spirits* cause swings in economic activity by confronting a general equilibrium model to the Great Depression. In particular, we follow recent research in macroeconomics that focuses on models in which business cycles are driven by self-fulfilling changes in agents’ beliefs.² In such models, a continuum of rational expectations equilibria, indexed by these beliefs, are possible. In the model we study, with varying capital utilization and externalities in production, indeterminacy obtains at empirically reasonable departures from constant returns.

To assess the accuracy of the model at replicating the facts, we feed in a series of sunspot shocks that we find to best reflect the behavior of nonfundamental confidence during the Great Depression era. As indices of confidence are not available for this period, we argue that an interest rate spread, which widens when a recession is expected, serves as a proxy. We then construct a vector autoregression model (VAR) in which the residual from a regression of the spread on fundamentals is taken to measure nonfundamental confidence. Given these shocks, we find that our theory can account for important historical facts. In particular, our model predicts a fall in confidence starting in 1930, followed by a drastic decline in output.

²Early work includes Woodford (1991); and Benhabib and Farmer (1999) offer an excellent survey of the literature.

That is, in 1929 the economy appears to have been in a typical recession, and the fall in confidence caused the recession to deepen into a depression. The model also outperforms those in previously cited work by replicating well the recovery and the 1937-1938 recession.

Perhaps the most closely related work is that of Cooper and Ejarque (1995) and Cooper and Corbae (2000), who analyze the Great Depression from the standpoint of a monetary economy with increasing returns in the intermediation process. These scale economies imply multiple equilibria; and confidence in the financial system determines which of the solutions is realized. These authors find that nonfundamental shocks may have played an important role, particularly in connection with bank panics. However, the final banking crisis took place in January 1933. Thus their models also do not provide much insight for the slowness of the recovery. Furthermore, the above papers neither estimate sunspots nor do they simulate the model.

The rest of this paper proceeds as follows. In section 2 we outline the model. In sections 3 and 4 we compute the sunspot shocks and feed them into the model. Section 5 examines the robustness of our results and section 6 concludes.

2 The model

In this section we present the model, discuss the calibration we use, and briefly report on qualitative dynamics. The model is based on Greenwood, Hercowitz and Huffman (1988) and Wen (1998). It is a standard one-sector dynamic general equilibrium model with variable capital utilization and production externalities.³ Given a certain degree of externalities, the equilibrium of the model is indeterminate and the economy is subject to extrinsic uncertainty. We assume that the economy is populated by a large number of identical consumer-worker households, each of which lives forever. The problem faced by a representative household is

$$\begin{aligned} \max_{\{c_t, l_t, u_t, k_{t+1}\}} \quad & E \sum_{t=0}^{\infty} \beta^t [(1 - \eta) \log c_t - \eta l_t] \\ \text{s.t.} \quad & c_t + x_t = y_t = A_t^\gamma (u_t k_t)^\alpha l_t^{1-\alpha}, \end{aligned}$$

³Bresnahan and Raff (1991) suggest that at least twenty percent of the aggregate capital stock was idle between 1929 and 1933. Thus, variable capital utilization may be an important factor for any model of the Great Depression.

$$A_t = (\bar{u}_t \bar{k}_t)^\alpha \bar{l}_t^{1-\alpha},$$

$$k_{t+1} = (1 - \delta_t)k_t + x_t,$$

and

$$\delta_t = \frac{1}{\theta} u_t^\theta,$$

given an initial stock of capital, $k(0) > 0$. We restrict the parameters $0 < \alpha < 1$, $0 < \beta < 1$, $\gamma \geq 0$, $0 < \eta < 1$, and $\theta > 1$. The variables c_t , l_t , x_t , k_t , and u_t denote consumption, labor, investment, capital, and the capital utilization rate. As in most studies of variable capital utilization, the rate of depreciation, δ_t , is an increasing function of the utilization rate. The economy as a whole is affected by organizational synergies that cause the output of an individual firm to be higher if all other firms in the economy are producing more. A_t stands for these aggregate externalities where bars over variables denote average economy-wide levels. The production complementarities are taken as given for the individual optimizer and they cannot be priced or traded. Deviations from constant returns to scale are measured by γ . All markets are perfectly competitive.

In symmetric equilibrium, the first order conditions entail

$$\frac{\eta}{1-\eta} l_t = (1-\alpha) \frac{y_t}{c_t},$$

$$u_t^\theta = \alpha \frac{y_t}{k_t},$$

$$\frac{1}{c_t} = E_t \frac{\beta}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta} u_{t+1}^\theta \right),$$

$$k_{t+1} = \left(1 - \frac{1}{\theta} u_t^\theta \right) k_t + x_t,$$

$$c_t + x_t = y_t = (u_t k_t)^\alpha l_t^{(1-\alpha)(1+\gamma)}$$

and a transversality condition.

Next we calibrate the model using parameter values, found in Table 1, that are typically found in the real business cycle and indeterminacy literatures. The fundamental period in the model is a quarter. The capital share, α , is 36 percent and the steady state rate of depreciation is 2.5 percent. The discount factor, β , is set at $1.03^{-1/4}$ which implies an annual steady state return of about three percent. The weight on utility of labor, η , has no influence on equilibrium dynamics and therefore need not be calibrated. Finally, increasing returns must be calibrated. Bernanke and Parkinson

(1991) conclude that data suggest significant increasing returns during the interwar years.⁴ Burns (1936) also points to some evidence for increasing returns during the depression years. Recent estimates for the U.S. economy (Basu and Fernald, 1997) indicate that scale economies are small. Our value of 1.25 is likely near the ceiling of empirically plausible values.⁵ In Section 5 we demonstrate the robustness of our results with respect to choice of this parameter.

