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An Economical Approach to Estimate a Benchmark Capital Stock. An Optimal Consistency Method

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Abstract. There are alternative methods of estimating capital stock for a benchmark year. However, these methods are costly and time-consuming, requiring the gathering of much basic information as well as the use of some convenient assumptions and guesses. In addition, a way is needed of checking whether the estimated benchmark is at the correct level. This paper proposes an optimal consistency method (OCM), which enables a capital stock to be estimated for a benchmark year, and which can also be used in checking the consistency of alternative estimates. This method, in contrast to most current approaches, pays due regards both to potential output and to the productivity of capital. It works well, and it requires only small amounts of data, which are readily available. This makes it virtually costless in both time and funding.

Key words: Benchmark Capital, Perpetual Inventory Method (PIM), Potential Output, Capital Productivity, Optimal Consistency Method (OCM).

JEL Classification: O4, B4

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1. Introduction

In working with production functions in the context of international and domestic studies, it is necessary to have estimates of fixed capital stock. Production functions, which are incorporated in models that are used in studying productivity and technological change, require estimates of the fixed capital stock in benchmark years (OECD, 2001; Hofman, 2000a; Maddison, 1993; Denison, 1993). Such estimates can be used for making snapshot comparisons or as the bases for building usable capital stock series.

There are two ways of estimating capital stock for a benchmark year. The first way is to build up a capital stock by aggregating scattered data for a given year, obtained via surveys, balance sheets, insurance reports, censuses and the like. The second way is to estimate the capital stock by accumulating recorded investments up to a given benchmark year, subject to an appropriate discount to reflect the depreciation of the capital. The former demands a major effort, and it can be very costly. Therefore, it is liable to be pursued only in an irregular manner. The quality of the data and the compiling methods will vary widely across countries, which makes comparisons uncertain.

The second way of estimating capital stock demands less effort and is currently preferred. Most OECD and other countries use it, which facilitates international comparisons, as the procedures are standard and therefore transparent. This is normally known as the "perpetual inventory method" or PIM (OECD, 2001; Hofman, 2000a, 2000b; Blades, 1993; Goldsmith, 1951). But when historical investments are not fully recorded and when their sources and definitions are inconsistent over time, the results are bound to depend on rough estimates, on rules of thumb, on the experiences of other countries, and so forth. All this makes the resulting benchmark-capital stock estimates accurate only within an unknown confidence interval, which cannot be determined.

Despite such shortcomings, neither method allows for an independent check, which could establish whether the estimated benchmark capital level is too high or too low. I propose here a method that optimises the economic relation between the patterns of actual investment and potential output, which allows both estimating a capital stock level for a benchmark year and checking the consistency of alternative estimates. I term this method the "optimal consistency method" or OCM. This third approach is based on a PIM-derived equation, optimised via linear programming, and it requires only a small amount of readily available data. In addition, the OCM estimate can be made to converge towards an actual capital series by combining it with an actual PIM. This would reduce the deviation of OCM results from any PIM-calculated reference series, requiring no additional information, as shown below. In contrast to other methods, the OCM takes account of measures of the productivity of capital and output at potential levels. This also contributes to dampen productivity fluctuations due to actual capital use or idleness, which can make it a more accurate estimate of the capital stock. The new method works well and it is virtually costless to implement.

2. **Reference Cases and Application**

The present study uses the capital stock data from Hofman (2000a) and OECD (1997) as references against which to check the proposed method. Hofman acknowledges that, for certain periods, especially before 1925, his data “may well be substantially revised when further research is done” (p.183). This indicates that his data is subject to an unknown level of errors. For the purpose of this paper, it can only be used as a notional reference. However, I intend to show how close Hoffman's results and those from the OECD dataset are from the OCM results. The latter may well profit from incorporating a measure of optimal productivity associated to potential output, as indicated later.

Hofman (2000a) applies a PIM to six Latin American countries to set up a 1950-benchmark for the Gross and Net Fixed Capital Stocks, disaggregated into Machinery & Equipment, Structures and Residential Capital. In turn, the OECD publishes flows and stocks of fixed capital, with similar disaggregations, also calculated via a PIM for a number of developed countries (OECD, 1997). I will apply my proposed method (OCM) to the six Latin American countries and to nine OECD countries, for which there is a usable dataset, considering three systematic years for each country. This will generate 45 estimates for the benchmark stock. These results will be compared with those in the two notional references above, under two alternatives: an OCM benchmark for a base year, corresponding to the beginning of an 11-year series, and a combined OCM-PIM benchmark for the end year of such series, as explained later.

