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The BIP Trilogy (Bipolarization, Inequality and Polarization): One Saga but Three Different Stories

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Abstract Inequality, bi-polarization and polarization are related but distinct concepts aiming at analysing the income distribution. This paper first recalls the main differences between these three notions of inequality, bipolarization and polarization. It then shows that a close look at the impact of various income sources on these three types of indicators confirms that indeed they measure three different features of an income distribution. The effect of the different income components on inequality, bipolarization and polarization is analyzed via what is known as the Shapley decomposition and the empirical illustration is based on 2008 data for Luxembourg.

JEL D31, D63, I31

Keywords Bi-polarization; income sources; inequality; Luxembourg; polarization; Shapley decomposition procedure

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1. Introduction

A growing literature attempted during the past twenty years to propose measures of polarization and to apply them to the income data of various countries. It is however very important to make a distinction not only between the study of inequality and that of polarization, but also between the concept of income *bi-polarization* and that of income *polarization*. As stressed by the title of this paper, the 'saga' ('a long detailed account'[2]) is the same because the three notions refer to a specific aspect of the same income distribution but the "story" (' narrative designed to instruct the reader'[3]) is not necessarily identical.

A similar distinction seems to have been made in the field of international relations. Mansfield (1993, pages 109-110) thus wrote that 'Hegemony (or unipolarity) is characterized by a 'wide' power disparity between the largest state in the system and all other states; bipolarity is characterized by the 'approximate' equality of the two largest states and a 'wide' power disparity between the smallest pole and any remaining state; and multipolarity is characterized by the 'approximate' equality of more than two particularly powerful states and a 'wide' power disparity between the smallest pole and any other state in the system.' Mansfield (1993) stressed however that an analysis of the distribution of power in international relations should not be limited to counting the number of major powers because 'for the purposes of explaining patterns of balancing behavior, the onset of war, and many aspects of the international political economy, scholars are likely to find it useful to analyze both features of the distribution of power that have been used repeatedly in studies of international relations: (1) polarity; and (2) concentration.'

As far as income distributions are concerned, the focus of inequality, as is well known, is on disparities in the distribution of income. However since inequality is maximal when one individual 'eats the whole pie', the relative frequency of the observations at income zero will be close to one so that there should be quite a strong link between the idea that a distribution is very unequal and that of viewing a distribution as being unipolar.

In contrast bi-polarization refers to the case where there is a significant number of individuals who are very poor but there exists also a non negligible share of the population that is quite rich. Such a gap between the "poor" and the "rich" implies evidently that there is no sizeable

[2] See www.merriam-webster.com/dictionary/saga
[3] See http://dictionary.reference.com/browse/story

middle class. The analysis of bi-polarization is hence linked to that of the importance of the middle class.

Finally the concept of polarization concerns quite a different issue, that of the potential for social conflict. As a first approximation one may want to link polarization with multimodality. In fact for Esteban et al. (1994) and Duclos et al. (2004) the concept of polarization is derived from the combination of the two notions of *identification* and *alienation*. In the case of income polarization, identification is related to the idea that individuals identify themselves with those with similar income levels. Alienation on the contrary assumes that an individual will feel more alienated with respect to another individual, the greater the distance between their incomes. There is thus a clear connection between the notions of polarization and multipolarity.

As mentioned previously, bipolarization, inequality and polarization (BIP) refer to the same 'saga' but are theoretically supposed to highlight different aspects of the income distribution. However, are such theoretical differences observed empirically? Answering this question is in fact the main aim of this paper where decomposition by income component will be the empirical tool used to determine whether these three concepts tell similar or different stories. The focus of this paper is thus on the differential impact of various income sources on inequality, bi-polarization and polarization.

Numerous studies have been devoted during the past thirty years to the study of the ways various income sources affect income inequality (see, Lerman, 1999, for a survey of the literature on this topic) and this is why we will not cover income inequality decomposition by factor components in our methodological section but simply report the results. There have on the contrary been very few attempts to estimate the contribution of various income sources to the bi-polarization or polarization of income. Deutsch and Silber (2010) analyzed the impact of various income sources on bi-polarization in Israel on the basis of the bi-polarization index P_G that had been introduced by Deutsch et al. (2007). Araar (2008) proposed some analytical methods to decompose the Duclos, Esteban, and Ray (2004) polarization index DER by income sources and population subgroups and applied his approach to data from China and Nigeria. Finally Apouey (2010) quantified the contribution of various determinants to social polarization in health.

The approach taken in the present paper, in analyzing the impact of different income sources on inequality, bi-polarization and polarization, is based on the systematic use of the concept of Shapley decomposition that has been proposed by Chantreuil and Trannoy (1999,

forthcoming), Shorrocks (1999) and Sastre and Trannoy (2001, 2002). More precisely we will apply the Shapley decomposition procedure to analyze the marginal impact of each income source on inequality, bi-polarization and polarization.

