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Abstract

In this paper we provide tests for the unit root hypothesis against the occurrence of an unspecified number of breaks which may be larger than 2 but smaller than the maximum allowed number of breaks, m , in univariate time series models. The advocated procedure is considerably less computationally intensive than those widely used in the literature. We provide critical values for the test, examine its small sample properties through Monte Carlo experiments and apply the new test to the Nelson and Plosser macroeconomic series.

JEL Classification: C13, C15, C32

Keywords: Unit root, Structural Break

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1 Introduction

During the last two decades the unit root hypothesis has underlain the investigation of persistence in economic phenomena. Following the work of Perron (1989) in which the author provided evidence against the unit root hypothesis and in favour of structural breaks using the Nelson and Plosser (1982) annual macroeconomic series, considerable work has concentrated on investigated Perron's claims. Perron suggested that a structural break may account for the inability to reject the unit root hypothesis from the data. He specified the Great Depression as the time when the structural break occurred. Subsequent research concentrated on providing evidence for or against the unit root hypothesis without assuming a known break point. Work by Banerjee, Lumsdaine, and Stock (1992) and Zivot and Andrews (1992) provided tests that allowed for a single break as an alternative to the unit root hypothesis. Lumsdaine and Papell (1997) extended part of their results to allow for two breaks.

In this paper we further extend previous results by providing tests for the unit root hypothesis against an unspecified number of breaks which may be larger than 2 but smaller than or equal to the maximum allowed number of breaks m . The advocated procedure is computationally less intensive than that suggested by Lumsdaine and Papell (1997). Additionally, it moves away from testing the null of a unit root against a specified number of structural breaks, which is a narrow consideration as Lumsdaine and Papell (1997) point out, and towards model selection strategies which are less dependent on an assumed number of breaks. We further provide critical values for the tests, consider their small sample properties through Monte Carlo experiments and apply them to the Nelson and Plosser macroeconomic series.

The paper is organised as follows: Section 2 presents the test and its theoretical properties. Section 3 provides a Monte Carlo study of its small

sample properties. The test is applied to the Nelson and Plosser series in Section 4. Section 5 concludes.

2 Unit root test against a m break alternative hypothesis

A test of the unit root hypothesis against the alternative of one or more structural breaks when the break dates are unknown is a nonstandard test both because it involves nonstationarity under the null hypothesis and because nuisance parameters, i.e. the break dates, are not identified under the null hypothesis. The problem of unidentifiability under the null hypothesis was first discussed by Davies (1977) where a general solution was proposed. This and many subsequent solutions proposed in the literature essentially involve integrating out the nuisance parameters (e.g. see Hansen (1996)). This is achieved by constructing appropriate test statistics for each of a set of values for the nuisance parameters and then choosing a summary statistic from the set of test statistics such as the maximum, minimum or average. The main difficulty in this approach is that the null distribution of the summary statistic is difficult to obtain analytically and therefore researchers usually resort to simulation methods.

The test we propose follows from the sequential DF t-statistics proposed by Banerjee, Lumsdaine, and Stock (1992) and Zivot and Andrews (1992) for the case of a single break. The following model forms the basis of our investigation.

$$y_t = \mu_0 + \mu_1 t + \alpha y_{t-1} + \sum_{i=1}^k \gamma_i \Delta y_{t-i} + \sum_{i=1}^m \phi_i DU_{i,t} + \sum_{i=1}^m \psi_i DT_{i,t} + \epsilon_t \quad (1)$$

$1 - \gamma(L)$ has all its roots outside the unit circle, where $\gamma(L) = \gamma_1 L + \dots + \gamma_k L^k$. We denote the probability limit of the estimated covariance matrix of the vector $(\Delta y_{t-1}, \dots, \Delta y_{t-k})$ by Σ . $DU_{i,t}$ and $DT_{i,t}$ are intercept and trend

break dummy variables respectively defined by :

$$DU_{i,t} = 1(t > T_{b,i}), \quad DT_{i,t} = 1(t > T_{b,i})(t - T_{b,i})$$

where $T_{b,i} + 1$ denotes the date of the i -th structural break and $1(\cdot)$ is the indicator function taking the value of 1 if the argument of the function is true and 0 otherwise.