In this paper, I will only estimate a benchmark for the aggregate Net Fixed Capital

Stock (NFCS), but main disaggregations of it or the Gross Fixed Capital Stock (GFCS) can also be estimated in the same way.

3. **Perpetual Inventory Method (PIM)**

To clarify, the PIM for the NFCS of a benchmark year starts by defining this year's capital as equal to last year's capital minus this year's depreciation (*Dep*) plus this year's gross fixed capital formation (GFCF):

$$K_0 = K_{-1} - Dep_0 + I_0 \qquad (1)$$

Where K = NFCS (Net Fixed Capital Stock) and I = GFCF (Gross Fixed Capital Formation) and the sub-indexes "-1" and "0" stand for the "previous year" and "this end-of-year", respectively. We can then assume alternative patterns of depreciation. Hofman (2000a), for example, uses a straight-line depreciation, while the OECD (1997, 2001) uses a geometrical depreciation pattern(1). The latter will normally be a good approximation of the former, especially for short periods, as in our application, so equation (1) can be restated as:

$$K_0 = K_{-1}(1 - \lambda) + I_0 \qquad (2)$$

where λ is the rate of depreciation. The capital is then accumulated retrospectively over its service life up to the benchmark year. For example, following Hofman (2000a), the service life of Residential Capital is assumed to be 50 years on average. Thus, if we require a benchmark for 1950, then the accumulation should start in 1901 and end in 1950, the capital of 1901 being the gross fixed capital formation (GFCF) on residential capital of that year alone. In turn, Structure Capital is assumed to live an average of 40 years and Business Fixed Capital 15 years. Hence, these two series should respectively begin accumulating in 1911 and 1936 and end in 1950. Therefore, the three types of capital can be accumulated according to the following series:

$$K_{1950} = \sum_{i=1901/11/36}^{1950} I_i \, (1-\lambda)^{(1950-i)} \qquad (3)$$

Where the sum subindex i ranges from either 1901 or 1911 or 1936 to 1950, according to capital type, so the equation will produce three different series. This equation represents a weighted average of historical investments, the weights of which are the reciprocal of the depreciation, powered by the number of remaining years to 1950. To arrive at the net fixed capital stock (NFCS) for the benchmark year, the three series should be added up. It can be seen that the minimum requirement to achieve this is the availability of the three types of GFCF from their start year to the benchmark year. In most countries, especially developing ones, this is either not always available or not normally consistent with modern definitions of GFCF, especially before the Second World War. This is serious drawback, as gaps have to be filled with a variety of estimating methods (see Hofman 2000a, especially Appendix D), which may not produce accurate figures and which can create significant deviations from the "true values", whatever these might be. This would also be true for the gross fixed capital stock (GFCS)[(2)]. Here is another reason why my proposed method may be a valuable alternative to the existing methods.

Capital stock is usually calculated without any explicit reference to its productivity.

That is, the aggregation of gross fixed capital formation over time only considers the market value of investment in constant prices. Since prices are not necessarily associated with productivity, the productivity content of this investment might be misleading. The price of investment goods often depends more on its demand than on its technical efficiency. That is, prices might be high on introduction, but lower when demand takes hold, e.g. units of computing power.

Implicitly, the measure of depreciation contains a productivity element, but this is mostly an average service life based on a conventional rule about capital service and retirements, rather than on an actual technical or economic measure, e.g. residential capital is supposed to live for 50 years. Therefore, the PIM takes account of neither the quality of capital over time (i.e. embodied productivity), nor the institutional conditions within which such capital is deployed (i.e. disembodied productivity) in given periods. The latter may vary significantly over time, as a result of significant changes of policy regime and ensuing capital usage. By optimising capital formation in association with output, the new method explicitly accounts for the optimal productivity of capital over given periods. In addition, the optimisation exercise implicitly takes care of capital idleness. Hence, the aggregation of capital of different qualities over different institutional periods, with different levels of capital employment, is catered for via an estimated optimal average productivity associated to a measure of potential output, as shown below.

