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## Long Memory and FIGARCH Models for Daily and High Frequency Commodity Prices

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**LONG MEMORY AND FIGARCH MODELS FOR DAILY AND HIGH  
FREQUENCY COMMODITY PRICES**

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## **Abstract**

Daily futures returns on six important commodities are found to be well described as FIGARCH fractionally integrated volatility processes, with small departures from the martingale in mean property. The paper also analyzes several years of high frequency intra day commodity futures returns and finds very similar long memory in volatility features at this higher frequency level. Semi parametric Local Whittle estimation of the long memory parameter supports the conclusions. Estimating the long memory parameter across many different data sampling frequencies provides consistent estimates of the long memory parameter, suggesting that the series are self-similar. The results have important implications for future empirical work using commodity price and returns data.

**Keywords:** Commodity returns, Futures markets, Long Memory, FIGARCH

**JEL Classification:** C4, C22.

## 1. Introduction

This paper is concerned with the stochastic properties of commodity futures prices and applies some recent developments in volatility modeling, in particular the FIGARCH long memory volatility model, to commodity futures returns. The volatilities of daily futures returns are found to be well described by the FIGARCH model, with relatively similar estimates of the long memory parameter across commodities. The conditional means of the daily returns are close to being uncorrelated with small departures from martingale behavior being represented by low order moving average models. We also estimate FIGARCH models for high frequency commodity futures returns based on intra day tick data. These high frequency commodity returns are dominated by strong intra day periodicity, hypothesized to be a result of repeated trading day cycles resulting from the institutional features of futures exchanges where trades are taking place. The intra day periodicity is removed using a deterministic Flexible Fourier Form (FFF) filter. The filtered high frequency futures returns are also well described by the FIGARCH process. The results of the paper have important implications for our understanding of the stochastic properties of commodity prices, and hence for empirical applications such as optimal hedge ratio estimation, tests for futures market efficiency, tests for the announcement effect of market news, option valuation, farm risk portfolio management, etc.

The FIGARCH model has already been applied gainfully to exchange rates, stock returns, inflation rates, and a range of other economic data; for example see Baillie, Bollerslev and Mikkelsen (1996), Bollerslev and Mikkelsen (1996), Baillie, Han and Kwon (2002), etc. However, there have been few applications of the model to commodities. Crato and Ray (2000) study long memory in the daily volatilities of several agricultural commodity futures returns, along with a stock index return, currencies, metals, and heating oil. They find strong evidence of long memory in daily commodity futures prices, though they do not explicitly estimate FIGARCH models. Jin and Frechette (2004) estimate FIGARCH volatility models for 14 agricultural futures series and find that FIGARCH fits the data significantly better than a traditional GARCH volatility model. While these studies have provided valuable information on the long memory properties of commodity futures price volatilities, much more work remains to be done.

This paper adds to our understanding of long memory in commodity price volatilities in three main ways. First, while Jin and Frechette (2004) argue in favor of the FIGARCH model over the GARCH model for commodity futures volatilities, they did not undertake a formal statistical test comparing the two models. Here we undertake a robust Wald test, which formally compares the fit of the GARCH and FIGARCH models. Second, in addition to the standard quasi-maximum likelihood estimator (QMLE), we also apply the semi parametric Local Whittle estimator of the long memory parameter. This provides additional information on the robustness of long-memory inferences concerning daily commodity price volatilities. Third, in addition to daily returns we study high frequency returns on futures contracts using intra day tick data. This study is the first to systematically examine volatility using high frequency commodity futures data.<sup>1</sup> We find that estimated models at different sampling frequencies are consistent with the theory that commodity futures returns are “self similar” processes, and hence have long memory parameters that are invariant to the sampling frequency; see Beran (1994). The “self-similarity” of the estimates of the long memory volatility parameter across relatively short spans of high frequency data strongly suggests that the long memory property is an intrinsic feature of the system rather than being due to exogenous shocks or regime shifts.

The plan of the rest of the paper is as follows. Section 2 discusses the application of the long memory FIGARCH volatility model to daily futures returns. Similar to Jin and Frechette (2004), we find the FIGARCH models to be econometrically superior to regular stable GARCH models. Section 3 describes the results from the analysis of high frequency futures returns and compares them to the daily return results. Section 4 presents an analysis of semi parametric Local Whittle estimation of the long memory parameter as a robustness check, and also compares estimates of the long memory parameter across a range of different sampling frequencies. This shows that the commodity return series display self similarity. Section 5 offers a brief conclusion.

## **2. Analysis of Daily Commodity Returns**

This section is concerned with the analysis of daily futures returns for different

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<sup>1</sup> Cai, Cheung and Wong (2001) have analyzed high frequency gold futures. However, their approach is somewhat

commodities. We examine six commodities; corn, soybeans, cattle, hogs, gasoline, and gold. Corn and soybeans are major annual crops that are of critical importance to U.S. agriculture. These crops are related in the sense they can be substitutes in production and both are used heavily as animal feed. They are different, however, in that most corn is produced in the northern hemisphere while soybeans have a significant southern hemisphere harvest in Brazil and Argentina. This southern hemisphere harvest may influence seasonal price and volatility patterns. Cattle and hogs are both important livestock commodities in U.S. agriculture but their different life cycles mean different inherent price dynamics, even though we would expect a lot of similarity in the stochastic properties of prices for these two livestock commodities. Gasoline is included to see if results are markedly different for a natural resource based commodity, and gold is included as a commodity that has a central role as a store of wealth.