To facilitate the analysis we follow Banerjee, Lumsdaine, and Stock (1992) and Lumsdaine and Papell (1997) and define the following vector of regressors: $\mathbf{z}_t = (1, t + 1, y_t - \bar{\mu}t, DU_{1,t+1}, \dots, DU_{m,t+1}, DT_{1,t+1}, \dots, DT_{m,t+1}, \Delta y_t - \bar{\mu}, \dots, \Delta y_{t-k+1} - \bar{\mu})'$, where $\bar{\mu} = E(\Delta y_t)$. Then, $y_t = \mathbf{z}'_{t-1} \boldsymbol{\theta}$ where $\boldsymbol{\theta} = (\mu_0 + (\gamma(1) - \alpha)\bar{\mu}, \mu_1 + \alpha\bar{\mu}, \alpha, \phi_1, \dots, \phi_m, \psi_1, \dots, \psi_m, \gamma_1, \dots, \gamma_k)'$. The sequence of errors is assumed to be a martingale difference sequence with finite conditional $4 + \xi$, $\xi > 0$, moments. The second conditional moment is denoted by σ^2 . Denoting the number of observations for model (1) by T , we rewrite the break dates as $T\delta_1, \dots, T\delta_m$ where $0 < \delta_i < 1$, $i = 1, \dots, m$ are the break fractions. We also define the scaling matrix

$$\Xi_T = \text{diag}(T^{1/2}, T^{3/2}, T, \underbrace{T^{1/2}, \dots, T^{1/2}}_m, \underbrace{T^{3/2}, \dots, T^{3/2}}_m, \underbrace{T^{1/2}, \dots, T^{1/2}}_k)$$

partitioned conformably to \mathbf{z}_t . We define the OLS estimator for model (1) and given break dates as

$$\hat{\boldsymbol{\theta}}(\delta_1, \dots, \delta_m) = \Psi_T(\delta_1, \dots, \delta_m)^{-1} \boldsymbol{\zeta}_T(\delta_1, \dots, \delta_m)$$

where $\boldsymbol{\zeta}_T(\delta_1, \dots, \delta_m) = \Xi_T^{-1} \sum_{i=1}^T \mathbf{z}_{t-1}(\delta_1, \dots, \delta_m) y_t$ and $\Psi_T(\delta_1, \dots, \delta_m) = \Xi_T^{-1} \sum_{i=1}^T \mathbf{z}_{t-1}(\delta_1, \dots, \delta_m) \mathbf{z}'_{t-1}(\delta_1, \dots, \delta_m) \Xi_T^{-1}$. We also define $\boldsymbol{\varphi}_T(\delta_1, \dots, \delta_m) = \Xi_T^{-1} \sum_{i=1}^T \mathbf{z}_{t-1}(\delta_1, \dots, \delta_m) \epsilon_t$

The following proposition follows directly from the results in Banerjee, Lumsdaine, and Stock (1992) and provides asymptotic representations for the coefficients for given break fractions $(\delta_1, \dots, \delta_m)$.

Proposition 1 *Suppose that y_t is generated according to (1) with $\mu_1 = \phi_1 = \dots = \phi_m = \psi_1 = \dots = \psi_m = 0$ and $\alpha = 1$. Let $W(\cdot)$ and $\mathbf{B}(\cdot)$ denote a scalar standard Brownian motion and a k -dimensional Brownian motion independent of $W(\cdot)$, respectively. Then, under the specification of the error sequence $\{\epsilon_t\}$, given above the following result holds*

$$\Psi_T(\delta_1, \dots, \delta_m)^{-1} \varphi_T(\delta_1, \dots, \delta_m) \Rightarrow \Psi(\delta_1, \dots, \delta_m)^{-1} \varphi(\delta_1, \dots, \delta_m)$$

uniformly over $(\delta_1, \dots, \delta_m)$, where

$$\begin{aligned} \varphi(\delta_1, \dots, \delta_m) = \sigma \left(W(1), \int_0^1 r dW(r), \int_0^1 H(r) dW(r), W(1) - W(\delta_1), \dots, \right. \\ \left. W(1) - W(\delta_m), \int_{\delta_1}^0 (r - \delta_1) dW(r), \dots, \int_{\delta_m}^0 (r - \delta_m) dW(r), \mathbf{B}(1) \right) \\ \Psi(\delta_1, \dots, \delta_m) = \begin{pmatrix} \Lambda & O \\ O' & \Sigma \end{pmatrix} \end{aligned}$$

$H(\cdot) = \sigma(1 - \gamma(1))^{-1}W(\cdot)$, O denotes a $2m + 3 \times k$ matrix of zeros. Λ is defined in the appendix.