The paper is organized as follows. Section 2 reviews the literature on the measurement of bi-polarization and polarization and Section 3 shows how to apply the Shapley decomposition procedure to analyze the impact of income sources on inequality, bi-polarization and polarization. Section 4 applies then this technique to a detailed analysis of the differential impact of various income sources on inequality, bi-polarization and polarization in Luxembourg in 2008. Concluding comments are finally given in Section 5.

2. On the measurement of bi-polarization and polarization:

2.1. Measures of bi-polarization:

The concept of bi-polarization is clearly related to that of the middle class (see, Wolfson, 1994) and there have been quite a few attempts at measuring the importance of the middle class, since a sizable middle class is supposed to be an important factor in economic development (see, Thurow, 1984, Foster and Wolfson, 1992 and 2010, Landes, 1998, Easterly, 2001, Birdsall, 2007a and b, and Pressman, 2007). There is however no agreement on how the relative importance of the middle class should be measured. Thurow (1984), for example, defined the middle class as including those households whose income ranges from 75% to 125% of the median household income. Blackburn and Bloom (1985) used a wider interval (60% to 225% of the median). Other ranges were also proposed (e.g., Davis and Huston, 1992, Lawrence, 1984). Birdsall et al. (2000) adopted Thurow's approach but for Birdsall (2007a) the middle class should include people at or above the equivalent of $10 day in 2005, and at or below the 90^{th} percentile of the income distribution in their own country.

The concept of bi-polarization stresses in fact two notions. The first one, "increasing spread", implies that moving from the middle position (the median) to the tails of the income distribution makes an income distribution more polarized. More precisely a rank preserving increment in incomes above the median or a rank preserving reduction in income below the median will widen the distribution, that is, extend the distance between the two groups (those above and below the median) and hence increase the degree of bi-polarization (the rich become richer and the poor poorer). The second concept, "increased bipolarity", refers to the

case where the incomes below the median or those above the median get closer to each other. What is happening here is a "bunching" of the two groups in the sense that the gaps between the incomes below the median (or those above the median) have been reduced and such a "bunching" is assumed to increase bi-polarization.

To measure bi-polarization Foster and Wolfson (1992; 2010) recommended using an index P_{FW} which is defined as

$P_{FW} = (G^B - G^W)(\mu / m)$	(1)

where μ, m, G^B and G^W are respectively the mean, the median, the between and within groups Gini indices of the distribution and where it is assumed that there are only two income groups, those with an income below or above the median income.

While P_{FW} is a relative bi-polarization index, there have also been propositions to define absolute (see, Chakravarty et al., 2007) or intermediate bi-polarization indices (see, Chakravarty and D'Ambrosio, 2010, and Lasso de la Vega et al., 2010). Some suggestions have also been made to extend Foster and Wolfson's Bi-polarization Index P_{FW} (see, Wang and Tsui, 2000, and Rodriguez and Salas, 2003).

More recently, Deutsch et al. (2007) proposed an alternative approach to the measurement of bi-polarization, related to previous work by Berrebi and Silber (1989) on the measurement of the flatness of an income distribution. They then derived a bi-polarization index P_G defined as

$P_G = (G_B - G_W)/G = (\Delta_B - \Delta_W)/\Delta$	(2)

where Δ, Δ_B and Δ_W are the mean differences for the whole distribution and the between and within groups mean differences. Deutsch et al. (2007) assumed that there were two groups, those with an income below or above the median income, so that these two groups do not overlap and as a consequence the overall Gini index G is equal to the sum of the indices G_B and G_W.

They also proved that P_G had the two most desirable properties of a polarization index: it obeys the axioms of *Non-Decreasing Spread* and *Non-Decreasing Bipolarity* (see, Appendix A, for a detailed proof). Moreover it is easy to show that the bi-polarization index P_G is invariant not only to a multiplication of all incomes by a constant but also to equal additions

to all incomes since the mean differences Δ, Δ_B and Δ_W that appear in (2) are invariant to equal additions to all incomes.

Finally one may note that there is quite an important similarity between the definition of the bi-polarization index P_G given in (2) and the Foster and Wolfson (1992; 2010) bi-polarization index P_W defined in (1).

2.2. On the measurement of polarization:

Here we are interested in studying the extent to which a population is clustered around a small number of distant poles so that we do not limit ourselves any more to two groups. Various measures of polarization have appeared in the literature, that correspond to this case (see, for example, Esteban and Ray, 1994, Zhang and Kanbur, 2001, Reynal-Querol, 2002, Duclos, Esteban and Ray, 2004, Lasso de la Vega and Urrutia, 2006, Esteban, Gradin and Ray, 2007, Gigliarano and Mosler, 2009 and Poggi and Silber, 2010).

