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What Kind of Preference Maximization Does the Weak  
Axiom of Revealed Non-inferiority Characterize?

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What kind of preference maximization does the  
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## **Abstract**

A multi-valued choice is weak justified if no chosen alternative is dominated by any other obtainable alternative, and for each discarded alternative there is *some* chosen alternative which dominates it. This definition allows us to build a connection between the behavioral property expressed by the weak axiom of revealed non-inferiority and a weak notion of rationality. Our notion of rationality is weaker than that characterized by the weak axiom of revealed preference.

**J.E.L. codes:** D0.

**Keywords:** Weak axiom of revealed non-inferiority.

# 1 Introduction

Eliasz and Ok (2006) accommodate preference incompleteness in revealed preference theory by studying the implications of weakening the fundamental choice-consistency condition of the weak axiom of revealed preference (WARP) in the weak axiom of revealed non-inferiority (WARNI).<sup>1</sup> This behavioural postulate entirely corresponds to maximizing behaviour on suitable domains. However, a choice function rationalized by the maximization of a preference relation (not necessarily complete) may fail to satisfy WARNI on an arbitrary choice domain. This is due to the fact that WARNI characterizes a particular type of rationality. Our concern is to spell out the form of maximality of choice characterized by this behavioural postulate on an arbitrary choice domain, and then we contrast this form of maximality with that characterized by WARP.

A choice function is weak justified if there exists a binary relation  $J$  (dubbed weak justification) such that, for all feasible sets, no available alternative is  $J$ -related to any chosen alternative, for each rejected alternative there is *some* chosen alternative which is  $J$ -related to it. Therefore, the binary relation  $J$  is a strict (not necessarily complete) preference relation.

A decision-maker makes weak justified choices if she can assert that no chosen alternative is dominated by any other obtainable one, and for each discarded alternative there is some chosen alternative which dominates it.

Our rationality hypothesis differs from that provided by Mariotti (2007), according to which choices are justified if there exists a binary relation  $J$  such that, for all feasible sets, no two chosen alternatives are  $J$ -related to each other, and each chosen alternative is  $J$ -related to *all* of the rejected alternatives. Mariotti (2007) shows that choices satisfy WARP if and only

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<sup>1</sup>On a finite universal set WARNI is identical to one of the behavioral properties suggested by Bandyopadhyay and Sengupta (1993).

if they are justified by an asymmetric relation.

The result of this note is that choices satisfy WARNI if and only if they are weak justified by an asymmetric relation.

## 2 Analysis

Let  $X$  be a non-empty set of alternatives. Let  $\mathcal{S}$  be a collection of non-empty subsets of  $X$ . By a *choice function*  $C$  on  $\mathcal{S}$  we mean a map  $C$  which assigns a non-empty subset  $C(S)$  of  $S$  to every  $S \in \mathcal{S}$ .

A binary relation  $J \subseteq X \times X$  is said to be asymmetric if, for all  $x, y \in X$ ,  $(x, y) \in J$  implies  $(y, x) \notin J$ .

If there exists a binary relation  $J$  on  $X$  such that, for all  $S \in \mathcal{S}$ :

- 1)  $\forall x \in C(S), \forall y \in S : (y, x) \notin J$
- 2)  $\forall y \in S \setminus C(S) : (x, y) \in J$  for some  $x \in C(S)$

then we say that  $J$  is a weak justification for  $C$ . If  $C$  has a weak justification, we say that  $C$  is weak justified. We will call  $J$  an asymmetric weak justification if  $J$  is asymmetric. Our notion is weaker than that provided by Mariotti (2007), according to which choices are justified if, for all  $S \in \mathcal{S}$ , for all  $x, y \in C(S)$  it holds that  $(x, y) \notin J$ , and for all  $x \in C(S)$  and for all  $y \in S \setminus C(S)$  it holds that  $(x, y) \in J$ .

Eliaz and Ok (2006) suggested to read the statement “ $x \in C(S)$ ” as “ $x$  is revealed not to be inferior to any other obtainable alternative in  $S$ ” rather than to follow the classic interpretation of “ $x$  is revealed to be at least as good as all other available alternatives in  $S$ ”. Under this interpretation of revealed preferences, they propose the weak axiom of revealed non-inferiority (WARNI). The idea behind this behavioural regularity is quite mild. It asserts that if an obtainable alternative from a set  $S$  is revealed not to be inferior to all of other chosen alternatives from  $S$ , then it must be chosen from  $S$  as well.

**WARNI:**  $\forall S \in \mathcal{S}, y \in S : [\forall x \in C(S) \exists T \in \mathcal{S} : y \in C(T) \text{ and } x \in T] \Rightarrow y \in C(S)$ .