4. Proposed Method

Let us start with the definition of the first difference for income or output (i.e. ΔY =

Y_1-Y_0), which can be re-arranged as

$$Y_1 = Y_0 + \Delta Y \quad (4)$$

Where "Δ" means variation and the sub-indexes "*1*" and "*0*" represent the terminal and the initial years, respectively. Y_1 is Gross Domestic Product (GDP) and ΔY represents a variation of GDP between two given years. Let us now assume that the production function of the economy is well proxied by capital alone or alternatively that there is a relatively stable relationship between average output and average capital and also between medium-term variations in output and medium-term variations in capital. The long-term and medium-term stability of capital-output ratios or their inverse, the productivity of capital, is well supported by empirical studies that use actual data, when allowance is made for capital idleness (Thirwall, 2003). Empirical evidence also shows that, even without correcting for capital idleness, both the yearly variations of capital-output ratios are normally smooth and moderate over the short and the medium term and the correlation coefficients between GDP and NFCS are very high. But notice that, even if over our medium-term periods the uncorrected *actua*l average productivity were either less stable, or their trend higher or lower than desired, (e.g. Brazil 1952-1962 or France 72-82) that should not affect the *optimal* average capital productivity, which is our focus.

The proposed method only requires that the variations of the capital-output ratios be reasonably smooth and stable over our 11-year periods, which is normally the case[(3)]. Therefore, given that we are aiming at establishing approximate NFCS levels for a benchmark year of, say, around 20 percent of the reference notional value, then the uncorrected *actual* capital-output ratios can be considered as fairly constant for the sample period. But whatever their actual variability, this proved to be no obstacle for obtaining good results from our method, as is shown later.

The long-term relationship between capital and output is normally represented by the average capital-output ratio, while the medium-term one is represented by the incremental capital-output ratio. That is,

$$k_0 = K/Y \qquad (5)$$

$$k_1 = \Delta K/\Delta Y \qquad (6)$$

where k_0 is the average capital-output ratio and k_1 is the incremental capital-output ratio. These two ratios represent the inverse of the average productivity of capital of the economy in the long- and medium-terms, respectively. But notice that these ratios do not represent the contribution of capital to output alone, as output is also made up of other contributions, such as those coming from labour and technical change. Hence, the inverse of the capital-output ratio more properly represents the amount of output that can be "sustained" by a unit of capital, allowing for the existence of other contributions. But, following normal practice, we will refer to the inverse of the capital-output ratio as the average productivity of capital.

Assuming that capital depreciates at a λ rate and that investment becomes productive with one year lag, then substituting (5) and (6) into (4):

$$Y_1 = (1/k_0)\ (1 - \lambda)\ K_{-1} + (1/k_1)\ \Delta K_0 \qquad (7)$$

Letting $(1/k_0) = \alpha_0$, $(1/k_1) = \alpha_1$, $\Delta K_0 = I_0$ and $(1 - \lambda) = \beta$, then

$$Y_1 = \alpha_0 \beta K_{-1} + \alpha_1 I_0 \tag{8}$$

Notice that I_0 and β are normally available. The latter is subject to a variety of estimating models, but it can always be produced (OECD, 2001). We attempt to estimate α_1 and the product $\alpha_0 K_{-1}$ and therefore Y_1 at optimum levels. The latter will constitute a measure of potential output, as shown later.

Therefore, if the product $\alpha_0 K_{-1}$ can be estimated, then the benchmark capital K_{-1} could also be estimated under different assumptions for α_0. A first assumption could be that $\alpha_0 = \alpha_1$. That is, the long-term and the medium-term average productivities of capital are the same. This is compatible with a Harrod-Domar production function (Jones, 1975) and with the AK endogenous growth model (Aghion & Howitt, 1998; Solow, 1994). A second, more general, assumption would be that $\alpha_0 \leq \alpha_1$, i.e. that the long-term average productivity is smaller than or equal to the medium-term productivity of capital. This would allow for the normal expectation that capital formation of later vintages is likely to have a higher productive quality than that of earlier vintages (see Denison, 1993; Kendrick, 1993; Hulten, 1992).