From the above proposition it is straightforward to obtain the asymptotic distribution of the t-statistic on the coefficient of α for given $\delta_1, \dots, \delta_m$. However, extending the grid search scheme proposed by Lumsdaine and Papell (1997) to m breaks is clearly computationally very demanding. Additionally, obtaining the critical values of the minimum t-statistic over the proposed m -dimensional grid by simulation would be prohibitive for $m > 3$. A further drawback of such an approach is that a given m has to be assumed. In order to construct our test we define the following alternative hypotheses:

$$H_i : \alpha < 1, \phi_{i+1} = \dots = \phi_m = \psi_{i+1} = \dots = \psi_m = 0, \quad i = 1, \dots, m - 1$$

$$H_m : \alpha < 1$$

As usual, we denote the null hypothesis $\alpha = 1, \mu_1 = \phi_1 = \dots = \phi_m = \psi_1 = \dots = \psi_m = 0$ by H_0 . Clearly, previous testing procedures concentrated on

testing H_0 against H_1 or H_2 . Our aim is to construct a test of H_0 against $\cup_{i=1}^m H_i$. The most straightforward method involves constructing the relevant t-statistics on the estimate of α for all possible break partitions for a given break number and all break numbers from 1 to m and taking the infimum of the set of these t-test statistics. Let us denote the set of all possible break partitions for a given number of breaks by \mathcal{T}_i , $i = 1, \dots, m$ and their union over i by \mathcal{T} . The distribution under the null hypothesis for a t-test statistic given the number of breaks and the break fractions follows from Proposition 1 and Remark 1 of Lumsdaine and Papell (1997). The distribution of the infimum of the t-test statistics, over \mathcal{T} , under the null hypothesis follows directly from Lemma A.4 of Zivot and Andrews (1992). The consistency of the test is guaranteed by the consistent estimation of the break fractions and the other coefficients under the alternative of structural breaks proven by, among others, Bai and Perron (1998). Note that the results of Bai and Perron (1998) concerning consistency of the estimated coefficients allows for deterministic trends. Nevertheless, such an approach is unnecessarily computationally intensive¹. By Bai and Perron (1998, pp. 64) we have that a sequential procedure would allow consistent estimation of break fractions, and therefore consistent estimation of the whole model under the alternative hypothesis, with only $O(T)$ least squares operations for any given number of breaks. We can therefore construct a consistent and less computationally intensive test using the t-statistics from these least squares operations.

We therefore propose constructing a test using the following grid search scheme following Bai and Perron (1998).

1. For a given maximum number of breaks, m , start by searching for a single break and store the t-statistics of the hypothesis $\alpha = 1$ for all possible partitions over the sample. Denote the set of all possible partitions as \mathcal{T}_1^a . Also, denote the set of t-test statistics by τ^1 .

¹An alternative procedure to estimate multiple breaks with reduced computational burden has recently been suggested by Bai and Perron (2000). This procedure could be used instead of the sequential procedure we suggest in this context.

- Choose the break date associated with the minimum sum of squared residuals (SSR) given by

$$SSR = \sum_{t=k+2}^T (y_t - \hat{\mu}_0 - \hat{\mu}_1 t + \hat{\alpha} y_{t-1} + \sum_{i=1}^k \hat{\gamma}_i \Delta y_{t-i} + \hat{\phi}_1 DU_{1,t} + \hat{\psi}_1 DT_{1,t})^2$$

where k is assumed known.

- Imposing the estimated break date on the sample, start looking for the next break over all possible partitions in the resulting subsamples. Denote the set of all possible partitions by \mathcal{T}_2^a . Obtain the set of t-statistics of the hypothesis $\alpha = 1$ over all possible partitions and denote this by $\boldsymbol{\tau}^2$. Append $\boldsymbol{\tau}^2$ to $\boldsymbol{\tau}^1$ to obtain $\boldsymbol{\tau}_1^2 = \boldsymbol{\tau}^1 \cup \boldsymbol{\tau}^2$.
- Choose the break with the minimum SSR as the next estimated break.
- Repeat steps 3 and 4 until m break dates have been estimated. Denote the resulting sets of all possible partitions as \mathcal{T}_i^a , $i = 3, \dots, m$.
- Adopt as the test statistic, τ_{\min}^m , the minimum t-statistic over the set $\boldsymbol{\tau}_1^m = \boldsymbol{\tau}^1 \cup \boldsymbol{\tau}^2 \cup \dots \cup \boldsymbol{\tau}^m$.

