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**Working Paper**

## Prioritizing public health expenditures when there is a private alternative

Memorandum, No. 2005,16

**Provided in Cooperation with:**

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*Suggested Citation:* Hoel, Michael (2005) : Prioritizing public health expenditures when there is a private alternative, Memorandum, No. 2005,16, University of Oslo, Department of Economics, Oslo

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# MEMORANDUM

No 16/2005

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**Michael Hoel**

ISSN: 0801-1117

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University of Oslo

This series is published by the  
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# **Prioritizing public health expenditures when there is a private alternative\***

By

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May 25, 2005

## **Abstract**

Cost-effectiveness analysis often plays an important role in prioritization among different types of public health expenditures. Cost-effectiveness is defined as the maximal health benefits for given expenditures on health care. With a private health sector as a supplement to the public sector, the socially optimal ranking of treatments to be included in the public health program is changed. The larger are the costs per treatment for a given benefit-cost ratio, the higher priority should the treatment be given. The more heterogeneous preferences for a particular treatment are, the lower priority should this treatment be given. If the health budget does not exceed the socially optimal size, treatments with sufficiently low costs should not be performed by the public health system if there is a private alternative.

*JEL classification:* H42, H51, I10, I18

*Keywords:* public health, prioritization, cost-effectiveness analysis

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\* Useful comments have been given by participants at a seminar at the *Ragnar Frisch Centre for Economic Research*, in particular by Kjell Arne Brekke, and by Per-Olov Johansson and Albert Ma. I gratefully acknowledge financial support from the Research Council of Norway through HERO – Health Economic Research Programme at the University of Oslo.

## 1. Introduction

To prioritize among different types of health expenditures, health economists have often argued that cost-effectiveness analysis should play an important role. Cost-effectiveness is defined as the minimum cost for a given health benefit, or equivalently, maximal health benefits for given expenditures on health care.<sup>1</sup> There is a large literature that is critical to this type of analysis. One line of criticism is that cost-effectiveness analysis requires an aggregate measure of health benefits. Whether this measure is “quality adjusted life years” (QALYs) or some other measure, one needs severe restrictions on a general preference ordering over life years and health quality of each life year to be able to represent preferences by any simple aggregate measure.<sup>2</sup> A second line of criticism has been that whatever aggregate health benefit measure one uses to represent preferences at the individual level, the summation of health benefits across individuals lacks a good ethical or welfare theoretical basis.<sup>3</sup>

The present paper ignores the above-mentioned problems with cost-effectiveness analyses of prioritization issues. The focus is instead on a different important issue: In most of the literature that discusses how a public health budget should be allocated across potential medical interventions, it is explicitly or implicitly assumed that the health interventions that are not funded by the public budget are not carried out. This may be a good assumption for treatments such as heart surgery or cancer treatment. However, for many treatments there is a private alternative to public treatment. The private alternative may be potential in the sense that it is only relevant if the treatment is not offered by the public system, or it may exist in parallel with the public alternative, e.g. due to waiting time for public treatment. Examples of treatments that typically are offered outside the public system, at least if not offered by the latter, are

- surgical sterilization
- assisted fertilization
- cataract surgery
- dental care
- prescription medicine

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<sup>1</sup> For a further discussion of analyses based on the cost-effectiveness see e.g. Weinstein and Stason (1977), Johannesson and Weinstein (1993), Gabler and Phelps (1997) and Gabler (2000).

<sup>2</sup> See e.g. Broome (1993), Mehrez and Gafni (1989), Culyer and Wagstaff (1993), Bleichrodt and Quiggin (1999) and Gafni et al. (1993).

<sup>3</sup> See e.g. Harris (1987), Wagstaff (1991), Nord (1993), Olsen (1997) and Dolan (1998).

Comparing different countries that all have a predominantly public health system, one will find that countries differ with respect to what is covered by the public system and what is not. In the countries where a treatment of the above type is not offered by the public system, this treatment will typically be offered by the private sector.

