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## A nonparametric test of the leverage hypothesis

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# A nonparametric test of the leverage hypothesis

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**cemmap** working paper CWP24/12

# A Nonparametric Test of the Leverage Hypothesis

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## Abstract

The so-called leverage hypothesis is that negative shocks to prices/returns affect volatility more than equal positive shocks. Whether this is attributable to changing financial leverage is still subject to dispute but the terminology is in wide use. There are many tests of the leverage hypothesis using discrete time data. These typically involve fitting of a general parametric or semiparametric model to conditional volatility and then testing the implied restrictions on parameters or curves. We propose an alternative way of testing this hypothesis using realized volatility as an alternative direct nonparametric measure. Our null hypothesis is of conditional distributional dominance and so is much stronger than the usual hypotheses considered previously. We implement our test on a number of stock return datasets using intraday data over a long span. We find powerful evidence in favour of our hypothesis.

*Key words:* Distribution function; Leverage Effect; Gaussian Process;

*Journal of Economic Literature Classification:* C14, C15.

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# 1 Introduction

The so-called leverage hypothesis is that negative shocks to stock prices affect their volatility more than equal magnitude positive shocks. Whether this is attributable to changing financial leverage is still subject to dispute (Engle and Ng (1993) and Figlewski and Wang (2000)), but the terminology is in wide use. There are many tests of the leverage hypothesis using discrete time data. These typically involve fitting of a general parametric or semiparametric model to conditional volatility and then testing the implied restrictions on parameters or curves, see for example Nelson (1991), Engle and Ng (1993), and Linton and Mammen (2005). There is also an important recent literature on leverage effects in continuous time. Wang and Mykland (2009, 2012) propose a nonparametric estimator of a class of leverage parameters inside a very general class of continuous time stochastic processes. They propose an estimator that is quite simple and easily studied and provide its limiting properties. They extend the theory to allow for measurement error and therefore more complicated estimators of volatility and leverage. Their modified procedure is consistent and asymptotically mixed normal in this case too, although the rate of convergence is slower. They provide the means to conduct inference about the leverage parameter, although their application is more towards prediction of volatility and they demonstrate the value added that their leverage effect has in this purpose. Ait-Sahalia, Fan, and Li (2011) also investigate the leverage effect “puzzle” within the continuous time framework. The puzzle is that natural estimators of the leverage effect based on high frequency data are usually very small and insignificant. They take apart the sources of this finding and interpret it as bias due to microstructure noise issues, and they propose a solution to this based on a bias correction. Empirically their method seems to uncover a stronger leverage effect.

We propose a way of testing the leverage hypothesis nonparametrically without requiring a specific parametric or semiparametric model. Our null hypothesis is that the conditional distribution of volatility given negative returns and past volatility dominates in the first order sense the distribution of volatility given positive returns and past volatility. This hypothesis is in some sense more general than those considered previously since we refer to the distribution of outcomes rather than just correlations, although Wang and Mykland (2012) also allow for the leverage effect to be defined through any (given) function  $F$  of volatility. Our distribution theory builds on work of Linton, Maasoumi, and Whang (2005) who considered tests of unconditional dominance for time series data. Linton, Song, and Whang (2010) consider conditional dominance tests but inside specific semiparametric models. We obtain the limiting distribution, which is a functional of a Gaussian process. Since the

limit distribution depends in a complicated way on nuisance parameters, we propose an inference method based on subsampling. Our test is consistent against a general class of alternatives. We use realized volatility as a direct nonparametric measure of volatility. This allows us to avoid specifying any model for volatility and makes our test model free. We do assume some high level conditions like stationarity and mixing properties of the data, but these can be generalized in various directions. Our methodology sits between discrete time econometrics and continuous time econometrics. If the volatility measure we use can be interpreted as an unbiased estimator of ex ante volatility, then our hypothesis can be interpreted inside the typical discrete time framework.

We apply our testing methodology to real data. We focus on whether there is a leverage effect between daily conditional variances and daily lagged returns in S&P500 (cash) index and individual stocks. The stocks we consider are five constituents of the Dow Jones Industrial Average: Microsoft, IBM, General Electronic, Procter& Gamble and 3M. The sample period covers 1993 to the end of 2009, which includes several very volatile episodes as well as some more tranquil ones. We find strong evidence in favour of our leverage effect in this data. We also cast several robustness checks and compare our results with some newly developed methods, and find these further results lend supports to our conclusions.

## 2 Hypotheses of interest

We suppose that we observe a process  $\{y_t, x_t, r_t\}_{t=1}^T$  that is either stationary or has stationary transitions, where  $x_t \in \mathbb{R}^{d_x}$  for some  $d_x$ . Let

$$\begin{aligned} F^+(y|x) &= \Pr(y_t \leq y \mid r_{t-1} \geq 0, x_t = x) \\ F^-(y|x) &= \Pr(y_t \leq y \mid r_{t-1} < 0, x_{t-1} = x). \end{aligned}$$

We consider the hypothesis

$$\begin{aligned} H_0 &: F^+(y|x) \geq F^-(y|x) \text{ a.s. for all } (y, x) \in \mathcal{Y} \times \mathcal{X} \\ H_1 &: F^+(y|x) < F^-(y|x) \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X}, \end{aligned}$$

where  $\mathcal{Y} \subset \mathbb{R}$  denotes the support of  $y_t$  and  $\mathcal{X} \subset \mathbb{R}^{d_x}$  denotes the support of  $x_t$ . A leading example would be to take  $y_t = \sigma_t^2$  and  $x_t = \sigma_{t-1}^2$ , in which case the hypothesis is effectively that bad news on returns leads to a bigger effect on the conditional distribution of future volatility than good news

whatever the current level of volatility. In this case, we can take  $\mathcal{Y} = \mathcal{X} \subset \mathbb{R}_+$ . Suppose that  $\sigma_t^2$  was generated from a GJR GARCH(1,1) process, i.e.,

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma_+r_{t-1}^2\mathbf{1}(r_{t-1} > 0) + \gamma_-r_{t-1}^2\mathbf{1}(r_{t-1} \leq 0). \quad (1)$$

The case where  $\gamma_- > \gamma_+$  corresponds to the presence of a leverage effect. In this case the distribution  $F^-(y|x)$  first order dominates  $F^+(y|x)$  for all  $x$ .

We allow for the more general formulation than this example for practical reasons. In view of the possible strong dependence in volatility, we might consider conditioning on  $x_t = (\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) \in \mathbb{R}^p$  instead of just on  $\sigma_{t-1}^2$ . In practice, however, this is likely to work poorly for large  $p$  because of the curse of dimensionality. We consider a compromise approach in which we condition on a lower dimensional transform of a vector of lagged volatilities. Specifically, let  $h : \mathbb{R}^p \rightarrow \mathbb{R}^{d_x}$  for  $d_x < p$  be a measurable function and replace  $\sigma_{t-1}^2$  by  $x_t = h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$ . For example,  $h(x_1, \dots, x_p) = \sum_{j=1}^p c_j x_j$  for known  $c_1, \dots, c_p$ . In this case we consider the conditional distributions  $F^+(y|x) = \Pr(\sigma_t^2 \leq y \mid r_{t-1} \geq 0, h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = x)$  and  $F^-(y|x) = \Pr(\sigma_t^2 \leq y \mid r_{t-1} < 0, h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = x)$ .

Letting

$$\begin{aligned} \pi_0^+(x) &= \Pr(r_{t-1} \geq 0 \mid x_t = x) \\ \pi_0^-(x) &= \Pr(r_{t-1} < 0 \mid x_t = x), \end{aligned}$$

we can write the above hypotheses by the conditional moment inequalities:

$$\begin{aligned} H_0 &: E \left[ \mathbf{1}(y_t \leq y) \left( \frac{\mathbf{1}(r_{t-1} < 0)}{\pi_0^-(x_t)} - \frac{\mathbf{1}(r_{t-1} \geq 0)}{\pi_0^+(x_t)} \right) \middle| x_t = x \right] \leq 0 \text{ for all } (y, x) \in \mathcal{Y} \times \mathcal{X}, \\ H_1 &: E \left[ \mathbf{1}(y_t \leq y) \left( \frac{\mathbf{1}(r_{t-1} < 0)}{\pi_0^-(x_t)} - \frac{\mathbf{1}(r_{t-1} \geq 0)}{\pi_0^+(x_t)} \right) \middle| x_t = x \right] > 0 \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X}, \end{aligned}$$

or equivalently,

$$\begin{aligned} H_0 &: E \left[ \mathbf{1}(y_t \leq y) \{ \pi_0^+(x_t) - \mathbf{1}(r_{t-1} \geq 0) \} \middle| x_t = x \right] \leq 0 \text{ for all } (y, x) \in \mathcal{Y} \times \mathcal{X} \\ H_1 &: E \left[ \mathbf{1}(y_t \leq y) \{ \pi_0^+(x_t) - \mathbf{1}(r_{t-1} \geq 0) \} \middle| x_t = x \right] > 0 \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X}, \end{aligned}$$

using the fact  $\pi_0^+(x) = 1 - \pi_0^-(x) > 0$  for all  $x$ . It is well known that the hypotheses of  $H_0$  and  $H_1$  can be equivalently stated using the unconditional moment inequalities

$$H_0 : E \left[ \mathbf{1}(y_t \leq y) g(x_t) \{ \pi_0^+(x_t) - \mathbf{1}(r_{t-1} \geq 0) \} \right] \leq 0 \text{ for all } (y, g) \in \mathcal{Y} \times \mathcal{G}, \quad (2)$$

$$H_1 : E \left[ \mathbf{1}(y_t \leq y) g(x_t) \{ \pi_0^+(x_t) - \mathbf{1}(r_{t-1} \geq 0) \} \right] > 0 \text{ for some } (y, g) \in \mathcal{Y} \times \mathcal{G}, \quad (3)$$

where  $g$  is an instrument that depends on the conditioning variable  $x_t$  and  $\mathcal{G}$  is the collection of instruments, see, e.g., Andrews and Shi (2011) and the references therein. In this paper, we take

$$\mathcal{G} = \{g_{a,b} : g_{a,b}(x) = \prod_{i=1}^{d_x} 1(a_i < x_i \leq b_i) \text{ for some } a, b \in \mathcal{X}\},$$

see Andrews and Shi (2011) for more examples of instruments. We will use the relation (2) to generate a test statistic.

The case of first order dominance between the conditional distributions  $F^-(y|x), F^+(y|x)$  may be quite strong, and one might consider the weaker concept of second order dominance. This would allow for more general functional forms for volatility than considered above and subtler relationships between the distributions than hitherto assumed.

We emphasize the null hypothesis of a leverage effect. Instead, one might take the hypothesis to be the absence of a leverage effect. Specifically, we might consider the conditional independence hypothesis, i.e.,

$$\sigma_t^2 \text{ is independent of } \text{sign}(r_{t-1}) \text{ given } \sigma_{t-1}^2. \quad (4)$$

This hypothesis would be consistent with a GARCH(1,1) process for  $\sigma_t^2$ , namely,  $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma r_{t-1}^2$  for positive parameters  $\omega, \gamma, \beta$ . The GJR process (1) is incompatible with this hypothesis. In fact, the GJR process is incompatible with (4) whenever  $\gamma_- \neq \gamma_+$ . In general, the alternative hypothesis to (4) contains many processes that do not represent what we think a leverage effect should be, and we should properly define the null hypothesis in a much more complicated way (in the parametric case this is straightforward, but not so in the nonparametric case), which is why we do not pursue this hypothesis here further.<sup>1</sup>

### 3 Test Statistic

Let  $\hat{\pi}^+$  be nonparametric kernel estimators of  $\pi_0^+$ , i.e.,

$$\hat{\pi}^+(x) = \frac{\sum_{t=2}^T 1(r_{t-1} \geq 0) K_h(x - x_t)}{\sum_{t=2}^T K_h(x - x_t)},$$

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<sup>1</sup>This hypothesis could be tested using the methods developed in Linton and Gozalo (1996) for example.

where  $K : \mathbb{R}^{d_x} \rightarrow \mathbb{R}$  is a kernel function and  $K_h(\cdot) = K(\cdot/h)/h$  and  $h$  is a bandwidth parameter satisfying the assumptions below. Now the hypothesis can be tested based on the following statistic

$$\bar{m}_T(y, g, \pi) = \frac{1}{T} \sum_{t=1}^T 1(y_t \leq y) g(x_t) \{\pi(x_t) - 1(r_{t-1} \geq 0)\}.$$

We consider Kolmogorov Smirnov-type (KS) and Cramér-von Mises-type (CM) test statistics, defined by

$$\begin{aligned} S_T &= \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \sqrt{T} \bar{m}_T(y, g, \hat{\pi}^+) \text{ and} \\ S_T^* &= \int \max \left\{ (\sqrt{T} \bar{m}_T(y, g, \hat{\pi}^+), 0) \right\}^2 dQ(y, g), \end{aligned}$$

respectively, where  $Q$  is a weighting function (i.e., probability measure) on  $\mathcal{Y} \times \mathcal{G}$  and the integral is over  $\mathcal{Y} \times \mathcal{G}$ . Below, for brevity, we mainly discuss the asymptotic properties of  $S_T$ , but we expect that analogous results hold for  $S_T^*$ .

## 4 Asymptotic Theory

We suppose that we observe a process  $\{y_t, x_t, r_t\}_{t=1}^T$ . In practice, we have only an estimate  $\hat{\sigma}_t^2$  of  $\sigma_t^2$  computed from high frequency data. Barndorff-Nielsen and Shephard (2002) have shown that the realized volatility consistently estimates  $\sigma_t^2$  at rate  $n_t^{-1/2}$ , where  $n_t$  is the number of high frequency observations within day  $t$ . We expect that  $n_t$  is very large relative to  $T$ . In the sequel we shall ignore the effect that estimation of volatility from high frequency data has.

### 4.1 The null distribution

Let  $\mathcal{X}_\varepsilon$  be an  $\varepsilon$ -neighborhood of  $\mathcal{X}$  for some  $\varepsilon > 0$ . For some constant  $B < \infty$ , let

$$\Pi = \left\{ \pi : \|\pi(\cdot)\|_{q, \mathcal{X}_\varepsilon} \leq B \right\}, \quad (5)$$

where  $q$  is an integer that satisfies  $q > d_x/2$ . For nonnegative integers  $k, \lambda$  and  $\omega$  with  $\omega \geq \lambda$ , we define the following class of kernels:



$$\begin{aligned}
\mathcal{K}_{k,\lambda,\omega} = \{ & K(\cdot) : \mathbb{R}^k \rightarrow \mathbb{R} : \int K(x)dx = 1, \int x^\mu K(x)dx = 0 \\
& \forall 1 \leq |\mu| \leq \omega - \lambda - 1, \int |x^\mu K(x)|dx < \infty \forall |\mu| = \omega - \lambda, \\
& D^\mu K(x) \rightarrow 0 \text{ as } \|x\| \rightarrow \infty \forall \mu \text{ with } |\mu| < \lambda, \\
& \sup_{z \in \mathbb{R}^k} |D^{\mu+e_j} K(x)|(\|x\| \vee 1) < \infty \forall \mu \text{ with } |\mu| \leq \lambda \forall j = 1, \dots, k, \\
& \text{and } K(\cdot) \text{ is zero outside a bounded set in } \mathbb{R}^{d_x}, \\
& \text{where } e_j \text{ denotes the } j\text{-th elementary } d_x\text{-vector.} \} .
\end{aligned}$$

#### ASSUMPTION A

1. (i)  $\{(y_t, x_t, r_t) : t \geq 1\}$  is a sequence of strictly stationary strong mixing random variables with mixing numbers of size  $-2(4d_x + 5)(d_x + 2)$ . (ii)  $\mathcal{X}$  is an open bounded subset of  $\mathbb{R}^{d_x}$  with minimally smooth boundary.
2. (i) The distribution of  $x_t$  is absolutely continuous with respect to Lebesgue measure with density  $f(x)$ . (ii)  $\inf_{x \in \mathcal{X}_\varepsilon} f(x) > 0$ ,  $D^\mu f(x)$  exists and is continuous on  $\mathbb{R}^{d_x}$  and  $\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu f(x)| < \infty \forall \mu$  with  $|\mu| \leq \max\{\omega, q\}$ , where  $\omega$  is a positive integer that also appears in the other assumptions below. (iii) The conditional distribution  $F(y|x)$  of  $y_t$  given  $x_t = x$  has bounded density  $f(y|x)$  for almost all  $x \in \mathbb{R}^{d_x}$ .
3.  $D^\mu [\pi_0^+(x)f(x)]$  exists and are continuous on  $\mathbb{R}^{d_x}$  and  $\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu [\pi_0^+(x)f(x)]| < \infty \forall \mu$  with  $|\mu| \leq \max\{\omega, q\}$ .
4.  $K(\cdot) \in \mathcal{K}_{d_x,0,\omega} \cap \mathcal{K}_{d_x,q,q}$ .
5. The bandwidth parameter  $h$  satisfies  $T^{\min\{\frac{1}{2(d_x+q)}, \frac{1}{4d_x}\}} h \rightarrow \infty$  and  $T^{\frac{1}{2\omega}} h \rightarrow 0$ .

Assumption A1 requires that  $x_t$  lies in an open bounded set with minimally smooth boundary. Examples of sets with minimally smooth boundaries include open bounded sets that are convex or whose boundaries are  $C^1$ -embedded in  $\mathbb{R}^{d_x}$ . Finite unions of aforementioned type whose closures are disjoint also have minimally smooth boundaries. The boundedness assumption is not restrictive, because, if needed, we can transform the values of  $x_t$  into a compact interval, say  $[0, 1]^{d_x}$ , via strictly

increasing transformation. Assumptions A2 and A3 imposes smoothness on  $f$  and  $\pi_0^+$ . They are needed to ensure that the realization of  $\hat{\pi}^+$  are smooth with probability tending to one and therefore the stochastic equicontinuity condition of a stochastic process  $\{\bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1\}$  that appears in our proof can be verified. The use of higher-order kernel  $K(\cdot)$  in Assumption A4 is due to the need to establish  $T^\kappa$  convergence of the kernel estimators  $\hat{f}$ ,  $\hat{\pi}^+(x)$  (see (20) and (21) in Appendix) for some sufficiently large  $\kappa \geq 1/4$ . Assumption A5 imposes some conditions on the rate of convergence of bandwidth to zero. The conditions are compatible if  $\omega$  is sufficiently large. These conditions can be relaxed, if needed, to allow for data-dependent methods of choosing bandwidth parameters, e.g. cross-validation or plug-in procedures.