α	β	δ	γ
0.36	$1.03^{-1/4}$	0.025	0.25

2.1 Steady state and dynamics

Next we derive the steady state. Denoting steady state values with no time subscripts, the Euler equation becomes

$$\frac{1}{\beta} = \alpha \frac{y}{k} + 1 - \delta,$$

which allows us to compute y/k . Given our parameterization, on an annual basis, capital is 2.78 times that of output. This value conforms to the findings in Maddison (1991) who reports ratios of gross non-residential capital stock to GDP of 2.91 in 1913 and of 2.26 in 1950. The first order condition with respect to capital utilization together with the Euler equation imply:

$$\frac{1}{\beta} = 1 - \delta(1 - \theta),$$

so that we have $\theta = 1.30$. This is essentially the same value as in Wen (1998). The law of motion of the capital stock in steady state gives

$$\delta = \frac{x}{k},$$

which yields a steady state investment share of 28 percent. This is close to the U.S. investment share in 1929.

⁴In addition, they find some evidence of varying capital utilization.

⁵Note that Cole and Ohanian (1999b) suggest a basic problem which implies that measuring increasing returns must remain imprecise: insufficient variations in factor inputs. They conclude that currently available methods are not adequate to return estimates of scale economies such that we can eventually draw a conclusive diagnosis against or in favor of models with indeterminacy.

Turning to dynamics, it is straightforward to show that our model possesses a unique interior steady state. We then take log-linear approximations to the equilibrium conditions to obtain the following dynamic system:

$$\begin{bmatrix} \hat{x}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{x}_t \\ \hat{k}_t \end{bmatrix} + \mathbf{R} \begin{bmatrix} \omega_{t+1} \\ 0 \end{bmatrix} \quad (1)$$

where hat variables denote percent deviations from their steady-state values; and \mathbf{J} is the 2×2 Jacobian matrix of partial derivatives of the transformed dynamic system.⁶ Here $\omega_{t+1} \equiv E_t \hat{x}_{t+1} - \hat{x}_{t+1}$ is the expectational error, which is by definition serially uncorrelated and mean zero. Mathematically, indeterminacy requires then that both eigenvalues of \mathbf{J} are inside the unit circle. In our model calibration, indeterminacy arises for external effects exceeding $\gamma = 0.10$.

Indeterminacy in rational expectations implies that equilibria are possible in which fluctuations in economic activity are driven by arbitrary and self-fulfilling changes in people's expectations. It should be stressed that such sunspot equilibria are not based on agent irrationality – under the circumstances it is perfectly rational to believe in *crowd psychology*. The economic mechanism that creates the continuum of solutions in our particular model can be understood as follows. In the presence of increasing returns to scale and upon optimistic expectations about the future return to capital, the household will increase today's investment and lower today's consumption. This shifts out the labor supply curve, increasing output and therefore capital utilization. Labor therefore increases even more as a consequence of an outward shift of the equilibrium labor demand schedule. Accordingly, the household will find itself with an augmented future capital stock and higher output; and its initial optimistic expectations are self-fulfilled.

3 Computing Sunspots

The goal of this paper is to determine whether nonfundamental changes in expectations can explain the fluctuations that occurred during the Great Depression era, which we define as encompassing the decline from 1929-1933, the subsequent slow recovery from 1934-1936, and the recession of 1937-1938. In the context of our model, in other words, among the infinite number of possible sequences of the expectational error, ω_{t+1} , in (1), we seek the one

⁶See the Appendix for a more complete description of the linear model.

that best describes the behavior of agents' extrinsic uncertainty during this historic period. We then evaluate the validity of our model by comparing the resulting model-generated sequences of output and other variables to true data from this era. Note that we are therefore working under the assumption that fluctuations are driven only by sunspot shocks.⁷

By definition, the shocks we choose must satisfy several properties. First, since we define them as sunspots, they must be purely nonfundamental. In other words, they are a measure of expectations or confidence that is determined independently of economic fundamentals. Second, they must be serially uncorrelated and mean zero. In order to find a measure of nonfundamental confidence, we follow methods similar to those used in Matsusaka and Sbordone (1995) and Chauvet and Guo (2001), who work with postwar data. In particular, we construct a vector autoregression model (VAR) with a measure of confidence and several measures of fundamentals. The residual from the confidence equation will serve as our sunspot shock. Since the residuals from a regression are by definition mean zero, this property is easily satisfied. In addition, testing for serial correlation will be straightforward.

3.1 Data

In this subsection we describe our data. Though we seek to evaluate the role of sunspots in only the Great Depression Era, we theorize that the fundamental and nonfundamental determinants of confidence are the same during the period as both before and after. We were able to find consistent quarterly data on all of our variables for the period 1921 to 2000. In particular, we work with data from 1921:II to 2000:II for a total of 317 observations.⁸

The first variable of interest is a measure of confidence to use in the VAR system. While Matsusaka and Sbordone and Chauvet and Guo use the index published by the Survey Research Center at the University of Michigan, unfortunately such data is not available for the period of the Great Depression. Therefore, we must instead use another proxy.