Before we discuss the asymptotic distribution of this test statistic we note that we do not look for consecutive breaks or for breaks at the end or beginning of the sample. Each estimated break is assumed to lie between two subsamples whose size goes to infinity with rate T as the sample size increases. In other words we impose a nonzero trimming parameter, ε on each break search. Under the null hypothesis of a unit root, the test statistic will have a well defined distribution which will be the same as that of the minimum of $\int_0^1 W_i^*(\hat{\boldsymbol{\delta}}_i, r) dW(r) / \left(\int_0^1 W_i^*(\hat{\boldsymbol{\delta}}_i, r) dr \right)^{1/2}$ over $\hat{\boldsymbol{\delta}}_i$ where $\hat{\boldsymbol{\delta}}_1 = \hat{\delta}_1$, $\hat{\boldsymbol{\delta}}_i = (\hat{\delta}_1, \dots, \hat{\delta}_{i-1}, \delta_i)$, $i = 2, \dots, m$ and $W_i^*(\boldsymbol{\delta}_i, r)$, $\boldsymbol{\delta}_1 = \delta_1$, $\boldsymbol{\delta}_i = (\delta_1, \dots, \delta_i)$, $i = 2, \dots, m$, is the continuous time residual from the projection of a Brownian motion onto the functions $[1, r, 1(r > \delta_1), (r - \delta_1)1(r > \delta_1), \dots, 1(r > \delta_i), (r - \delta_i)1(r > \delta_i)]$. Note that in $\hat{\boldsymbol{\delta}}_i$ the only parameter that varies with the

minimization is δ_i . The rest of the break fractions are given and have been estimated from previous SSR minimisations. This distribution merits further discussion. We firstly note that obviously the set over which we take the infimum, $\mathcal{T}^a \equiv \cup_{i=1}^m \mathcal{T}_i^a$, is a subset of the set \mathcal{T} , over which the infimum would have been taken had we simply extended the method used by Lumsdaine and Papell (1997) to more than two breaks. Therefore, the uniform convergence in distribution of the test statistics over \mathcal{T}^a follow straightforwardly from extending the results of Zivot and Andrews (1992) and Lumsdaine and Papell (1997). The asymptotic behaviour of the estimates $\hat{\delta}_i$ depend crucially on whether $\varepsilon = 0$ or not. If $\varepsilon = 0$, $\hat{\delta}_1 = 0$ or 1 with equal probability. Otherwise, $\hat{\delta}_1$ converges to some random variable. For more details see Nunes, Kuan, and Newbold (1995) and Bai (1998). It is clear that the conditional distribution of $\hat{\delta}_i$ given $\hat{\delta}_1, \dots, \hat{\delta}_{i-1}$ is the same as that of $\hat{\delta}_1$. The marginal distribution is however clearly not the same. In any case the distribution of break fractions and the test statistic is likely to depend on the trimming parameter, ε . In conclusion, the asymptotic distribution is quite complex and will be approximated by simulation similarly to previous work in the literature. Under the alternative hypothesis of up to m structural breaks, the break fractions and therefore the coefficients of the model are estimated consistently according to Bai and Perron (1998) and consequently the statistic goes off to minus infinity providing a consistent test.

We note the following. Firstly, we distinguish between three cases. The first assumes that $\psi_1 = \dots = \psi_m = 0$ under both the null and the alternative. This case will be denoted as case A. The second assumes the same for ϕ_1, \dots, ϕ_m . This will be denoted as B. The third considers the general model (1) under the alternative and will be denoted as C. Secondly, we assume that k is known. This assumption is not crucial to the analysis and may easily be dropped if the results of Ng and Perron (1995) are taken into account. Their work assumes that the error term in the unit root model fol-

lows an ARMA process but that ADF tests are used. Then, it is shown that if a data dependent procedure is used to determine k and this data dependent procedure allows k to rise within specified rates then the distribution of the ADF tests do not change. Both standard information criteria (AIC, BIC) and sequential testing procedures are shown to satisfy the required conditions.

The critical values of the test for cases A,B and C are presented in Table 1 for up to $m = 5$ and $\varepsilon = 0.05$. For higher m , results are available upon request. The critical values have been computed by simulation where standard random walks are generated and used to estimate the relevant model for each case. The errors are standard normal and generated using the GAUSS pseudo-random number generator. For all simulations the number of observations for the random walks is set to 250 and the number of replications to 1000.