We now derive the asymptotic of the test statistic under the null hypothesis. Define the empirical processes in  $(y, g) \in \mathbb{R} \times \mathcal{G}$

$$\nu_T(y, g) = \sqrt{T} \{ \xi_T(y, g) - E \xi_T(y, g) \}, \quad (6)$$

where

$$\xi_T(y, g) = \frac{1}{T} \sum_{t=1}^T \{ 1(y_t \leq y) - F(y|x_t) \} g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}. \quad (7)$$

Let  $\nu(y, g)$  be a mean zero Gaussian process with covariance function given by

$$C((y_1, g_1), (y_2, g_2)) = \lim_{T \rightarrow \infty} \text{cov}(\nu_T(y_1, g_1), \nu_T(y_2, g_2)).$$

The limiting null distribution of our test statistic is given in the following theorem.

**Theorem 1.** *Suppose that Assumption A holds. Then, under the null hypothesis  $H_0$ ,*

$$S_T \Rightarrow \begin{cases} \sup_{(y,g) \in \mathcal{B}} [\nu(y, g)] & \text{if } \mathcal{B} \neq \emptyset \\ -\infty & \text{if } \mathcal{B} = \emptyset \end{cases},$$

where  $\mathcal{B} = \{(y, g) \in \mathcal{Y} \times \mathcal{G} : E [1(y_t \leq y)g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}] = 0\}$ .

Theorem 1 shows that our test statistic has a non-degenerate limiting distribution on the boundary of the null hypothesis, i.e. the case where the "contact set" (i.e., the subset of  $\mathcal{Y} \times \mathcal{G}$  where the null hypothesis (2) holds with equality) is non-empty. Since the distribution depends on the true data generating process, we cannot tabulate it once and for all. We suggest estimating the critical values by a subsampling procedure.

## 4.2 Critical Values and Consistency

We first define the subsampling procedure. With some abuse of notation, the test statistic  $S_T$  can be re-written as a function of the data  $\{W_t : t = 1, \dots, T\}$  :

$$S_T = \sqrt{T} \tau_T(W_1, \dots, W_T),$$

where  $\tau_T(W_1, \dots, W_T)$  is given by  $\sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \bar{m}_T(y, g, \hat{\pi}^+)$ . Let

$$G_T(\cdot) = \Pr \left( \sqrt{T} \tau_T(W_1, \dots, W_T) \leq \cdot \right) \quad (8)$$

denote the distribution function of  $S_T$ . Let  $\tau_{T,b,t}$  be equal to the statistic  $\tau_b$  evaluated at the subsample  $\{W_t, \dots, W_{t+b-1}\}$  of size  $b$ , i.e.,

$$\tau_{T,b,t} = \tau(W_t, W_{t+1}, \dots, W_{t+b-1}) \text{ for } t = 1, \dots, T - b + 1.$$

We note that each subsample of size  $b$  (taken *without replacement* from the original data) is indeed a sample of size  $b$  from the true sampling distribution of the original data. Hence, it is clear that one can approximate the sampling distribution of  $S_T$  using the distribution of the values of  $\tau_{T,b,t}$  computed over  $T - b + 1$  different subsamples of size  $b$ . That is, we approximate the sampling distribution  $G_T$  of  $S_T$  by

$$\hat{G}_{T,b}(\cdot) = \frac{1}{T - b + 1} \sum_{t=1}^{T-b+1} 1 \left( \sqrt{b} \tau_{T,b,t} \leq \cdot \right).$$

Let  $g_{T,b}(1 - \alpha)$  denote the  $(1 - \alpha)$ -th sample quantile of  $\hat{G}_{T,b}(\cdot)$ , i.e.,

$$g_{T,b}(1 - \alpha) = \inf \{ w : \hat{G}_{T,b}(w) \geq 1 - \alpha \}.$$

We call it the *subsample critical value* of significance level  $\alpha$ . Thus, we reject the null hypothesis at the significance level  $\alpha$  if  $S_T > g_{T,b}(1 - \alpha)$ . The computation of this critical value is not particularly onerous, although it depends on how big  $b$  is. The subsampling method has been proposed in Politis and Romano (1994) and is thoroughly reviewed in Politis, Romano, and Wolf (1999). It works in many cases where the standard bootstrap fails: in heavy tailed distributions, in unit root cases, in cases where the parameter is on the boundary of its space, etc.

We now show that our subsampling procedure works under a very weak condition on  $b$ . In many practical situations, the choice of  $b$  will be data-dependent, see Linton, Maasoumi and Whang (2005,

Section 5.2) for some methodology for choosing  $b$ . To accommodate such possibilities, we assume that  $b = \hat{b}_T$  is a data-dependent sequence satisfying

**Assumption B:**  $P[l_T \leq \hat{b}_T \leq u_T] \rightarrow 1$  where  $l_T$  and  $u_T$  are integers satisfying  $1 \leq l_T \leq u_T \leq T$ ,  $l_T \rightarrow \infty$  and  $u_T/T \rightarrow 0$  as  $T \rightarrow \infty$ .

The following theorem shows that our test based on the subsample critical value has asymptotically correct size:

**Theorem 2.** *Suppose Assumptions A and B hold. Then, under the null hypothesis  $H_0$ ,*

$$\lim_{T \rightarrow \infty} \Pr[S_T > g_{T, \hat{b}_T}(1 - \alpha)] \leq \alpha,$$

*with equality holding if  $\mathcal{B} \neq \emptyset$ , where  $\mathcal{B}$  is defined in Theorem 1.*

Theorem 2 shows that our test based on the subsampling critical values has asymptotically valid size under the null hypothesis and has asymptotically exact size on the boundary of the null hypothesis. Under additional regularity conditions, we can extend this pointwise result to establish that our test has asymptotically correct size uniformly over the distributions under the null hypothesis, using the arguments of Andrews and Shi (2011) and Linton, Song and Whang (2010). For brevity, we do not discuss the details of this issue in this paper.

We next establish that the test  $S_T$  based on the subsampling critical values is consistent against the fixed alternative  $H_1$ .

**Theorem 3.** *Suppose that Assumptions A and B hold. Then, under the alternative hypothesis  $H_1$ ,*

$$\lim_{T \rightarrow \infty} \Pr[S_T > g_{T, \hat{b}_T}(1 - \alpha)] = 1.$$

## 5 Empirical Results

In this section we perform the conditional leverage hypothesis test using real data. We focus on whether there is a leverage effect between daily conditional variances and daily lagged returns in S&P500 (cash) index and individual stocks. The stocks considered here are five constituents of Dow Jones Industrial Average (DJIA): Microsoft (MSFT), IBM (IBM), General Electronic (GE), Procter&Gamble (PG) and 3M (MMM). The samples used for the test span from Jan-04-1993 to Dec-31-2009 (4,283 trading days). We first introduce two estimators for estimating the daily conditional variance  $\sigma_t^2$ . Then we detail how to construct the test statistic with the estimated  $\sigma_t^2$  and present the empirical results.

## 5.1 Estimating The Daily Conditional Variance

For estimating the daily conditional variance  $\sigma_t^2$ , we first consider the realized variance estimator  $RV_t$ :

$$RV_t = \sum_{i=1}^{n_t} r_{i,t}^2,$$

where

$$r_{i,t} = \log P \left( t - 1 + \frac{i}{n_t} \right) - \log P \left( t - 1 + \frac{i-1}{n_t} \right)$$

is the  $i$ th intraday log return on day  $t$ ,  $n_t$  is the total number of intraday log return observations on day  $t$ , and  $P(t - 1 + i/n_t)$  is the intraday asset price at time stamp  $t - 1 + i/n_t$ .

The second estimator we consider for estimating  $\sigma_t^2$  is the squared intraday range estimator  $RG_t^2$  (Garman and Klass, 1980; Parkinson, 1980):

$$RG_t^2 = \frac{IG^2}{4 \log 2},$$

where

$$IG = \max_{t-1 \leq \tau < t} \log P(\tau) - \min_{t-1 \leq \tau < t} \log P(\tau),$$

and  $P(\tau)$  is the intraday asset price at time stamp  $\tau$  on day  $t$ ,  $t - 1 \leq \tau < t$ . The constant  $4 \log 2$  is an adjustment factor to scale  $IG^2$  in order to obtain an unbiased estimation for  $\sigma_t^2$ . Together with other mild regular conditions,  $RG_t^2$  is a conditionally unbiased estimator for  $\sigma_t^2$ .

Let  $RV_t^{1min}$  and  $RV_t^{5min}$  denote 1-min and 5-min realized variances<sup>2</sup>. Figure 1 shows time series plots of  $RV_t^{1min}$ ,  $RV_t^{5min}$ ,  $RG_t^2$  and daily return  $R_t$  for S&P500 index and the five constituents of DJIA. Here the  $R_t$  of S&P500 is daily index return, and of the DJIA stocks are daily holding period returns excluding dividends. From the Figure, during the 2008 financial crisis period, all the five stocks and S&P500 show huge fluctuations in daily returns and conditional variances, while in other periods, these fluctuations are relatively mild.

Table 1 shows some summary statistics of these daily returns and estimated daily conditional variances. Since the initial estimations on the conditional daily variances are very small, we scale them by  $10^4$  before we calculate the summary statistics. From the Table, for individual stocks, it can be seen that their  $RG_t^2$  have lower mean values but higher standard deviations than the realized variance estimators, which may suggest that the squared intraday range estimator has a

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<sup>2</sup>Detail discussions on the data used for the estimations can be found in the Appendix B.

downward biasness in these cases. The squared intraday range estimator also has a lower first order autocorrelation (denoted by  $ACF(1)$ ) than the realized variances. Comparing the realized variances of the five stocks, we can find ranks of magnitudes of the price fluctuations are consistent: MSFT and MMM are the most volatile and least volatile stocks among the five stocks. The squared intraday range estimator, however, shows that PG is the least volatile stock, even though it still indicates that MSFT has the largest volatility. Comparing with the individual stocks, S&P500 has less volatile returns during the sample period: its estimated daily conditional variances have smaller mean values than those of the five individual stocks. Overall, the mean values of the three estimated conditional daily variances suggest that daily returns of S&P500 and the five stocks have substantially different degrees of variations. In addition, all the estimated  $\sigma_t^2$  have large sample standard deviations, which indicates that the volatilities themselves also fluctuate significantly during the sample period. Finally, as shown by previous research, the shapes of unconditional distributions of the estimated  $\sigma_t^2$  are often right-skewed and fat-tailed. From the high sample skewness and kurtosis shown in the table, the two stylized empirical properties are also evident for our cases.

## 5.2 Unconditional Cross Correlation

We then have a look of how the daily conditional variances correlate with the daily returns. Figure 2 shows the sample (unconditional) cross correlations  $\rho_j$  between the three estimated  $\sigma_t^2$  and  $R_{t-j}$  for the five stocks of DJIA and S&P500 index. We set lag length  $j = -10, \dots, 10$  (negative  $j$  means lead return and lagged conditional variances). The horizontal dash lines in these plots are the 95% confident bands for the cross correlations under the null of zero correlation.

From Figure 2 it can be seen that as  $j < 0$ , the sample cross correlation  $\rho_j$  gradually increases as  $j$  approaches to zero, and then drops sharply from positive to negative as  $j$  becomes positive. As for  $j > 0$ , the negative  $\rho_j$  instead converges slowly to zero as  $j$  grows. It also can be seen that the  $\rho_j$ 's for the cases of  $j < 0$  are overall larger and more statistically significant than those for the cases of  $j > 0$ , and the result suggests that the cross correlations are asymmetric and the unconditional leverage effects exist for the five DJIA stocks and S&P500 index. In addition, as for the contemporaneous sample cross-correlations ( $j = 0$ ), except for the cases of PG and S&P500, their values for the other four DJIA stocks are almost closer to zero than the lead-lag cross correlations, and are also not statistically significant. These insignificant contemporaneous correlations may suggest that volatility feedback effects on contemporaneous returns are weak for these stocks on a daily basis.

### 5.3 Conditional Hypothesis Test

As shown in the previous subsection, the unconditional cross correlations  $\rho_j$  between  $\sigma_t^2$  and  $R_{t-j}$  for the five stocks of DJIA and S&P500 index demonstrate asymmetric patterns. Will such leverage effects still exist when we explicitly take the lag  $\sigma_t^2$  into account? With the estimated daily conditional variances at hand, we now use them to empirically construct the test statistic for the conditional leverage hypothesis test. As shown in Section 3, the following test statistic can be used for testing null hypothesis of (3):

$$\tau_j^C = \sup_{(y,g) \in \mathcal{X} \times \mathcal{G}} m_j(y, g),$$

where

$$m_j(y, g) = \mathbb{E} [\mathbf{1} \{ \sigma_t^2 \leq y \} g(\sigma_{t-1}^2) (\pi_j^+ \{ \sigma_{t-1}^2 \} - \mathbf{1} \{ r_{t-j} \geq 0 \})]$$

and

$$\pi_j^+(x) = \Pr(r_{t-j} \geq 0 | \sigma_{t-1}^2 = x),$$

Under the null of (3),  $\tau_j^C \leq 0$ .

To practically evaluate  $\tau_j^C$ , we set the instrument function  $g(\sigma_{t-1}^2) = \mathbf{1} \{ \underline{\sigma} < \sigma_{t-1}^2 \leq \bar{\sigma} \}$ , and plug  $\hat{\sigma}_t^2$ ,  $\hat{\sigma}_{t-1}^2$  and  $R_{t-j}$  into the function  $\bar{m}_T$  shown in Section 3. An empirical version of  $m_j(y, g)$  can be

$$\hat{m}_{T_j}(y, \underline{\sigma}, \bar{\sigma}) = \frac{1}{T_j} \sum_{t=j+1}^T \mathbf{1} \{ \hat{\sigma}_t^2 \leq y \} \mathbf{1} \{ \underline{\sigma} < \hat{\sigma}_{t-1}^2 \leq \bar{\sigma} \} \{ \hat{\pi}_j^+(\hat{\sigma}_{t-1}^2) - \mathbf{1} (R_{t-j} \geq 0) \}.$$

Here  $T_j = T - j$  and

$$\hat{\pi}_j^+(x) = \frac{\sum_{t=j+1}^T K_h(x - \hat{\sigma}_{t-1}^2) \mathbf{1} (R_{t-j} \geq 0)}{\sum_{t=j+1}^T K_h(x - \hat{\sigma}_{t-1}^2)},$$

for  $j \geq 1$  and  $K_h(\cdot) = K(\cdot/h)/h$  is a univariate kernel function. Moreover, we can decompose  $\hat{m}_{T_j}(y, \underline{\sigma}, \bar{\sigma})$  as the difference between the following two terms:

$$\begin{aligned} \hat{m}_{T_j}^-(y, \underline{\sigma}, \bar{\sigma}) &= \frac{1}{T_j} \sum_{t=j+1}^T \mathbf{1} \{ \hat{\sigma}_t^2 \leq y, r_t < 0 \} g(\hat{\sigma}_{t-1}^2) \hat{\pi}_j^+(\hat{\sigma}_{t-1}^2), \\ \hat{m}_{T_j}^+(y, \underline{\sigma}, \bar{\sigma}) &= \frac{1}{T_j} \sum_{t=j+1}^T \mathbf{1} \{ \hat{\sigma}_t^2 \leq y, r_t \geq 0 \} g(\hat{\sigma}_{t-1}^2) \hat{\pi}_j^-(\hat{\sigma}_{t-1}^2). \end{aligned}$$

A feasible test statistic is

$$\widehat{\tau}_{T_j}^C = \sup_{(y, \underline{\sigma}, \bar{\sigma}) \in \mathcal{X} \times \mathcal{X} \times \mathcal{X}} \widehat{m}_{T_j}(y, \underline{\sigma}, \bar{\sigma}).$$

The kernel function  $K_h(x)$  used here is the Gaussian kernel  $K(x) = (2\pi)^{-\frac{1}{2}} \exp(-(1/2)^{-1}x^2)$ .

To practically evaluate  $\widehat{\tau}_j^C$ , we apply the following settings. First, we fix the lower bound  $\underline{\sigma} = 0$  for reducing computational burden. Then to the intervals  $[\min_{t=2, \dots, T} \widehat{\sigma}_t^2, 1.1 \times \max_{t=2, \dots, T} \widehat{\sigma}_t^2]$  and  $[\min_{t=2, \dots, T} \widehat{\sigma}_{t-1}^2, 1.1 \times \max_{t=2, \dots, T} \widehat{\sigma}_{t-1}^2]$ , we assign each of them 100 grid points. From the  $100 \times 100$  points, we seek an optimal combination of  $(y, \bar{\sigma})$  to maximize  $\widehat{m}_j(y, 0, \bar{\sigma})$ .