The notion of polarization in a multi-group context, which was originally proposed by Esteban and Ray (1994), was in fact an attempt at capturing the degree of potential conflict inherent in a given distribution. The idea is that political or social conflict is more likely, the more homogenous, separate and of a similar size the groups are, so that it is polarization, and not inequality, that matters for conflict. For Esteban and Ray (1994) society may be considered as an amalgamation of groups, where two individuals drawn from the same group are assumed to be "similar" whereas two persons belonging to different groups are supposed to be "different" with respect to a given set of attributes.

To describe the potential for social conflict Esteban and Ray (1994) have introduced the notions of "identification" and "alienation". The first notion refers to the fact that if each group consists of very similar individuals, it is then likely that their objectives will also be very similar, and, as a consequence, they will form a stronger unit because of their mutual sense of identification. In addition, if there is a clear difference between groups, then this heterogeneity across groups will in a certain way contribute to tensions by increasing the probability that the objectives of two or more groups will be conflicting. This is therefore the identification-alienation framework introduced by these authors.

A distinction can be made between the case where the groups are defined a priori, whether on the basis of their income or of some other criterion (e.g. ethnicity) or that where we let the data define the groups. Defining the groups from the onset was the approach taken by Esteban and Ray (1994), Zhang and Kanbur (2001) or Poggi and Silber (2010). Such an

approach raises however several questions: how many groups should be defined? How big should a group be to be considered as relevant? To overcome these difficulties Duclos et al. (2004) suggested computing the degree of polarization on the basis of the density function so that the number of groups considered would be endogenous. Their approach is based on the use of continuous distributions and they approximate the strength of group identification of a person via the value of the density function evaluated at the person's income. Let f be such a density. The effective antagonism of an individual with income x towards an individual with income y is a nonnegative function of the identification $I = f(y)$ and of the alienation $a = |x - y|$. Polarization is, then, assumed to be proportional to the sum of all effective antagonisms and the authors derive axiomatically a new measure of polarization DER defined as

$DER = P_\alpha(f) = \int\int f(x)^{1+\alpha} f(y) \mid x - y \mid dxdy$	(3)

with $0.25 \leq \alpha \leq 1.$

To compute DER in applied work Duclos et al. (2004) first note that expression (3) may also be written as

$DER = P_\alpha(F) = \int_y f(y)^\alpha a(y) dF(y)$	(4)

with

$a(y) = \mu + y(2F(y) - 1) - 2\int_{-\infty}^{y} x dF(x)$	(5)

Assume now a random sample of n i.i.d. observations on the income y_i $(i = 1,\ldots,n)$ which are drawn from the distribution $F(y)$ and which are classified in increasing ordered $(y_1 \leq \ldots \leq y_i \leq \ldots \leq y_n)$. Duclos et al. (2004) then state that an natural estimator of $P_\alpha(F)$ is

$P_\alpha(\hat{F}) = (1/n)\sum_{i=1}^{n} \hat{f}(y_i)^\alpha \hat{a}(y_i)$	(6)

with

$\hat{a}(y_i) = \hat{\mu} + y_i[(1/n)(2i-1)-1] - (1/n)[2\sum_{j=1}^{i-1} y_j + y_i]$	(7)

In (7) $\hat{\mu}$ is the sample mean while in (6) $\hat{f}(y_i)^{\alpha}$ is estimated non-parametrically using kernel estimation procedures.

3. Using the Shapley decomposition procedure to measure the marginal contribution of various income sources to inequality, bi-polarization and polarization:

Although when introduced in the literature (see, Chantreuil and Trannoy, 1999, forthcoming, and Sastre and Trannoy, 2001, 2002) the so-called Shapley decomposition procedure was applied to the breakdown of income inequality, Shorrocks (1999; forthcoming) has shown that this decomposition could be applied to any function.[4] For example, this technique was subsequently applied to the study of variations in poverty across Russian regions (Kolenikov and Shorrocks, 2005), the decomposition of the sources of disparities in the relative wealth position of Mexican Americans (Cobb-Clark and Hildebrand, 2006), the breakdown of the R-square of a regression (Israeli, 2007) or of the Likelihood Ratio Index of a logit regression (D'Ambrosio et al., 2011), the decomposition of changes in the wage distribution (Devicienti, 2010) or in poverty/inequality indices due to changes in tax-benefit policy (Bargain and Callan, 2010). Another application, directly relevant for this paper, is the one by Deutsch and Silber (2010) who used the Shapley decomposition procedure to measure the exact impact of an income source on the bi-polarization index P_G.

Before explaining briefly the mechanism underlying the Shapley decomposition rule, let us recall that, in the case where the total income variable is the sum of a set of income components, a straightforward and intuitive way to assess the contribution of each component taken separately to, say, inequality is to apply a before/after calculation. Each source contribution is then equal to its first round marginal impact corresponding to the difference between overall inequality and the inequality obtained when the income component (or when the inequality from that income component) is eliminated.

[4] See Auvray and Trannoy (1992) and Rongve (1994) for the first applications.

