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Working Paper No. 563 July 2006 ISSN 1473-0278

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July 4, 2006

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Abstract

I study necessary and sufficient conditions for a choice function to be rationalised in the following sense: there exists a complete asymmetric relation T (a *tournament*) such that for each feasible (finite) choice situation, the choice coincides with the uncovered set of T. This notion of rationality explains not only cyclical and context dependent choices observed in practice, but also provides testable restrictions on observable choice behavior.

Keywords: Rationalizability, Uncovered Set, Intransitive Choice.

J.E.L. Classification: D01.

Acknowledgement 1 *I wish to thank P. Manzini and M. Mariotti for critical comments and helpful discussions. All errors and responsibilities are mine.*

Acknowledgement 2 *Financial support from Queen Mary Research Studentship is gratefully acknowledged.*

1 Introduction

A large body of experimental findings show that choices may be "cyclic" and/or "context-dependent", see, *inter alia*, Camerer, [2]; Loomes et al., [12]; and Tversky, [26]. More surprisingly, these patterns of choice do not seem correlated in any way to the complexity of the decision situation that a decision-maker (DM) may face when called upon (Loomes et al., [12]; and Roelofsma et Read, [20]; Simon et Tversky, [24]).

The considerations that these kinds of choice are not rare phenomena in every day life, and so may have considerable relevance in studies of economic, social, and political behavior, motivated a substantial analytical rethink of the behavioral regularities (consistency properties) postulated by standard models of choice theory.

Standard models (under certainty) posit the *Weak Axiom of Revealed Preference* (WARP), and so view a choice as the outcome of the maximisation of a fixed binary relation (rationale) for every feasible set.[1] This postulate precisely rules out context-dependent choices (an alternative, say x, is chosen while a distinct one, say y, is rejected from a set, whereas a reverse pattern of choice is made from a different set to which they both belong -see, Ehlers et Sprumont, [6]- as well as cyclic choices.[2]

Many alternatives to the standard choice models have been proposed to account for one or both kinds of observed choice behavior. Preserving some form of maximality of a single asymmetric and complete rationale (a *tourna-*

[1] See, *inter alia*, (Suzumura, [25]), (Moulin, [17]). For a recent survey on standard choice theory see (Varian, [27]).

[2] A choice is cyclic whenever only x is chosen from $\{x, y\}$, only y from $\{y, z\}$, and only z from $\{x, z\}$.

ment) defined over all feasible sets it has been shown that a DM may exhibit cyclic choices whenever his choosable alternative dominates at least "indirectly" every other available one (*top-cycle choice rule*) (Ehlers et Sprumont, [6]). This approach, however, leaves out the problem of context-dependent choices, and, in particular, on the one hand, it leaves unexplained the reasoning followed by a DM when he deems rejectable an alternative directly dominating the choosable one, and, on the other hand, the proposed rule does not seem easily usable in practice.[3]

On the line of research that a DM uses simple mental mechanism based on just some of the available information, a two-stage choice procedure which is able to accounts for both kinds of violations has been proposed (but not fully axiomatised): if a DM eliminates unpreferred alternatives by means of the sequential application of a fixed pair of asymmetric and transitive rationales, then the choice coincides with alternative which has the property to be "uncovered" (Manzini et Mariotti, [14]).[4]

Is it possible to explain these kinds of violations of WARP by means of one-reason decision making which is 'naively simple', and picks the uncovered alternatives from every feasible set using all available information? In this paper I provide an affirmative answer to this question.

I propose the following choice rule dubbed *uncovered set rule*: a DM

[3]See, *inter alia*, (Gigerenzer et al., [8]).

[4]This is an 'extension' of a two-stage choice procedure where unpreferred alternatives are eliminated by the sequential implementation of a fixed ordered pair of asymmetric rationales (Manzini et Mariotti, [13]). Other two-stage choice procedures can be found in the literature (see, Houy, [9]; Manzini et Mariotti, [15]; Ok, [19]). A different and interesting approach is to explain the observed data with the least number of rationales with no choice consistency requirements (Apesteguia et et Ballester, [1]; Kalai et al., [10]).

deems the alternative x choosable from every feasible set whenever it has the property that it dominates every other available one, say z, either directly (x is chosen over z) or in two pairwise comparisons (it exists a distinct alternative y such that x is chosen over y which in turn is chosen over z - two steps, so indirect, dominance of x over z).[5] This choice rule 'rationalises' the choice if it elicits the alternatives specified by the DM, for any feasible set. When this happens the choice is said to behave according to the *uncovered set choice rule*.