Our idea is similar to that of Temin (1976), who quantifies the pessimism felt during the beginning years of the Great Depression. He constructs an index of expectations by taking into account changes in ratings of outstanding bonds. In particular, he interprets extensive net downgradings as reflecting

⁷Of course, we acknowledge that other shocks occurred this time, but this method allows us to isolate the effects of sunspot shocks.

⁸Note that we checked the robustness of our results to sample size. Eliminating later data has a negligible effect on our results.

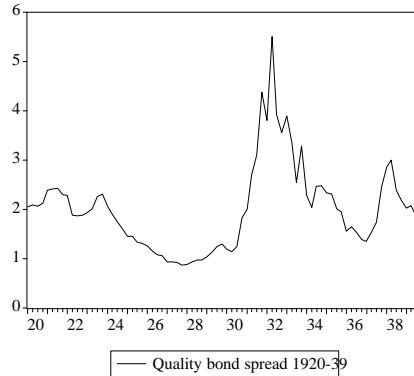


Figure 1: Baa-Aaa bond spread, 1920-1939

anticipated greater risk and therefore predicting bad economic times. Similarly, Kindleberger (1989) suggests that interest spreads quantify confidence:

“As it happened, interest rates declined sharply after the 1929 crash, except for instruments like second-grade bonds, which measure not the rate of interest but confidence.” [Kindleberger, 1989, p. 78]

We therefore argue that an interest rate spread is an ideal candidate to measure confidence. In particular, if people fear a recession, for example, the spread would widen, since the anticipated risk of default on average hits lower rated companies first and foremost. Therefore, a rise in the spread represents a fall in confidence.⁹ By turning to financial markets, we believe that we can extract a conclusive measure of investors’ attitudes about the economy, and therefore their propensity to invest.¹⁰

⁹Though confidence data is available after 1952, we still use the interest rate spread over our entire sample. There are two reasons for this. First, examining the interest rate spread during only the Great Depression Era, we found it to be nonstationary. Second, using the interest rate spread for the first part of the sample and a confidence index for the second would result in an inconsistent series.

¹⁰Our strategy is related to that of Salyer and Sheffrin (1998), in that they argue that financial markets involve expectations and therefore use financial data to compute sunspot shocks.

In order to determine which interest rate spread to use, we focus on finding the difference between the returns on a low and a high risk asset. We therefore use the difference between the return on Baa rated bonds and Aaa rated bonds. In this way, we eliminate any potential noise that may result from using different types (e.g. government and private) of assets. Indeed, Bernanke (1990) and Friedman and Kuttner (1993) use the same quality spread as an instrument for perceived default risk. Figures 1 and 2 plot this bond quality differential.¹¹ The first Figure displays the Baa-Aaa bond quality differential for the period 1920-1939. Here we see that the spread increases after 1929 and it remains high during the 1930s. The delayed upward move may reflect the fact that confidence did not fall immediately following the stock market crash, but about a year later. In fact, the New York stock market leveled off in the first few months of 1930 and employment actually picked up from its December 1929 level. Along these lines, Dominguez, Fair and Shapiro (1988) find that even professional forecasters did not become pessimistic right away:

“Harvard and Yale forecasting services [...] failed to anticipate the Depression and remained optimistic about economic performance following the crash.” [Dominguez, Fair and Shapiro, 1988, p. 595]

Figure 2 displays the bond quality spread over the entire sample, 1921-2000. We see that this spread’s increase of more than 400 basis points during the Great Depression does not reoccur in any other recessions in recent times.¹² Our evidence therefore suggests that Baa-Aaa interest rate spreads during the Great Depression era may contain important information about non-fundamentals during this unparalleled episode in economic history.

We choose our measures of fundamentals with the following in mind. First, our variables must be good predictors of the spread. Second, if the residuals from the spread equation are to be taken to measure sunspot shocks, they must, by definition, be orthogonal to past values of the chosen fundamentals. We must therefore choose variables that capture as thoroughly as possible the state of economic activity over the sample period, so

¹¹All data is from the NBER and the Federal Reserve.

¹²This stands in contrast to results in Cole and Ohanian (2000), who examine the behavior of several other short and long term spreads such as the spread between the short-run commercial paper rate and the rate on matched-maturity U.S. Treasury Bills (see Bernanke, 1990, and Friedman and Kuttner, 1993).

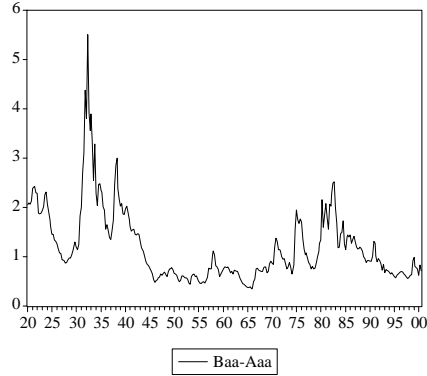


Figure 2: Interest Rate Spread, 1920 to 2001

that changes in expectations due to fundamentals are accounted for in the regressions.