The paper discusses this issue using a simple model where maximizing the sum of some aggregate measure of health benefits for a given budget is socially optimal (given the budget constraint) provided there is no alternative to the public health care. It is shown that when a private alternative to the public system is introduced, a simple cost-effectiveness criterion of the type above no longer gives a socially optimal outcome. The reason for this is that the benefit of including a particular treatment in the public program can no longer be measured simply by the gross health improvement this public treatment gives: Some of the health care would otherwise have been performed by the private sector, so the net health increase is lower than the gross increase. On the other hand, by including a treatment in the public system, there are reduced costs in the private sector. This cost reduction must be included in the benefit side of including the treatment in the public program. In order to add the benefits of improved health with the cost reduction in the private sector one is thus forced to make a monetary valuation of the net increase in health benefits. The paper shows that when this is correctly done, the social optimum (given the public health budget) no longer implies that the public health system should maximize gross or net health benefits for the given public health budget. A comparison is also given between the socially optimal priority ranking and the standard cost-effective ranking of different treatments.

The rest of the paper is organized as follows. In Section 2 the basic model is introduced, and it is shown that with this model the socially optimal ranking of different treatments is the standard cost-effective ranking, provided there is no private alternative. The consequence of introducing a private alternative is derived in Section 3. An important result in this section is that in addition to the ratio of health benefits to costs, the costs per treatment are important for the optimal ranking. The larger are these costs for a given benefit-cost ratio, the higher priority should the treatment be given. Heterogeneity in preferences is introduced in Section 4. The results of Section 3 remain valid, and in addition it is shown that the more heterogeneous preferences

for a particular treatment are, the lower priority should this treatment be given. Some concluding remarks are given in Section 5.

## 2. Prioritizing when there is no alternative to public treatment

I use a simple static model, where the utility of a healthy individual with net income  $y$  is given by the increasing and strictly concave function  $u(y)$ . The utility loss due to illness is denoted  $h$  so that total utility is  $u(y)-h$ . With this notation, the marginal willingness to pay for an improvement in health is given by  $\frac{1}{u'(y)}$ , which is increasing in  $y$ .

There are  $n$  mutually exclusive potential illnesses, and each individual gets illness  $j$  with probability  $\pi_j$  giving a utility loss  $h_j$ . The utility loss is assumed to be completely eliminated if the individual is treated with a cost  $c_j$ .<sup>4</sup> Standard cost-effectiveness analyses recommends that treatments are ranked according to the ratio  $h_j/c_j$ , and that the public health sector includes all treatments with this ratio above some threshold determined by the size of the health budget. I shall now show that with the preference function given above, this result is also the social optimum. Let  $F(y)$  be the income distribution function, i.e.  $F(y)$  tells us what share of the population that has net income below  $y$ . Without loss of generality we assume that no one has an income above 1, i.e.  $F(1)=1$ . The density function corresponding to  $F$  is  $f(y)=F'(y)$ . At the level of the society, the probabilities  $\pi_i$  are shares of persons with each of the  $n$  illnesses. Social welfare is given by the sum of utilities of all individuals, and we have a vector of policy variables  $\delta_1, \dots, \delta_n$ , where  $\delta_j=1$  if treatment for illness  $j$  is included in the public health program and  $\delta_j=0$  otherwise. The social optimum is defined as the solution to the following maximization problem<sup>5</sup>:

$$(1) \quad \begin{aligned} \max \quad & \sum_i \delta_i \pi_i \int_0^1 u(y) f(y) dy + \sum_i (1 - \delta_i) \pi_i \int_0^1 [u(y) - h_i] f(y) dy \\ \text{s. t.} \quad & \sum_i \delta_i \pi_i c_i \leq B \end{aligned}$$

<sup>4</sup> Nothing would be changed if we instead assumed that the utility loss was  $h_i^0 + h_i$  without treatment and  $h_i^0$  with treatment.

<sup>5</sup> With this setup we have implicitly assumed that  $\sum_i \pi_i = 1$ . If there is a state of no illness, this state is included in the list of  $n$  "illnesses" with  $h_i=0$  for this state.

where  $B$  is the exogenously given budget (per capita).