Let  $\widehat{\tau}_{T_j, b, l}^C$  be the statistic  $\widehat{\tau}_{T_j}^C$  evaluated at the following subsample of size  $b$ ,

$$\{(\widehat{\sigma}_l^2, \widehat{\sigma}_{l-1}^2, R_{l-j}), (\widehat{\sigma}_{l+1}^2, \widehat{\sigma}_l^2, R_{l+1-j}), \dots, (\widehat{\sigma}_{l+b-1}^2, \widehat{\sigma}_{l+b-2}^2, R_{l+b-j-1})\},$$

for  $l = j + 1, \dots, T - b + 1$ . We approximate the sampling distribution  $G_{T_j}^C$  of  $\sqrt{T_j} \widehat{\tau}_j^C$  by

$$\widehat{G}_{T_j, b}^C(\cdot) = \frac{1}{T - b - j + 1} \sum_{l=j+1}^{T-b+1} \mathbf{1}(\sqrt{b} \widehat{\tau}_{T_j, b, l}^C \leq \cdot).$$

Let  $p_{T_j} = 1 - G_{T_j}^C(\sqrt{T_j} \widehat{\tau}_{T_j}^C)$  be the p-value of  $\sqrt{T} \widehat{\tau}_{T_j}^C$ . In practice the p-value can be empirically obtained by using

$$\widehat{p}_{T_j, b} = 1 - \widehat{G}_{T_j, b}^C(\sqrt{T} \widehat{\tau}_{T_j}^C).$$

Let  $g_{T_j}^C(1 - \alpha)$  denote the  $(1 - \alpha)$ -th quantile of  $G_{T_j}^C(\cdot)$ :

$$g_{T_j}^C(1 - \alpha) = \inf \left\{ \omega : G_{T_j}^C(\omega) \geq 1 - \alpha \right\}.$$

The  $g_{T_j}^C(1 - \alpha)$  can be empirically approximated by

$$\widehat{g}_{T_j, b}^C(1 - \alpha) = \inf \left\{ \omega : \widehat{G}_{T_j, b}^C(\omega) \geq 1 - \alpha \right\}.$$

To illustrate how the test statistic behaves, we use MSFT as an example. Figure 3 shows plots of the surfaces  $\widehat{m}_{T_j}^-(y, 0, \bar{\sigma})$ ,  $\widehat{m}_{T_j}^+(y, 0, \bar{\sigma})$ , and  $\widehat{m}_j(y, 0, \bar{\sigma})$  against  $(y, \bar{\sigma})$  for  $j = 1$  and  $5$  as  $RV_t^{5min}$  is used as the estimate of  $\sigma_t^2$ . The surfaces  $\widehat{m}_{T_j}^-(y, 0, \bar{\sigma})$  and  $\widehat{m}_{T_j}^+(y, 0, \bar{\sigma})$  are smooth and monotonically increasing with  $y$  and  $\bar{\sigma}$ , and visually they look almost the same. It also can be seen that the surfaces of  $\widehat{m}_j(y, 0, \bar{\sigma})$  are not everywhere nonnegative, and after searching for their maximum values, we find  $\sqrt{T_1} \widehat{\tau}_{T_1}^C$  and  $\sqrt{T_5} \widehat{\tau}_{T_5}^C$  are around 0.0916 and 0.1101 respectively. With the same configurations as in



Figure 3, we show the empirical critical values  $\widehat{g}_{T_j,b}^C(1-\alpha)$  over different subsample sizes in Figure 4. The subsample  $(1-\alpha)$ -th quantile  $\widehat{g}_{T_j,b}^C(1-\alpha)$  at different significant levels  $\alpha$  often decrease as the subsample size  $b$  increases. They also become more concentrated as  $b$  becomes large. It is expected, since as the subsample size approaches the full sample size  $T_j$ , the approximated subsample distribution  $\widehat{G}_{T_j,b}(\omega)$  will converge to the point mass  $\mathbf{1}(\sqrt{b}\widehat{\tau}_{T_j} \leq \omega)$ . Figure 5 shows a plot for the corresponding empirical p-values over different subsample sizes. The empirical p-value of  $j=5$  is larger and more stable than that of  $j=1$  as the subsample size varies. The empirical p-values for both cases also show decreasing trends as  $b$  increases, which is consistent with the property that  $\widehat{G}_{T_j,b}(\omega)$  becomes more concentrated as  $b$  approaches  $T_j$ . Over different subsample sizes, the empirical p-value ranges from 0.977 to 0.340 for  $j=1$  and from 0.986 to 0.851 for  $j=5$ . The results support the claim that the leverage effects exist for MSFT.

As for more empirical results, Table 2 to 4 show the subsample critical values for the five stocks and S&P500 when different estimated  $\sigma^2$  are used. We focus on the cases when the subsample size  $b=500, 1000$  and  $2000$  and lag lengths of returns  $j=1$  and  $5$ . In each table we also report the test statistic  $\sqrt{T_j}\widehat{\tau}_{T_j}^C$ . Table 5 shows their corresponding empirical p-values. From these tables, it can be seen that only in a few cases the conditional leverage hypothesis can be rejected. For instance, as  $RV_t^{1min}$  is used, GE has  $\sqrt{T_1}\widehat{\tau}_{T_1}^C=0.1996$  and the null hypothesis of (3) can be rejected at  $\alpha=0.05$  under  $\widehat{G}_{T_1,2000}^C$ . Another example is IBM. When  $RV_t^{1min}$  and  $RG^2$  are used, its  $\sqrt{T_5}\widehat{\tau}_{T_5}^C$  are around  $0.248$  and  $0.186$  and the corresponding empirical p-values are around  $0.062$  and  $0.074$  when  $b=2000$ . For the rest cases, the results are consistent: the null hypothesis that  $F_j^-(y|x)$  first order dominates  $F_j^+(y|x)$  cannot be rejected for each  $j$  at  $\alpha=0.1$ , suggesting that given the lag conditional daily variances, the leverage effects may still exist for the DJIA stocks and S&P500 index.

## 5.4 Robustness Checks

We then cast some robustness checks for our empirical results. Due to similar results from using different integrated variance estimators and lag length of returns and to save space, we only present results for the case of  $RV_t^{1min}$  and  $j=1$  in the following discussions.

In the previous subsection we showed the leverage effects may exist in our cases over the sample period from Jan-04-1993 to Dec-31-2009. To see whether our results still hold within a certain period of time, we divide the sample period into two subperiods: From Jan-04-1993 to Nov-30-2001 (2,248 trading days) and Dec-03-2001 to Dec-31-2009 (2,035 trading days), and cast the conditional test for

each of them. The division of sample period is based on the U.S. recessions identified by the NBER: The first subsample period spans the 2001 recession ending in Nov-2001, while the second one spans the 2007-2009 recession ending in June-2009. Due to a smaller sample size in each subperiod, here we set  $b = 500$  and  $1000$  for the subsampling scheme. We show the results in Table 6. It can be seen that except for MSFT with  $b = 1000$  (with p-value equal to 0.0753), the conditional leverage hypothesis cannot be rejected at the significant level 0.26 for the rest cases. The results for the subperiods overall are consistent with those for the whole sample period, and for the five DJIA stocks and S&P500 index, we conclude that the leverage effects may still exist within the two subperiods.

In the previous analysis we only set  $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \sigma_{t-1}^2$ , and we did not consider a more general functional form of lagged  $\sigma^2$ . We want to see whether a more complicated functional form of  $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$  affects the test results. Here we let

$$h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \frac{2}{p+1} \sum_{i=0}^{p-1} \left(\frac{p-1}{p+1}\right)^i \sigma_{t-1-i}^2, \quad (9)$$

which is an approximation of  $EMA(\sigma_{t-1}^2)$ , the exponential moving average of the conditional variances up to time  $t-1$ , i.e.,

$$EMA(\sigma_{t-1}^2) = \frac{2}{p+1} \sigma_{t-1}^2 + \left(1 - \frac{2}{p+1}\right) EMA(\sigma_{t-2}^2),$$

where the term  $2/(p+1)$  is called the smoothing ratio. We set the parameter  $p$  equal to 5, 10 and 22, which correspond to daily observations in previous one week, two weeks and one month respectively. Daily  $RV_{t-i}^{1min}$ ,  $i = 1, \dots, p$  are used to replace  $\sigma_{t-i}^2$ ,  $i = 1, \dots, p$  in (9) in the test. Up panel of Table 7 shows results of the leverage hypothesis test conditioning on  $h(RV_{t-1}^{1min}, \dots, RV_{t-p}^{1min})$  and one day lagged returns  $R_{t-1}$  with the subsample window length  $b = 2000$  for constructing the empirical distribution of the test statistic. It can be seen that except for MSFT with  $p = 10$  (which has empirical p-value 0.0039), the rest results still hold as in the previous subsection: The null hypothesis cannot be rejected at a moderately significant level and there is evidence for the presence of the leverage effects.

We can also specify  $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$  as a predictive equation on  $\sigma_t^2$  as  $\sigma_{t-1}^2, \dots, \sigma_{t-p}^2$  are used, i.e.,  $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \mathbb{E}_{t-1}(\sigma_t^2 | \sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$ . With the realized variances at hand, a practical way to constructing the predictive equation is to estimate the HAR-RV (heterogeneous autoregressive realized variance) regression:

$$RV_t = \alpha_D + \beta_{RD} RV_{t-1} + \beta_{RW} RV_{t-1, week} + \beta_{RM} RV_{t-1, month} + \varepsilon_t, \quad (10)$$

The variables in the above regression

$$\begin{aligned}
RV_{t-1,week} &= \frac{1}{5} \sum_{i=0}^4 RV_{t-1-i}, \\
RV_{t-1,month} &= \frac{1}{22} \sum_{i=0}^{21} RV_{t-1-i}
\end{aligned}$$

are called normalized weekly and monthly realized variances. We run the HAR-RV regression (10) in the fashion of real time forecasting with  $RV_t^{1min}$  as the inputs. The predictive regression at each period  $t$  is estimated by using an expanding window scheme with initial window length equal to 22. At time  $t-1$ , the real time projected realized variance for time  $t$ , denote by  $\widehat{RV}_t^{1min}$ , is a function of  $RV_{t-1}^{1min}, \dots, RV_{t-22}^{1min}$ . Bottom panel of Table 7 shows the leverage effect test conditioning on  $\widehat{RV}_t^{1min}$  and lagged daily  $R_{t-1}$  with the subsample window length  $b = 2000$ . The values of the test statistic are small (with range from 0.0087 to 0.1433) and the empirical p-values are all far larger than the frequently used rejection levels. The results show that the leverage effects may exist even when we include information for predictions of future realized variances.

## 5.5 Alternative Methods

In this subsection we use three alternative methods on detecting the leverage effects to check whether our conclusions in previous subsections still hold. The first one is based on estimating the leverage parameters in a parametric model. The other two are methods recently developed by Wang and Mykland (2012) and Ait-Sahalia et al. (2012), which are based on the framework of continuous time finance and high frequency data estimations. We find results from these methods further support our previous analysis.

We first adopt a simple and direct way to test whether the leverage effects exist: Estimating the HAR-RV model (heterogeneous autoregressive realized variance) with additional terms for the leverage effects:

$$\begin{aligned}
RV_t &= \alpha_D + \beta_{RD}RV_{t-1} + \beta_{RW}RV_{t-1,week} + \beta_{RM}RV_{t-1,month} + \\
&\quad \gamma_+ R_{t-1}^2 \times 1 \{R_{t-1} \geq 0\} + \gamma_- R_{t-1}^2 \times 1 \{R_{t-1} < 0\} + \varepsilon_t
\end{aligned} \tag{11}$$

and testing whether  $\gamma_- > \gamma_+$ . We specify the null hypothesis as  $H_0 : \gamma_- \leq \gamma_+$ , and if it is rejected, we have evidence to say that there is the presence of leverage effects.

In addition, it is well known that a realized variance can be decomposed into continuous (non-jump) and jump components (Barndorff-Nielsen and Shephard, 2006). One of the most commonly used estimators for the continuous component is the realized bi-power variation  $BV_t$ :

$$BV_t = \frac{\pi}{2} \left( \frac{n_t}{n_t - 1} \right) \sum_{i=1}^{n_t-1} |r_{i,t}| |r_{i+1,t}|.$$

With the realized bi-power variation  $BV_t$ , the jump variation  $JV_t$  is given by

$$JV_t = RV_t - BV_t.$$

One thus can modify the right hand side of the above HAR-RV model as a linear combination of  $BV$  and  $JV$ , and such modification may improve performances of the HAR-RV type regression on predicting the future realized variances. Andersen et al. (2007) find that the continuous component has a far better predict power than does the jump component on the future realized variances at different time horizons. To see whether such separation of the realized variances in the HAR-RV type model really affects the above results, we incorporate the leverage effects into the following HAR-RV-CJ (heterogeneous autoregressive realized variance model controlling for the continuous and jump components) model:

$$\begin{aligned} RV_t = & \alpha_D + \beta_{BD}BV_{t-1} + \beta_{BW}BV_{t-1,week} + \beta_{BM}BV_{t-1,month} + \\ & \beta_{JD}JV_{t-1} + \beta_{JW}JV_{t-1,week} + \beta_{JM}JV_{t-1,month} + \\ & \gamma_+ R_{t-1}^2 \times 1 \{R_{t-1} \geq 0\} + \gamma_- R_{t-1}^2 \times 1 \{R_{t-1} < 0\} + \varepsilon_t, \end{aligned} \quad (12)$$

where  $BV_{t-1,week}$  ( $JV_{t-1,week}$ ) and  $BV_{t-1,month}$  ( $JV_{t-1,month}$ ) are the normalized weekly and monthly realized bi-power (jump) variation.

For the terms of the leverage effects, besides  $R_{t-1}^2$ , we also use  $|R_{t-1}|$  in the regressions. The OLS estimation results for the HAR-RV and HAR-RV-CJ augmented with the terms for the leverage effects are shown in Table 8 and 9. In the parenthesis under the estimated coefficients are t-statistics obtained using Newey-West standard errors with 18 lag periods. The in-sample model fittings have high adjusted  $R^2$  (all above 0.41), which is one of the most documented features of the HAR-RV type model in literatures. For our cases, it can be seen that the estimated  $\gamma_-$  shows strong statistical significance in MSFT, IBM, and GE, moderately significance in S&P500, and no significance in PG and MMM. The estimated  $\gamma_+$ , however, are not statistically significant over the six cases, and some

of them are even negative. The results of the statistical significance of the estimated  $\gamma_-$  and  $\gamma_+$  are consistent over different specifications for the regressions, no matter whether  $R_{t-1}^2$  or  $|R_{t-1}|$  is used. Comparing the estimated  $\gamma_-$  under the HAR-RV and HAR-RV-CJ type models, we find their values are very similar within each case. The same results also hold for the estimated  $\gamma_+$ , but they are much smaller than  $\gamma_-$ . As  $R_{t-1}^2$  is used, the estimated  $\gamma_-$  ranges from 0.0176 (PG) to 0.2016 (GE) under the HAR-RV model and from 0.0197 (PG) to 0.2038 (GE) under the HAR-RV-CJ model. Turning to the cases of  $|R_{t-1}|$ , the same estimates range from 0.0029 (IBM) to 0.0115 (GE) under the HAR-RV model and from 0.0032 (IBM) to 0.0119 (GE) under the HAR-RV-CJ model. These results suggest that among our cases, volatility of GE reacts most when its lag return receives a negative impact.

We show results of testing  $H_0 : \gamma_- \leq \gamma_+$  in the last two columns of the Table 8 and 9. For the cases considered here, except PG and MMM, the  $t$ -statistics are all above 2.37, suggesting the hypothesis can be rejected at the significant level 0.009. As for PG and MMM, the hypothesis can also be well rejected at the significant level 0.05. Overall, the evidence shown here indicates that the hypothesis at least can be rejected at a moderate significant level. There is evidence to say that negative shocks to asset returns have more impacts on the realized variances than positive shocks with the same magnitudes have, and the leverage effects may exist.

We then use the quadratic co-variation approach proposed by Wang and Mykland (2012) to verify the existence of leverage effects in intraday log returns and volatilities. Considering the following data generating process for the log price  $X_t := \log P_t$  and volatility  $\sigma_t$ :

$$\begin{aligned} dX_t &= \mu_t dt + \sigma_t dW_t, \\ d\sigma_t &= a_t dt + f_t dW_t + g_t dB_t, \end{aligned} \tag{13}$$

where  $W_t$  and  $B_t$  are two mutually independent Brownian motions. Wang and Mykland (2012) propose to use the quadratic co-variation between  $X_t$  and  $F(\sigma_t^2)$  as a quantitative measure of the contemporaneous leverage effect:

$$\langle X, F(\sigma^2) \rangle_T = 2 \int_0^T F'(\sigma_t^2) \sigma_t^2 f_t dt. \tag{14}$$

The function  $F(\cdot)$  is twice differentiable and monotonic on  $(0, \infty)$ , and in the following discussion we assume either  $F(x) = x$  or  $F(x) = 1/2 \log(x)$ .

Recall that in this paper we define the leverage effects as negative shocks to prices/returns affect volatility more than equal positive shocks, which is somehow different from the contemporaneous

leverage effect defined in (14). While (14) only evaluates covariation between  $X_t$  and  $F(\sigma_t^2)$ , it does not tell us whether the negative or positive shocks have more effects on the volatility. One way to link (14) and our definition of leverage effects is to require the parameter  $f_t$  to be negative: If  $f_t < 0$ , negative (positive) shocks to the log returns increase (decrease) volatility.

Suppose within the time interval  $[0, T]$ , the log price process  $X_t$  is observed at equally spaced time stamps; i.e.,  $r_t$  is observed every  $\Delta t_{n,i+1} = T/n$  units of time. To empirically estimate  $\langle X, F(\sigma^2) \rangle_T$ , we first divide the observed  $X_t$ 's into different blocks. Suppose the number of such blocks is  $K_n$  and each block contains  $M_n = \lfloor c\sqrt{n} \rfloor$  observations, where  $c$  is some constant. Wang and Mykland (2012) use

$$\langle X, \widehat{F}(\sigma^2) \rangle_T = 2 \sum_{i=0}^{K_n-2} (X_{\tau_{n,i+1}} - X_{\tau_{n,i}}) \left( F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)$$

to estimate  $\langle X, F(\sigma^2) \rangle_T$ , where  $\tau_{n,i}$ ,  $i = 0, \dots, K_n - 1$  is the lower bound of the  $i$ th block, and

$$\widehat{\sigma}_{\tau_{n,i+1}}^2 = \frac{n}{M_n \times T} \sum_{t_{n,j} \in (\tau_{n,i}, \tau_{n,i+1}]} (X_{t_{n,j+1}} - X_{t_{n,j}})^2.$$

is an estimate for the integrate variance within the block  $(\tau_{n,i}, \tau_{n,i+1}]$ . They show that

$$n^{\frac{1}{4}} \left( \langle X, \widehat{F}(\sigma^2) \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)$$

converges in law to  $Z \times B(c, T)$ , where  $Z$  is a standard normal random variable and independent of any information up to time  $T$  and

$$B(c, T) = \sqrt{\frac{16}{c} \int_0^T (F'(\sigma_t^2))^2 \sigma_t^6 dt + cT \int_0^T (F'(\sigma_t^2))^2 \sigma_t^4 \left( \frac{44}{3} f_t^2 + \frac{22}{3} g_t^2 \right) dt}.$$

To estimate  $B(c, T)$ , one can use  $\sqrt{G_n^1 + G_n^2}$ , where

$$\begin{aligned} G_n^1 &= 2\sqrt{n} \sum_i (X_{\tau_{n,i+1}} - X_{\tau_{n,i}})^2 \left( F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)^2, \\ G_n^2 &= 2 \frac{M_n T}{\sqrt{n}} \sum_i \widehat{\sigma}_{\tau_{n,i}}^2 \left( F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)^2. \end{aligned}$$

It can be shown that  $G_n^1 + G_n^2$  converges in probability to  $B(c, T)^2$ . One can also use

$$\widetilde{\sigma}_{\tau_{n,i+1}}^2 = \frac{n}{M_n \times T} \sum_{t_{n,j} \in (\tau_{n,i}, \tau_{n,i+1}]} (X_{t_{n,j+1}} - \overline{\Delta X}_{\tau_{n,i+1}})^2.$$

to replace  $\widehat{\sigma}_{\tau_{n,i+1}}^2$  in the estimation, where  $\overline{\Delta X_{\tau_{n,i+1}}} = 1/M_n (X_{\tau_{n,i+1}} - X_{\tau_{n,i}})$  is an average of log return changes within the time interval  $(\tau_{n,i}, \tau_{n,i+1}]$ . Let  $\langle X, \widehat{F(\sigma^2)} \rangle_T$  denote the leverage estimate when  $\widehat{\sigma}_{\tau_{n,i+1}}^2$  is used. As shown in Wang and Mykland (2012),  $\widehat{\sigma}_{\tau_{n,i+1}}^2$  and  $\widetilde{\sigma}_{\tau_{n,i+1}}^2$  are asymptotic equivalent, and therefore  $\langle X, \widehat{F(\sigma^2)} \rangle_T$  and  $\langle X, \widetilde{F(\sigma^2)} \rangle_T$  are also asymptotic equivalent. Furthermore, the following two test statistics can be used to detect local leverage effects:

$$L_1 := \frac{n^{\frac{1}{4}} \left( \langle X, \widehat{F(\sigma^2)} \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)}{\sqrt{G_n^1 + G_n^2}}, \quad (15)$$

$$L_2 := \frac{n^{\frac{1}{4}} \left( \langle X, \widetilde{F(\sigma^2)} \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)}{\sqrt{G_n^1 + G_n^2}}. \quad (16)$$

It can be shown that  $L_1$  and  $L_2$  both converge in law to the standard normal distribution.