For Sastre and Trannoy (2002), this approach, to which they refer as a 'local method'[5] suffers from two drawbacks. First, the sum of the first round marginal impacts of the various factors does not result in an exact decomposition of the overall index (see also Shorrocks, 1999). A solution to this problem would be to eliminate each factor in turn. In this case, the decomposition is exact; however, and this is the second drawback of this method, a path dependency problem arises: the contribution of each factor (except when there are only two income sources) clearly depends on their order in the elimination process.

The Shapley decomposition allows overcoming this path dependency issue. Indeed, the idea of the Shapley decomposition procedure is precisely to average the contribution of each income component over all the possible sequences allowing us to eliminate the different income sources. Before giving a concrete example, it is important to note that Sastre and Trannoy (2002) mention two possibilities to 'eliminate a variable'. The first one simply consists in removing a variable from the computation, assuming that the income from that source is equal to zero for all observations. We then compare a situation where a given income source does not exist and that where it is present. They refer to this option as the *zero income decomposition*. The second one amounts to supposing that a given income source is equally distributed, implying that each individual (household) is attributed the mean value of this income source. Here the two situations compared are that where all the individuals receive the same amount of income from a given source and that where the distribution of this income source is assumed to be unequally distributed. This method is named the *equalized income decomposition*. When decomposing inequality, Sastre and Trannoy (2002) tend to recommend not using the zero income decomposition as it yields highly volatile results, depending on the level of aggregation of the income components.

Let us now explain the algorithm in more details. We take as illustration the case of the equalized income assumption and suppose, to simplify, that there are only two income sources, x and y with means $\bar{x}$ and $\bar{y}$. The inequality index I_G may therefore be written as $I_G(x,y)$, x and y being the vectors of the two incomes sources received by the various individuals. We first want to compute the contribution of income source x. The inequality linked to this component may be that existing when this component is the first or the second

[5] Sastre and Trannoy (2002) use this term by contrast to 'global methods' where "contributions must be computed for all income components defined at the outset". The sum of the components should also add up to the inequality to be "explained". Shorrocks's (1982) decomposition rule or Lerman and Yitzhaki's (1985) decomposition method belong to this global method.

to be eliminated. If it is eliminated first, the function $I_G(x,y)$ will become equal to $I_G(x=\bar{x},y)$ since the inequality of variable *x* has been eliminated, so that in this case the contribution $C(x)$ of *x* to the function $I_G(x,y)$ may be expressed as $C(x)=[I_G(x,y)-I_G(x=\bar{x},y)]$. If the variable *x* is the second one to be eliminated, the function $I_G(x,y)$ will be written as $I_G(x,y=\bar{y})$. Since both elimination sequences are possible and assuming the probability of these two sequences is the same, we may conclude that the contribution *C(x)* of income source *x* to the function $I_G(x,y)$ is equal to

$$C(x)=(1/2)\{I_G(x;y)-I_G(x=\bar{x};y)\} + (1/2)\{I_G(x;y=\bar{y})-I_G(x=\bar{x};y=\bar{y})\} \quad (8)$$

Similarly one can prove that the contribution *C(y)* of the variable *y t*o the function $I_G(x,y)$ may be expressed as

$$C(y)=(1/2)\{I_G(x;y)-I_G(x;y=\bar{y})\} + (1/2)\{I_G(x=\bar{x};y)-I_G(x=\bar{x};y=\bar{y})\} \quad (9)$$

Combining (10) and (11) we observe that

$$I_G(x,y)=C(x)+C(y) \quad (10)$$

since $I_G(x=\bar{x};y=\bar{y})$ is assumed to be equal to 0.

The same type of decomposition may be extended to the case of more than two determinants and it is then possible to determine the exact marginal impact of each of the different variables (income sources) on the inequality index I_G. The zero income Shapley decomposition follows the same algorithm but instead of replacing the value of an income source by its mean, we simply give it a value of 0.

As already mentioned, Shorrocks (1999) has shown that the Shapley decomposition can be applied to any function. In this paper we applied it also to the analysis of bi-polarization and polarization.

As far as the bi-polarization index P_G is concerned, one should note that the equalized income Shapley decomposition procedure would give the same result as the zero income Shapley decomposition procedure. The reason is that the bi-polarization index P_G, as indicated previously, is invariant not only to a multiplication of all incomes by a constant but also to equal additions to all incomes.

4. On the differential impact of various income sources on inequality, bi-polarization and polarization: the case of Luxembourg

The empirical analysis uses data from the Socio-Economic Panel "Liewen zu Lëtzebuerg" (PSELL-3) which is the component for Luxembourg of the EU *Statistics on Income and Living Conditions* (EU-SILC). PSELL-3 was launched in 2003, with an initial sample of 3500 households that were representative of the population living in private households in Luxembourg. In this paper, we analyse the data for the year 2008 composed of 3779 households.