We use the following variables in the VAR: the growth rate in real gross national product (y); the growth rate in real money supply, as measured by M2 (m); the rate of change of the GNP deflator (p); and the absolute change in the nominal return on prime commercial paper (cp).¹³ Data on y provide a measure of the overall performance of the real economy. We use the widely defined real money supply to capture the effects of monetary policy. In particular, much of the literature on the Great Depression focuses on this variable, building on the work of Friedman and Schwartz (1963), who blame (non-)actions taken by the Federal Reserve as prime culprits. Money supply also reflects the workings of the intermediaries sector (i.e. the banking crises) as do interest rates (Bernanke, 1983).¹⁴

¹³Data on y , are from Balke and Gordon (1986) and NIPA. Data on m , p and cp are from Balke and Gordon (1986) and the Federal Reserve.

¹⁴Note that we also considered including government purchases and the leading economic indicators. However, these data are not available for the period 1940-1948, and so were eliminated from consideration. We considered using this data only for the period ending in 1939, but problems of nonstationarity arose.

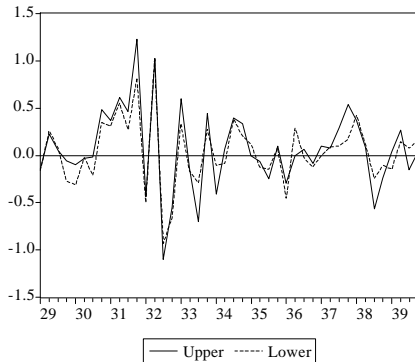


Figure 3: Two spread equation residuals

3.2 Model and results

Our model is as follows:

$$\begin{bmatrix} m_t \\ cpt \\ p_t \\ y_t \\ S_t \end{bmatrix} = [\mathbf{P}_1(\mathbf{L})] \begin{bmatrix} m_t \\ cpt \\ p_t \\ y_t \\ S_t \end{bmatrix} + [\mathbf{P}_2] \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^c \\ \varepsilon_t^p \\ \varepsilon_t^y \\ \varepsilon_t^s \end{bmatrix}, \quad (2)$$

where S is the interest rate spread. Dickey-Fuller-tests indicate that each of these variables exhibits stationarity over the considered sample. The matrix $\mathbf{P}_1(\mathbf{L})$ is of polynomials of length 4 so that we include four lags of each variable.¹⁵ We also include a constant in each regression.

We consider two different specifications of \mathbf{P}_2 , both of which are commonly applied in the empirical literature.¹⁶ In the first, the matrix is upper triangular.¹⁷ In other words, the innovations to the spread are first in the causal chain so that other variables may respond contemporaneously to them, but they are exogenous. Here the spread innovations are in a sense the primary cause of fluctuations. In the alternative set up, \mathbf{P}_2 is lower

¹⁵Inclusion of 4 lags was determined using the Akaike information criterion.

¹⁶See for example Christiano, Eichenbaum and Evans (1999) for a recent survey.

¹⁷With respect to the confidence equation, this is the standard approach taken by Matsusaka and Sbordone (1995) and Chauvet and Guo (2001).

triangular. In other words, the innovation to the money supply is first in the causal chain and the spread is last. Here one can argue that the effects of any potential omitted variables are included in the spread equation. Figure 3 compares the two residuals from the spread equations over the Great Depression era. Clearly the results are robust to specification of \mathbf{P}_2 . The correlation between the two series is 0.91. In addition, their correlation over the entire sample is 0.90. Moreover, both series show negative animal spirits from 1930:IV onwards. Therefore we assume that innovations to the spread can indeed be characterized as exogenous with respect to the other innovations and we use the first series throughout the rest of the paper. That is, we run the VAR assuming that \mathbf{P}_2 is an upper triangular matrix.

The results of the spread regression are displayed in Table 2. Every variable is significant at the 5% level and the \overline{R}^2 indicates a very good fit.¹⁸

Variable or Statistic	Significance or Value
S	0.00
y	0.00
m	0.00
cp	0.00
p	0.04
\overline{R}^2	0.91
Durbin-Watson	1.96

The residuals from this regression are taken to measure nonfundamental confidence. The Durbin-Watson statistic clearly indicates that we cannot reject the null hypothesis of no first-order serial correlation.

The Depression-era residuals $\{\varepsilon_t^s\}$ are plotted in Figure 4. A fall in nonfundamental confidence is indicated by a positive innovation. It is useful to divide the analysis of the measured shocks between the decline of 1929-1933, the slow recovery of 1934-1936 and the recession of 1937-1938. With regards to the decline, we see that the residuals are positive from the last quarter of 1930 until 1932, peaking again in 1933. This again reflects the delayed fall of confidence, and also its continued persistence. Our sunspot sequence also indicates a slow initial decline in spirits, which should not be puzzling since a number of professional forecasters, including the Harvard Economic Society and Yale's Irving Fisher, remained optimistic well into

¹⁸We also estimated a version of the model excluding inflation. Results were similar.