But we are dealing with the "sustainability" of capital, as indicated above, rather than its confined productivity. The data coming from both Hofman (2000a) and OECD (1997) show that the capital-output ratio often increases over time, so a relation like $\alpha_0 \geq \alpha_1$ appears more likely. That is, when uncorrected for idleness, the actual average productivity or sustainability of capital, Y/K, appears to decrease softly over time. Therefore, if the trend in output-capital ratios, corrected for idleness, can be estimated from our 11-year series, or from any other source, then a correction coefficient could be applied, as $\alpha_0 = c\alpha_1$, where c is a correction coefficient that can be equal, larger or smaller than one. Assuming that this is a useful aim, this will produce output-capital ratios closer to the ratios that come from other sources, when the latter are *corrected* for idleness. Our combined OCM-PIM final benchmark year does this, but for the *idleness-uncorrected* average productivity of capital that comes from our reference series, as shown below.

5. **Estimation Procedure**

With a view to estimating α_1 and the product $\alpha_0 K_{-1}$ at optimal levels, i.e. avoiding fluctuation-affected estimates, we use a linear programming model based on the generalisation of equation (8) as $Y^*_t = \alpha_0 K_{t-2}\beta + \alpha_1 I_{t-1}$. Let $K_{t-2} = K_b$ be the capital base-year, which would correspond to the year before the 11-year time-series for I (e.g. 1950 when the GFCF series start in 1951). Then the iterative solution of the above equation for any year "t" is

$$Y_t^* = \alpha_0 K_b \beta^t + \alpha_1 \sum_{i=1}^{t} I_{i-1} \beta^{(t-i)} \qquad (9)$$

where the year "t" ranges from 1 to n, K_b is the base-year capital stock, "*" denotes "optimal" and the summation index i ranges from 1 to t. The initial or base-year product $\alpha_0 K_b$ and the productivity coefficient α_1 are the two parameters to estimate. The linear programme takes the following form (see also Albala-Bertrand, 1999):

Minimise:

$$Z = \sum_{t=1}^{n}(Y_t^* - Y_t) = (\alpha_0 K_b \sum_{t=1}^{n}\beta^t + \alpha\sum_{t=1}^{n}\sum_{i=1}^{t} I_{i-1}\beta^{(t-i)}) - (\sum_{t=1}^{n} Y_t) \qquad (10)$$

Subject to:

$Y^*_t \geq Y_t$

$\alpha_0 K_b$ and $\alpha_1 \geq 0$

where the model calculates the series Y^*_t via equation (9), "*n*" is the last year of our series (e.g. 1962 when the base year is 1950) and "t" is any year in the series.

Once we have obtained the base-year result for an initial capital, which we call the OCM benchmark capital, we can use this as the starting year for a PIM, applied to the same11-year series used in our optimisation exercise. The capital value at the end of such series would constitute our final benchmark capital. We call this value the OCM-PIM benchmark capital. Notice this would incorporate capital formation uncorrected for idleness. As indicated earlier, given that this optimisation method includes measures of optimal capital productivity and potential output, the initial or base capital may already produce an acceptable benchmark capital. But to correct for the possibility of significant departures from our reference values, if this is what is aimed at, a PIM applied to our initial OCM capital will normally reduce any deviation to only a small percentage, as shown below. This method can also be applied to the gross fixed capital stock (GFCS), if required(4).

6. **Application and Results**

We can now apply the above methodology, using the same data for investment and depreciation rates as Hofman (2000a, 2000b) and the OEDC (1997). This will allow us to assess whether the benchmark estimates coming from their perpetual inventory method (PIM) are consistent with those coming from our optimal consistency method (OCM). For the nine OECD countries, we use as *initial* or *base* benchmark years 1970, 1975 and 1980, as not all countries here have calculations for net fixed capital stocks (NFCS) before 1970. In turn, for the six Latin American countries, we use 1950, 1965 and 1980, as these have net fixed capital stock series from 1950 to 1994 from Hofman (2000a). Therefore, our *final* benchmark years will be 1982, 1987 and 1992 for the OECD countries, and 1962, 1977 and 1992 for the Latin American countries. That is, to estimate the OCM parameters, all we require is 11-year series for both GDP and GFCF, as well as an average depreciation rate. For example, following equation (9), when the chosen final benchmark year is 1962, the initial benchmark year, $K_{b,}$ will be 1950. And the initial year for the required GDP and GFCF series should be 1952 and 1951, respectively.