The Lagrangian to the problem (1) is

$$(2) \quad L = \int_0^1 [u(y) - \sum_i (1 - \delta_i) \pi_i h_i] f(y) dy + \lambda (B - \sum_i \delta_i \pi_i c_i)$$

A treatment  $j$  should be included in the public program if  $L$  is increasing in  $\delta_j$  but not included if  $L$  is declining in  $\delta_j$ . It follows from (2) that

$$(3) \quad \frac{\partial L}{\partial \delta_j} = \pi_j h_j \int_0^1 f(y) dy - \lambda \pi_j c_j$$

Since the integral in this expression is equal to 1, we should include all treatments satisfying<sup>6</sup>

$$(4) \quad \frac{h_j}{c_j} > \lambda$$

where the value of  $\lambda$  follows from the size of the budget.

Although we are assuming that the budget is exogenously given, it is useful to consider for a moment the case where it is optimally chosen, i.e. where  $B$  is chosen to solve the optimization problem (1). Since  $B$  is the budget per capita, we assume that an increase in  $B$  reduces all net incomes by the same amount<sup>7</sup>, i.e.  $\frac{du}{dB} = u'(y)(-1)$ .

Using this and setting the derivative of (2) with respect to  $B$  equal to zero gives

$$(5) \quad \lambda = \int_0^1 u'(y) f(y) dy$$

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<sup>6</sup> Due to the integer problem, there will typically be one marginal treatment, given by equality instead of the inequality in (4). This treatment should be partially included, i.e. some but not all persons should be offered treatment. The proportion of persons treated is determined so that the budget is exactly used up. This integer problem is from now on ignored.

<sup>7</sup> This can be justified by assuming that the initial tax system is optimally designed, where this optimization has taken into consideration possible distributional preferences. Starting at such an optimum, it makes no difference which element of the tax system one changes in order to finance a small increase in the required tax revenue. See e.g. Hoel and Sæther (2003) for a further discussion.



which together with (4) implies

$$(6) \quad \frac{h_j}{c_j} > \int_0^1 u'(y) f(y) dy$$

The interpretation of this is straightforward: Treatments for the different illnesses should be included in the public health system if and only if the health gain (in utility units) per Euro exceeds the population average of the marginal utility of income (i.e. the inverse of the marginal willingness to pay for a health improvement).

### 3. Prioritizing when there is a private alternative

Assume that if a particular treatment  $j$  is not included in the public program, each person has the option to buy treatment in the private sector at the same price as the cost would have been for public treatment. Moreover, it is assumed that there is no private insurance, so that any treatment is paid upon treatment.<sup>8</sup> We define the surplus from private treatment, denoted  $s(y, c_j, h_j)$ , by

$$(7) \quad s(y, c_j, h_j) = u(y - c_j) - [u(y) - h_j]$$

It follows from the concavity of  $u$  that this function is increasing in  $y$ . If  $s(y, c_j, h_j) < 0$  for a person who gets illness  $j$ , this person will prefer to be untreated than to pay for private treatment. If  $s(y, c_j, h_j) > 0$  for a person who gets illness  $j$ , this person will prefer to pay for private treatment rather than go untreated. If  $s(y, c_j, h_j) < 0$  for all  $y$ , then no one will buy this type of private treatment. If  $s(y, c_j, h_j) > 0$  for all  $y$ , everyone will buy private treatment if this treatment is not included in the public health program. The most interesting case is the case where there exists a critical value  $y_j$  (with  $0 < F(y_j) < 1$ ) such that  $s(y_j, c_j, h_j) = 0$ . In this case those persons who have incomes below  $y_j$  will choose to go untreated if public treatment is not offered, while persons with incomes above  $y_j$  will buy private treatment.

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<sup>8</sup> See e.g. Blomquist and Johansson (1997) and references given there for analyses of the interaction between public compulsory and private voluntary health insurance.