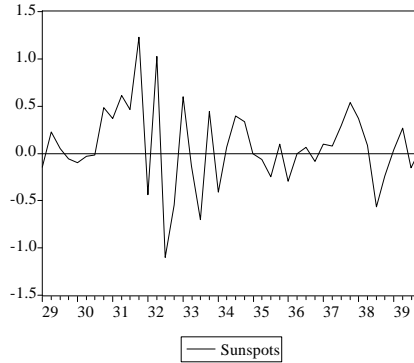


Figure 4: Depression-era residuals

1930. Similarly, Hart (1933), again summarizing contemporaneous journal opinions, finds:

”The idea that the depression was due to a buyers’ strike, or to unjustified withholding of buying power by consumers followed naturally from the conviction that conditions were essentially sound, and that the depression was psychological.” [Hart, 1933, p. 678]

Overall, the tepid rise in the spread and the interpreted fall in extrinsic confidence coincides with Temin’s (1976) and Romer’s (1993) observations that expectations first turned “uncertain” following the months after the stock market crash and only later confidence became bruised to what we coin pessimistic animal spirits. As Kindleberger (1986) puts it:

“From August 1930 [...] the divergence between high and low quality issues reflects a drastic change in expectations and loss of confidence.” [Kindleberger, 1986, p. 122]

We also see mostly positive residuals, and therefore pessimism, throughout the period of the recovery. This offers a possible explanation for the sluggishness of the recovery. Lastly, the residuals increase again at the onset of the 1937-1938 recession. This parallels Roose’s (1954) interpretation of the 1937-1938 episode which attributes considerable importance to the uncertainty of business expectations partially based on a

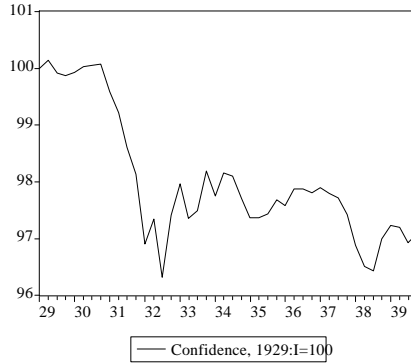


Figure 5: Computed confidence index

”[...] serious political conflict between New Deal and business.” [Roose, 1954, p. 238]

In Figure 5 we display an index of confidence constructed from the residuals, which is computed by chaining the measured innovations from quarter to quarter. Here we clearly see confidence falling through the sample. As with the residuals, we see that confidence recovers only slightly after passing through a trough mid-1932; and it reaches a low-level plateau during 1933. Confidence continues to fall through the mid-1930s and takes another dive in 1937.

Figure 6 plots the residuals for the entire sample. We observe that sunspot shocks are significantly smaller during the postwar period. This parallels findings reported by DeLong and Summers (1986) and by Farmer and Guo (1995) that the volatility of demand shocks becomes remarkably smaller in the postwar period.

3.3 Causality

Given the evidence that both confidence itself and our empirical measure of it did not fall until 1930, we now present results from Granger causality tests on output and our measure of nonfundamental confidence. That is, we examine the null hypotheses that our spread residuals (ε_t^s) do not Granger cause output growth (y) and vice versa. We carry out each test over the

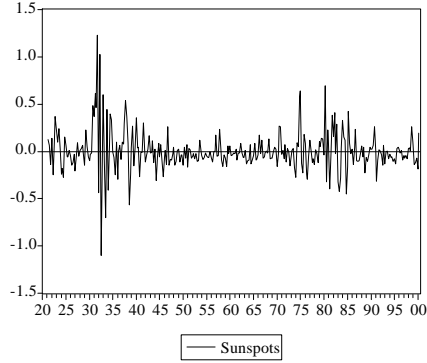


Figure 6: Entire sample residuals

period 1929:I to 1939:IV using both 4 and 8 lags. The results of these tests are reported in Table 3. Results clearly indicate that the residuals Granger cause output growth, and not vice versa. Therefore we conclude that though confidence initially fell with a lag, over the entire Depression era causality ran from confidence to output.¹⁹

Null Hypothesis (number of lags)	F-statistic	Significance
ε_t^s does not Granger cause y (4)	2.53	0.05
ε_t^s does not Granger cause y (8)	2.81	0.02
y does not Granger cause ε_t^s (4)	1.10	0.37
y does not Granger cause ε_t^s (8)	1.08	0.40

In the preceding analysis, we identified a series of animal spirits shocks implied by the econometric model. In the next section, we will ask if these shocks explain the Great Depression era in deepness and in duration. We will particularly stress the sluggish recovery in the sense that detrended per capita output was still far from trend in 1939.

¹⁹It can be shown that (i) using our full sample or (ii) using real balances instead of output yields similar results.

4 The Great Depression in the model

In this section, we use the sunspot shocks generated above to compute implied series for output, consumption, investment, the investment share, hours worked and labor productivity in our model. We then compare the results to data from the Great Depression era in order to determine the plausibility of the hypothesis that sunspot shocks were an important driving force behind the fluctuations that occurred during this period.