We use 1-min equally spaced data, and set  $T =$  one day and  $M_n = 30$  for the estimations. Figure 6 shows time series plots of daily standardized quadratic co-variations, which are just daily  $L_1$  and  $L_2$  with  $\langle X, F(\sigma^2) \rangle_T = 0$ . Table 10 reports some statistics of these standardized quadratic co-variations. It can be seen that except MMM, all the other four DJIA stocks and S&P500 index on average have negative  $L_1$  and  $L_2$ . Without considering signs of the estimates, we compare absolute values of the daily  $L_1$  and  $L_2$  with two critical values 1.96 and 2.58 (corresponding to significant levels  $\alpha = 0.05$  and 0.01 of standard normal). We find very few days have significant  $L_1$  and  $L_2$ : Among the 4,283 days, the number of significant days ranges from 18 to 257 for  $\alpha = 0.05$  and 0 to 13 for  $\alpha = 0.01$ . If considering significant negativity only, the numbers of significant days range from 60 to 392 for  $\alpha = 0.05$  and 0 to 42 days for  $\alpha = 0.01$  (with critical values equal to -1.64 and -2.33 of standard normal). S&P500 index on average has lower standardized quadratic co-variations and more significant days than the five DJIA stocks. Overall, the results suggest that the leverage effects may still exist in the five DJIA stocks and S&P500 index when log returns and volatilities are estimated at the intraday level. But only in certain periods, these leverage effects are strong enough to be detected by the quadratic co-variation approach.

We finally turn to the method proposed by Ait-Sahalia et al. (2012). In their method, the following CIR process are considered for the squared volatility:

$$dv_t = \alpha_v (\theta - v_t) + \kappa \sqrt{v_t} dB_t,$$

where  $v_t := \sigma_t^2$ , and  $2\alpha_v \theta > \kappa^2$ . For the log price process, they assume it follows the same process as in (13) except now  $\mathbb{E}(dB_t dW_t) = f dt$ , where  $f$  is a constant. It can be shown that the parameter  $f$

is the limit of correlation between  $v_{t+l} - v_t$  and  $X_{t+l} - X_t$  when the time interval  $l$  approaches zero, i.e.,

$$f = \lim_{l \rightarrow 0} \text{Corr}(v_{t+l} - v_t, X_{t+l} - X_t). \quad (17)$$

Ait-Sahalia et al. (2012) use the limit correlation above as a measure for the leverage effect, which is different from Wang and Mykland (2012) and our definition. To link the limit correlation (17) to our definition of the leverage effect, again we may assume  $f$  to be negative.

Let  $\hat{f}_k$  and  $f_k$  be the sample and true correlations between the difference of the estimated integrated variances and difference of log prices at time  $t + k\Delta$  and  $t$ . Here  $\Delta$  is the time unit and we follow Ait-Sahalia et al. (2012) to let  $\Delta$  as one day for our cases<sup>3</sup>. Under some regular conditions, Ait-Sahalia et al. (2012) show that  $f_k$  and  $f$  satisfy the following linear relationship:

$$f_k = f + b \times k + o(k\Delta),$$

which provides an easily-implemented way to identify the limit correlation  $f$ . That is, running a linear regression of  $f_k$  (or  $\hat{f}_k$  if  $f_k$  is unknown) on the intercept term and  $k$ , and the estimated intercept term can be used as an estimate of  $f$ . Furthermore, Ait-Sahalia et al. (2012) propose the following data driven procedures for practically estimating the linear regression:

1. For each  $k = 1, \dots, K$ , we calculate  $\hat{f}_k$ , and rank the  $\hat{f}'_k$ s for  $k = k_0, \dots, \lfloor K/2 \rfloor$ . Then we take the three smallest values of these ranked  $\hat{f}'_k$ s. Let  $k_{(1)}$ ,  $k_{(2)}$ , and  $k_{(3)}$  be the corresponding indices  $k$ 's that the three smallest  $\hat{f}'_k$ s have. Let  $\underline{k}^* = \max(k_{(1)}, k_{(2)}, k_{(3)})$ .
2. Regressing  $\hat{f}_k$  on  $k$  with  $k = \underline{k}^*, \dots, \underline{k}^* + m$ , where  $m = a_0, \dots, K - \underline{k}^*$ . Let  $m^*$  denote the value of  $m$  at which the regression yields the highest (unadjusted)  $R^2$ . Then let  $\bar{k}^* = \underline{k}^* + m^*$ . The estimated intercept term of the regression with the data  $\left\{k, \hat{f}_k\right\}_{k=\underline{k}^*}^{\bar{k}^*}$  is the final estimate of  $f$ .

We follow Ait-Sahalia et al. (2012) to set  $K = 252$ ,  $k_0 = 6$  and  $a_0 = 11$ . As shown in Ait-Sahalia et al. (2012), the sample correlation  $\hat{f}_k$  is a bias estimation for the true  $f_k$ . To improve performance of the data driven approach above, we can replace the sample correlation  $\hat{f}_k$  with the following bias corrected estimation in the above data driven procedures:

$$\hat{f}_k^{bc} = \gamma \frac{2\sqrt{k^2 - k/3}}{2k - 1} \hat{f}_k, \quad (18)$$

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<sup>3</sup>Note that in Ait-Sahalia et al. (2012), the basic time unit for  $t$  is one year, so  $\Delta = \text{one day} = 1/252$ .



where

$$\gamma = \left( 1 - \frac{4\Delta^2 \mathbb{E}(v_t^2)}{n \text{Var}(RV_{t+k\Delta} - RV_t)} \right)^{-\frac{1}{2}}.$$

Here the parameter  $n$  is the number of log return observations used to estimate the realized variance. For estimating  $\Delta^2 \mathbb{E}(v_t^2)$ , we can first estimate the realized quarticity:

$$QV_t = \frac{n_t}{3} \sum_{i=1}^{n_t} r_{i,t}^4,$$

and then calculate its sample mean. In the following we use

$$\hat{\gamma} = \left( 1 - \frac{4 \times \text{sample mean of } QV_t}{\text{sample mean of } n_t \times \text{sample variance of } RV_t} \right)^{-\frac{1}{2}} \quad (19)$$

as an estimate of  $\gamma$  in  $\hat{f}_k^{bc}$ .

All the following empirical analysis is carried out on a daily basis: We use daily  $RV_t^{1min}$  and  $R_t$  to calculate  $\hat{f}_k$ , and the 1-min log returns to estimate the daily realized quarticities in  $\hat{\gamma}$ . Figure 7 plots  $\hat{f}_k$  and  $\hat{f}_k^{bc}$  against  $k$  for the five DJIA stocks and S&P500 index. We use 1-min log returns to estimate the realized quarticities. For all the cases, the two estimated correlations are negative over  $k$ , and the bias corrected correlations are constantly lower than the uncorrected ones. Except for small  $k$ , the two correlation estimations have extremely similar patterns in each case. Comparing the results over different cases, however, the patterns of the correlation estimations are somehow different, but they often have a higher value as  $k = 1$  and then suddenly drop to a lower value as  $k$  deviates from one. It also can be seen that as  $k > 100$ , the estimations gradually become stable and all of them steadily move either up or down as  $k$  becomes large. Overall, the estimated correlations vary substantially as the time interval  $k$  changes, no matter they are bias corrected or not; and the smaller the time interval  $k$ , the more possible that we will get a higher estimated correlation.

To estimate  $f$ , we may use  $\hat{f}_1$  or  $\hat{f}_1^{bc}$ . As shown above, however, it is very likely that we get a higher estimated correlation with  $k = 1$  than with  $k > 1$ , and such high  $\hat{f}_1$  or  $\hat{f}_1^{bc}$  used for estimating  $f$  perhaps result in an upward bias estimate. To obtain a more accurately estimated  $f$ , we use the data driven procedures introduced above. In Table 11 we report the final estimate of  $f$  based on the estimated intercept term from running linear regressions of  $\hat{f}_k$  (or  $\hat{f}_k^{bc}$ ) on  $k \in [\underline{k}^*, \bar{k}^*]$ , and the unadjusted  $R^2$  of the regression. The value of  $R^2$  is very high for all of the six cases, and  $\underline{k}^*$  and  $\bar{k}^*$  in each case are similar when either  $\hat{f}_k$  or  $\hat{f}_k^{bc}$  is used. For all the six cases, the estimated  $f$  is

moderately negative, with range from -0.24 (IBM) to -0.48 (GE)<sup>4</sup>. The negativity of the estimated  $f$  implies that on average a positive shock to the log price has a smaller impact on the volatility than a negative shock with the same magnitude has, and the leverage effects may exist. It also can be seen that the estimated  $f$  from using  $\hat{f}_k^{bc}$  and  $\hat{f}_k$  are qualitatively similar, but the former is slightly lower than the later.

## 6 Conclusion

We have found strong evidence in favour of a leverage effect in daily stock returns both at the individual stock level and the index level. The null hypothesis we consider is quite strong, namely first order distributional dominance. Therefore, it is quite powerful that the data do not reject this hypothesis. Our empirical evidence is robust along a number of directions. For example, our results still hold for subperiods and for different specifications on the functional form of the lagged conditional variances. In addition, several recently developed alternative methods are used on our data and their results also support our findings.

On the theoretical side, we have considered stationary processes, but this can be relaxed along the lines of Dahlhaus (1997) and Dahlhaus and Subba Rao (2006). We may also weaken the restrictions on the amount of dependence to be consistent with some evidence on the time series properties of realized volatility, see for example Andersen, Bollerslev, Christoffersen, and Diebold (2011).

## Appendix A: Proofs

Consider the empirical process  $\bar{\nu}_T(y, g, \pi)$  indexed by  $(y, g, \pi) \in \mathbb{R} \times \mathcal{G} \times \Pi$

$$\bar{\nu}_T(y, g, \pi) = \sqrt{T} \{ \bar{m}_T(y, g, \pi) - E \bar{m}_T(y, g, \pi) \}.$$

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<sup>4</sup>One thing worth to note is that, the estimated  $f$  of MSFT and S&P500 shown here are different from those shown in Ait-Sahalia et al. (2012). It is mainly because we use different sample periods and different realized variance estimators. In Ait-Sahalia et al. (2012), they used the pre-averaging approach (Jacob et al., 2009) to estimate the integrated variances. With  $RV_t^{1min}$ , we re-estimate the  $f$  of MSFT with the data driven method over the same sample period as theirs (Jan-2005 to June-2007). The estimated  $f$  from using  $\hat{f}_k^{bc}$  ( $\hat{f}_k$ ) is -0.90 (-0.87) and  $[\underline{k}^*, \bar{k}^*] = [125, 165]$  ( $[125, 165]$ ). As for S&P500 (sample period is from Jan-2004 to Dec-2007), the estimated  $f$  from using  $\hat{f}_k^{bc}$  ( $\hat{f}_k$ ) is -0.62 (-0.60), and  $[\underline{k}^*, \bar{k}^*] = [22, 101]$  ( $[24, 101]$ ).

The following Lemma establishes stochastic equicontinuity of  $\{\bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1\}$ .

**Lemma 1.** Suppose that Assumption A holds. Then, for each  $\varepsilon > 0$  and  $\eta > 0$ , there exists  $\delta > 0$  such that

$$\overline{\lim}_{T \rightarrow \infty} P^* \left[ \sup_{\rho((y_1, g_1, \pi_1), (y_2, g_2, \pi_2)) < \delta} |\bar{\nu}_T(y_1, g_1, \pi_1) - \bar{\nu}_T(y_2, g_2, \pi_2)| > \eta \right] < \varepsilon,$$

where  $\text{Pr}^*$  denotes  $P$ -outer measure and

$$\begin{aligned} \rho((y_1, g_1, \pi_1), (y_2, g_2, \pi_2)) &= \rho_a((y_1, g_1), (y_2, g_2)) \vee \rho_b(\pi_1, \pi_2) \\ \rho_a((y_1, g_1), (y_2, g_2)) &= \left( \int_{\mathcal{Y}} \int_{\mathcal{Y}} \{[1(w_1 \leq y) - F(y|w_2)] g(w_2) \right. \\ &\quad \left. - [1(w_1 \leq y) - F(y|w_2)] g(w_2)\} dw_1 dw_2 \right)^{1/2} \\ \rho_b(\pi_1, \pi_2) &= (E(\pi_1(x_t) - \pi_2(x_t))^2)^{1/2} \end{aligned}$$

**PROOF OF LEMMA 1.** The result of Lemma 1 follows from the stochastic equicontinuity results of Andrews (1989, Theorem 7) that are applicable to classes of functions that are products of smooth functions from an infinite dimensional class and a Type IV class of uniformly bounded functions. It suffices to verify Assumption E of the latter paper. (Assumption) E(i) holds by taking  $W_{aTt}$ ,  $W_{bTt}$ ,  $\tau_a(\cdot)$ ,  $\tau_b(\cdot)$ ,  $m_a(W_{aTt}, \tau_a)$ , and  $m_b(W_{bTt}, \tau_b)$  to be  $x_t$ ,  $(y_t, x_t)$ ,  $\pi(\cdot)$ ,  $1(\cdot \leq y)g(\cdot)$ ,  $\pi(x_t)$  and  $1(y_t \leq y)g(x_t)$ , respectively. E(ii) holds by Assumption A1(ii) with  $\mathcal{W}_a^*$  given by  $\mathcal{X}$ . E(iii) follows from Assumptions A2-A3 and the definition of  $\Pi$  in (5). E(iv) is irrelevant to our case. E(v) holds since  $\{1(\cdot \leq y)g(\cdot) : y \in \mathcal{Y}, g \in \mathcal{G}\}$  is a type IV class of uniformly bounded functions with index  $p = 2$ , constant  $\psi = 1/2$ , and dimension  $d = d_x + 1$ . Finally, E(vi) holds by Assumption A1(i).  $\blacksquare$

**PROOF OF THEOREM 1.** To prove Theorem 1, we first establish the following results:

$$T^{1/4} \sup_{x \in \mathcal{X}} |\hat{\pi}^+(x) - \pi_0^+(x)| \xrightarrow{P} 0, \quad (20)$$

$$T^{1/4} \sup_{x \in \mathbb{R}^{d_x}} |\hat{f}(x) - f(x)| \xrightarrow{P} 0, \quad (21)$$

$$\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu \hat{\pi}^+(x) - D^\mu \pi_0^+(x)| \xrightarrow{P} 0 \quad \forall \mu \text{ with } 1 \leq \mu < q, \quad (22)$$

$$\sup_{(y, g) \in \mathcal{Y} \times \mathcal{G}} |\bar{\nu}_T(y, g, \hat{\pi}^+) - \bar{\nu}_T(y, g, \pi_0^+)| \xrightarrow{P} 0, \quad (23)$$

$$\sup_{(y, g) \in \mathcal{Y} \times \mathcal{G}} \left| \sqrt{T} E \bar{m}_T(y, g, \pi) |_{\pi = \hat{\pi}^+} - \sqrt{T} R_T(y, g) \right| \xrightarrow{P} 0, \quad (24)$$

where

$$R_T(y, g) = \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} F(y|x_t) g(x_t) \\ + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx.$$

Equations (20)- (22) can be established using Theorem 1 of Andrews (1994) by verifying its Assumptions NP1-NP5. Notice that NP1-NP3 are implied by Assumptions A1-A3 with  $\eta = \beta = \infty$  and  $|\lambda| = \mu$  and  $Y_t$ ,  $X_t$ ,  $f_t(x)$ , and  $g(x)$  given by  $1(r_{t-1} \geq 0)$ ,  $x_t$ ,  $f(x)$ , and  $\pi_0^+(x)$ , respectively. NP4 (a) and (c) holds by Assumption A4 with  $\Omega = \hat{\Omega} = 1$  and NP4(b) is not relevant in our case, see Comment 5 to Theorem 1 of Andrews (1994). Finally NP5 is implied by Assumption A5. This establishes (20) - (22).

Equation (23) holds by a standard argument (see p.2257 of Andrews (1994)) because  $\{\bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1\}$  is stochastically equicontinuous by Lemma 1 and  $\Pr(\hat{\pi}^+ \in \Pi) \rightarrow 1$  and  $\rho_b(\hat{\pi}^+, \pi_0^+) \xrightarrow{P} 0$  using (22) and Assumption A3.