The uncovered set rule has sociological grounds, and it corresponds to the idea of the existence of a dominance hierachy among elements of a given set (i.e. humans societies, institutions, etc.), which depends on a number of attributes and, above all, on what are the elements of the set under consideration.[6]

Since it is not hard to imagine a choice situation where alternatives conflict, it is plausible to think that a DM having pairwise inconsistent choices (i.e. cyclical pairwise choices) when called upon to make a choice from larger set may follow the suggested mental mechanism to construct a hierarchy of dominance depending on the alternatives under consideration.

To see how this rule is able to account for context-dependent choices, suppose that a DM is called upon to make a choice from $X = \{v, w, x, y, z\}$, and in pairwise choices he deems v choosable over x, y, and z, w choosable over v, y, and z, whereas x over w, y over x and z, and finally z over x. If

[5] The suggested rule has been extensively studied in social science disciplines (see, for instance, Fishburn, [7]; Landau, [11]; and Miller, [16]).

[6] Thorleif Schjelderupp-Ebbe was the first scholar who discovered the existence of a such rule among birds in the early twentieth century (Schjelderupp-Ebbe, [21]).

he behaves according to suggested choice rule he would deem v, w, and x choosable from X while rejectable the remaining alternatives (nevertheless the fact that y is choosable from $\{x, y\}$ and z from $\{x, z\}$), whereas when called to make a choice from $\{x, y, z\}$ he would deem y choosable while x and z rejectable. From this example it is clear that the uncovered set choice rule accounts for cyclic choices as well. The maximality of the choice is assured by the hierarchy of dominance that this rule produces among alternatives.[7]

Of the existing literature on the suggested choice rule two works are particular relevant here. A full characterization of the uncovered set rule has been provided by means of a family of rationales, one for each feasible set (Duggan, [4]). Yet, using a tournament it has been shown that the uncovered set choice rule is the least choice rule obeying properties relating choices across tournaments and across sets (Moulin, [18]).[8] The goal of this paper is to identify the structure of the uncovered set choice rule by using only consistency properties across sets.

In the next section I formalise and characterise the uncovered set choice rule while the last section concludes.

[7] See, *inter alia*, (Miller, [16]).

[8] A companion paper of Moulin's work shows that the minimal covering set choice rule corresponding to some alternatives of the uncovered set choice rule is the least choice rule obeying consistency requirements of different nature. (Dutta, [5]).

2 Uncovered Set Choice Rule

2.1 Nomenclature

Let X be a non-empty finite set made up of all distinct alternatives, and denote by $\mathcal{X}$ the set of all non-empty subsets of X. Given a set $A \in \mathcal{X}$, and $x, y \in A$, $\mathcal{A}_{x,y}(A)$ denotes the class of all non-empty proper subsets of A with the property that x and y belong to them [i.e. $\mathcal{A}_{x,y}(A) = \{B \subset A : x, y \in B\}$].

A *choice rule* f is a multivalued function $f : \mathcal{X} \to \mathcal{X}$ associating a *choice set* $f(A) \subseteq A$ to every $A \in \mathcal{X}$. As usual the *decisive axiom* is assumed to be part of the definition of f [i.e. $A \neq \emptyset \Rightarrow |f(A)| \geqslant 1$]. Saying f is *resolute* means that the choice set from binary sets is a singleton [i.e. $|A| = 2 \Rightarrow |f(A)| = 1$].

A binary relation T on X is a *tournament* if it is asymmetric [for all $x, y \in A$, $xTy \Rightarrow \neg(yTx)$] and complete [for all $x, y \in A$, xTy or yTx]. As usual xTy is read as follows: x dominates y in the pairwise comparison between x and y. Given $x, y, z \in X$, y is dominated by x in two steps if xTz and zTy (i.e. $xTzTy$). Note that a restriction of T to a set $A \in \mathcal{X}$, denoted by $T|A$, is a tournament. A $T-$*cycle* in X is a sequence $(x_1, ..., x_k)$ of distinct alternatives of X such that $x_i T x_{i+1}$ for $i = 1, ..., k-1$ and $x_k T x_1$, for every $3 \leq k \leq |X| + 1$.