4.1 Output

Figure 7 displays quarterly output in the US data with that implied by the model from 1929:I to 1939:IV. The U.S. data are detrended to allow for the absence of long-run technological progress and of population growth, as we abstract from these in the model. Cole and Ohanian (1999a) point out that the economy was at or very near trend in 1929 and the interpolated geometric trend computed by Balke and Gordon (1986) supports this. We therefore assume that our economy is in steady state in 1929:I and feed in sunspot shocks starting at this time. For ease of presentation, we also rescale both model and U.S. data so that output is equal to 100 in 1929:I. Lastly, we are left with a degree of freedom in choosing the variance of our sunspot shock. This is due to the fact that models with indeterminacy and sunspots give definite predictions about relative variabilities as well as serial and cross correlations, but not about the amplitude of fluctuations.²⁰ It is common in the indeterminacy literature to choose the parameter so that the variance of model output is equal to that in the data (see for example Farmer and Guo, 1994). Here, instead, we use a similar methodology such that we scale the data so that the absolute decline of output at the trough coincide in model and in data.

Three important results emerge. First, the model economy predicts well the size and duration of the Depression. It is worth noting that output in the model falls with a lag. In particular, it is not until 1930:IV that the economic run-down effectively sets in. In the preceding year, the artificial economy appears to be leveling off. This is most likely due to the previously cited evidence that confidence did not fall immediately following the 1929 crash and to the probability that other factors are chiefly responsible for the

²⁰In terms of our our model, we do not have any guide in deciding the elasticity of investment with respect to sunspot shocks. Qualitatively, we set this elasticity such that a 100 basis point sunspot shock reduces investment by 0.4 percent.

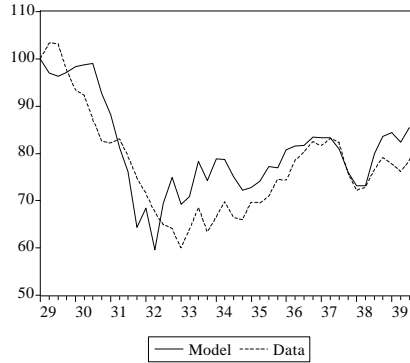


Figure 7: U.S. data and model-generated data (output), 1929:I=100

initial phase of the decline. This finding suggests the following interpretation of the role of sunspots in the Great Depression. The initial stages of the decline did not differ much from other recessions, but intensified pessimism during the summer of 1930 produced the slide into the abyss. This parallels Temin’s narrative description of 1930:

“People responded to the fall in business activity and prices in 1930 in roughly the same way they reacted to the roughly similar fall in 1921. They knew that business was bad, but they expected it to recover soon. It was only when business failed to show signs of recovery in the fall of 1930 that expectations changed. As far as one can see, it was the failure of business to pick up in the fall of 1930 rather than the decline of stock-market prices in 1929 that produced the change.” [Temin, 1976, p. 82]

The second finding is that the model also predicts a tepid recovery. This stands in sharp contrast to the prediction of the real business cycle model, in which there is a much faster recovery, due to the presence of large positive technology shocks (Cole and Ohanian, 1999a). Likewise, sticky price monetary models also predict a comparatively rapid recovery (Bordo et al., 2000) as the result of expansionary actions taken by the Federal Reserve and/or the abandonment of the Gold Standard in early 1933. In contrast, our results suggest that self-fulfilling changes in agents’ beliefs played a significant

role in much of the economic dive as well as in the lukewarm recovery. In 1939:IV, the U.S. and model economies remain 17 and 14 percent below trend respectively.

Lastly, the model does very well in predicting the recession from 1937:II to 1938:II. Note that it virtually coincides in both timing and deepness with the U.S. economy. In this respect the model is also superior to those mentioned above. Moreover, recall that when Cole and Ohanian (2001) take into account institutional changes arising from New Deal cartelization policies, which they claim as being essential for our understanding of the second half of the 1930s, their technology-driven model still misses the 1937-1938 recession.

There are some differences between the behavior of the model economy and the behavior of the U.S. economy during this episode. Most notably, the model predicts that the main cycle's trough occurs three quarters too soon. We interpret this as reflecting the fact that in the model, a change in expectations has an immediate effect on output whereas in the true economy, there may very well be a delay in the time it takes for such a shock to work its way through the real economy. One simple idea to improve the model in this respect would be to introduce adjustment costs.

4.2 Intuition

Why does our model predict well the decline and especially the sluggish recovery? In order to understand this, let us reconsider why both the real business cycle model and the sticky price model do not. The real business cycle model is driven by technology shocks; and Cole and Ohanian (1999a) find that these were strong and positive during the recovery. Bordo et al. (2000) cite evidence of a large monetary expansion. Simply put, in this paper, we measure a sequence of negative domestic shocks that hit the U.S. economy during exactly the period in which existing theories fail to uncover any such effects.

In addition, our model exhibits more persistence than the real business cycle model. This is due to the presence of increasing returns to scale, which encourage “bunching” of periods of low output when agents are pessimistic. That is, once the economy is in state of low economic activity, agents prefer to wait until the economy moves to a higher productivity state to increase labor input, and therefore output. As a consequence, output can stay persistently low. We argue that this is another reason for the slow adjustments during the second half of the thirties. Thus, identified sunspots coupled with

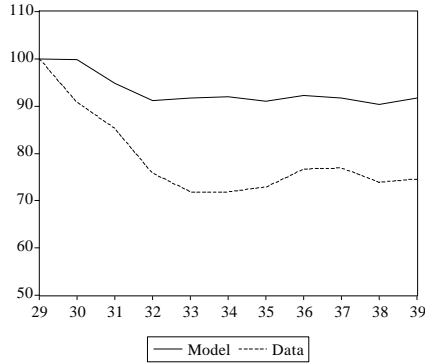


Figure 8: U.S. data and model-generated data (consumption), 1929=100

modest increasing returns appear to constitute an important ingredient of a theory of the Great Depression.