Table 1: Critical values for τ_{\min}^m for models A, B and C.

Model	m	Significance Level			
		0.10	0.05	0.025	0.01
A	1	-4.661	-4.938	-5.173	-5.338
	2	-5.467	-5.685	-5.965	-6.162
	3	-6.265	-6.529	-6.757	-6.991
	4	-6.832	-7.104	-7.361	-7.560
	5	-7.398	-7.636	-7.963	-8.248
B	1	-4.144	-4.495	-4.696	-5.014
	2	-4.784	-5.096	-5.333	-5.616
	3	-5.429	-5.726	-6.010	-6.286
	4	-5.999	-6.305	-6.497	-6.856
	5	-6.417	-6.717	-6.998	-7.395
C	1	-4.820	-5.081	-5.297	-5.704
	2	-5.847	-6.113	-6.344	-6.587
	3	-6.686	-7.006	-7.216	-7.401
	4	-7.426	-7.736	-7.998	-8.243
	5	-8.016	-8.343	-8.593	-9.039

3 Monte Carlo study

In this Section we investigate the small sample properties of the new tests. Both the size and power properties are investigated. To retain the brevity of the analysis we concentrate on model A. Results for models B and C are similar. The size properties are investigated by simulating standard random walks. We undertake four data generation processes (DGP) to investigate the power properties. These processes are stable AR(1) processes with intercept and trend. Breaks occur in the intercept, μ_0 . μ_1 is set to 0.2 for all DGPs. We impose 1,2 and 3 breaks to the processes. The coefficients are given in Table 2.

Table 2: Coefficients for DGPs using model A.

DGP	α	μ_0	ϕ_1	ϕ_2	ϕ_3
1	0.9	0.5	0.5	0.5	0.5
2	0.9	0.5	1	1	1
3	0.7	0.5	0.5	0.5	0.5
4	0.7	0.5	1	1	1

The disturbances for all simulated processes are drawn from the standard normal distribution. Samples of sizes 50,100,150 and 200 are examined for the investigation of the size properties and samples of size 50,150 and 250 are examined for the investigation of the power properties. For the stable AR processes, the effect of initial conditions which are set to 0 is minimised by rejecting the first 200 observations of the simulated sample. For all experiments 1000 replications are carried out. m takes values from 1 to 5. For these size experiments and the power experiments that follow we assume that k is known and equal to 0. The estimated sizes for all tests are given in Table 3.

As expected, given that simulated critical values are used, all tests have good size properties. The finite sample power for the tests is presented in Figure 1. Each row of graphs presents the power of the tests for a given DGP. Each column presents the power for DGPs with 1,2 and 3 breaks respectively.

Table 3: Estimated Size of τ_{\min}^m , $m = 1, \dots, 5$.

m	Sample size			
	50	100	150	200
1	0.068	0.067	0.068	0.071
2	0.073	0.050	0.060	0.056
3	0.079	0.058	0.064	0.048
4	0.088	0.061	0.061	0.045
5	0.095	0.070	0.080	0.057

Each graph presents the power of τ_{\min}^m for different sample sizes.

The power properties of the tests are in accord with expectations. The further away the alternative is from the unit root null, in terms of coefficient values, the higher the power of the tests. The same holds for larger numbers of observations. It is interesting to note that the higher the maximum number of breaks, m the lower the power of the test in general. The reduction in power with m is more pronounced for larger sample sizes. Clearly, as we successively allow for alternative models closer to the null hypothesis, as is the case when more breaks are allowed, the lower the power of the test. This phenomenon is not apparent for samples of 50 observations. Finally, we note the reduction in power when more breaks are introduced into the DGPs.

We carry out a final and crucial simulation experiment to determine the size properties of the tests when k is not assumed known but is determined by sequential testing as in Zivot and Andrews (1992) and with maximum possible $k = 4$. The rest of the experiment design is as before. Estimated sizes are presented in Table 4. Clearly, although there is considerable over-rejection in small samples, the test improves its performance rapidly as the number of observations increases.