Social welfare  $W$  is given by the following expression:

$$(8) \quad W = \sum_i \delta_i \pi_i \int_0^1 u(y) f(y) dy + \sum_i (1 - \delta_i) \pi_i \int_0^{y_i} [u(y) - h_i] f(y) dy \\ + \sum_i (1 - \delta_i) \pi_i \int_{y_i}^1 u(y - c_i) f(y) dy$$

The first term in this expression gives the welfare level in the states of illness for which the public health system offers treatment. The second and third term give the welfare level in the states for which no public treatment is offered. The second term covers the persons who choose to be untreated when there is no public treatment.<sup>9</sup> The second term is the welfare of those who choose to be untreated, and the third term is the welfare of those who choose private treatment. Using (7) we can rewrite (8) as

$$(9) \quad W = \sum_i \delta_i \pi_i \int_0^1 u(y) f(y) dy + \sum_i (1 - \delta_i) \pi_i \int_0^1 [u(y) - h_i] f(y) dy \\ + \sum_i (1 - \delta_i) \pi_i \int_{y_i}^1 s(y, c_j, h_j) f(y) dy$$

The Lagrangian corresponding to the maximization of this expression subject to the budget constraint given in (1) is

$$(10) \quad M = \sum_i \delta_i \pi_i \int_0^1 u(y) f(y) dy + \sum_i (1 - \delta_i) \pi_i \int_0^1 [u(y) - h_i] f(y) dy \\ + \sum_i (1 - \delta_i) \pi_i \int_{y_i}^1 s(y, c_j, h_j) f(y) dy + \mu [B - \sum_i \delta_i \pi_i c_i]$$

and

$$(11) \quad \frac{\partial M}{\partial \delta_j} = \pi_j h_j - \pi_j \int_{y_j}^1 s(y, c_j, h_j) f(y) dy - \mu \pi_j c_j$$

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<sup>9</sup> As explained above,  $y_j$  is defined by  $s(y_j, c_j, h_j) = 0$  provided  $0 < F(y_j) < 1$ . If  $s(y, c_j, h_j) < 0$  for all  $y$  satisfying  $0 < F(y) < 1$  we set  $y_j = 1$ , and if  $s(y, c_j, h_j) > 0$  for all  $y$  satisfying  $0 < F(y) < 1$  we set  $y_j = 0$ .

which is positive if and only if

$$(12) \quad \frac{h_j}{c_j} - \int_{y_j}^1 \frac{s(y, c_j, h_j)}{c_j} f(y) dy > \mu$$

This equation is easier to interpret if we insert (7) and rearrange:

$$(13) \quad \frac{1}{c_j} \left\{ h_j F(y_j) + \int_{y_j}^1 [u(y) - u(u - c_j)] f(y) dy \right\} > \mu$$

If no one had chosen private treatment (i.e.  $y_j = F(y_j) = 1$ ), the term in the curly brackets would simply be  $h_j$ , so that (13) would be identical to (4). When  $y_j < 1$ , the term in brackets is a weighted average of the health benefits for those who otherwise would be untreated (a proportion  $F(y_j)$  of the population) and the cost saving for those who would be privately treated if no public treatment was offered. Since  $s(y, c_j, h_j) > 0$  for those who would choose private treatment it follows from (7) that this average is lower than  $h_j$  when  $y_j < 1$ .

It has so far implicitly been assumed that all illnesses can be treated in the private sector if they are not offered by the public health system. In practice, there may for various reasons be some treatments that will not be offered by the private sector even if the public sector does not offer these treatments. For such treatments, we can simply set  $y_j = 1$  instead of being determined by  $s(y_j, c_j, h_j) = 0$ . It is straightforward to verify that all of the analysis above remains valid, so that for treatments that are not offered by the private sector the second term on the LHS of (12) is zero. The following Proposition follows:

*Proposition 1: If there are treatments that are not offered by the private sector, such treatments should be given higher priority as a candidate for inclusion in the public health program than treatments that have the same ratio  $h_j/c_j$  of health benefits to costs but are offered by the private sector.*