4.3 Other variables

Next, we check the behavior of other variables. Since not all variables are available at quarterly frequency, we present only annualized values here.²¹ Figures 8 to 12 display the patterns of consumption, investment, the investment share, employment and labor productivity.

Starting with consumption, a striking aspect of the data is that after its initial drop in 1932, it remains at its new level until 1939. Cole and Ohanian (1999a) speculate that this behavior may reflect a convergence to a new growth path. However, the sunspot model can reproduce this general pattern while maintaining the single steady state assumption: model consumption was only 0.5% above its 1932 trough level in 1939. While the initial fall in consumption is less in the model than in the data, the smoothness and stability is replicated. Note that the smoothness of consumption in our model is enhanced by the inclusion of variable capacity utilization (Benhabib and Wen, 2000). That is, given the ability to leave some capital idle, agents do not have to change consumption as much when a given shock hits.

²¹Data for consumption, investment, and hours worked are from Cole and Ohanian (1999a).

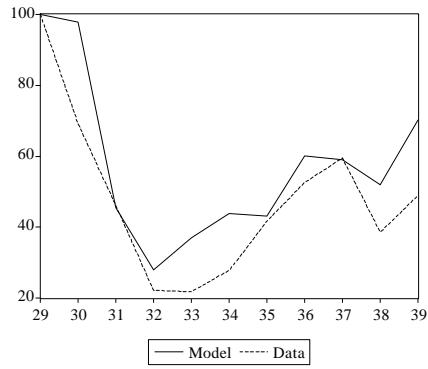


Figure 9: U.S. data and model-generated data (investment), 1929=100

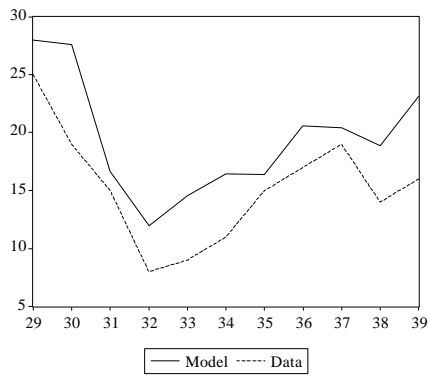


Figure 10: U.S. data and model-generated data (investment share), 1929=100

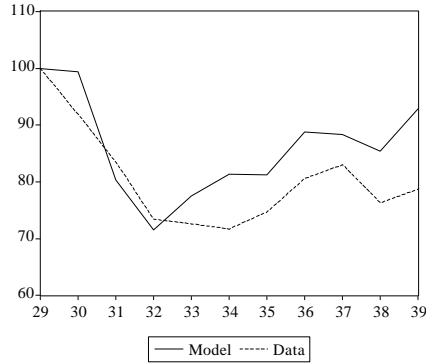


Figure 11: U.S. data and model-generated data (labor input), 1929=100

All other model variables track quite well the Great Depression in timing and duration: all follow the “double dip character” that we see in the data. The behaviors of hours and investment are replicated especially well.²² The model can also reproduce the procyclicality of the investment share. Given the degree of increasing returns in the model, we can even explain a large share of the decline in labor productivity. This is another noteworthy feature of the sunspot economy: the 18 percent decline in factor productivity has become a conundrum for the real business cycle strategy, as it is unlikely the result of technological regress (Ohanian, 2001).

5 Robustness

In this section we test the robustness of our results with respect to the assumption of large increasing returns. We also check the ability of our model to reproduce the behavior of output over our entire sample.

In Figure 13 and 14 we compare model-generated output to U.S. data using returns to scale of 1.2 and 1.15. Examining these figures, it becomes clear that increasing returns are key to achieving persistence in the fall in output, and therefore matching the anemic recovery. We see that persistence

²²Note that the model-generated sequences of consumption and investment cannot be reconciled with that of output. This results because we approximate the model around steady state, while the Great Depression was likely an episode when the economy was far from steady state.