Table 4: Estimated Size of τ_{\min}^m , $m = 1, \dots, 5$ using sequential testing to determine k .

m	Sample size			
	50	100	150	200
1	0.151	0.138	0.113	0.094
2	0.149	0.121	0.104	0.066
3	0.144	0.113	0.097	0.063
4	0.148	0.126	0.104	0.059
5	0.150	0.132	0.118	0.056

4 Application

We apply the new tests to the extended Nelson and Plosser series². In doing so, we follow Zivot and Andrews (1992) and Lumsdaine and Papell (1997) who applied their tests of unit root tests to the original Nelson and Plosser data. The tests for models A and C are applied. We also apply the tests by Zivot and Andrews (1992) and Lumsdaine and Papell (1997) to the series so as to compare the outcomes of the tests. For all tests and series we use a maximum of $k = 8$ for the order of the polynomial of the lagged differences of the series entering the models. Then, the lag is chosen by successively decreasing the order of the polynomial as long as the t-statistic of the last lag is insignificant. The critical value for the sequential tests is 1.6. The results are presented in Tables 5 and 6.

Broadly speaking the ZA, LP, and τ_{\min}^2 produce similar results. However as m increases the rejection of the null becomes more difficult. This is in accordance with the results of the Monte Carlo exercise. More specifically for model A we note that τ_{\min}^2 rejects the null of a unit root for 8 series. All other tests reject the null hypothesis for a smaller number of series. The situation changes slightly for model C where allowing for more breaks leads to fewer rejections.

²The data have been obtained from Ramsey and Rothman (1996).

Table 5: τ_{\min}^m for model A on extended Nelson and Plosser series

Series ^a	ZA ^b	LP ^c	τ_{\min}^2	τ_{\min}^3	τ_{\min}^4
Real GNP	-5.026**	-7.549***	-7.549***	-7.700***	-7.700***
Nominal GNP	-5.422***	-6.696**	-6.696***	-6.943**	-6.943*
Real Per capita GNP	-4.728*	-7.459***	-7.459***	-7.724***	-7.724***
Industrial Production	-4.363	-6.234*	-5.583*	-5.583	-5.583
Employment	-5.620***	-7.449***	-7.449***	-7.730***	-7.730***
Unemployment	-5.416***	-7.374***	-7.034***	-8.449***	-8.842***
GNP deflator	-3.258	-4.560	-4.024	-5.534	-5.891
CPI	-2.861	-3.664	-3.038	-3.038	-3.658
Nominal wage	-5.633***	-6.144*	-6.144**	-6.144	-6.144
Real wage	-3.158	-4.810	-4.159	-4.159	-4.159
Money supply	-4.739*	-5.591	-5.591*	-5.889	-5.889
Velocity	-2.810	-3.757	-3.308	-3.308	-3.308
Bond yields	-3.439	-4.244	-4.244	-4.585	-4.792
S&P 500	-3.554	-4.962	-4.613	-5.463	-5.463
Number of Rejections	7	7	8	5	5

^aSingle, double and triple starred entries indicate rejection of the unit root null hypothesis at the 10%, 5% and 1% significance level

^bZivot and Andrews (1992) statistic

^cLumsdaine and Papell (1997) statistic

5 Conclusions

In this paper we provide a new computationally efficient test for the null hypothesis of a unit root against the alternative hypothesis of an unspecified number of structural breaks. Critical values are provided and a Monte Carlo study of the small sample properties of the test is carried out.

The new test provides a substantial extension of existing techniques. As Lumsdaine and Papell (1997) point out, testing the null of a unit root against a specified number of structural breaks is a narrow consideration which should be abandoned in favour of model selection strategies which are less dependent on an assumed number of breaks. The present test fills this need. Of course, as the study of the power properties of the test indicate

Table 6: τ_{\min}^m for model C on extended Nelson and Plosser series

Series ^a	ZA ^b	LP ^c	τ_{\min}^2	τ_{\min}^3	τ_{\min}^4
Real GNP	-5.548**	-7.023**	-6.694***	-7.148*	-7.148
Nominal GNP	-5.831***	-7.358***	-7.358***	-7.366**	-7.366
Real Per capita GNP	-6.1046***	-7.463***	-6.531**	-6.668	-7.310
Industrial Production	-4.835*	-7.071**	-6.448**	-6.448	-7.133
Employment	-5.606***	-7.028**	-7.028***	-7.028**	-7.028
Unemployment	-5.502**	-7.234**	-7.167***	-7.439***	-7.439*
GNP deflator	-3.633	-4.824	-4.561	-5.833	-8.177**
CPI	-3.183	-4.889	-3.686	-5.549	-5.549
Nominal wage	-5.498**	-6.308	-5.751	-5.751	-5.751
Real wage	-3.229	-6.145	-5.513	-5.513	-5.513
Money supply	-4.542	-6.168	-5.764	-5.764	-5.764
Velocity	-4.540	-6.898**	-4.825	-6.030	-8.224**
Bond yields	-3.917	-5.002	-4.408	-4.513	-4.513
S&P 500	-5.098**	-5.719	-5.719	-5.719	-5.719
Number of Rejections	8	7	6	4	3