Data were obtained from the Futures Industry Institute data center.<sup>2</sup> The daily data are daily closing futures prices on major U.S. futures markets for the relevant commodity; in particular, the Chicago Board of Trade for corn and soybeans, the Chicago Mercantile Exchange for live cattle and hogs, and the New York Mercantile Exchange for unleaded gasoline and gold. Returns are defined in the conventional manner as continuously compounded rates of return and calculated as the first difference of the natural logarithm of prices. To compute the futures returns, nearby contracts were used and then the data switched to the next available contract nearby on the first day of the month in which the current nearby contract expires. For consistency, returns are always defined using the same futures contract.<sup>3</sup> The use of nearby futures contracts to define our futures return series, has the advantage that we are using the most actively traded contracts to generate our return data. However, if volatility depends on time to maturity, as might be expected in at least some instances, then switching from an expiring futures contract to the next nearby maturity may introduce jumps into the volatility process (because of jumps in time to maturity at the switch

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informal and does not include either FIGARCH or Local Whittle estimation of the long memory parameter.

<sup>2</sup> The Futures Industry Institute is now called the Institute for Financial Markets. For more information and data availability see <http://www.theifm.org>.

<sup>3</sup> That is, at each point when the data switch to the next nearby maturing contract the futures return is defined as the difference between the natural logarithm of today's futures price for a contract maturing at the next nearby and yesterday's futures price for a contract with exactly the same maturity date. In this way, daily returns are never defined using prices from two different contracts with different maturity dates.

points). We will discuss how we allowed for the effects of these jumps in time to maturity when we outline the econometric model further below.

The details of the sample periods used for each commodity are provided in Table 1, along with some summary statistics for daily returns over these periods. All the daily data begin at the first trading day of January 1980, except for gasoline. For gasoline, we exclude data from January 1980 through December 1990 and begin the sample period the first trading day of January 1991. This is to avoid two periods of exceptional volatility in gasoline prices that we argue are a result of structural shifts in the volatility process for this commodity. The first period is 1986-87, a period in which Saudi Arabia expanded its oil production significantly to discipline other OPEC countries. The second period extends from August 1990 to December 1990 and is due to the Iraqi invasion of Kuwait and the subsequent Gulf War. By starting the gasoline price series in January of 1991 we avoid having to model these structural breaks in the volatility process. All of the daily data end at the last trading day of December 2000, except for corn which ends the last trading day in March of 2001. In all cases we used the most recent data that was provided in the data set obtained from the Futures Industry Institute.

Previous studies by Cecchetti, Cumby and Figlewski (1988), Baillie and Myers (1991), and Yang and Brorsen (1992) have argued that most daily cash and futures commodity returns are well described as martingales with GARCH effects. The possibility of mixed diffusion-jump processes has also been suggested as a way to characterize volatility in commodity prices. Yang and Brorsen (1992) compared GARCH, mixed diffusion-jump, and deterministic chaos models of cash commodity prices and concluded that the GARCH volatility process provided the best fit. It is only more recently that studies such as Crato and Ray (2000) and Jin and Frechette (2004) have begun to investigate the long memory properties of commodity volatilities.

Figures 1 and 2 plot the sample autocorrelations for the returns, squared returns and absolute returns in daily futures prices for two representative commodities; namely live cattle and corn. There is one noticeable difference between the crop commodity and the livestock commodity, namely that both squared and absolute daily returns for corn exhibit strong yearly seasonality in their sample autocorrelations while this does not occur for live cattle. To conserve space the corresponding graphs for the other commodities are not shown. However, it was observed that

soybeans also display seasonality in volatility (though not as pronounced as in the case of corn, perhaps because of the influence of a southern hemisphere harvest for soybeans) while live hogs, gasoline and gold display no seasonality in volatility (similar to live cattle). In order to analyze the intrinsic stochastic properties of the daily corn and soybean return volatilities we filter out the seasonality by using a FFF filter.<sup>4</sup> The sample autocorrelations for the returns, squared returns and absolute returns for the filtered daily corn futures price series is provided in Figure 3. Notice that the FFF filter has been quite effective in removing the seasonality in the squared and absolute corn futures returns. In all subsequent analysis of the corn and soybean return volatilities we use the filtered volatility models.

Plots of the live cattle sample autocorrelations (Figure 1), the FFF filtered corn sample autocorrelations (Figure 3), and other commodity return sample autocorrelations (not shown) reveal a familiar lack of autocorrelation in returns and the marked persistence in autocorrelations of squared and absolute returns that was first noticed by Ding, Granger and Engle (1993) for the case of stock market returns. In particular, the autocorrelation functions for the squared and absolute returns do not display the usual exponential decay associated with the stationary and invertible class of ARMA models, but rather appear to be generated by a long memory process with hyperbolic decay.