To establish (24), Write

$$\begin{aligned} \sqrt{T} E \bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} &= \sqrt{T} E 1(y_t \leq y) g(x_t) \{\pi(x_t) - 1(r_{t-1} \geq 0)\}|_{\pi=\hat{\pi}^+} \\ &= \sqrt{T} \int F(y|x) g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] f(x) dx \\ &\quad + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx, \end{aligned} \quad (25)$$

where the second equality holds by rearranging terms and applying law of iterated expectations. Consider the first term on the right hand side of (25). We have

$$\begin{aligned} &\sqrt{T} \int F(y|x) g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] f(x) dx \\ &= \sqrt{T} \int F(y|x) g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] \hat{f}(x) dx \\ &\quad + \sqrt{T} \int F(y|x) g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] [f(x) - \hat{f}(x)] dx \\ &= A_{1T} + A_{2T}, \text{ say.} \end{aligned} \quad (26)$$

The term  $A_{2T}$  is asymptotically negligible because it is bounded uniformly over  $(y, g) \in \mathcal{Y} \times \mathcal{G}$  by

$$T^{1/4} \sup_{x \in \mathcal{X}} |\hat{\pi}^+(x) - \pi_0^+(x)| \times T^{1/4} \sup_{x \in \mathbb{R}^{d_x}} |f(x) - \hat{f}(x)| \xrightarrow{P} 0 \quad (27)$$

using (20) and (21). Now, consider  $A_{1T}$ . Write

$$\begin{aligned} [\hat{\pi}^+(x) - \pi_0^+(x)] \hat{f}(x) &= \frac{1}{T} \sum_t K_h(x - x_t) \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \\ &\quad + \frac{1}{T} \sum_t K_h(x - x_t) \{\pi_0^+(x_t) - \pi_0^+(x)\}. \end{aligned}$$

Notice that

$$\begin{aligned} &\sqrt{T} \int F(y|x)g(x) \left[ \frac{1}{T} \sum_t K_h(x - x_t) \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \right] dx \\ &= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \int F(y|x)g(x) K_h(x - x_t) dx \\ &= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} [F(y|x_t)g(x_t) \\ &\quad + \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du] \\ &= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} F(y|x_t)g(x_t) + o_p(1), \end{aligned} \tag{28}$$

uniformly over  $(y, g) \in \mathcal{Y} \times \mathcal{G}$ , where the last equality holds by the following argument. For each  $(y, g) \in \mathcal{Y} \times \mathcal{G}$  and some  $\delta > 0$ , we have

$$\begin{aligned} &E \left( \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \right. \\ &\quad \left. \times \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du \right)^2 \\ &\leq C \left( E \left| \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du \right|^{2+\delta} \right)^{2/(2+\delta)} \\ &\rightarrow 0, \end{aligned} \tag{29}$$

where the inequality follows by the moment inequality for sums of strong mixing random variables (see Lemma 3.1 of Dehling and Philipp (2002)) and Assumption A1(i), and convergence to zero holds by the fact that  $F(y|\cdot)g(\cdot)$  is a bounded function using a well known convergence result for convolutions of functions in an  $L_p$ -space (with  $p = 2 + \delta$ ) (see Theorem 8.14 (a) of Folland (1984)). Using a stochastic equicontinuity argument as in Lemma 1, we can show that the convergence to

zero holds uniformly over  $(y, g) \in \mathcal{Y} \times \mathcal{G}$ . This establishes (28). Furthermore, we have

$$\begin{aligned}
& \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \left| \sqrt{T} \int F(y|x)g(x) \left[ \frac{1}{T} \sum_t K_h(x - x_t) \{ \pi_0^+(x_t) - \pi_0^+(x) \} \right] dx \right| \\
&= \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \left| \frac{1}{\sqrt{T}} \sum_t \int F(y|x_t + uh)g(x_t + uh) \{ \pi_0^+(x_t) - \pi_0^+(x_t + uh) \} K(u)du \right| \\
&\leq O_p(T^{1/2}h^\omega) \xrightarrow{p} 0,
\end{aligned} \tag{30}$$

where the equality holds by a change of variables, the inequality holds by an  $\omega$ -term Taylor expansion using Assumptions A3 and A4, and the last convergence to zero holds by Assumption A5. Now, the result (24) is established by combining (25)-(30).

We now establish Theorem 1. We have

$$\begin{aligned}
\sqrt{T}\bar{m}_T(y, g, \hat{\pi}^+) &= \bar{\nu}_T(y, g, \hat{\pi}^+) + \sqrt{T} E\bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} \\
&= \bar{\nu}_T(y, g, \pi_0^+) + \sqrt{T} E\bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} + o_p(1)
\end{aligned} \tag{31}$$

$$= \nu_T(y, g) + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx + o_p(1), \tag{32}$$

where the second equation hold by (23) and the last equality holds by (24). Notice that, under the null hypothesis, we have  $\int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx = 0$  for all  $(y, g) \in \mathcal{B}$ , while  $\int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx < 0$  if  $(y, g) \notin \mathcal{B}$ . Furthermore, we can show that

$$\nu_T(\cdot, \cdot) \Rightarrow \nu(\cdot, \cdot) \tag{33}$$

with the sample paths of  $\nu(\cdot, \cdot)$  uniformly continuous with respect to pseudometric  $\rho_a$  on  $\mathcal{Y} \times \mathcal{G}$  with probability one. The latter holds by a standard argument because Lemma 1 implies that the pseudometric space  $(\mathcal{Y} \times \mathcal{G}, \rho_a)$  is totally bounded,  $\{\nu_T(\cdot, \cdot) : T \geq 1\}$  is stochastically equicontinuous, and finite dimensional convergence in distribution holds using a CLT for bounded strong mixing random variables (see Corollary 5.1 of Hall and Heyde (1980)). Therefore, using the same arguments as those in Linton, Maasoumi and Whang (2005, Proof of Theorem 1) and continuous mapping theorem, Theorem 1 is now established as desired.  $\blacksquare$

**PROOF OF THEOREM 2.** The proof is similar to the proof of Theorem 2 of Linton, Maasoumi and Whang (2005).  $\blacksquare$

**PROOF OF THEOREM 3.** The proof is similar to the proof of Theorem 3 of Linton, Maasoumi and Whang (2005).  $\blacksquare$

## Appendix B: Data Descriptions and Constructions

The data for estimating  $\sigma_t^2$  come from different sources. For estimating the realized variances of S&P500, we use intraday high frequency data provided by tickdata.com, which consist of 1-minute and 5-minute index prices in regular trading time. The squared intraday range estimator  $RG_t^2$  of S&P500, however, is estimated by using data of the highest and lowest trading prices during a day, which come from yahoo finance. For the five DJIA stocks, their  $RG_t^2$  are estimated with daily highest and lowest price data from the CRSP, and their  $RV_t$  are estimated with the intraday trade prices from the TAQ database.

The raw data of the high frequency observations from the TAQ database contain noises. In order to obtain a more accurate realized variance estimation, we adopt the following procedures, which are suggested by Barndorff-Nielsen et al. (2009), to clean the high frequency data of the individual stocks:

- (1) We keep the data points between 09:30AM to 16:00PM (regular trading time), and delete data points with a time stamp outside this time interval.
- (2) We delete the data points which prices are zero.
- (3) We keep data points which the trade occurred on AMEX (A), NYSE (N), NASD (T/Q), and delete the rest data points.
- (4) Data points which are corrected traded are deleted (their Correction Indicator is not zero, CORR!= 0).
- (5) Data points which trades are not in abnormal sale condition are kept (The entries with the column COND which does not has a letter code, or has the letter "F" or "E").
- (6) If multiple trades have the same time stamp, we use their median price.
- (7) We delete the data point in which the absolute difference between its price and median of 50 neighborhood observations is larger than five times mean absolute deviation from the median.

Note that (7) is to replace rule T4 in Barndorff-Nielsen et al. (2009) for cleaning outliers in the high frequency trade data. The rule T4 uses the quote data to discipline the trade data: if the trade

prices are above the ask plus bid-ask spread or below the bid minus the bid-ask spread, they will be deleted. However, it can be shown that such rule in practice is not very activated. Since our raw data are the trade data, in order to more efficiently implement the cleaning procedures without using the quote data, we use a more viable rule such as (7) for dealing with the outliers.

The time unit of the cleaned data is one second, but the data points are not equally-spaced. We then transform the cleaned data to equally-spaced data. We set the time intervals for the equally-spaced data equal to two frequently used choices: 1 minute and 5 minute. Then the last-tick method is used to construct the equally-spaced data. Finally the equal-spaced data are used to calculate the log returns and realized variances. In US stock market, except on a few inactive trading days, one day has 6.5 trading hours (from 09:30AM to 16:00PM). Therefore in general, the numbers of log return observations  $n_t$  equal to 390 and 78 per day for the 1-min and 5-min cases respectively.



Table 1: The table shows summary statistics of daily 1-min realized variance  $RV_t^{1min}$ , 5-min realized variance  $RV_t^{5min}$ , squared intraday range estimator  $RG_t^2$ , and stock return  $R_t$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We scale  $RV_t^{1min}$ ,  $RV_t^{5min}$ , and  $RG_t^2$  by  $10^4$  before we calculate the statistics. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

MSFT							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.1327	5.3750	84.8100	5.6820	2.7469	14.6002	0.8598
$RV_t^{5min}$	0.1245	3.7010	60.3400	3.8738	4.3588	35.8134	0.7106
$RG_t^2$	0.0792	3.3990	99.5000	4.8744	6.4918	78.7447	0.4014
$R_t$	-0.1560	0.0008	0.1957	0.0221	0.2258	5.8996	-0.0333
IBM							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.1364	3.3960	112.1000	3.9539	8.1612	157.4227	0.6992
$RV_t^{5min}$	0.1077	2.8480	75.3800	3.6239	6.6502	79.9087	0.6176
$RG_t^2$	0.0309	2.7540	96.3000	4.3458	7.1945	97.5840	0.3966
$R_t$	-0.1554	0.0007	0.1316	0.0198	0.3747	6.3703	-0.0328
GE							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.1397	4.1300	163.4000	7.0400	9.6189	140.9916	0.7186
$RV_t^{5min}$	0.1102	3.3420	180.1000	6.8264	10.6219	179.1406	0.6840
$RG_t^2$	0.0381	3.0970	200.3000	7.2896	9.8209	167.5244	0.5708
$R_t$	-0.1279	0.0004	0.1970	0.0196	0.3141	8.7333	-0.0127
PG							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.1451	2.8810	116.2000	3.3935	13.6388	376.3909	0.5577
$RV_t^{5min}$	0.1078	2.3460	79.7400	3.0248	9.4751	171.7254	0.6229
$RG_t^2$	0.0460	1.9050	68.9800	3.4090	8.7980	121.1760	0.4916
$R_t$	-0.3138	0.0005	0.1021	0.0160	-1.8188	38.2772	-0.0546
MMM							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.1732	2.5600	104.2000	3.0787	12.2027	311.1090	0.6280
$RV_t^{5min}$	0.0803	2.3080	96.6000	3.1066	10.7258	234.8230	0.6051
$RG_t^2$	0.0194	2.0290	115.3000	3.5674	11.4230	268.8034	0.3725
$R_t$	-0.0959	0.0004	0.1107	0.0159	0.1628	4.3600	-0.0348
S&P500							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$RV_t^{1min}$	0.0163	0.6995	73.5700	1.9412	17.4148	515.4656	0.6517
$RV_t^{5min}$	0.0272	0.8876	57.8300	1.9839	11.6192	223.2035	0.6958
$RG_t^2$	0.0114	1.0660	42.8800	2.4713	8.9923	109.6879	0.6089
$R_t$	-0.0904	0.0003	0.1158	0.0122	-0.0024	9.1608	-0.0668

Table 2: The table shows the subsample critical value  $\widehat{g}_{T_j,b}^C(1-\alpha)$  for the conditional leverage hypothesis test at four different levels of  $\alpha$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns  $j = 1$  and 5 and subsample sizes  $b = 500, 1000$  and 2000.  $\sigma_t^2$  is estimated by the daily realized variance with 1-min log returns. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

	$b$	$j = 1$					$j = 5$			
		$\alpha$					$\alpha$			
		0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001
MSFT	500	0.2612	0.2810	0.3515	0.3735		0.3550	0.3727	0.4182	0.4598
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0825$	1000	0.2085	0.2345	0.2553	0.2683	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.0823$	0.2776	0.2857	0.3017	0.3170
	2000	0.1557	0.1641	0.1758	0.1807		0.1890	0.1944	0.2055	0.2069
IBM	500	0.2669	0.2966	0.3415	0.3585		0.4320	0.4743	0.5836	0.6580
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0138$	1000	0.2406	0.2550	0.2741	0.2991	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.2479$	0.3759	0.4355	0.4874	0.5087
	2000	0.1452	0.1539	0.1627	0.1671		0.2306	0.2551	0.2661	0.2813
GE	500	0.3275	0.3596	0.4135	0.4445		0.3314	0.3830	0.4819	0.5294
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.1996$	1000	0.2939	0.3365	0.4402	0.4707	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1885$	0.2953	0.3223	0.3406	0.3556
	2000	0.1779	0.1853	0.2185	0.2471		0.3167	0.3248	0.3496	0.3989
PG	500	0.2813	0.3021	0.3311	0.3506		0.2896	0.3231	0.3917	0.4121
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0622$	1000	0.2318	0.2795	0.3575	0.3769	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1073$	0.2453	0.2748	0.3274	0.3833
	2000	0.1554	0.1887	0.2155	0.2240		0.177	0.1897	0.2127	0.2157
MMM	500	0.3360	0.3872	0.5014	0.5454		0.3548	0.4010	0.4813	0.5120
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0595$	1000	0.3256	0.3529	0.3823	0.4055	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1242$	0.2773	0.3074	0.3552	0.3660
	2000	0.1813	0.1927	0.2062	0.2245		0.1695	0.1802	0.1947	0.2038
S&P500	500	0.2203	0.2615	0.3288	0.3547		0.3675	0.4464	0.498	0.5252
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0096$	1000	0.1997	0.2152	0.2371	0.2508	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1480$	0.3481	0.3777	0.4177	0.4689
	2000	0.2041	0.2140	0.2241	0.2292		0.2613	0.3255	0.3958	0.4243

Table 3: The table shows the subsample critical value  $\widehat{g}_{T_j,b}^C(1-\alpha)$  for the conditional leverage hypothesis test at four different levels of  $\alpha$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns  $j = 1$  and 5 and subsample sizes  $b = 500, 1000$  and 2000.  $\sigma_t^2$  is estimated by the daily realized variance with 5-min log returns. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

	$b$	$j = 1$					$j = 5$			
		$\alpha$					$\alpha$			
		0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001
MSFT	500	0.3407	0.3635	0.3981	0.4522		0.3825	0.4282	0.4821	0.4996
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0916$	1000	0.2622	0.2829	0.3106	0.3228	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.1101$	0.2911	0.3148	0.3548	0.3709
	2000	0.1454	0.1590	0.1630	0.1692		0.1845	0.1966	0.2085	0.2136
IBM	500	0.2849	0.3361	0.3875	0.4905		0.4443	0.4936	0.5745	0.6150
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0247$	1000	0.2510	0.3144	0.3687	0.3881	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.1970$	0.3549	0.4161	0.4809	0.5051
	2000	0.1096	0.1130	0.1214	0.1438		0.2238	0.2347	0.2572	0.2764
GE	500	0.3433	0.3776	0.4344	0.4622		0.3466	0.3785	0.4565	0.4803
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0940$	1000	0.2465	0.2699	0.3045	0.3220	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1342$	0.2931	0.3240	0.3624	0.3769
	2000	0.1531	0.1823	0.2342	0.2489		0.1953	0.2104	0.2212	0.2279
PG	500	0.2646	0.2963	0.3311	0.3535		0.2862	0.3252	0.3718	0.3859
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0544$	1000	0.1785	0.2019	0.2547	0.2967	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.0776$	0.2079	0.2359	0.2843	0.291
	2000	0.1287	0.1402	0.1539	0.1601		0.1438	0.1669	0.2002	0.2185
MMM	500	0.3679	0.5158	0.6259	0.7201		0.3430	0.3817	0.4175	0.4508
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0637$	1000	0.4355	0.5008	0.5299	0.5677	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1058$	0.2587	0.3023	0.3734	0.3938
	2000	0.2514	0.2727	0.3131	0.3401		0.1412	0.1509	0.1660	0.1741
S&P500	500	0.1903	0.2110	0.2739	0.2947		0.3524	0.4850	0.5389	0.5830
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0000$	1000	0.1158	0.1242	0.1556	0.1922	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.1160$	0.3561	0.3858	0.4409	0.4660
	2000	0.0897	0.0953	0.1146	0.1248		0.1886	0.2116	0.2571	0.2706

Table 4: The table shows the subsample critical value  $\widehat{g}_{T_j,b}^C(1-\alpha)$  for the conditional leverage hypothesis test at four different levels of  $\alpha$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns  $j = 1$  and 5 and subsample sizes  $b = 500, 1000$  and 2000.  $\sigma_t^2$  is estimated by the intra-daily range variance estimator. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$j = 1$				$j = 5$				
		$\alpha$				$\alpha$				
	$b$	0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001
MSFT	500	0.4644	0.5648	0.7774	0.8167	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.1534$	0.4478	0.5330	0.5944	0.6220
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0940$	1000	0.4412	0.4959	0.5720	0.5954		0.3699	0.4190	0.5083	0.5238
	2000	0.3001	0.3102	0.3308	0.3403		0.3140	0.3222	0.3328	0.3389
IBM	500	0.2890	0.3374	0.3864	0.4246	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.1855$	0.3381	0.3781	0.4443	0.4841
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.1354$	1000	0.2349	0.2517	0.2879	0.3031		0.2573	0.2841	0.3264	0.3433
	2000	0.1853	0.1916	0.2055	0.2092		0.1809	0.1923	0.2090	0.2301
GE	500	0.5426	0.6059	0.7410	0.8486	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.0912$	0.3487	0.4119	0.5107	0.5821
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0575$	1000	0.4323	0.4668	0.5725	0.5999		0.2726	0.2891	0.3174	0.3351
	2000	0.2902	0.3095	0.3328	0.3493		0.2197	0.2472	0.3121	0.3254
PG	500	0.2644	0.3006	0.3464	0.3733	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.1117$	0.3586	0.3913	0.4558	0.5071
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0837$	1000	0.1468	0.1595	0.1762	0.1865		0.3018	0.3405	0.3952	0.4364
	2000	0.1110	0.1175	0.1265	0.1281		0.1986	0.2075	0.2201	0.2233
MMM	500	0.3645	0.4466	0.6223	0.6821	$\sqrt{T_5}\widehat{\tau}_{T_5}^C = 0.0609$	0.4895	0.536	0.5965	0.6332
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0773$	1000	0.2584	0.2792	0.3107	0.3545		0.3465	0.3777	0.4176	0.4383
	2000	0.2150	0.2213	0.2315	0.2383		0.1694	0.1856	0.2193	0.2291
S&P500	500	0.3126	0.3525	0.4024	0.4240	$\sqrt{T}\widehat{\tau}_{T_5}^C = 0.0751$	0.4060	0.4707	0.5518	0.5825
$\sqrt{T_1}\widehat{\tau}_{T_1}^C = 0.0392$	1000	0.2606	0.2812	0.3033	0.3144		0.3203	0.4497	0.4980	0.5078
	2000	0.1843	0.1980	0.2301	0.2416		0.1216	0.1741	0.2233	0.2340