To prevent a single rogue year from having undue influence on the optimal point, we apply a three-year moving average to both series over the sample period. This 11-year period is considered long enough to cover a cycle. But, in so far as a cycle is contained, a shorter series can also be used, if need be. Therefore, we do not expect that either a particular odd year or an odd sample could over-influence the estimations, which appears to be the case with our results. Finally, to make the notional reference capital and our OCM capital estimates comparable with each other, we use the same rates of depreciation as those used by the above sources, i.e. we calculate average rates of depreciation for our sample periods from OECD and Hofman's data. That is, we only use one depreciation rate for each 11-year sample, which also economise information.

Table 1 below presents the estimates from our Optimal Consistency Method (OCM) together with those from Hofman's and OECD's Perpetual Inventory Methods (PIM) for all the 15 countries and 45 benchmark years, showing our results for benchmarks in the *initial* and *final* years of our series. The first column indicates the country, the monetary unit at constant prices of a given year, and the three selected years for the *initial* benchmark capital valuation, i.e. for OECD: 1970, 75 and 80; and for Latin America: 1950, 65 and 80. The benchmark capital valuation for *final* year will then correspond to 12 years after the initial year, which is not indicated in the table, i.e. for OECD: 1982, 87 and 92; and for Latin America: 1962, 77 and 92). The remaining columns are as follow: columns 1 and 2 present the benchmarks for the OCM capital and the reference capital for the initial year, respectively. As said, the independent reference capital for OECD countries was taken from OECD (1997), while that for Latin American ones was taken from Hofman (2000a).

Column 3 is the percentage departure, in term of surplus or deficit, between the two capital valuations in the previous two columns (a positive sign indicates that the optimal capital is larger than the reference capital, and vice versa). Columns 4 and 5 present the benchmarks for the OCM-PIM capital and the reference capital for the final year, respectively. Column 6 is the percentage departure, in terms of surplus and deficit, between the *benchmark capital* and the reference capital.

TABLE 1: SUMMARY RESULTS

	[1]	[2]	[3]	[4]	[5]	[6]
	OptKi	**RefKi**	**Surp/Def**	**OptKf**	**RefKf**	**Surp/Def**
	OCM	REF	**Benchmark K**	OCM-PIM	REF	**Benchmark K**
	Initial	Initial	**(%)**	Final	Final	**(%)**
Australia 70	595858	526206	**13**	874315	838800	**4**
75	637702	659190	**-3**	966594	975602	**-1**
80	722613	786243	**-8**	1071243	1103203	**-3**
A$M (1990)						
Belgium 70	13490	9773	**38**	17570	15301	**15**
75	14678	12234	**20**	18066	16524	**9**
80	13630	14678	**-7**	18290	18844	**-3**
BFrM (1990)						
Canada 70	408283	406131	**1**	651932	646962	**1**
75	545613	496361	**10**	757993	730533	**4**
80	541100	599960	**-10**	839678	852637	**-2**
C$M (1986)						
Finland 70	1115417	849496	**31**	1544641	1373453	**12**
75	1088086	1099796	**-1**	1589037	1575543	**1**
80	1201827	1291772	**-7**	1729707	1758955	**-2**
MrkaM(1990)						
France 70	5541538	3611206	**53**	7823602	6804084	**15**
75	6992432	5095023	**37**	8769563	7718005	**14**
80	6965207	6366066	**9**	8822449	8930058	**-1**
FrM (1980)						
Norway 70	1082789	946654	**14**	1721147	1631288	**6**
75	1227610	1221694	**0**	1917870	1916778	**0**
80	1452037	1525758	**-5**	1987074	2051033	**-3**
KrM (1985)						
UK 70	854	660	**29**	1031	914	**13**
75	733	788	**-7**	975	1009	**-3**
80	685	888	**-23**	1037	1153	**-10**
£B (1985)						
Germany* 70	4966254	4186530	**19**	6685587	6188380	**8**
75	6248013	5123567	**22**	7507601	6804780	**10**
80	5242188	5906760	**-11**	6932651	7633400	**-9**
MrkM(1991)						
USA 70	9781	7939	**23**	13891	12390	**12**
75	11884	9856	**21**	15520	14093	**10**
80	10669	11801	**-10**	14618	15349	**-5**
US$B (1992)						
Argentina 50	24289	26189	**-7**	40218	41097	**-2**
65	62523	46352	**35**	94777	86328	**10**
80	94896	100565	**-6**	109284	112407	**-3**
Aust.M (1980)						
Brazil 50	2328	1623	**43**	5117	4739	**8**
65	5449	5634	**-3**	16808	17011	**-1**
80	25820	21884	**18**	34971	32923	**6**
CrzrsM(1980)						
Chile 50	967957	863205	**12**	1473534	1408516	**5**
65	1878161	1606185	**17**	2281624	2130859	**7**
80	1670809	2238184	**-25**	2848116	3116350	**-9**
$M (1980)						
Colombia 50	650491	662760	**-2**	1080804	1087339	**-1**
65	1015115	1180037	**-14**	2007013	2091001	**-4**
80	2666998	2434170	**10**	4029898	3920450	**3**
$M (1980)						
Mexico 50	1245963	839689	**48**	2351735	2118792	**11**
65	3111993	2626154	**19**	6834471	6590191	**4**
80	11515366	8177824	**41**	14359402	12556857	**14**
$M (1980)						
Venezuela 50	115442	94606	**22**	251792	244292	**3**
65	266617	263888	**1**	533551	529965	**1**
80	543312	679287	**-20**	673640	733542	**-8**
BlvrsM (1980)						