The concept of income used in the PSELL dataset is quite broad as it comprises earnings from work including company cars, all social benefits received in cash, income from investment and property and inter-households payments. It is however not comprehensive as it excludes non-monetary income components such as imputed rents, the value of goods produced for own consumption and non-cash employee income (with the exception of company car). In the first stage of our analysis we make a distinction between three broad income sources which together constitute total unadjusted gross income: income from work, income from capital and social transfers.[6] In a second stage we further refine our analysis and use seven income sources (see Table B1 in Appendix B).

Table 1 displays the relative contribution of each of the three broad income sources to the values of the Gini index I_G, the bi-polarization index P_G and the polarization index DER, following the methodology presented in the previous sections. The Shapley contributions of the different income sources (in percentage terms) to these three indices are quite different.

[6] The definition of each component can be found in the EU-SILC 2008 guidelines. See http://circa.europa.eu/Public/irc/dsis/eusilc/library?l=/guidelines_questionnaire/operation_guidelines/silc065_version/_EN_1.0_&a=d

Table 1: Shapley decomposition procedure giving the contribution of three aggregated income sources to the Gini inequality index, the bi-polarization index P_G and the DER polarization index, 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
Gini inequality index, zero income decomposition				
Work	-0.011	-3.0	73.2	-0.04
Capital	0.297	80.7	3.3	24.5
Transfers	0.082	22.4	23.5	0.95
Total	0.368	100	100	
Gini inequality index, equalized income decomposition				
Work	0.292	79.4	73.2	1.08
Capital	0.02	5.3	3.3	1.61
Transfers	0.056	15.3	23.5	0.65
Total	0.368	100	100	
Bi-polarization P_G index				
Work	0.467	136	73.2	1.86
Capital	-0.008	-2.2	3.3	-0.66
Transfers	-0.116	-33.8	23.5	-1.44
Total	0.343	100	100	
Polarization DER (α=0.5) index				
Work	0.04	25.2	73.2	0.34
Capital	0.06	37.3	3.3	11.31
Transfers	0.06	37.5	23.5	1.6
Total	0.159	100	100	
Polarization DER (α=1) index				
Work	0.029	39	73.2	0.53
Capital	0.016	21.1	3.3	6.39
Transfers	0.029	39.9	23.5	1.7
Total	0.074	100	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.

Regarding the decomposition of the Gini inequality index, we examine the two types of Shapley decomposition, the zero income and the equalized income decompositions. The two cases give strikingly different results. In the first case it turns out that the marginal contribution of income from capital was 80.7% in 2008. In other words, had there not been any income from capital, inequality would have been much smaller (around 20% of the actual value of the Gini index). In the second case, what contributes most to inequality is income

from work whose relative contribution represents almost 80% of the value of the Gini index. In other words, had income from work been equally distributed, inequality would have been much smaller (around 20% of the actual value of the Gini index). The relative contribution of each source can be easily compared to its share in the income distribution when computing the ratio of the former to the latter. In the case of the zero decomposition, the relative contribution of income from capital is substantially higher than its share in total income (25 times!), whereas the contribution of income from work is extremely small compared to its income share. The equalized income decomposition gives results that are much more in line with the shares of the various income sources in total income. In 2008, the results show that income from work contributes to overall inequality slightly more than its share in the income distribution whereas income from capital contributes 61% more than its weight in total income and transfers contributes only 65% of their share in total gross income.

One should not be surprised to observe a dependence of the contributions of the income sources on the type of Shapley decomposition that is implemented. In the first case we are wondering what would have happened to inequality if a given income source did not exist. In the second case we do not simulate the disappearance of an income source. We simply ask what inequality would have been, had a given income source been equally distributed between the households. These two questions are totally different and hence one should not been amazed by the fact that the answers in the two cases are different.

We now examine the results of the Shapley decomposition of the bi-polarization index P_G. As mentioned previously the results of such a breakdown will be identical, whatever the type of Shapley decomposition that is implemented. It appears that here income from capital does not play an important role since its contribution is negative and around 2.2% of the value of the index P_G. This implies that if there had been no income from capital the index P_G would have been 2.2% higher. The main impact on bi-polarization turns out to be that of income from work. The Shapley contribution of this income source is greater than 100% (around 136%). As a consequence we can conclude that if this income source did not exist, the index P_G would have been negative and its magnitude would have been equal to 36% of the actual magnitude of the bi-polarization index. Finally, results show that income from work contribute to overall bi-polarization close to twice its share in the income distribution.

A negative value of the index P_G implies that the between groups Gini index is smaller than the within groups Gini index. Income from work has therefore a great impact on between

groups inequality (the two groups being those with an income higher and lower than the median income) and this is how it affects bi-polarization. Finally we also observe that income from transfers has a negative impact on bi-polarization (the absolute value of its Shapley contribution is equal more or less to 40%). This means that if there had been no transfers, bi-polarization would have been quite higher (around 135% of its actual value). We may therefore conclude that *bi-polarization is mainly due to income from work*.