Table 5: The table shows the empirical p-values of the conditional test statistic  $\sqrt{T_j} \hat{\tau}_{T_j}^C$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). Three different estimated  $\sigma_t^2$  are used. We set lag lengths of daily returns  $j = 1$  and  $5$  and subsample sizes  $b = 500, 1000$  and  $2000$ . The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

	$j$	$RV_t^{1min}$			$RV_t^{5min}$			$RG_t^2$		
		$b$			$b$			$b$		
		500	1000	2000	500	1000	2000	500	1000	2000
MSFT	1	0.9101	0.9138	0.8608	0.8956	0.7914	0.3936	0.9160	0.8916	0.9698
	5	1.0000	0.9936	1.0000	0.9849	0.9662	0.8507	0.7389	0.6653	0.9343
IBM	1	0.9834	0.9769	0.8643	1.0000	0.9568	0.8958	0.6847	0.4674	0.4435
	5	0.2994	0.1687	0.0617	0.5938	0.4141	0.2421	0.6155	0.4562	0.0744
GE	1	0.5264	0.2500	0.0201	0.8356	0.6544	0.6975	0.9884	0.8952	0.8148
	5	0.6580	0.4568	0.3914	0.8597	0.9354	0.7329	0.9318	0.9814	0.9186
PG	1	0.9625	0.9656	0.8481	0.9765	0.7667	0.8117	0.7149	0.5356	0.4374
	5	0.8776	0.8441	0.6856	0.9284	0.7174	0.5416	0.8198	0.5627	0.4247
MMM	1	0.9730	0.9900	0.8643	0.9514	0.9650	0.8783	0.9236	0.9062	0.8927
	5	0.9086	0.7485	0.3827	0.8338	0.7211	0.4002	0.9374	0.7619	0.9387
S&P500	1	0.9860	0.9939	1.0000	0.9987	0.9915	1.0000	0.9323	0.9178	0.7877
	5	0.6786	0.5612	0.8494	0.8890	0.6559	0.4952	0.9294	0.7555	0.5066

Table 6: The table shows value of test statistic  $\sqrt{T_j} \hat{\tau}_{T_j}^C$ , empirical p-value and the subsample critical value  $\hat{g}_{T_j, b}^C(1 - \alpha)$  for the conditional leverage hypothesis test at four different levels of  $\alpha$  for SP500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM).  $\sigma_t^2$  is estimated by the daily realized variance with 1-min log returns. We set lag lengths of stock returns  $j = 1$  and subsample sizes  $b = 500$  and  $1000$ . The sample period is divided into two subperiods: 1) from Jan-04-1993 to Nov-30-2001 (2,248 trading days). 2) from Dec-03-2001 to Dec-31-2009 (2,035 trading days).

		$\alpha$						
	Period	$\sqrt{T_1} \hat{\tau}_{T_1}^C$	$b$	$\hat{p}_{T_1, b}$	0.1	0.05	0.01	0.0001
MSFT	Jan.93-Nov.01	0.1401	500	0.6049	0.2257	0.2662	0.3446	0.3610
			1000	0.0753	0.1370	0.1434	0.1647	0.1724
	Dec.01-Dec.09	0.0754	500	0.9150	0.2722	0.2839	0.3566	0.3780
			1000	0.9828	0.2284	0.2430	0.2584	0.2703
IBM	Jan.93-Nov.01	0.1458	500	0.8834	0.3014	0.3251	0.3461	0.3634
			1000	0.8343	0.2576	0.2652	0.2829	0.3008
	Dec.01-Dec.09	0.0214	500	0.9651	0.2140	0.2340	0.2541	0.2725
			1000	0.9445	0.1517	0.1662	0.1880	0.1997
GE	Jan.93-Nov.01	0.1622	500	0.7216	0.3519	0.3900	0.4354	0.4448
			1000	0.2562	0.3391	0.3980	0.4555	0.4738
	Dec.01-Dec.09	0.1472	500	0.6388	0.3157	0.3342	0.3652	0.3810
			1000	0.5155	0.2464	0.3204	0.3389	0.3448
PG	Jan.93-Nov.01	0.1075	500	0.8159	0.2597	0.2878	0.3207	0.3329
			1000	0.7150	0.1814	0.1881	0.2002	0.2047
	Dec.01-Dec.09	0.0470	500	0.9764	0.2942	0.3075	0.3339	0.3511
			1000	0.9695	0.2712	0.3285	0.3621	0.3803
MMM	Jan.93-Nov.01	0.1459	500	0.9989	0.3949	0.4506	0.5296	0.5523
			1000	1.0000	0.3584	0.3720	0.3949	0.4069
	Dec.01-Dec.09	0.0238	500	0.9951	0.2018	0.2294	0.2776	0.2971
			1000	1.0000	0.1995	0.2282	0.2501	0.2549
S&P500	Jan.93-Nov.01	0.0902	500	0.5638	0.2712	0.3069	0.3406	0.3603
			1000	0.4516	0.2111	0.2243	0.2386	0.2526
	Dec.01-Dec.09	0.0598	500	0.9391	0.2025	0.2200	0.2401	0.2574
			1000	0.8634	0.1839	0.2045	0.2333	0.2454

Table 7: The table shows the empirical p-values, the test statistic  $\sqrt{T_j} \hat{\tau}_{T_j}^C$ , and the subsample critical values  $\hat{g}_{T_j, b}^C (1 - \alpha)$  for the conditional leverage hypothesis test at four different levels of  $\alpha$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). In addition to the lagged one day return  $R_{t-1}$ , the test is also conditioning on more general forms of  $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$ . Upper panel shows results of the test conditioning on  $h(\cdot)$  as a finite approximation for the exponential moving average in (9) and bottom panel shows results of the test conditioning on  $h(\cdot)$  as a real time forecast for  $RV_t^{1min}$  from the HAR-RV model in (10).  $\sigma_t^2$  is estimated by daily realized variance with 1-min log returns. We set lag lengths of stock returns  $j = 1$  and subsample size  $b = 2000$ . The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

Finite Exponential Moving Average							
	$p$	$\hat{p}_{T_1, 2000}$	$\sqrt{T_1} \hat{\tau}_{T_1}^C$	$\alpha$			
				0.1	0.05	0.01	0.001
MSFT	5	0.5236	0.1078	0.1804	0.1961	0.2059	0.2107
	10	0.0039	0.2082	0.1886	0.1928	0.1985	0.2162
	22	0.2194	0.2027	0.2129	0.2214	0.2284	0.2291
IBM	5	0.9098	0.0817	0.1784	0.2181	0.2381	0.2525
	10	0.8717	0.0716	0.1543	0.1736	0.1959	0.2043
	22	0.8126	0.0636	0.1913	0.1954	0.2186	0.2230
GE	5	0.4882	0.1274	0.2332	0.2416	0.2452	0.2455
	10	0.4593	0.1194	0.2271	0.2490	0.2714	0.2788
	22	0.4286	0.1401	0.2604	0.2918	0.3162	0.3254
PG	5	0.9904	0.0281	0.1099	0.1147	0.1409	0.1530
	10	0.7412	0.0450	0.1168	0.1416	0.1468	0.1504
	22	0.6668	0.0537	0.1247	0.1293	0.1396	0.1443
MMM	5	0.5263	0.1329	0.1731	0.1877	0.1981	0.2161
	10	0.6734	0.1163	0.2279	0.2340	0.2443	0.2650
	22	0.9934	0.1220	0.2009	0.2160	0.2237	0.2260
S&P500	5	0.7601	0.0361	0.0947	0.1022	0.1235	0.1336
	10	1.0000	0.0086	0.1001	0.1033	0.1178	0.1193
	22	0.9834	0.0148	0.1415	0.1449	0.1501	0.1812
Real Time Forecast from the HAR-RV model							
	$p$	$\hat{p}_{T_1, 2000}$	$\sqrt{T_1} \hat{\tau}_{T_1}^C$	$\alpha$			
				0.1	0.05	0.01	0.001
MSFT	22	0.6607	0.1166	0.1349	0.1658	0.1719	0.1912
IBM	22	0.7469	0.0710	0.1189	0.1405	0.1522	0.1672
GE	22	0.3529	0.1433	0.0928	0.2152	0.2269	0.2458
PG	22	0.9400	0.0117	0.0596	0.1057	0.1151	0.1236
MMM	22	0.9308	0.0442	0.0901	0.1795	0.1925	0.2111
S&P500	22	1.0000	0.0087	0.0502	0.1182	0.1258	0.1456

Table 8: The Table shows the OLS estimation results of the HAR-RV model augmented with the terms for the leverage effects (11) and results of testing the hypothesis  $H_0 : \gamma_- \leq \gamma_+$ . The 1-min realized variance  $RV_t^{1min}$  is used in the OLS fittings. In the parenthesis under the estimated coefficients are t-statistics obtained with Newey-West standard errors with 18 lag periods. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$\alpha_D$	$\beta_{RD}$	$\beta_{RW}$	$\beta_{RM}$	$\gamma_+$	$\gamma_-$	Adj $R^2$	$H_0 : \gamma_- \leq \gamma_+$	
									t-statistic	p-value
MSFT	$R_{t-1}^2$	0.0000 (1.5109)	0.3080 (3.4832)	0.4009 (3.4076)	0.2520 (4.8492)	0.0054 (1.0721)	0.0463 (3.1394)	0.7996	2.88	0.0020
	$ R_{t-1} $	-0.0000 (-2.1753)	0.2982 (3.3179)	0.4043 (3.4144)	0.2530 (4.8945)	0.0007 (1.9027)	0.0040 (5.5406)	0.8017	5.38	0.0000
IBM	$R_{t-1}^2$	0.0000 (3.1423)	0.1908 (1.2951)	0.5788 (2.7290)	0.1399 (2.1214)	-0.0131 (-1.4009)	0.0361 (2.9700)	0.5970	3.00	0.0014
	$ R_{t-1} $	0.0000 (1.8065)	0.1953 (1.4204)	0.5534 (2.9082)	0.1459 (2.4100)	0.0000 (-0.0355)	0.0029 (3.9841)	0.5974	3.67	0.0001
GE	$R_{t-1}^2$	0.0000 (2.7765)	0.2581 (3.7433)	0.4724 (6.0651)	0.1006 (1.5677)	-0.0140 (-0.7284)	0.2016 (3.4924)	0.6300	3.42	0.0003
	$ R_{t-1} $	-0.0000 (-0.7339)	0.2739 (3.6260)	0.4597 (5.8477)	0.0999 (1.6040)	0.0011 (1.0789)	0.0115 (3.8243)	0.6175	3.44	0.0003
PG	$R_{t-1}^2$	0.0000 (3.5922)	0.2021 (1.9651)	0.2902 (2.7418)	0.3967 (4.4417)	-0.0123 (-1.1327)	0.0176 (0.8961)	0.4187	1.91	0.0282
	$ R_{t-1} $	0.0000 (0.5923)	0.1732 (1.7321)	0.2616 (2.3486)	0.3915 (4.6627)	0.0014 (1.1906)	0.0065 (1.9860)	0.4433	2.33	0.0101
MMM	$R_{t-1}^2$	0.0000 (2.5989)	0.1103 (0.8507)	0.5569 (3.8863)	0.1881 (2.8876)	0.0074 (0.7038)	0.1306 (1.6649)	0.5429	1.66	0.0490
	$ R_{t-1} $	0.0000 (0.0432)	0.1142 (0.8813)	0.5667 (3.5723)	0.1707 (2.4630)	0.0012 (1.6301)	0.0056 (2.0489)	0.5360	2.02	0.0215
S&P500	$R_{t-1}^2$	0.0000 (1.1027)	0.1014 (0.8112)	0.5021 (3.3468)	0.1377 (2.6276)	0.0199 (1.2872)	0.1952 (2.8746)	0.6496	2.75	0.0030
	$ R_{t-1} $	-0.0000 (-1.6314)	0.0935 (0.6226)	0.5880 (3.1857)	0.1207 (1.7781)	0.0011 (1.6719)	0.0063 (2.3395)	0.5995	2.37	0.0089



Table 9: The Table shows the OLS estimation results of the HAR-RV-CJ model augmented with the terms for the leverage effects (12) and results of testing the hypothesis  $H_0 : \gamma_- \leq \gamma_+$ . The 1-min realized variance  $RV_t^{1min}$  is used in the OLS fittings. In the parenthesis under the estimated coefficients are t-statistics obtained with Newey-West standard errors with 18 lag periods. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$\alpha_D$	$\beta_{BD}$	$\beta_{BW}$	$\beta_{BM}$	$\beta_{JD}$	$\beta_{JW}$	$\beta_{JM}$	$\gamma_+$	$\gamma_-$	Adj $R^2$	$H_0 : \gamma_- \leq \gamma_+$	
												t-statistic	p-value
MSFT	$R_{t-1}^2$	0.0000 (2.1119)	0.2999 (2.8715)	0.4329 (3.5586)	0.2525 (4.2595)	-0.0313 (-0.3367)	0.3389 (2.0899)	0.9365 (4.7025)	0.0055 (1.0799)	0.0475 (3.0443)	0.7950	2.86	0.0021
	$ R_{t-1} $	0.0000 (0.0322)	0.2883 (2.7402)	0.4362 (3.5820)	0.2451 (4.1166)	-0.0287 (-0.3086)	0.3450 (2.1076)	0.9789 (4.8550)	0.0009 (1.8815)	0.0042 (5.1591)	0.7974	5.29	0.0000
IBM	$R_{t-1}^2$	0.0000 (3.9448)	0.1845 (1.1346)	0.5967 (2.6655)	0.1408 (2.1168)	0.0011 (0.0159)	0.2239 (0.5862)	0.7517 (2.0366)	-0.0106 (-1.1125)	0.0387 (3.0450)	0.5869	2.94	0.0016
	$ R_{t-1} $	0.0000 (2.5105)	0.1875 (1.2241)	0.5739 (2.8251)	0.1412 (2.2809)	0.0123 (0.1824)	0.1320 (0.2948)	0.8368 (2.0337)	0.0003 (0.5110)	0.0032 (4.1115)	0.5881	3.65	0.0001
GE	$R_{t-1}^2$	0.0000 (3.6073)	0.2816 (4.3197)	0.4995 (6.1839)	0.0842 (1.5749)	0.0133 (0.0515)	0.4664 (1.1606)	0.5403 (1.8748)	-0.0107 (-0.5542)	0.2038 (3.4998)	0.6373	3.37	0.0004
	$ R_{t-1} $	-0.0000 (-1.4288)	0.2981 (4.3399)	0.4864 (5.8685)	0.0805 (1.5416)	-0.0098 (-0.0373)	0.3522 (0.8862)	0.7051 (2.4516)	0.0016 (1.6488)	0.0119 (3.9520)	0.6256	3.43	0.0003
PG	$R_{t-1}^2$	0.0000 (3.8307)	0.1727 (1.2404)	0.3839 (6.7152)	0.3571 (2.6765)	0.1209 (3.6550)	-0.0834 (-0.3955)	0.8698 (4.1265)	-0.0090 (-0.7830)	0.0197 (0.9524)	0.4118	1.74	0.0412
	$ R_{t-1} $	0.0000 (0.6036)	0.1349 (1.0996)	0.3530 (5.8374)	0.3350 (2.9011)	0.1376 (4.2783)	-0.1197 (-0.5279)	0.8877 (4.1579)	0.0018 (1.4862)	0.0070 (2.0842)	0.4394	2.32	0.0101
MMM	$R_{t-1}^2$	0.0000 (3.9076)	0.1089 (0.7835)	0.6092 (4.0426)	0.1477 (2.2613)	0.0130 (0.0624)	-0.0012 (-0.0069)	0.8853 (3.0911)	0.0094 (0.9267)	0.1376 (1.7609)	0.5459	1.72	0.0430
	$ R_{t-1} $	0.0000 (0.0660)	0.1016 (0.7176)	0.6302 (3.7460)	0.1309 (1.8819)	0.1073 (0.7681)	-0.0501 (-0.2799)	0.8321 (3.5467)	0.0014 (1.9428)	0.0059 (2.1791)	0.5386	2.07	0.0191
S&P500	$R_{t-1}^2$	0.0000 (1.1751)	0.1096 (0.9227)	0.4774 (3.3274)	0.1542 (2.9598)	0.0212 (0.0974)	-0.4703 (-0.6704)	1.4289 (1.1810)	0.0220 (1.4051)	0.1972 (2.8945)	0.6479	2.78	0.0027
	$ R_{t-1} $	-0.0000 (-1.5487)	0.0978 (0.7030)	0.5675 (3.3248)	0.1374 (2.0635)	0.0702 (0.3511)	-0.5867 (-0.7494)	1.5699 (1.1615)	0.0012 (1.7194)	0.0064 (2.3504)	0.5971	2.39	0.0084