* West Germany Only

OCM: Optimal Consistency Method benchmark values

REF: PIM reference values

OCM-PIM: PIM applied to OCM benchmark values

To arrive at the initial or base capital, we have assumed that $\alpha_0 = \alpha_1$, with a correction coefficient c=1. That is, the average productivity of capital over our 11-year series is assumed to be the same as that of the accumulated capital until the year before such series. This assumption can be modified in the light of estimations of the average productivity trend, over our sub-periods, which could be derived from our series of GDP and GFCF.

The summary table below presents the percentage departure, in absolute numbers, between our OCM capital estimates and the independent reference values, for both the initial and final benchmark capitals, within certain useful intervals.

TABLE 2: PERCENTAGE DEPARTURE

OCM BENCHMARK CAPITAL				OCM-PIM BENCHMARK CAPITAL			
Interval (%)	**No**	**%**	**Acc %**	**Interval (%)**	**No**	**%**	**Acc %**
0-5.	8	18	18	**0-5.**	24	53	53
5-10.	11	24	42	**5-10.**	10	22	76
1-15.	5	11	53	**1-15.**	11	24	100
15-20.	5	11	64	**15-20.**	0	0	100
20-30.	8	18	82	**20-30.**	0	0	100
30-40.	4	9	91	**30-40.**	0	0	100
40-54.	4	9	100	**40-54.**	0	0	100
Total	45	100		**Total**	45	100	
Surplus	27	60		**Surplus**	27	60	
Deficit	18	40		**Deficit**	18	40	

OCM Results. It can be seen in Table 2 above that 53 percent of our OCM estimations of the initial benchmark capital are under 15 percent departure from our notional reference values. The maximum departure is an excess of 54 percent (France 1970). It also shows that when using $\alpha_0 = \alpha_1$ there is an overestimation in 60 percent of cases. This is due to the fact that, in most cases, the idleness-uncorrected trend in the actual average productivity of capital appears to be falling. Therefore, if α_0 had been larger than α_1, then the base benchmark capital would have been a smaller and closer to the notional reference series. But a correction for α_0 can only be justified if the trend in average productivity is estimated with allowance for capital idleness. Otherwise, the correction would only produce an OCM value closer to the reference values, which are not devoid of shortcomings.

Our estimate of a benchmark capital may well be better than the alternatives, as in contrast to them, it pays due regards to capital productivity associated to potential output, and implicitly to capital idleness. But if the aim is to produce benchmark values closer to our references, then estimates about productivity trend associated with our series of GDP and GFCF can be entertained. For example, the OECD reference series for France shows that the idleness-uncorrected productivity of capital was decreasing, implying that our optimal productivity for the accumulated capital should have been larger than the calculated α_1. So if we had some knowledge about this trend, then $\alpha_0 > \alpha_1$, making the uncorrected departure significantly smaller than the one above(5). In the absence of such an information, the recommendation would be to pick 11-year series for an economic period that can be considered "normal". But there is another solution, as shown below.