As far as polarization is concerned, we have made a distinction between the case where the polarization sensitivity parameter α is equal to 0.5 and that where it is equal to 1. When $\alpha = 0.5$, Table 1 shows that income from work contribute less than income from capital and transfers. However when $\alpha = 1$, it appears that the main Shapley contributions are those of income from work and income from transfers (these two contributions are of somehow similar magnitude and represent together around 80% of the value of the index DER), income from capital having a marginal contribution of around 20%. Since, when examining the case of zero income decomposition, we had found that when $\alpha = 0$ (the case of the Gini index), the main Shapley contribution was that of income from capital, we can state that as the polarization sensitivity parameter α increases, the contribution of income from capital decreases.

We may therefore conclude that, although the three income sources contribute to polarization, as we increase the sensitivity polarization parameter the two main influences on polarization become those of income from work and income from transfers. To understand these results we have to remember that the polarization index DER measures really the extent to which there are local poles in the income distribution. These local poles are hence mainly due to income from work and income from transfers, income from capital playing a smaller role.

These findings should help us understanding the differences that exist between the concepts of inequality, bi-polarization and polarization. We base our conclusions on the case where the alternative in the Shapley decomposition is the absence of a given income source. In such a case the main contribution to inequality in Luxembourg is income from capital. In the case of bi-polarization the main role is played by income from work whereas polarization whose degree is assumed to measure the extent to which there are local poles in the distribution of income tends to be mainly related to income form work and income from transfers.

These conclusions should be confirmed by a more detailed analysis based on seven income sources. Table 2 gives the Shapley contributions of these seven sources to the Gini index. When using the zero income decomposition the source which contributes most to inequality is income from self-employment. It is followed by unemployment benefits and income from capital. When using the equalized income decomposition, income from employment contributes the most to inequality.

Table 2: Shapley decomposition procedure giving the contribution of seven income sources to the Gini inequality index, 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
Gini inequality index, zero income decomposition				
income from employment	-0.194	-52.6	67.1	-0.8
income from self employment	0.155	42.1	6.09	6.9
income from unemployment benefits	0.126	34.2	1.67	20.5
income from old age and survivors allowances	0.028	7.7	13.69	0.6
income from rent and capital	0.112	30.5	3.33	9.2
income from family allowances	0.046	12.5	4.67	2.7
income from other transfers	0.094	25.6	3.45	7.4
Total	0.368	100.0	100	
Gini inequality index, equalized income decomposition				
income from employment	0.248	67.5	67.1	1.0
income from self employment	0.042	11.3	6.09	1.9
income from unemployment benefits	0.006	1.5	1.67	0.9
income from old age and survivors allowances	0.033	9.0	13.69	0.7
income from rent and capital	0.018	5.0	3.33	1.5
income from family allowances	0.014	3.8	4.67	0.8
income from other transfers	0.007	1.8	3.45	0.5
Total	0.368	100.0	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.

Table 3 gives the Shapley contributions of the seven income sources to bi-polarization. The Shapley contribution of income from employment is around 140% which means that if there had been no income from employment, the index P_G would have been negative and its magnitude in absolute value would have been around 40% of the value of

P_G. Since, as mentioned previously, a negative value of the index P_G implies that the between groups Gini index is smaller than the within groups Gini index, we can conclude that income from employment has a great impact on between groups inequality and this is how it affects bi-polarization. If we now look at the marginal impact of income from self employment we observe that its Shapley value is 16.6% which means that if there had been no income from self employment bi-polarization would have been smaller and equal to more or less 85% of its actual value. The Shapley contribution of income from rent or capital is around 8% which means that if this source did not exist, bi-polarization would have been a bit smaller (92% of its actual value).

Table 3: Shapley decomposition procedure giving the contribution of seven income sources to the bipolarization P_G index, 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
income from employment	0.483	140.8	67.1	2.1
income from self employment	0.057	16.6	6.09	2.7
income from unemployment benefits	-0.053	-15.5	1.67	-9.3
income from old age and survivors allowances	-0.036	-10.4	13.69	-0.8
income from rent and capital	0.027	8.0	3.33	2.4
income from family allowances	-0.054	-15.9	4.67	-3.4
income from other transfers	-0.081	-23.6	3.45	-6.8
Total	0.343	100.0	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.

If we now look at the Shapley contributions of the seven income sources to the value of the polarization index DER, we make, here also, a distinction between the case where the polarization sensitivity parameter α is equal to 0.5 and that where it is equal to 1. In the former case (see, Table 4) we observe that all the income sources have generally a positive Shapley contribution. Moreover all these contributions are smaller than 100% which means that, whatever the source, we can say that, had the source under study not existed, polarization would have been smaller. Table 4 shows also that, when the parameter α is equal to 0.5, income from family allowances contributes the most to polarization.