Definition 3 *Given a tournament T on X and $A \in \mathcal{X}$, the uncovered set of T on A, denoted by $uc(T|A)$, is the set made up of all distinct alternatives that dominate every other alternative in A in at most two steps: $x \in uc(T|A)$ if xTy or there exists $z \in A$ such that $xTzTy$ for every $y \in A$.*

Definition 4 *A choice rule f is an uncovered set rule if there exists a tournament T on X such that $f(A) = uc(T|A)$ for every $A \in \mathcal{X}$.*

2.2 Axiomatisation

In this section I characterise the uncovered set choice rule by means of the following axioms:

Axiom 5 *(Weak Expansion). $\cap_{i\in I} f(A_i) \subseteq f(\cup_{i\in I} A_i)$.*

The above axiom, denoted by WE, is also known as Sen's condition γ [Sen, [22] and [23]] and has a long history in axiomatic choice theory. This condition states that if an alternative is deemed choosable from every element of a given collection of non-empty sets, then it must still be deemed choosable from their union.

Axiom 6 *(Condorcet Consistency). If $A \in \mathcal{X}$, $x \in A$, and $f(\{x, y\}) = \{x\}$ for all $y \in A\backslash\{x\}$, then $\{x\} = f(A)$.*

This axiom, denoted by CC, asserts that if an alternative is always chosen in pairwise comparisons, then it should be uniquely chosen from a set containing all those alternatives. An alternative that is always chosen in pairwise comparisons is known as *Cordorcet winner.* Similar to WE, CC has a long history in axiomatic choice theory as well [Moulin, [18]; Sen, [22]].

Axiom 7 *(Restricted Chernoff). If $A \in \mathcal{X}$, $|A| > 3$, and $x \in f(A)$, then $\forall y \in A\backslash\{x\}, \exists B \in \mathcal{A}_{x,y}(A)$ such that $x \in f(B)$.*

The above axiom, denoted by RC, is new to the best of my knowledge, and it asserts that whenever an alternative x is chosen out of a set $A \in \mathcal{X}$, then x

should be itself chosen from some proper subsets of A, each including different available alternatives in A (or equivalently, given a set A, if x is never chosen out of all proper subset of A including itself and another available alternative, then x should not be chosen from A [i.e. if $A \in \mathcal{X}$ and $x \notin \cup_{B \in \mathcal{A}_{x,y}(A)} f(B)$ for some $y \in A \setminus \{x\}$, then $x \notin f(A)$]). This axiom is weaker than the canonical Chernoff axiom [i.e. if $A, B \in \mathcal{X}$, and $B \subseteq A$, then either $f(A) \cap B = \emptyset$ or $f(A) \cap B \subseteq f(B)$] (Chernoff, [3]).

Axiom 8 *(No-Discrimination). If $x, y, z \in X$, $f(\{x, y\}) = x$, $f(\{y, z\}) = y$, and $f(\{x, z\}) = z$, then $f(\{x, y, z\}) = \{x, y, z\}$.*

This condition, denoted by N-D, is new as well and states that if a DM has clear mind on three pairwise choices, but his pairwise choices are inconsistent, then when called upon to make a choice from a set including all those alternatives the DM deems each of them equally choosable.

Theorem 9 *A choice rule f is resolute and satisfies WE, CC, RC, and N-D if, and only if, it is an uncovered set rule.*

Proof. (*If*). Let f be an uncovered set rule. By asymmetry of T, it follows that f is resolute. Next, WE, CC, RC, and N-D are checked.

(WE). Let the premise hold. Since $x \in f(A_i)$ for every $i \in I$ and f is an uncovered set rule, then $x \in uc(T|A_i)$ for each $i \in I$. Thus $x \in uc(T| \cup_{i \in I} A_i)$, and so $x \in f(\cup_{i \in I} A_i)$. Observe that WE is vacuously satisfied if $\cap_{i \in I} f(A_i)$ is empty.

(CC). Let the premise hold. Assume, to the contrary, that $\{x\} \neq f(A)$. Then $\{x\} \neq uc(T|A)$, and so it exists $y \in A \setminus \{x\}$ such that $f(\{x, y\}) = \{y\}$, a contradiction.