OCM-PIM. If we aim at getting closer to alternative PIM-calculated reference values, and either we have little knowledge about productivity trends or do not need a benchmark capital around the starting year of our 11-year series, then we can use a combination of OCM and PIM. That is, we use the initial or *base capital* as the starting capital for a PIM carried over our 11-year series, whose outcome we call the OCM-PIM result. The benchmark capital can then be found towards the end of the

PIM-generated series. We have chosen the last year of the series, i.e.12 years after the base year, but a shorter period can also do. This takes advantage of the well-known fact that whatever initial arbitrary value for capital, there will always be a convergence towards PIM-calculated reference values, when applying a PIM to any initial capital and using the same series of GFCF and depreciation rates (6).

OCM-PIM Results. As can be seen, via the OCM-PIM, the results are significantly closer to our notional references. The divergence between the benchmark capital and the reference capital is now fully contained in the interval 0-15 percent, with over half of the cases falling in the interval 0-5 percent. That is, on average, the OCM-PIM divergence is reduced to around a third of the OCM divergence(7).

This is a good result, but given that our references do not necessarily constitute the actual or true values for the benchmark capital, then there is no reason to assume that our results are less accurate than those from our references. One should recall that the reference values contain a series of guesswork and rough estimates, so that they cannot be regarded as definitive. In addition, by incorporating a measure of both potential output and the associated productivity of capital, our OCM allows us to determine whether the estimates coming from other sources and methods, such as those from our reference values, are acceptable in terms of magnitude level. With some variance, this appears to be the case with our two sources.

7. **Conclusion**

Most methods currently used to estimate capital stock for a benchmark year depend heavily on both the availability of data and a variety of estimating methods to fill gaps. The former is often incomplete and inconsistent over time, while the latter are usually rough and depend more on convenient than on correct assumptions. The method proposed in this paper to estimate a benchmark for the Net Fixed Capital Stock (NFCS) avoids such shortcomings. All that it requires are good GDP and GFCF series for only 11 years or less.

Checking against two methodologically independent results for nine OECD and six Latin American countries, including 45 cases, we have shown how an alternative valuation for the benchmark capital stock could be produced. We have also shown that the results from our optimal consistency method (OCM) depart not more than 53 percent from the reference results, with about half of them departing not more than 15 percent from our alternative sources. We also showed that a combined OCM-PIM approach could produce estimates that are very close to our notional references for capital. Given that the alternative sources themselves are subject to various errors, our results may well be more accurate than theirs.

Therefore, this method, based on parametric coefficients of a stable equation, estimated via linear programming, appears to be strong enough to generate good estimates of a capital stock level for a benchmark year, which in turn allows checking the consistency of alternative estimates of the benchmark capital stock. In addition, it is quick and inexpensive to implement.

Notes

(1) There are various possible assumptions as can be seen in OECD (2001). For example, there could be a variable depreciation over time, diminishing at the beginning and accelerating after a trough or just a variable one according to other economic factors. In turn, a straight-line depreciation pattern assumes that efficiency declines linearly over the lifetime of capital, therefore, the same depreciation rate applies over the capital-service life. This means that the rate of depreciation for a given piece of capital stock is equal to the inverse of this capital life, i.e. if the capital stock life is 40 years, then the depreciation rate is 1/40 a year. When applied over the initial capital stock this should take into account the retrospectively accumulation to that benchmark. It can be seen that the results between a straight line-depreciation and a geometrical depreciation, as applied here, do not differ significantly. So the latter would normally be a good proxy for the former, in any case.

(2) The benchmark for a GFCS is simply the accumulation of gross fixed capital formation (GFCF) over its working life, without allowing for depreciation. That is, if the working capital life is, say, 40 years, and the benchmark year is, say, 1950, then the GFCS at the benchmark year will be the sum of GFCF from 1911 to 1950. After that, this capital will undergo the retirement of any GFCF in their 41-th year of age. Notice that this formulation, contrary to that of the Net Fixed Capital Stock, assumes that capital does not depreciate over its service life, but its switches off its service once its working life finishes, and therefore should be discounted (retired) from the capital stock. For example, a light bulb has about the same efficiency until it burst.

But most types of capital actually require a good deal of maintenance and repairs, over their service life, so as to delay efficiency losses, e.g. rolling stock or roads. The problem with using this capital for growth studies is that the expenditure in maintenance and repairs, which clearly is there to counteract economic depreciation, is not counted as (replacement/restoration) investment. Therefore, there is a massive amount of investment-like outlays that is simply ignored. This might make sense for taxation or accounting reasons, but it does not make much economic sense. In addition, most investment actually lose efficiency over their working lives, even allowing for maintenance and repairs, like dams, housing, and most machinery and equipment, which may make NFCS a more appropriate measure for economic studies.