Table 4: Shapley decomposition procedure giving the contribution of seven income sources to the Polarization DER index (with α=0.5 and α=1), 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
Polarization DER (α=0.5) index				
income from employment	0.021	13.2	67.1	0.2
income from self employment	0.001	0.3	6.09	0.1
income from unemployment benefits	0.022	13.5	1.67	8.1
income from old age and survivors allowances	0.025	15.6	13.69	1.1
income from rent and capital	0.029	18.0	3.33	5.4
income from family allowances	0.044	27.4	4.67	5.9
income from other transfers	0.019	11.9	3.45	3.5
Total	0.159	100.0	100	
Polarization DER (α=1) index				
income from employment	0.033	45.4	67.1	0.7
income from self employment	-0.008	-11.2	6.09	-1.8
income from unemployment benefits	0.005	6.8	1.67	4.1
income from old age and survivors allowances	0.007	9.8	13.69	0.7
income from rent and capital	0.009	11.8	3.33	3.6
income from family allowances	0.024	32.9	4.67	7.1
income from other transfers	0.003	4.4	3.45	1.3
Total	0.074	100.0	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.

If we now take the case where the polarization sensitivity parameter α is equal to 1 (see Table 4), we first observe that only one source has a negative contribution, income from self employment, but the magnitude of its contribution is small. We can nevertheless say that when the Shapley contribution is negative, this implies that had there been no income from self employment polarization would have been higher. Income from self employment seems therefore to smooth the income distribution in the sense that it decreases the extent to which there are local poles. This might indicate that self employed individuals may come from all strata of the population.

All the other income sources have a positive Shapley contribution, but smaller than 100%. This implies that, had one of these sources not existed, polarization would have been smaller. The most important contributions are those of income from employment and from family allowances which are therefore those that have an impact on the existence of local poles in the distribution of income.

To summarize, when using the zero income decomposition of the Gini inequality index, the main contribution to inequality in Luxembourg is income from self-employment and capital. In the case of bi-polarization the main role is played by income from work whereas polarization whose degree is assumed to measure the extent to which there are local poles in the distribution of income tends to be mainly related to income form work and income from family allowances. As a whole these conclusions confirm those drawn on the basis of only three broad income sources.

4. Concluding comments

This paper confirms that inequality, bi-polarization and polarization are indeed three different concepts. Such a conclusion is based on an empirical analysis that attempted to look at the impact of different income sources on inequality, bi-polarization and polarization on the basis of the concept of Shapley decomposition, using the "Liewen zu Lëtzebuerg" socio-economic panel for the year 2008. When making a distinction between only three broad income sources, income from work, from capital and from transfers, and implementing a Shapley breakdown, we found, when using the zero income decomposition, that inequality in Luxembourg is strongly related to income from capital. On the other hand when using the equalized income decomposition we concluded that income from work plays the main role. We also observed that the extent of bi-polarization is mainly a consequence of the distribution of income from work. Finally polarization whose degree is assumed to measure the extent to which there are local poles in the distribution of income tends to be mainly related to income form work and income from transfers. Similar conclusions were derived on the basis of seven types of income sources.

It thus appear that a given income source has generally a different impact on inequality, bi-polarization and polarization so that it seems to be of utmost importance to make a clear distinction between these three concepts.

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Appendix A: The index P_G and the two properties of "increasing spread" and "increasing bipolarity"

For non-overlapping groups the P_G index may be written as

$P_G = (G_B - G_W)(G_B + G_W)$	(A-1)

where G_B and G_W refer respectively to the between groups (of equal size) and within groups Gini indices.
Since a Gini index *G* may be written as

$G = (1/2)(\Delta / \bar{y})$	(A-2)

where Δ is the mean difference and $\bar{y}$ the mean income, we can also express (A-1) as

$P_G = \{[((1 / 2)(\Delta_B / \bar{y})) - ((1/2)(\Delta_W / \bar{y}))]/[\left((1/2)\left(\Delta_B/\bar{y}\right)\right) + ((1/2)(\Delta_W / \bar{y}))]\}$	(A-3)

$\leftrightarrow P_G = (\Delta_B - \Delta_W)/(\Delta_B + \Delta_W)$	(A-4)

Note that this result shows that P_G has an interesting property: it is invariant to both a multiplication of all incomes by a constant but also to equal additions to all incomes, since a mean difference is invariant to equal additions to all incomes. This result was not stressed in the original paper of Deutsch et al. (2007).
Since the groups are of equal size, the between groups Gini index G_B may be written as

$G_B = (1/2)((\bar{y}_R - \bar{y}_P)/(\bar{y}_R + \bar{y}_P))$	(A-5)

where $\bar{y}_R$ and $\bar{y}_P$ are the mean incomes of the "rich" (those whose income is higher than the median) and of the "poor" (those whose income is lower than the median).
Since

$G_B = (1/2)(\Delta_B/\bar{y})$	(A-6)

we end up, combining (A-5) and (A-6) with

$\Delta_B = 2G_B\,\bar{y}$ =(1/2)$(\bar{y}_R - \bar{y}_P)$	(A-7)

since $\bar{y}$ =(1/2)$(\bar{y}_R + \bar{y}_P)$.
In addition, we know (see, for example, Silber, 1989) that the within groups Gini index may be expressed as

$G_W = (1/2)(\Delta_W/\bar{y}) = (f_P s_P G_P) + (f_R s_R G_R)$	(A-8)

where f_P,f_R, s_P,s_R,G_P, G_R are respectively the shares in the total population of the poor and rich, the shares in total income of the poor and rich and the Gini indices among the poor and rich.