^aSingle, double and triple starred entries indicate rejection of the unit root null hypothesis at the 10%, 5% and 1% significance level

^bZivot and Andrews (1992) statistic

^cLumsdaine and Papell (1997) statistic

the maximum number of breaks still has a significant impact on the test outcome. Nevertheless, this simply reflects the fact that the distance between the null and the alternative hypotheses depends on the number of maximum breaks. Clearly no statistical technique can provide a valid analysis of the data without proper consideration of the underlying economic phenomena. Such consideration should provide an indication of the appropriate maximum number of breaks to use in the statistical analysis.

6 Appendix

For a matrix A , let $v(A)$ be defined by the relation $D_{2m+3}v(A) = \text{vec}(A)$ where D_n is the $n^2 \times 1/2n(n+1)$ duplication matrix as defined in Magnus

and Neudecker (1988, pp. 49). Let $\delta_{i,j} = \max(\delta_i, \delta_j)$, $i, j = 1, \dots, m$. Then

$$\begin{aligned}
v(\Lambda) = & \left(1, \frac{1}{2}, \int_0^1 H(r)dr, 1 - \delta_1, \dots, 1 - \delta_m, \frac{(1 - \delta_1)^2}{2}, \dots, \frac{(1 - \delta_m)^2}{2}, \right. \\
& \frac{1}{3}, \int_0^1 rH(r)dr, \frac{1 - \delta_1^2}{2}, \dots, \frac{1 - \delta_m^2}{2}, \frac{\delta_1^3 - 3\delta_1 + 2}{6}, \dots, \frac{\delta_m^3 - 3\delta_m + 2}{6}, \\
& \int_0^1 H(r)^2 dr, \int_{\delta_1}^1 H(r)dr, \dots, \int_{\delta_m}^1 H(r)dr, \int_{\delta_1}^1 (r - \delta_1)H(r)dr, \dots, \int_{\delta_m}^1 (r - \delta_m)H(r)dr, \\
& 1 - \delta_1, 1 - \delta_{1,2} \dots, 1 - \delta_{1,m}, \frac{(1 - \delta_1)^2}{2}, \frac{1 - \delta_{1,2}^2}{2} - \delta_1(1 - \delta_{1,2}), \dots, \frac{1 - \delta_{1,m}^2}{2} - \delta_1(1 - \delta_{1,m}), \\
& 1 - \delta_2, 1 - \delta_{2,3} \dots, 1 - \delta_{2,m}, \frac{1 - \delta_{2,1}^2}{2} - \delta_2(1 - \delta_{2,1}), \frac{(1 - \delta_2)^2}{2}, \dots, \frac{1 - \delta_{2,m}^2}{2} - \delta_2(1 - \delta_{2,m}), \\
& \dots, \\
& 1 - \delta_{m-1}, 1 - \delta_{m-1,m} \frac{1 - \delta_{m-1,1}^2}{2} - \delta_{m-1}(1 - \delta_{m-1,1}), \dots, \frac{1 - \delta_{m-1,m}^2}{2} - \delta_{m-1}(1 - \delta_{m-1,m}), \\
& 1 - \delta_m \frac{1 - \delta_{m,1}^2}{2} - \delta_m(1 - \delta_{m,1}), \dots, \frac{(1 - \delta_m)^2}{2}, \\
& \frac{(1 - \delta_1)^3}{3}, \frac{1 - \delta_{1,2}^3}{3} - \frac{(\delta_1 + \delta_2)(1 - \delta_{1,2}^2)}{2} + \delta_1\delta_2(1 - \delta_{1,2}), \dots, \\
& \frac{1 - \delta_{1,m}^3}{3} - \frac{(\delta_1 + \delta_m)(1 - \delta_{1,m}^2)}{2} + \delta_1\delta_m(1 - \delta_{1,m}), \\
& \dots, \\
& \frac{(1 - \delta_{m-1})^3}{3}, \frac{1 - \delta_{m-1,m}^3}{3} - \frac{(\delta_{m-1} + \delta_m)(1 - \delta_{m-1,m}^2)}{2} + \delta_{m-1}\delta_m(1 - \delta_{m-1,m}), \\
& \left. \frac{(1 - \delta_m)^3}{3} \right)
\end{aligned}$$