(3) For all our sub-periods in OECD countries, the standard deviation as percent of the mean (i.e. the variation coefficient) ranges from 1.2 percent (Belgium 82-92) to 6.3 percent (France 72-82), with an average of 2.8 percent. In turn, the correlation coefficients between capital and GDP vary from 0.89 (Finland 82-92) to 1.00 (Australia 72-82) with an average of 0.98. For Latin American countries, the variation coefficient ranges from 1.7 percent (Colombia 67-77) to 8.3 percent (Venezuela 82-92) with an average of 4.3 percent. The correlation coefficients here vary from 0.84 (Venezuela 82-92) to 1.00 (Mexico 67-77), with an average of 0.95. These high coefficients respond partly to the fact that GFCF is a component of both the capital stock and GDP. But they also respond to the possibility that capital may be an all-dominant factor or a good proxy for other productive factors that contribute to GDP, which are then dragged by capital and move in similar proportions and directions, within a range. This would be especially true when capital idleness is allowed for (Thirwall, 2003), which is what the optimisation in this paper implicitly sorts out.

(4) Following note (2) above, the Gross Capital Stock (GFCS) level for a benchmark year can be defined as $GFCS_b = \Sigma GFCF_i$, where "b" is the benchmark year, and the sum subindex i ranges from the year of the initial investment (GFCF) up to the benchmark year, matching its service life. For example if b = 1950 and capital life d = 50 years, then the sum index would accumulate all investment from 1901 to 1950. Thereafter, this capital will undergo retirements, corresponding to the cessation of the working life of the GFCF in their 51th year. Therefore, the GFCS for any year after the benchmark year would be $GFCS_{b+t} = GFCS_{b+t-1} - KR_{b+t-d} + GFCF_{b+t}$. Where KR is the value of capital retired due to service life exhaustion. Accordingly, the subindex "t" corresponds to additional years after the benchmark year and the sub-index "d" to the service life of GFCF at the moment of its inception. For example, if we want to know the residential GFCS for 1955, assuming that d=50 years and b=1950, then the equation above will look as $GFCS_{1955}=GFCS_{1954}-RK_{1905}+GFCF_{1955}$.

Following a similar iterative procedure to that for the NFCS, the equivalent equation would take the form of $GFCS_{b+t}= GFCS_b-\Sigma RK_i+\Sigma GFCFi$. Where the range of the first sum over RK is from b-d to b-d+t and that of the second sum over GFCF is from b to b+t. This can be also approximated via a geometrical depreciation rate, using equations (2) to (9) as in the NFCS. The retirement (RK) would now represent an average percentage of GFCS. So our method (OCM) can also be applied to the gross fixed capital stock, if need be. Lastly, we used GAMS (General Algebraic Modeling System) as a solver for the linear programme, but any other alternative would do.

(5) From Hofman (2000a) and OECD (1997) it can be seen that in periods in which the range of capital-out ratios is large the overestimation of the base capital might also be large. This is especially the case for France 1970 and 1975, Belgium 1970, Finland 1970, Mexico 1950 and Brazil 1950 and Argentina 1965. It can also be seen that in most cases there was a strong upward trend in idleness-uncorrected capital-output ratios or, its inverse, a strong downward trend in average capital productivity. So if the *base capital* should be used as *benchmark capital*, then we could devise some reliable test to correct the initial optimal α_0 in relation to α_1. This should be mostly based upon the 11-year series for GDP and GFCF in relation to the rule of capital accumulation. The asymptotic property of a PIM from any initial capital stock towards a reference PIM is likely to provide the answer, but this escapes the present paper.

(6) An acceptable convergence from an arbitrary value of initial capital may however take many decades of PIM accumulation, but there will be no independent way to judge how long that should be. The method proposed here, however, requires only 11-year series for GDP and GFCF, as our starting *optimal base capital* would normally be close enough to the reference or actual capital stock.

(7) Table 1 shows that, on average, the departure from the OCM-PIM benchmark capital to the reference capital narrows to one-third of the departure associated with the OCM benchmark capital. The speed of gap narrowing for individual countries would be associated to both the growth of capital via GFCF and the magnitude of its initial base capital departure. In equality of conditions, we would expect that the faster that growth, the larger the narrowing of the gap and *vice versa*.

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