(RC). Assume that $x \notin \left(\cup_{B \in \mathcal{A}_{x,y}(A)} f(B)\right)$ for some $y \in A \setminus \{x\}$. Recall that the axiom of decisiveness is postulated to be part of the definition of f. I show that $x \notin f(A)$. Assume, to the contrary, that $x \in f(A)$. So, $x \in uc(T|A)$. By assumption there exists $y \in A \setminus \{x\}$ such that $x \notin f(\{x, y\})$. Then $\{y\} = f(\{x, y\})$ by the decisive axiom, and so yTx. Then there exists $z \in A \setminus \{x, y\}$ such that $xTzTy$. Thus x dominates y in two steps, and so $x \in uc(T|\{x, y, z\})$. Therefore $x \in f(\{x, y, z\})$,[9] a contradiction.

(N-D). Since f is an uncovered set rule, the statement follows directly.

(*Only if*). Let f be resolute and satisfy WE, CC, RC, and N-D.

Define $T = \{(x, y) \in X \times X : \{x\} = f(\{x, y\})\}$. By construction and the resoluteness of f, T is asymmetric. Since f is defined on a universal domain, it follows that T is complete. Therefore T is a tournament on X.

Next, the following claim :

$$f(T|A) = uc(T|A) \tag{1}$$

is proved to hold for every $A \in \mathcal{X}$.

A proof by induction based on the cardinality of A is provided.

By definition of f, its resoluteness, CC, and N-D claim (1) is trivially true for all $A \in \mathcal{X}$ such that $|A| \leq 3$.

Assume that (1) is true for all $A \in \mathcal{X}$ such that $|A| = k$, where $k > 3$. I prove that the result is true for $|A| = k + 1$. Let me proceed along the following two steps: (*1*) $x \in f(A) \Rightarrow x \in uc(T|A)$, (*2*) $x \in uc(T|A) \Rightarrow x \in f(A)$.

Step 1: $x \in f(A) \Rightarrow x \in uc(T|A)$.

[9] It comes from the fact that f is an uncovered set choice rule.

Assume, to the contrary, that $x \in A \backslash uc(T|A)$. There exists $y \in A \backslash \{x\}$ such that yTx and (for all $z \in A \backslash \{x, y\}$, if xTz, then yTz).

Since yTx, by definition of T and the resoluteness of f it follows that $\{x\} \neq f(\{x, y\})$. Take any $B \in \mathcal{A}_{x,y}(A)$ with $3 \leq |B| < |A|$. Since x does not dominate y in at most two steps, it follows that $x \notin uc(T|B)$. By the inductive hypothesis it must be that $x \notin f(B)$. Since this is true for every $B \in \mathcal{A}_{x,y}(A)$, then $x \notin \cup_{B \in \mathcal{A}_{x,y}(A)} f(B)$. Hence, by RC $x \notin f(A)$, a contradiction.

Step 2: $x \in uc(T|A) \Rightarrow x \in f(A)$.

Let $x \in uc(T|A)$. I prove that $x \in f(A)$.

First let me introduce some more notation.

Let $\overline{A}_x$ denote the set made of all elements of A which are chosen over x according to f in pairwise comparisons, i.e.

$$\overline{A}_x = \{y \in A \backslash \{x\} : f(\{x, y\}) = y\};$$

similarly, let $\underline{A}_x$ denote the set made of all elements of A over which x is chosen according to f in pairwise comparisons, i.e.

$$\underline{A}_x = \{y \in A \backslash \{x\} : f(\{x, y\}) = x\}.$$

Partition A in $\{x\}$, $\overline{A}_x$ and $\underline{A}_x$. I proceed according to whether $\overline{A}_x = \emptyset$ or $\overline{A}_x \neq \emptyset$.

Case 1: $\overline{A}_x = \emptyset$

By CC it follows directly that $\{x\} = f(A)$.