Returning to treatments with a private alternative, we now have a second term (the integral term in (12)) compared with the case of no private treatment (see (4)). This term varies among treatments. In Appendix A the following Lemma is proved:

*Lemma 1: The LHS of (12) is increasing in  $h_j$  and decreasing in  $c_j$ , and increases with a proportional increase in  $h_j$  and  $c_j$ .*

From this Lemma we immediately get the following proposition:

*Proposition 2: A treatment  $j$  is given higher priority as a candidate for inclusion in the public health program*

- *the higher is the health benefit  $h_j$  of the treatment for a given cost*
- *the lower is the cost  $c_j$  of the treatment for a given health benefit*
- *the higher is the cost  $c_j$  of the treatment for a given ratio  $h_j/c_j$  of benefits to costs*

The two first properties of Proposition 2 are the same as one has for the case of no private alternative. The third property implies an important difference between the present case and the case with no private alternative. One can no longer simply rank treatments by their benefit-cost ratios, the cost per treatment is also an important factor to take into consideration.

Let us finally see what the optimal public budget is for the case in which there is a private alternative to the public health sector. Proceeding as we did when we derived (5), we now find

$$(14) \quad \mu = \int_0^1 u'(y) f(y) dy + \sum_i (1 - \delta_i) \pi_i \int_{y_i}^1 s_y(y, c_i, h_i) f(y) dy$$

Since  $s_y$  is positive (cf. the discussion after (7)), it is clear that  $\mu > \lambda$ , i.e. the RHS of (12) is larger than the RHS of (4) when the budget is optimally chosen in both cases. Since the terms on the LHS of (12) are smaller than the corresponding terms on the RHS of (4), it follows that fewer treatments pass the criterion for inclusion in the

public health program in the present case than in the case when there is no private alternative. In other words:

*Proposition 3: The optimal budget of the public health system is smaller when treatment also is offered by the private sector than it is when there is no private alternative.*

It is shown in Appendix B that if  $c_j$  is sufficiently small, the LHS of (13) is smaller than the value of  $\mu$  defined by (14). Since the shadow price  $\mu$  of the budget constraint is larger the smaller is the budget, the following proposition follows:

*Proposition 4: If the health budget is equal to or smaller than its socially optimal size, treatments that may be supplied by the private sector should not be included in the public health program if the costs of these treatments are sufficiently low, no matter how high the ratio of health benefits to treatment costs are.*

#### **4. Heterogeneous preferences**

In the previous section, the reason why different persons made different choices with regard to getting private treatment or not was completely determined by differences in incomes. In reality people may also have different preferences even if they have identical incomes. To focus on such heterogeneity of preferences, it is assumed in this section that everyone has the same income. For an illness  $j$ , a person of type  $\theta_j$  has a utility loss  $\theta_j h_j$  if untreated. The distribution of  $\theta_j$  in the population is given by the distribution function  $G_j(\theta_j)$  with a corresponding density function  $g_j(\theta_j) = G_j'(\theta_j)$ . The distribution function  $G_j(\theta_j)$  tells us what share of the population that has a utility loss less than  $\theta_j h_j$  for an untreated illness of type  $j$ . It is assumed that the average value of  $\theta_j$  in the population is 1.

With the assumptions above, it is straightforward to verify that without any private alternative, we get exactly the same result as we found in Section 2, i.e. prioritization should be done according to a ranking of the ratios  $h_j/c_j$ . When there is a private alternative, we define the surplus from private treatment in the same way as before:

$$(15) \quad r(y, c_j, h_j, \theta_j) = u(y - c_j) - [u(y) - \theta_j h_j]$$

Private treatment will be chosen if and only if this surplus is positive, i.e. if and only if  $\theta_j$  exceeds a threshold value given by

$$(16) \quad \theta_j^* = \frac{u(y) - u(y - c_j)}{h_j}$$

Social welfare  $V$  is given by the following expression:

$$(17) \quad V = \sum_i \delta_i \pi_i u(y) + \sum_i (1 - \delta_i) \pi_i \int_{\theta_i < \theta_i^*} [u(y) - \theta_i h_i] g_i(\theta_i) d\theta_i \\ + \sum_i (1 - \delta_i) \pi_i [1 - G_i(\theta_i^*)] u(y - c_i)$$