Table 10: The table shows minimum, mean and maximum values of daily standardized quadratic co-variations between intraday log price  $X_t$  and function of spot variance  $F(\sigma_t^2)$  obtained by using the method in Wang and Mykland (2012), and number of significant days when comparing the daily standardized quadratic co-variations with different critical values. The standardized quadratic co-variations are defined as  $L_1$  and  $L_2$  in (15) and (16) with  $\langle X, F(\sigma^2) \rangle_T = 0$ . We use 1-min equally spaced data, and set  $T =$  one day and  $M_n = 30$  for the estimations. We assume  $F(x) = x$  and  $F(x) = 1/2 \log(x)$ . The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

MSFT					IBM			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.807	-2.561	-2.906	-2.520	-2.421	-2.691	-2.509	-2.709
Mean	-0.059	-0.065	-0.058	-0.065	-0.014	-0.017	-0.015	-0.019
Max.	2.224	2.498	2.429	2.716	2.584	2.728	2.763	2.728
$ \cdot  \geq 1.96$	29	46	32	47	39	51	40	54
$\cdot \leq -1.64$	85	102	88	98	66	94	70	99
$ \cdot  \geq 2.58$	2	0	1	1	1	2	1	3
$\cdot \leq -2.33$	2	3	3	5	1	3	1	3
GE					PG			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.322	-2.613	-2.51	-2.601	-2.416	-2.354	-2.426	-2.403
Mean	-0.038	-0.039	-0.042	-0.042	-0.037	-0.038	-0.041	-0.043
Max.	2.815	2.765	2.790	2.851	2.263	2.475	2.260	2.501
$ \cdot  \geq 1.96$	42	63	42	70	18	35	20	37
$\cdot \leq -1.64$	83	108	86	109	60	95	66	101
$ \cdot  \geq 2.58$	1	2	1	2	0	0	0	0
$\cdot \leq -2.33$	0	1	1	2	1	2	1	2
MMM					S&P500			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.362	-2.638	-2.383	-2.646	-2.605	-2.735	-2.620	-2.987
Mean	0.001	0.003	0.000	0.003	-0.191	-0.186	-0.186	-0.186
Max.	2.496	2.471	2.504	2.434	2.850	2.785	2.781	2.969
$ \cdot  \geq 1.96$	43	64	50	71	148	244	157	257
$\cdot \leq -1.64$	88	99	87	101	308	379	307	392
$ \cdot  \geq 2.58$	0	1	0	1	2	8	4	13
$\cdot \leq -2.33$	1	6	1	6	15	39	19	42

Table 11: The table shows estimated  $f = \lim_{l \rightarrow 0} Corr(v_{t+l} - v_t, X_{t+l} - X_t)$  by using the data driven method in Ait-Sahalia et al. (2012). Here  $X_t := \log P_t$  and  $v_t := \sigma_t^2$ . The estimates are based on sample correlation and bias corrected sample correlation (denoted by  $\hat{f}_k$  and  $\hat{f}_k^{bc}$ ) between daily returns  $R_t$  and the difference of the realized variances  $RV_t^{1min}$ . We also report the upper and lower bounds  $\bar{k}^*$  and  $\underline{k}^*$  for  $\left\{k, \hat{f}_k\right\}_{k=\underline{k}^*}^{\bar{k}^*}$  used for estimating the regression, and the (unadjusted)  $R^2$  of the regression. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

Uncorrected $\hat{f}_k$						
	IBM	MSFT	GE	PG	MMM	S&P500
$f$	-0.24	-0.31	-0.48	-0.32	-0.28	-0.36
$\bar{k}^*$	15	21	44	29	17	17
$\underline{k}^*$	55	53	87	91	54	94
$R^2$	0.97	0.92	0.96	0.98	0.94	0.89
Bias Corrected $\hat{f}_k^{bc}$						
	IBM	MSFT	GE	PG	MMM	S&P500
$f$	-0.25	-0.32	-0.48	-0.34	-0.30	-0.37
$\bar{k}^*$	15	20	31	29	17	15
$\underline{k}^*$	55	53	88	88	54	93
$R^2$	0.97	0.93	0.97	0.98	0.95	0.91

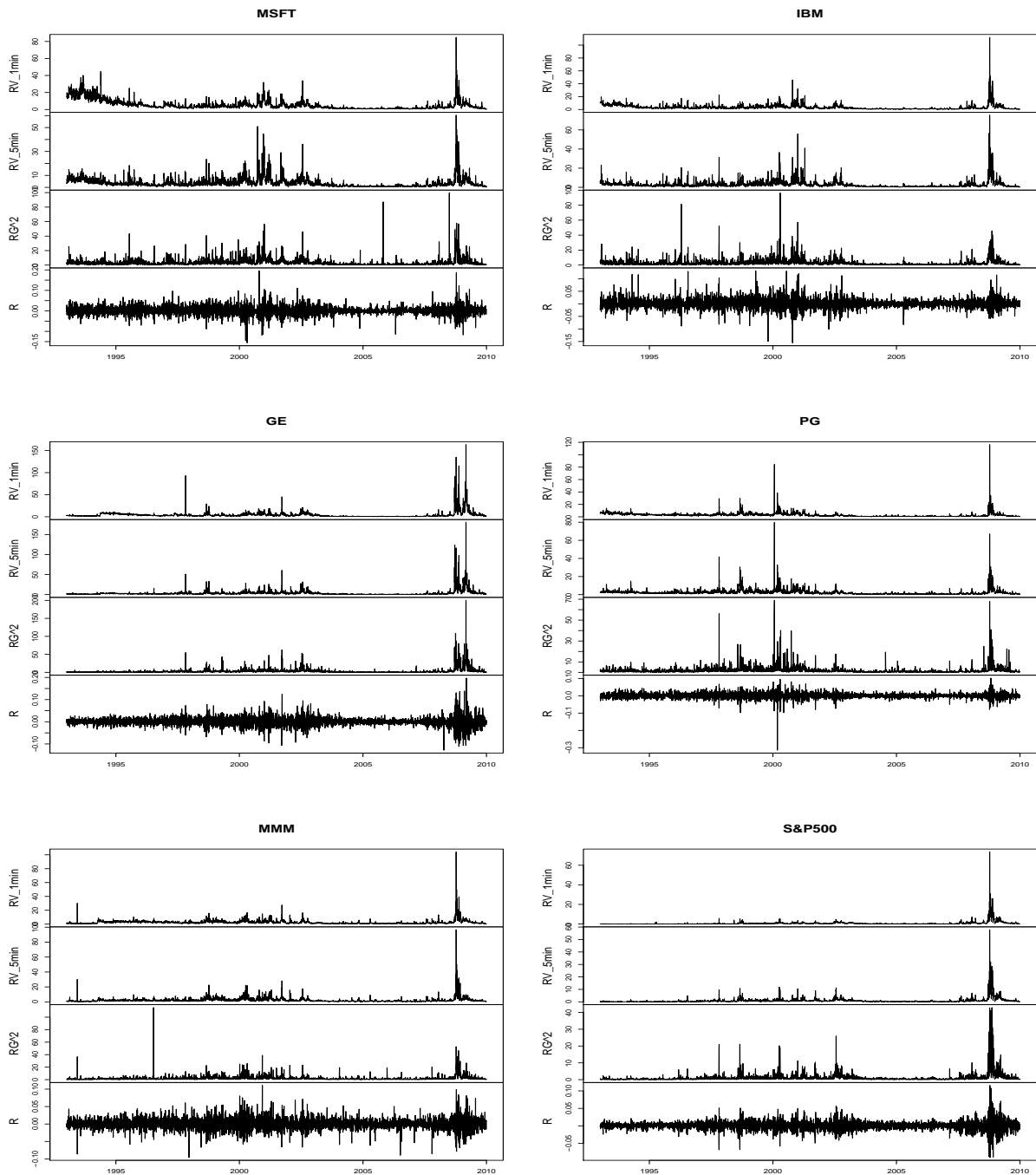


Figure 1: Time series plots of daily  $RV_t^{1min}$ ,  $RV_t^{5min}$ ,  $RG_t^2$ , and  $R_t$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The quantities of  $RV_t^{1min}$ ,  $RV_t^{5min}$ , and  $RG_t^2$  shown here are scaled by  $10^4$ . The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

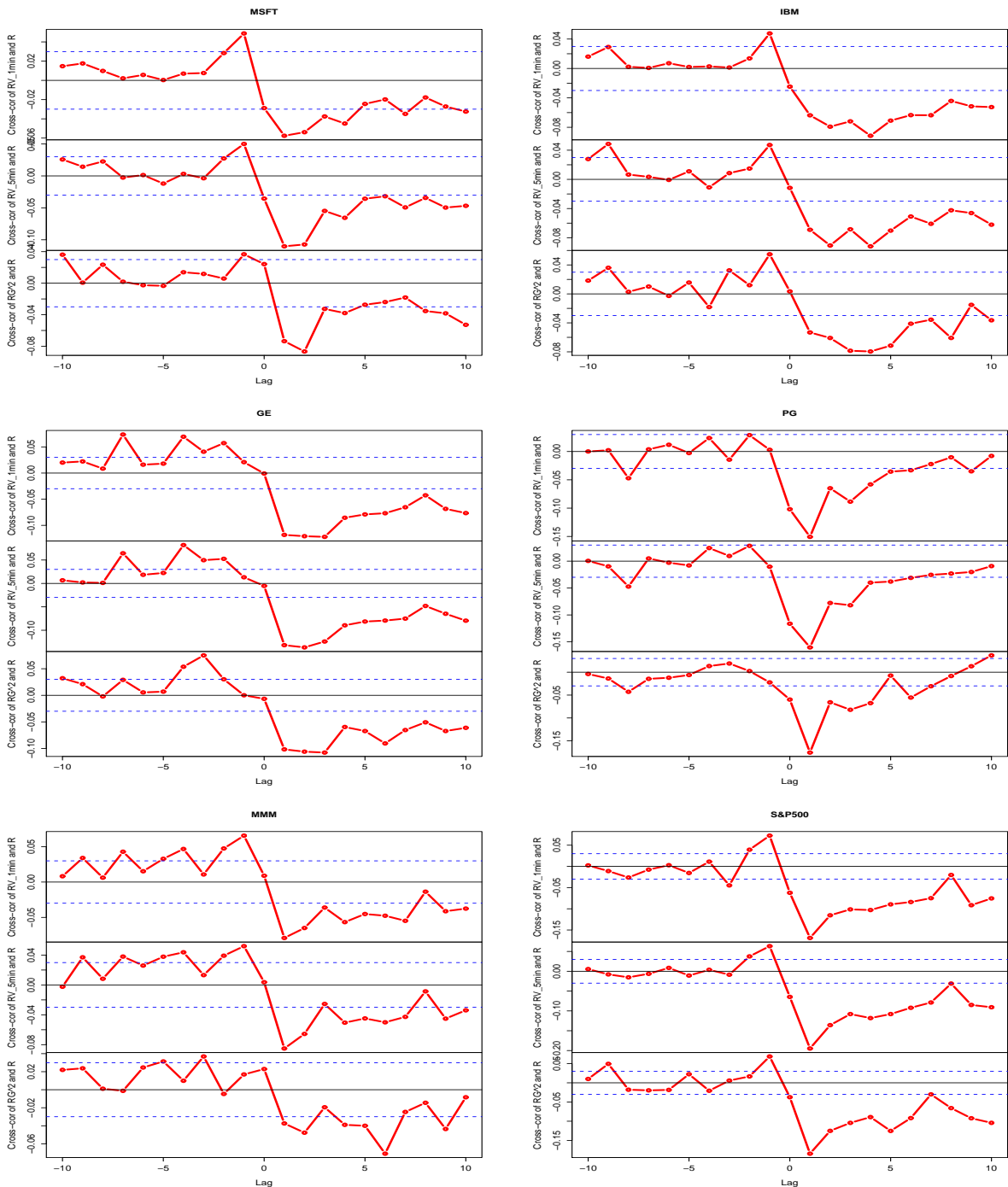


Figure 2: Cross correlations of daily  $RV_t^{1min}$ ,  $RV_t^{5min}$ ,  $RG_t^2$ , and  $R_{t-j}$  for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag length  $j = -10, \dots, 10$ , and the sample period for the calculations is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

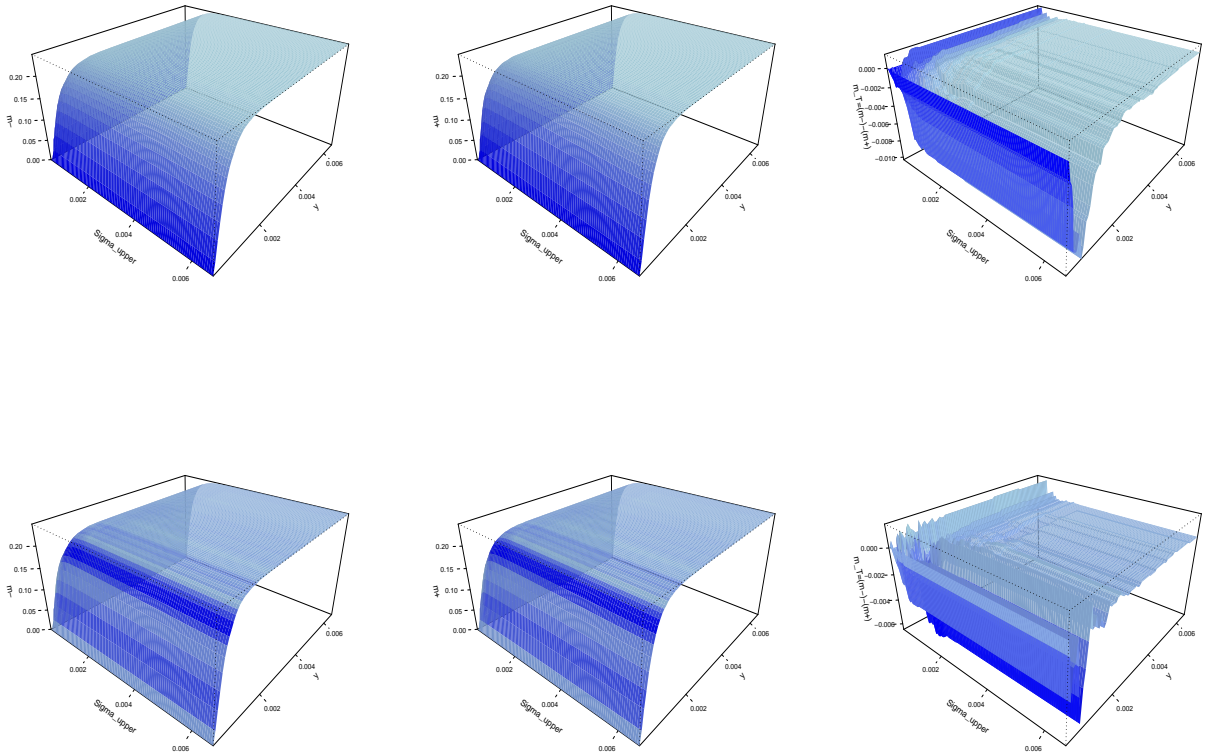


Figure 3: Plots of surfaces  $\widehat{m}_{T_j}^-(y, 0, \bar{\sigma})$  (left),  $\widehat{m}_{T_j}^+(y, 0, \bar{\sigma})$  (middle) and  $\widehat{m}_j(y, 0, \bar{\sigma})$  (right) of MSFT. Note that  $\widehat{m}_j(y, 0, \bar{\sigma}) = \widehat{m}_{T_j}^-(y, 0, \bar{\sigma}) - \widehat{m}_{T_j}^+(y, 0, \bar{\sigma})$ . Upper:  $j = 1$ . Bottom:  $j = 5$ . Here  $RV_t^{5min}$  is used as the estimate of daily  $\sigma_t^2$ .

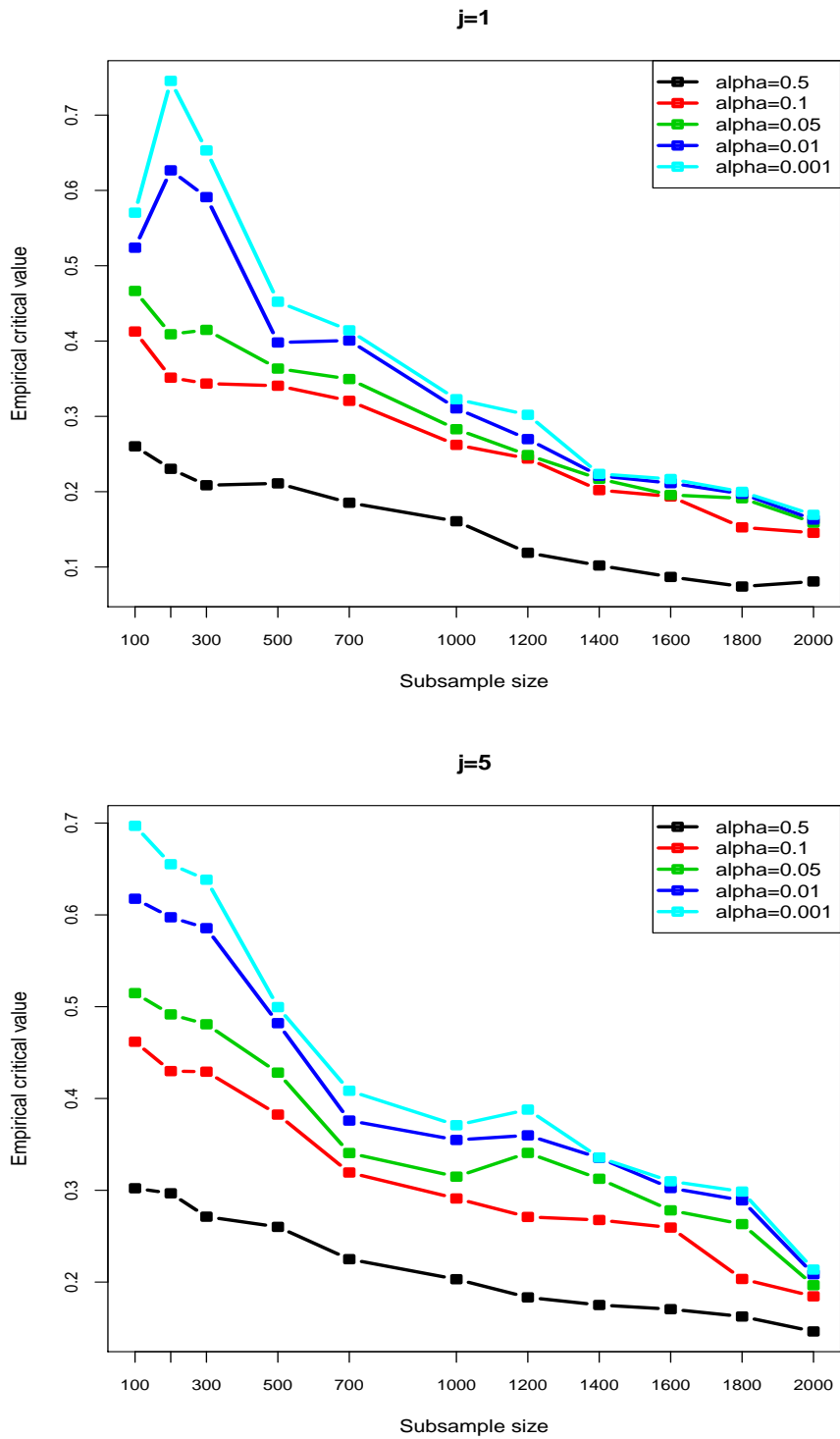


Figure 4: The subsample critical values  $\widehat{g}_{T_j,b}^C(1-\alpha)$  of MSFT. Upper:  $j = 1$ . Bottom:  $j = 5$ . Here  $RV_t^{5min}$  is used as the estimate of daily  $\sigma_t^2$ . 45