This behavioral postulate is weaker than WARP which asserts that if  $x \in C(S)$  and there exists a feasible set  $T$  such that  $y \in C(T)$  and  $x \in T$ , then  $y \in C(S)$ . Furthermore, WARNI implies the canonical Property  $\alpha$  (also known as Chernoff choice-consistency condition or basic contraction consistency), according to which an alternative that is deemed choosable from a feasible set  $T$  and belongs to a subset  $S$  of  $T$  must be deemed choosable from  $S$  (i.e.,  $x \in T \subseteq S$  and  $x \in C(S) \Rightarrow x \in C(T)$ ).<sup>2</sup>

**Theorem 1** *There exists a choice that is not weak justified.*

**Proof.** Let  $X$  be the set consisting of three distinct alternatives:  $x, y$ , and  $z$ . Let  $\mathcal{S} = \{\{x, y\}, X\}$ , and suppose that  $C(\{x, y\}) = \{x\}$  and  $C(X) = \{x, y\}$ . It is easy to see that  $C$  cannot be weak justified. For suppose that  $C$  is weak justified. Then, since  $C(X) = \{x, y\}$ , we must have  $(x, y) \notin J$ . Since  $C(\{x, y\}) = \{x\}$ , we must have  $(x, y) \in J$ , contradicting the definition of weak justification of  $C$ . ■

**Theorem 2** *A choice function  $C$  on  $\mathcal{S}$  is asymmetric weak justified if and only if it satisfies WARNI.*

**Proof.** Assume that  $C$  is asymmetric weak justified. We show that  $C$  satisfies WARNI. Suppose that for all  $S \in \mathcal{S}$ , with  $y \in S$ , it holds that for every  $x \in C(S)$  there exists  $T \in \mathcal{S}$  such that  $y \in C(T)$  and  $x \in T$ . As  $C$  is asymmetric weak justified it follows that for all  $x \in C(S)$  it holds that  $(x, y) \notin J$ . By way of contradiction, let  $y \notin C(S)$ . Because  $C$  is asymmetric

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<sup>2</sup>See Eliaz and Ok (2006, lemma 1, p. 81).

weak justified it follows that there exists  $x \in C(S)$  such that  $(x, y) \in J$  yielding a contradiction.

For the converse, let  $C$  satisfy WARNI. We show that  $C$  is asymmetric weak justified. Define for all distinct  $x, y \in X$ :

$$(x, y) \in J \Leftrightarrow \exists S \in \mathcal{S} : x \in C(S), y \in S \setminus C(S), \text{ and } \nexists T \in \mathcal{S}, x \in T : y \in C(T).$$

Then  $J$  is asymmetric. To show that  $C$  satisfies property 1), let  $x \in C(S)$  and  $y \in S$  for some  $S \in \mathcal{S}$ . By way of contradiction, let  $(y, x) \in J$ . Then there exists  $S' \in \mathcal{S}$  such that  $y \in C(S')$ ,  $x \in S' \setminus C(S')$ , and for no  $T \in \mathcal{S}$ , with  $y \in T$ , it holds that  $x \in C(T)$ , which contradicts that  $x \in C(S)$  and  $y \in S$ . Finally, we show that  $C$  meets property 2). Suppose that  $y \in S \setminus C(S)$  for some  $S \in \mathcal{S}$ . WARNI implies that there exists  $x \in C(S)$  such that for all  $T \in \mathcal{S}$  it holds true  $y \notin C(T)$  if  $x \in T$ . It follows that  $(x, y) \in J$ . ■

Theorem 2 clarifies how much rationality in terms of preference maximization we give up in passing from WARP to WARNI. Both properties require no chosen alternative is dominated by any other available alternative. However, while WARP demands that each chosen alternative has to dominate *all* of the discarded alternatives, WARNI requires that for each rejected alternative there exists *some* chosen alternative which dominates it. Obviously there are choices which are weak justified but not justified, as argued next.

**Theorem 3** *There exists a choice that is weak justified but not justified.*

**Proof.** Let  $X$  be the set consisting of three distinct alternatives:  $x, y$ , and  $z$ . Suppose that  $\mathcal{S} = \{\{x, y\}, \{z, y\}, X\}$ . Define the choice  $C$  on  $\mathcal{S}$  by  $C(X) = \{x, z\}$ ,  $C(\{x, y\}) = \{x\}$ , and  $C(\{z, y\}) = \{z, y\}$ . It is easy

to see that  $C$  is weak justified, but not justified. For suppose that  $C$  is justified. Then, since  $C(X) = \{x, z\}$ , we must have  $(x, y), (z, y) \in J$ . But  $C(\{z, y\}) = \{z, y\}$  implies that  $(z, y) \notin J$  yielding a contradiction. ■

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