The first term in this expression gives the welfare level in the states of illness for which the public health system offers treatment. The second and third term give the welfare level in the states for which no public treatment is offered. The second term is the welfare of those who choose to be untreated, and the third term is the welfare of those who choose private treatment. The Lagrangian corresponding to the maximization of this expression subject to the budget constraint given in (1) is

$$(18) \quad N = \sum_i \delta_i \pi_i u(y) + \sum_i (1 - \delta_i) \pi_i \int_{\theta_i < \theta_i^*} [u(y) - \theta_i h_i] g_i(\theta_i) d\theta_i \\ + \sum_i (1 - \delta_i) \pi_i [1 - G_i(\theta_i^*)] u(y - c_i) + \gamma (B - \sum_i \delta_i \pi_i c_i)$$

and

$$(19) \quad \frac{\partial N}{\partial \delta_j} = \pi_j u(y) - \pi_j G_j(\theta_j^*) u(y) + \pi_j h_j \int_{\theta_j < \theta_j^*} \theta_j g_j(\theta_j) d\theta_j \\ - \pi_j [1 - G_j(\theta_j^*)] u(y - c_j) - \pi_j c_j \gamma$$

which is positive if and only if

$$(20) \quad \frac{1}{c_j} \left\{ h_j \int_{\theta_j < \theta_j^*} \theta_j g_j(\theta_j) d\theta_j + [1 - G_j(\theta_j^*)] [u(y) - u(y - c_j)] \right\} > \gamma$$

The interpretation of this expression is the same as it was for (13). Integrating by parts yields

$$(21) \quad \frac{h_j}{c_j} \left[ 1 - \int_{\theta_j < \theta_j^*} G_j(\theta_j) d\theta_j \right] > \gamma$$

For a given distribution function, the term in square brackets in LHS of (21) is smaller the larger is  $\theta_j^*$ . From (16) we immediately see that a larger  $h_j$  or a smaller  $c_j$  give a smaller value of  $\theta_j^*$ , i.e. a larger term in square brackets. The whole expression on the LHS of (21) is thus higher the higher is  $h_j$  and the smaller is  $c_j$ . To see how a proportional increase in  $h_j$  and the  $c_j$  affect the LHS of (21) we insert  $h_j = \alpha c_j$  into (16) and differentiate with respect to  $c_j$ :

$$(22) \quad \begin{aligned} \frac{\partial}{\partial c_j} \left[ \frac{u(y) - u(y - c_j)}{\alpha c_j} \right] &= \frac{1}{(\alpha c_j)^2} \left[ \alpha c_j u'(y - c_j) - (u(y) - u(y - c_j)) \right] \\ &= \frac{1}{\alpha c_j} \left[ u'(y - c_j) - \frac{u(y) - u(y - c_j)}{c_j} \right] > 0 \end{aligned}$$

from the concavity of  $u$ . A proportional increase in  $h_j$  and the  $c_j$  thus increases  $\theta_j^*$ , i.e. reduces the LHS of (21).

From the discussion above, it is clear that the results given in Propositions 1 and 2 remain valid also when heterogeneous preferences are introduced.

The LHS of (21) depends on the distribution function  $G_j$ . More precisely, we have:

*Proposition 5: Consider two illnesses  $j$  and  $k$  that are identical with respect to average severity ( $h_j = h_k$ ) and treatment costs ( $c_j = c_k$ ). If preferences are more*

heterogeneous for  $k$  than for  $j$  in the sense that the difference between  $G_k$  and  $G_j$  is a mean preserving spread in the terminology of Rothschild and Stiglitz (1970), the LHS of (21) is lower for  $k$  than for  $j$ . Treatment  $k$  should thus be given lower priority as a candidate for inclusion in the public health program than treatment  $j$ .