Case 2: $\overline{A}_x \neq \emptyset$

Assume, to the contrary, that $x \in A \backslash f(A)$. Since $\overline{A}_x \neq \emptyset$, take any $\overline{y} \in \overline{A}_x$. Then $x \in uc(T|A \backslash \{\overline{y}\})$, and so by the inductive hypothesis

$x \in f(A \backslash \{\overline{y}\})$. Since $\{\overline{y}\} = f(\{x, \overline{y}\})$, by definition of T it follows that $\overline{y} T x$. Since $x \in uc(T|A)$, there exists $\underline{y} \in \underline{A}_x$ such that $xT\underline{y}T\overline{y}$. Then $x \in uc\left(T|\left\{x, \underline{y}, \overline{y}\right\}\right)$, and so by inductive hypothesis $x \in f\left(\left\{x, \underline{y}, \overline{y}\right\}\right)$. By WE it follows that $x \in f(A)$, a contradiction. ■

Remark 10 *The axioms are independent, as is argued next.*

For an example violating only WE, let $X = \{x, y, z, w\}$, define $f(\{x, y\}) = f(\{x, z\}) = \{x\}$, $f(\{y, z\}) = f(\{y, w\}) = \{y\}$, $f(\{x, w\}) = \{w\}$, $f(\{z, w\}) = \{z\}$, $f(A) = uc(T|A)$ for $A \in \mathcal{X} \backslash X$, and $f(X) = \{x, y\}$. Note that f is not an uncovered set rule since $uc(T|X) = \{x, y, w\}$, but $w \notin f(X)$. WE is violated because $w \in f(\{x, y, w\})$ and $w \in f(\{x, z, w\})$, but $w \notin f(X)$.

For an example violating only CC, let $X = \{x, y, z\}$, define $f(\{x, y\}) = f(\{x, z\}) = \{x\}$, $f(\{y, z\}) = \{y\}$, and $f(X) = \{x, y\}$. The choice rule f is not an uncovered set rule because $\{x\} = uc(T|X)$, but $\{x, y\} = f(X)$. CC is violated because $f(\{x, y\}) = f(\{x, z\}) = \{x\}$, but $y \in f(X)$.

For an example violating only RC, let $X = \{x, y, z, w\}$, define $f(\{x, y\}) = f(\{x, z\}) = \{x\}$, $f(\{y, z\}) = f(\{y, w\}) = \{y\}$, $f(\{x, w\}) = \{w\}$, $f(\{z, w\}) = \{z\}$, $f(A) = uc(T|A)$ for $A \in \mathcal{X} \backslash X$, and $f(X) = X$. The choice rule f is not an uncovered set rule because $z \notin uc(T|X)$ while $f(X) = X$. RC is violated because z is not revealed preferred in any proper subset including the alternative y, but $z \in f(X)$.

For an example violating only N-D, let $X = \{x, y, z\}$, define $f(\{x, y\}) = \{x\}$, $f(\{y, z\}) = \{y\}$, $f(\{x, z\}) = \{z\}$, and $f(X) = \{x, y\}$. The choice rule f is not an uncovered set rule because $z \in uc(T|X)$, but $z \notin f(X)$. N-D is violated because it exists a cycle among the pairwise choices, but $z \notin f(X)$.

For an example violating only the resoluteness condition, let $X = \{x, y, z\}$, define $f(\{x, y\}) = \{x, y\}$, $f(\{x, z\}) = \{x\}$, $f(\{y, z\}) = \{y\}$, and $f(X) = \{x, y\}$. The choice rule f is not an uncovered set rule because $uc(T|X) = \emptyset$ while $f(X) = \{x, y\}$. The resoluteness condition is violated because $|f(\{x, y\})| = 2$.[10]

3 Concluding comments

This paper formalised a one-reason decision making which has been extensively studied in mathematical sociology (Landau, [11]) and in voting theory (Fishburn, [7]; Miller, [16]): *uncovered set rule*.

I identified necessary and sufficient condition for a choice function to coincide with the uncovered set of a fixed asymmetric and complete binary relation (*tournament*). Only consistency property relating choices across set were used.

Its central features are simplicity and reasonableness. It is simple to use because it corresponds to the naive idea that the choosable alternative should have the property to dominate every other available one in at most two pairwise choices. It is a reasonable choice rule because it can be tested by some mild consistency properties. Moreover, not only it has the advantage to account for cyclic and context-dependent choices preserving some form of maximality, but also to explain what kind of psychological mechanism a decision-maker may follows when he express rejectable an alternative dominating the choosable one.

These anomalies in observed choice behavior can go with rather different

[10]Note that here CC does not apply because it does not exist a Condorcet winner.

psychological motivations and may have different alternative explanations. Yet, the uncovered set choice rule is a simple and reasonable rule that can shed some light on the complex underlying links between the commonly observed violations of WARP and the psychological mechanisms driving decision behaviors.

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