But

$G_P = (1/2)(\Delta_P/\bar{y}_P)$	(A-9)

And

$G_R = (1/2)(\Delta_R/\bar{y}_R)$	(A-10)

where Δ_R and Δ_P are respectively the mean income differences among the "poor" and the "rich".

Combining (A-8), (A-9) and (A-10) we get

$\Delta_W = 2\bar{y}\, G_W =$ $(2\bar{y})\{(f_P)\left(f_P\left(\frac{\bar{y}_P}{\bar{y}}\right)\right)\left((1/2)(\Delta_P/\bar{y}_P)\right) + (f_R)\left(f_R\left(\frac{\bar{y}_R}{\bar{y}}\right)\right)\left((1/2)(\Delta_R/\bar{y}_R)\right)\}$	(A-11)

since $s_P = \left(f_P\left(\frac{\bar{y}_P}{\bar{y}}\right)\right)$ and $s_R = \left(f_R\left(\frac{\bar{y}_R}{\bar{y}}\right)\right)$

so that we end up with

$\Delta_W = {f_P}^2\Delta_P + {f_R}^2\Delta_R = (\frac{1}{4})\Delta_P + (\frac{1}{4})\Delta_R$	(A-12)

since $f_P = f_R = (1/2)$ (the two groups of "poor" and "rich" are of equal size).

Combining (A-4), (A-7) and (A-12) we get

$P_G = \{[(1/2)(\bar{y}_R - \bar{y}_P)] - [(\frac{1}{4})\Delta_P + (\frac{1}{4})\Delta_R]\}/\{[(1/2)(\bar{y}_R - \bar{y}_P)] + [(\frac{1}{4})\Delta_P + (\frac{1}{4})\Delta_R]\}$	(A-13)

$\leftrightarrow P_G = [2(\bar{y}_R - \bar{y}_P) - (\Delta_P + \Delta_R)]/[2(\bar{y}_R - \bar{y}_P) + (\Delta_P + \Delta_R)]$	(A-14)

Expression (A-14) shows clearly that P_G is a function of the mean incomes of the "rich" and of the "poor" and of the mean differences of the incomes of the "rich" and of the "poor".

The axiom of increasing spread (IS) states that bi-polarization will increase if either $\bar{y}_R$ increases or $\bar{y}_P$ decreases. It is best to assume that this increase in the mean income of the "rich" or this decrease in the mean income the poor occurs, for a given value of the mean differences of the income of the "rich" and of the "poor", so that IS does not take place at the same time as there is a change in bipolarity.

Similarly there will be increased bi-polarity (IB) if Δ_R or/and Δ_P decreases. It is here again best to assume that such a change occurs without any change in the mean income of the "rich" and the "poor" so that increased bipolarity (IB) will occur without affecting the spread (IB).

We can now easily compute the derivative of the bi-polarization index P_G with respect to the mean income of the "rich" and of the "poor" as well as with respect to the mean difference of the incomes of the "rich" and of the "poor" and, as expected, we can easily prove that $\frac{\partial P_G}{\partial \bar{y}_R}>0$; $\frac{\partial P_G}{\partial \bar{y}_P}<o$;$\frac{\partial P_G}{\partial \Delta_R} < 0$; $\frac{\partial P_G}{\partial \Delta_P} < 0$;

The bi-polarization index P_G therefore obeys the principles of "increasing spread" and "increasing bipolarity".

Appendix B.

Table B1: Income Components in the PSELL dataset.

3 sources	7 components	single component
income from work	Employment	gross employee cash or near cash income
		gross non cash employee income
	self-employment	gross cash benefits or losses from self-employment (including royalties)
capital	Capital	income from rental of a property or land
		interests/dividends from capital investments
transfers	unemployment benefits	unemployment benefits
	old age and survivors benefits	old-age benefits
		survivors benefits
	family allowances	family/children related allowances
	other transfers	sickness benefits
		disability benefits
		education related allowances
		social exclusion not elsewhere classified
		housing allowances
		regular inter-household cash transfers received
		income received by under 16

Reading note: 'income from work' is the sum of 'income from employment' and 'income from self-employment'. 'Income from employment' is the sum of two income components, namely 'gross employee cash or near cash income' and 'gross non cash employee income'; 'Income from self-employment' includes only one component, 'gross cash benefits or losses from self-employment'. The sum of each column is equal to the 'total unadjusted gross income'.