Proof: The integral in the LHS of (21) is higher for  $k$  than for  $j$ , see Theorem 1(a) of Rothschild and Stiglitz (1970).

To interpret Proposition 5, it is useful to consider the simple case where there are only two types of persons:  $L$ -persons have low valuation of the health benefits while  $H$ -persons have high valuation. In obvious notation, for the two treatments  $k$  and  $j$  we have  $\theta_k^L < \theta_j^L$  and  $\theta_k^H > \theta_j^H$ . The first of these inequalities makes the health benefits lower for  $k$  than for  $j$ . If there were no private alternative, this effect would be exactly offset by the second inequality. However, with a private alternative the second inequality is irrelevant: The  $H$ -persons are in any case going to be treated, so for these the benefits of public treatment are their cost savings, which are identical for  $k$  and  $j$ .

Using (15) and the fact that the expected values of all  $\theta_i$  are equal to 1, we can rewrite (18) as

$$(23) \quad N = u(y) - \sum_i (1 - \delta_i) \pi_i h_i + \sum_i (1 - \delta_i) \pi_i \int_{\theta_i > \theta_i^*} r(y, c_i, h_i, \theta_i) g_i(\theta_i) d\theta_i + \gamma(B - \sum_i \delta_i \pi_i c_i)$$

Proceeding as we did when we derived (5) and (14), we now find

$$(24) \quad \gamma = u'(y) + \sum_i (1 - \delta_i) \pi_i \int_{\theta_i > \theta_i^*} r_y(y, c_i, h_i, \theta_i) g_i(\theta_i) d\theta_i$$

where  $r_y$  is positive due to the concavity of  $u$ . The reasoning after (12) thus remains valid, so Proposition 3 is true also in the present case.



It is shown in Appendix B that if  $c_j$  is sufficiently small, the LHS of (20) is smaller than the value of  $\gamma$  defined by (24). The reasoning leading up to Proposition 4, and thus Proposition 4 itself, therefore is valid also in the present case.

## 5. Concluding remarks

The preceding analysis has shown that the existence of a private alternative has important consequences for the ranking of treatments in a cost-effectiveness analysis. Perhaps the two most important conclusions are that for a given ratio of health benefits to treatment costs, lower priority should be given to a treatment the lower is the cost per treatment and the more heterogeneous are the population's preferences regarding the desirability of the treatment. In the examples above, surgical sterilization is a good example of a treatment with low cost (at least for men). This is a once in ones lifetime treatment and the cost is only a small fraction of 1 per cent of the income of most people. The present analysis therefore gives a good justification for not including this treatment among the treatments covered by the public system. Prescription medicines for chronic diseases are on the other hand often quite costly (over a life time). However, some prescription medicines are good examples of a large heterogeneity in the population regarding the benefits of the medicine. In many cases a new and more costly medicine will have the same primary medical effect as a medicine already in use. However, the new medicine may have weaker unpleasant side effects. Such side effects very often vary strongly among different patients, being non-existent or weak for some, and very severe for others. The preceding analysis suggests that covering the costs of such medicines through the public budget should not necessarily be given high priority even if average benefits are large relative to the costs.

Although the focus of the present paper has been on prioritization within a given public health budget, the analysis is relevant also for a system of private health insurance. If an insurance company wishes to maximize the welfare of its members given the amount of money spent on their health, it faces the same issues as the present paper has discussed. The main difference is that the term "private alternative" must be replaced by "out-of-pocket alternative".