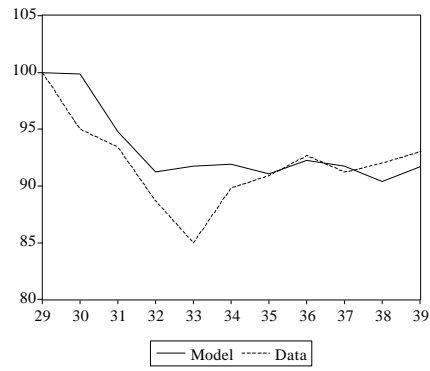


Figure 12: U.S. data and model-generated data (labor productivity), 1929=100

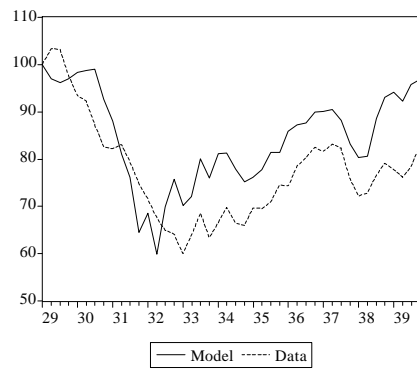


Figure 13: U.S. data and model-generated data (output), $\gamma = 0.2$

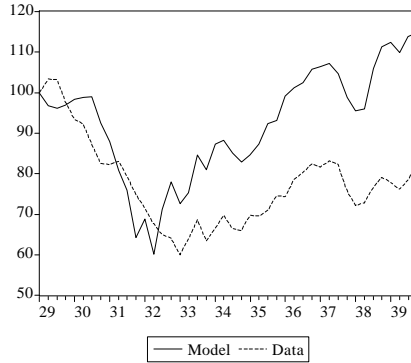


Figure 14: U.S. data and model-generated data (output), $\gamma = 0.15$

rises and data matching improves as returns to scale increase. In particular, when returns to scale are 1.15, while the model still produces the sequential pattern of the Great Depression including the sharp recession of 1937-38, the recovery in the artificial economy is too fast and too strong. Put another way, we require that the identified shocks be coupled with relatively large increasing returns in order to reproduce the economy's persistence.

Finally, we look past 1939 and simulate the model accordingly up to 2000:II. Over this entire period, the correlation of annual output growth in the model with that in the data is 0.31.²³ During the Depression era decade the corresponding correlation is 0.69. This again confirms findings reported by DeLong and Summers (1986) and by Farmer and Guo (1995) that demand shocks were either less important or smaller in the postwar period or were partially neutralized by active fiscal and monetary policies. We take these results, however, as evidence that our model can be applied outside the time frame of the Great Depression era with some success.²⁴

Overall, our findings suggest that shocks to expectations may have played an important role during the 1930s. The results can be interpreted as an compelling alternative to approaches that charge inept monetary policy or technology shocks, as these are reportedly unable to explain the sum of facts

²³Here we use our original specification, with $\gamma = 0.25$.

²⁴In addition, for example, we find pessimistic animal spirits shocks dating from 1989:IV to 1991:I which essentially coincides with the NBER recession (1990:III to 1991:I) and parallels Blanchard's (1993) interpretation of the 1990-91 recession.

over the whole Great Depression era.

6 Summary and Conclusion

In this paper, we have taken a novel approach to modelling the Great Depression. Instead of relying on technology or monetary shocks, we test the hypothesis that this historic episode can be explained by a deterioration in confidence. In the context of a neoclassical model in which indeterminacy of equilibria results with modest increasing returns to scale, we believe we have found the “other shock” needed to explain the slow recovery. Using an interest rate spread as a proxy for confidence, and a VAR to extract its nonfundamental part, we have constructed a series of sunspot shocks that, when fed into the model, predict a large and persistent fall in output. In particular, our model replicates well both the decline of 1929-1932 and the recession of 1937-1938. As such we feel our model represents a vast improvement over previous work that examines only fundamental and/or banking shocks.

Our results suggest the following interpretation of the Great Depression. The 1929 stock market crash was followed for about a year by what appeared to be the start of a normal recession. Only later, during the summer of 1930, did confidence began to deteriorate dramatically. Hence the recession was transformed into a depression. In 1932, faith in the economy hit bottom; and the continuing sequence of pessimistic animal spirits are a prime candidate in the quest to explain the subsequent stagnation that only ended with the onset of World War II.

Clearly, there were other forces at work during 1929 to 1939 as well, and a useful extension of the model would be to allow for this. In addition, an examination of the relationship between the pessimism we observe here and the banking panics seems in order. While the results here are certainly not the final word on the origins of this unique episode, we believe that we have shed some light on this seemingly insoluble mystery.

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7 Appendix

Let us denote $\hat{y}_t \equiv \log(y_t/y)$ et cetera, then the linear model is given by

$$\hat{y}_t = \alpha(1 + \gamma)\hat{u}_t + \alpha(1 + \gamma)\hat{k}_t + (1 - \alpha)(1 + \gamma)\hat{l}_t \quad (\text{A1})$$

$$\hat{l}_t = \hat{y}_t - \hat{c}_t \quad (\text{A2})$$

$$\hat{\delta}_t = \hat{y}_t - \hat{k}_t \quad (\text{A3})$$

$$-\hat{c}_t = -E_t\hat{c}_{t+1} + \alpha\beta\frac{y}{k} \left[E_t\hat{y}_{t+1} - \hat{k}_{t+1} \right] - \beta\delta E_t\hat{\delta}_{t+1} \quad (\text{A4})$$

$$\hat{k}_{t+1} = (1 - \delta)\hat{k}_t - \delta\hat{\delta}_t + \frac{x}{k}\hat{x}_t \quad (\text{A5})$$

$$\hat{\delta}_t = \theta\hat{u}_t \quad (\text{A6})$$

and

$$\frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t = \widehat{y}_t. \quad (\text{A7})$$

The model equations (A1)-(A7) can be compactly written as

$$\begin{bmatrix} \widehat{y}_t \\ \widehat{c}_t \\ \widehat{l}_t \\ \widehat{u}_t \\ \widehat{\delta}_t \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{x}_t \\ \widehat{k}_t \end{bmatrix} \quad (\text{A8})$$

and

$$\begin{bmatrix} E_t \widehat{x}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{x}_t \\ \widehat{k}_t \end{bmatrix}. \quad (\text{A9})$$