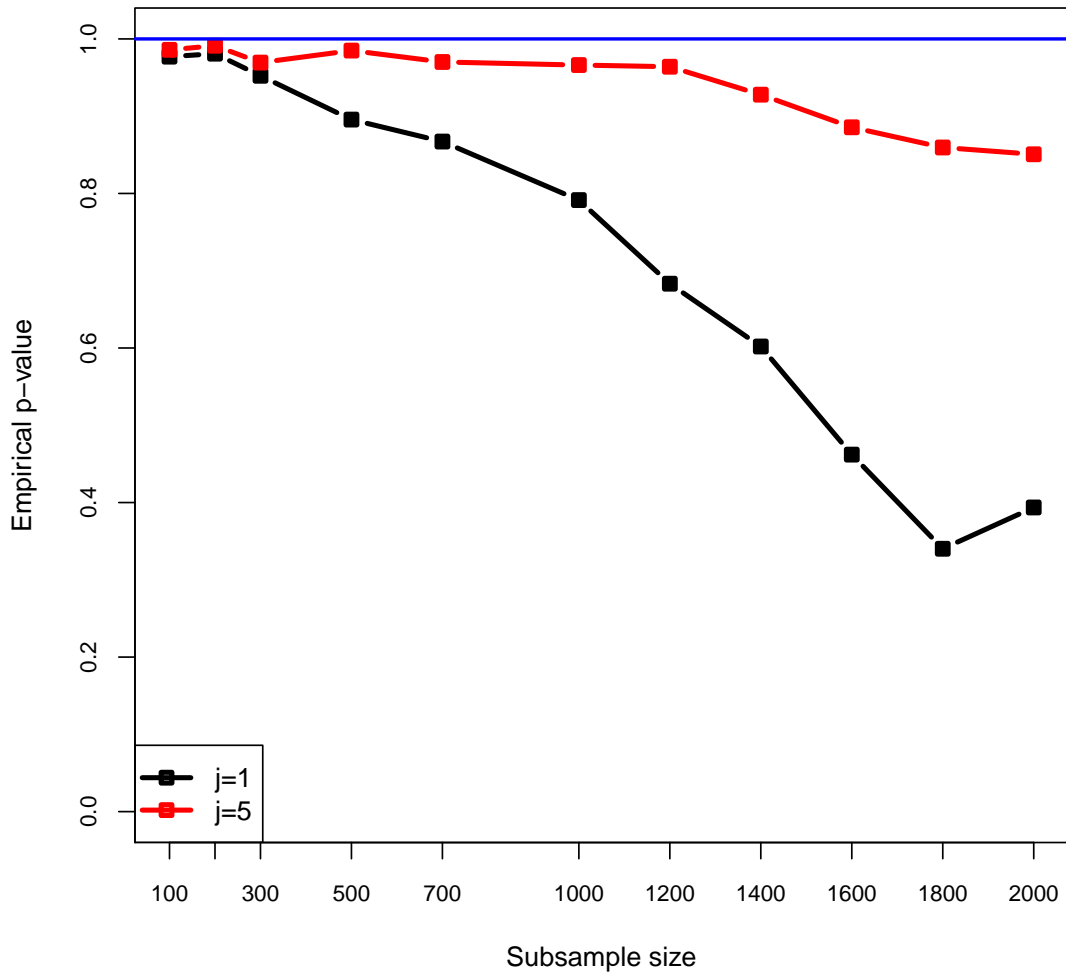


Figure 5: The subsample empirical p-values for  $\sqrt{T_j} \hat{\tau}_{T_j}^C$  of MSFT. Here  $RV_t^{5min}$  is used as the estimate of daily  $\sigma_t^2$ .



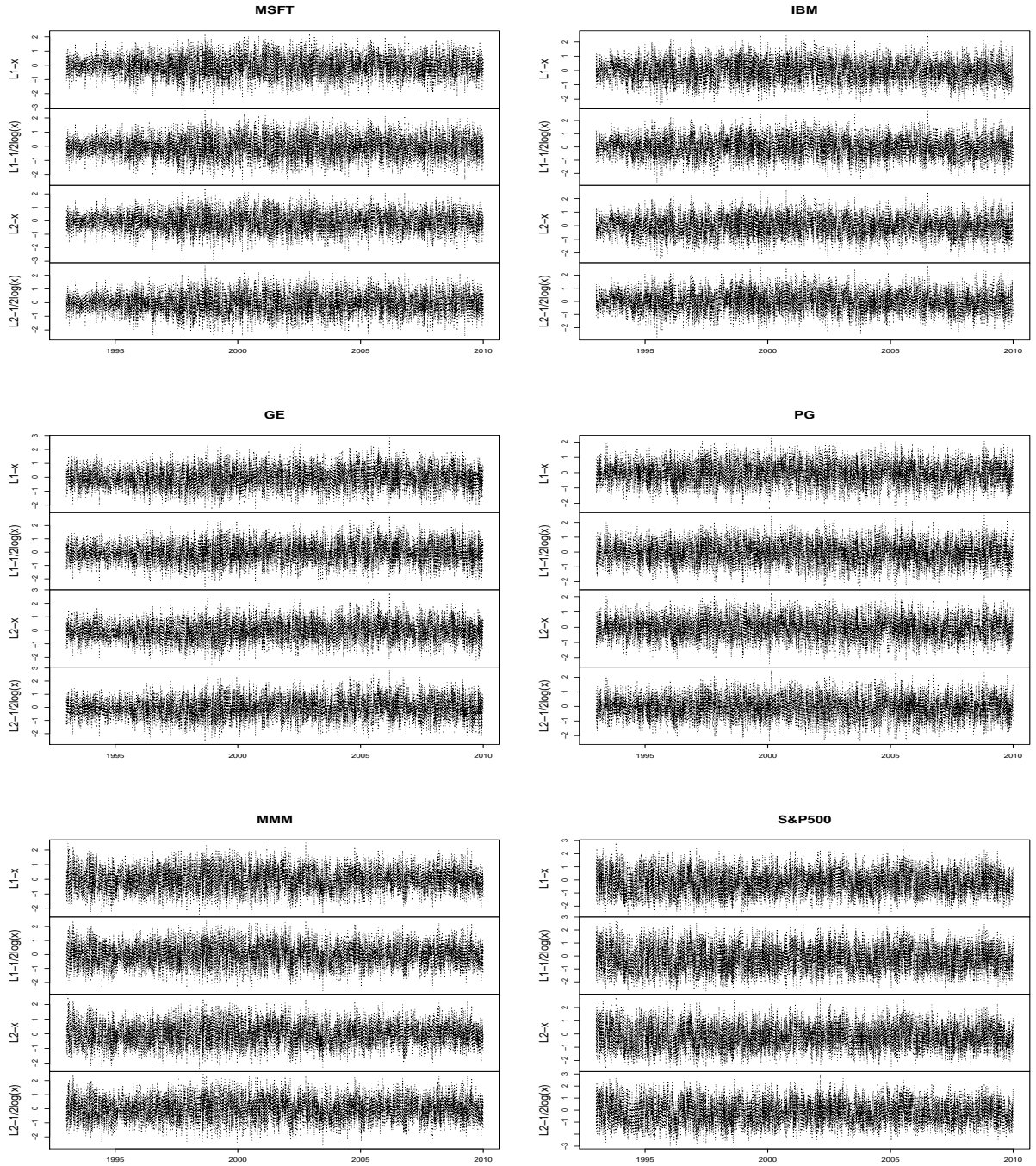


Figure 6: Time series plots of daily standardized quadratic co-variations between intraday log return  $X_t$  and function of spot variance  $F(\sigma_t^2)$  obtained by using the method in Wang and Mykland (2012). The standardized quadratic co-variations are defined as  $L_1$  and  $L_2$  in (15) and (16) with  $\langle X, F(\sigma^2) \rangle_T = 0$ . We use 1-min equally spaced data, and set  $T =$  one day and  $M_n = 30$  for the estimations. We assume  $F(x) = x$  and  $F(x) = 472 \log(x)$ . The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

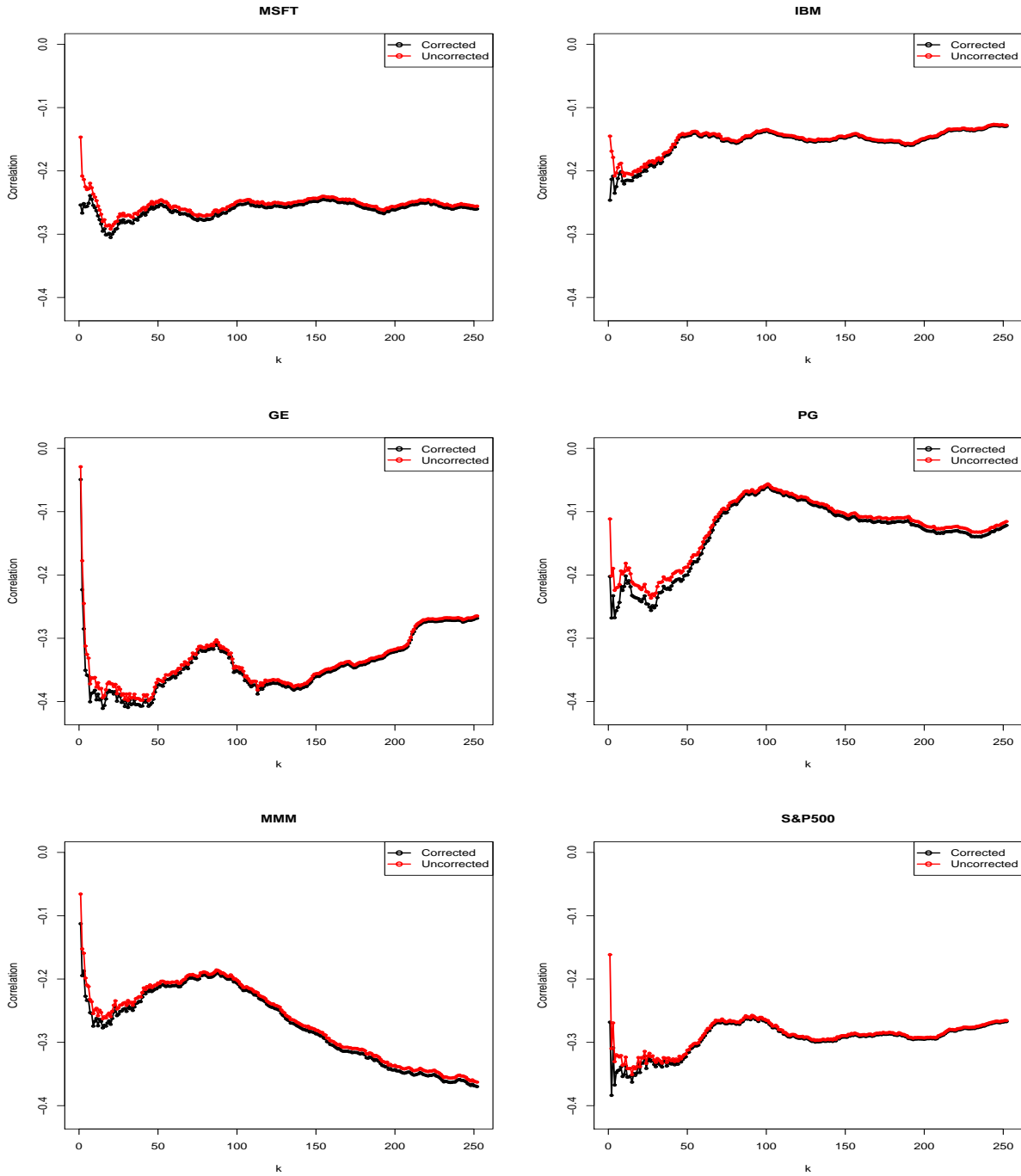


Figure 7: Plots of sample correlation between returns and difference of the daily realized variances,  $\hat{f}_k$  and bias corrected sample correlation  $\hat{f}_k^{bc}$  in (18) against  $k$ . The bias corrected sample correlation  $\hat{f}_k^{bc}$  is obtained by using method in Ait-Sahalia et al. (2012). We replace  $\gamma$  in (18) with  $\hat{\gamma}$  in (19) which is estimated with mean of the realized quartilities and sample variance of the realized variances. We use 1-min equally spaced log returns to estimate the daily realized variances and quartilities. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

## References

- [1] Ait-Sahalia, Y., J. Fan, and Y. Li (2012). The leverage effect puzzle: disentangling sources of bias a high frequency. NBER working paper series 17592.
- [2] Andersen, T.G., T. Bollerslev, P. F. Christoffersen, and F. X. Diebold (2011). Financial Risk Measurement for Financial Risk Management Forthcoming in Handbook of the Economics of Finance. <http://public.econ.duke.edu/~boller/>
- [3] Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007), Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility, ” *The Review of Economics and Statistics* **89**, 701-720.
- [4] Andrews, D.W.K. (1989), “Stochastic Equicontinuity for Semiparametric Models.” Unpublished manuscript, Cowles Foundation, Yale University.
- [5] Andrews, D.W.K. (1994), “Asymptotics for Semiparametric Econometric Models via Stochastic Equicontinuity.” *Econometrica* **62**, 43-72.
- [6] Andrews D.W.K and X. Shi (2011), ”Inference based on Conditional Moment Inequalities,” Cowles Foundation Discussion Paper No. 1761R, Yale University.
- [7] Barndorff-Nielsen, O.E., and Shephard, N., (2002), ”Econometric Analysis of Realized Volatility and its Use in Estimating Stochastic Volatility Models,” *Journal of the Royal Statistical Society, Series B* 64, 253-280.
- [8] Barndorff-Nielsen, O. E. and N. Shephard, (2006), Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation, ” *Journal of Financial Econometrics* **4**, 1-30.
- [9] Barndorff-Nielsen, O. E., P. R. Hansen, A. Lunde, and N. Shephard (2009), Realized kernels in practice: trades and quotes,” *Econometrics Journal*, 12, 1-32.
- [10] Dahlhaus, R. (1997): “Fitting time series models to nonstationary processes,” *Annals of Statistics* 25, 1-37.
- [11] Dahlhaus, R., and S. Subba Rao (2006), “Statistical inference for time-varying ARCH processes,” *The Annals of Statistics* 34, 1075-1114.

- [12] Dehling, H. and W. Philipp (2002), "Empirical Process Techniques for Dependent Data," in *Empirical Process Techniques for Dependent Data*, ed. by H. Dehling, T. Mikosch, M. Sørensen. Cambridge University Press.
- [13] Engle, R.F. and V.K. Ng (1993), "Measuring and Testing the impact of news on volatility," *The Journal of Finance* XLVIII, 1749-1778.
- [14] Engle, R.F. and Rangel (2006): "The spline GARCH model for unconditional volatility and its global macroeconomic causes,". Mimeo, NYU.
- [15] Figlewski, S. and X. Wang (2000), Is the "Leverage Effect" a Leverage Effect? Manuscript, NYU.
- [16] Folland, G.B. (1984), *Real Analysis: Modern Techniques and their Applications*, John Wiley: New York.
- [17] Garman, M. B. and M. J. Klass (1980), On the Estimation of Security Price Volatilities from Historical Data," *The Journal of Business*, 53, 67-78.
- [18] Hall, P. and Heyde, C. C. (1980), *Martingale limit theory and its application*, Academic Press, New York.
- [19] Jacod, J., Y. Li, P. A. Mykland, M. Podolskij, and M. Vetter (2009), Microstructure Noise in the Continuous Case: The Pre-Averaging Approach, " *Stochastic Processes and Their Applications* **119**, 2249-2276.
- [20] Linton, O. and P. Gozalo (1996). Conditional independence restrictions: testing and estimation. Cowels Foundation Discussion paper no 1140.
- [21] Linton, O. , E. Maasoumi and Y.-J. Whang (2005), "Consistent Testing for Stochastic Dominance under General Sampling Schemes," *Review of Economic Studies* 72, 735-765.
- [22] Linton, O.B., and E. Mammen (2005): "Estimating semiparametric ARCH( $\infty$ ) models by kernel smoothing methods," *Econometrica*.
- [23] Linton, O., K. Song and Y.-J. Whang (2010), "An Improved Bootstrap Test of Stochastic Dominance," *Journal of Econometrics* 154, 186-202.

- [24] Nelson, D.B. (1991), "Conditional heteroskedasticity in asset returns: A new approach," *Econometrica* 59, 347-370.
- [25] Parkinson, M. (1980): The Extreme Value Method for Estimating the Variance of the Rate of Return," *The Journal of Business*, 53, 61-65.
- [26] Politis, D. N. and J. P. Romano (1994), "Large sample confidence regions based on subsamples under minimal assumptions." *Annals of Statistics* 22, 2031-2050.
- [27] Politis, D. N., J. P. Romano and M. Wolf (1999), *Subsampling*, Springer-Verlag, New York.
- [28] Wang, Dan Christina and Mykland, Per A., The Estimation of Leverage Effect with High Frequency Data (November 10, 2011). Available at SSRN: <http://ssrn.com/abstract=1919138> or <http://dx.doi.org/10.2139/ssrn.1919138>