## Appendix A: Proof of Lemma 1

To simplify notation, subscripts are omitted (except for the critical value  $y_j$ ). From (7) it follows that

$$(A1) \quad s_y = u'(y-c) - u'(y) > 0$$

$$(A2) \quad s_c = -u'(y-c) < 0$$

$$(A3) \quad s_h = 1$$

We also want to know how  $s/c$  is affected by a proportional increase in  $c$  and  $h$ . We therefore set  $h = \alpha c$  and define

$$(A4) \quad z(c) = \frac{s(y, c, \alpha c)}{c}$$

giving

$$(A5) \quad z'(c) = \frac{c(s_c + \alpha s_h) - s}{c^2}$$

Inserting from (A2) and (A3) and utilizing the concavity of  $u$  gives

$$(A6) \quad z'(c) = \frac{-cu'(y-c) + h - s}{c^2} < \frac{-[u(y) - u(y-c)] + h - s}{c^2}$$

The numerator of the last expression is zero from (7), we thus have  $z'(c) < 0$ . A proportional increase in  $c$  and  $h$  therefore reduces  $s/c$ . A proportional increase in  $c$  and  $h$  must therefore increase the LHS of (12). To see how a partial increase in  $h$  affects the LHS of (12), we differentiate:

$$(A7) \quad \frac{\partial}{\partial h} \left\{ \frac{h}{c} - \int_{y_j}^1 \frac{s(y, c, h)}{c} f(y) dy \right\} = \frac{1}{c} + s(y_j, c, h) \frac{\partial y_j}{\partial h} - \int_{y_j}^1 \frac{1}{c} f(y) dy$$

$$= \frac{1}{c} \left( 1 - \int_{y_j}^1 f(y) dy \right) = \frac{1}{c} F(y_j) > 0$$

This proves that the LHS of (12) is increasing in  $h$ . Finally, the LHS of (12) must be declining in  $c$ , since an increase in  $c$  is equivalent to a combination of a proportional increase in  $c$  and  $h$ , which decreases the LHS if (12), and a reduction in  $h$ , which also decreases the LHS of (12).

#### Appendix B: Proof of Proposition 4

Denoting the LHS of (13) by  $K_j$  and using (7) we have

$$(B1) \quad K_j = \int_0^1 \frac{u(y) - u(y - c_j)}{c_j} f(y) dy + \int_0^{y_j} \frac{s(y, c_j, h_j)}{c_j} f(y) dy$$

Using a first order Taylor expansion of  $u(y)$  this can be written as

$$(B2) \quad K_j = \int_0^1 u'(y) f(y) dy + \int_0^1 \frac{c_j}{2} [-u''(x_j(y))] f(y) dy + \int_0^{y_j} \frac{s(y, c_j, h_j)}{c_j} f(y) dy$$

where  $x_j(y) \in (y - c_j, y)$ . Notice that in the last integral,  $s(y, c_j, h_j) < 0$  since  $y < y_j$ .

Combining (B2) with (14) gives (when the LHS of (14) is denoted by  $\mu^*$ )

$$(B3) \quad K_j - \mu^* = \int_0^1 \frac{c_j}{2} [-u''(x_j(y))] f(y) dy + \int_0^{y_j} \frac{s(y, c_j, h_j)}{c_j} f(y) dy$$

$$- \sum_i (1 - \delta_i) \pi_i \int_{y_i}^1 s_y(y, c_i, h_i) f(y) dy$$

The RHS of (B3) consists of three terms. The first of these is positive, but approaches zero as  $c_j$  approaches zero. The second and third terms are both negative and bounded away from zero as  $c_j$  approaches zero. Both sides of (B3) are therefore negative for a

sufficiently small value of  $c_j$ . If  $\mu \geq \mu^*$ , the inequality (13) does therefore not hold for sufficiently small values of  $c_j$ .

Consider next the case treated in Section 4. Denoting the LHS of (20) by  $H_j$  and using (15) and the fact that the expected values of all  $\theta_i$  are equal to 1, we have

$$(B4) \quad H_j = \frac{u(y) - u(y - c_j)}{c_j} + \int_{\theta_j < \theta_j^*}^{\theta_j} \frac{r(y, c_j, h_j, \theta_j)}{c_j} g_j(\theta_j) d\theta_j$$

This expression corresponds completely to (B1), and it is straightforward to verify that an analysis similar to the one following (B1) leads to the conclusion that if  $\gamma \geq \gamma^*$  (defined by (24)), the inequality (20) does not hold for sufficiently small values of  $c_j$ .

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