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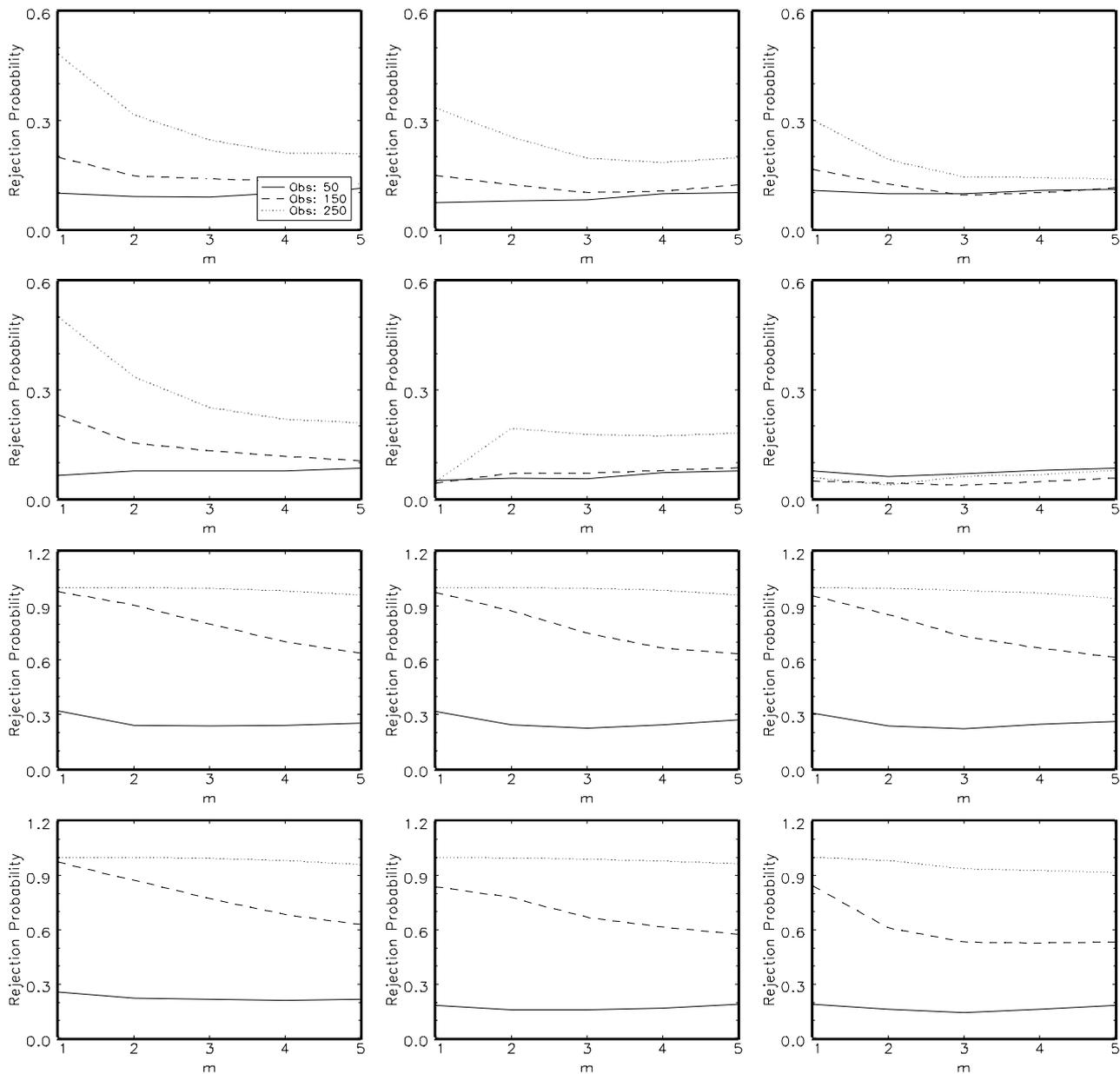
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Figure 1: Finite sample power of τ_{\min}^m , $m = 1, \dots, 5$, Model A.



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