More formally, the autocorrelation at lag  $k$ ,  $\rho_k$ , tends to satisfy  $\rho_k \approx ck^{2d-1}$  as  $k$  gets large, where  $c$  is a constant and  $d$  is the long memory parameter. This type of persistence is consistent with the notion of hyperbolic decay and is sometimes called the “Hurst phenomenon.” The Hurst coefficient is defined as  $H = d + 0.5$ . If  $d = 1$ , so that  $H = 1.5$ , then the autocorrelation function does not decay and the series has a unit root. If  $d = 0$ , so that  $H = 0.5$ , then the autocorrelation function decays exponentially and the series is stationary. But for  $0 < d < 1$ , i.e.  $0 < H < 1.5$ , the series is sufficiently flexible to allow for slower hyperbolic rates of decay in the autocorrelations.

While many stochastic processes could potentially exhibit the long memory property, the most widely used process is the ARFIMA( $p, d, q$ ) process of Granger and Joyeux (1980), Granger (1980), and Hosking (1981). In the ARFIMA process a time series  $x_t$  is modeled as

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<sup>4</sup> See the appendix for details of the FFF filter.



$a(L)(1-L)^d x_t = b(L)\varepsilon_t$  with  $a(L)$  and  $b(L)$  being  $p$ 'th and  $q$ 'th order polynomials in the lag operator  $L$ , with all their roots lying outside the unit circle, while  $\varepsilon_t$  is a white noise process. The ARFIMA process is stationary and invertible in the region of  $-0.5 < d < 0.5$ . At high lags the ARFIMA( $p, d, q$ ) process is known to have an autocorrelation function that satisfies  $\rho_k \approx ck^{2d-1}$ , so that the autocorrelations may decay at a slow hyperbolic rate, as opposed to the required exponential rate associated with the stationary and invertible class of ARMA models. The sample autocorrelation function of the squared and absolute daily filtered futures corn returns appears to be very consistent with the above properties and analogous plots for the other commodity returns were found to be extremely similar.

Virtually all studies of daily asset returns, including commodity assets, have found returns,  $y_t$ , to be stationary with small autocorrelations at the first few lags, which can be attributed to a combination of a small time-varying risk premium, bid-ask bounce, and/or non-synchronous trading phenomena; see Goodhart and O'Hara (1997) for a description of this issue in high frequency currency markets. On the other hand, volatility has been found to be very persistently autocorrelated with long memory hyperbolic decay. A model that is consistent with these stylized facts is the MA( $n$ )-FIGARCH( $p, d, q$ ) process,

$$y_t = 100\Delta \ln(P_t) = \mu + b(L)\varepsilon_t, \quad (1)$$

$$\varepsilon_t = z_t \sigma_t, \quad (2)$$

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \{1 - \beta(L)\} - \phi(L)(1-L)^d] \varepsilon_t^2 \quad (3)$$

where  $P_t$  is the asset price,  $z_t$  is an i.i.d.(0,1) random variable, the polynomial in the lag operator associated with the moving average process is  $b(L) = 1 + b_1L + b_2L^2 + \dots + b_nL^n$ . The FIGARCH model in equation (3) can be best motivated from noting that the standard GARCH( $p, q$ ) model of Bollerslev (1986) can be expressed as

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2,$$

where the polynomials are  $\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \dots + \alpha_q L^q$  and  $\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \dots + \beta_p L^p$ . The GARCH(p, q) process can also be expressed as the ARMA[max(p, q), p] process in squared innovations as

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]\nu_t$$

where  $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$ , and is a zero mean, serially uncorrelated process which has the interpretation of being the innovations in the conditional variance. The FIGARCH(p, d, q) process in equation (3) can also be written as

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]\nu_t, \quad (4)$$

where  $\phi(L) = [1 - \alpha(L) - \beta(L)](1-L)^{-1}$  is a polynomial in the lag operator of order [max(p, q)-1]. Equation (4) can be easily shown to transform to equation (3), which is the standard representation for the conditional variance in the FIGARCH(p, d, q) process. Further details concerning the FIGARCH process can be found in Baillie, Bollerslev and Mikkelsen (1996). The parameter  $d$  characterizes the long memory property of hyperbolic decay in volatility because it allows for autocorrelations to decay at a slow hyperbolic rate. The attraction of the FIGARCH process is that for  $0 < d < 1$ , it is sufficiently flexible to allow for intermediate ranges of persistence, between complete integrated persistence of volatility shocks associated with  $d = 1$  and the geometric decay associated with  $d = 0$ .

The volatility model in equation (3) has to be slightly adjusted to accommodate the potential jumps in volatility that can occur at contract switching points, when futures return data are computed from a sequence of nearby futures contracts. The long spans of daily futures returns are constructed from contracts with different maturities and the resulting variations (and jumps) in time to maturity may have an influence on the volatility process. To account for

possible time to maturity effects we introduce a time to maturity variable in the formulation of the FIGARCH(1, d, 1) model in (3), which then becomes,

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma TM_t + [1 - \beta L - (1 - \phi L)(1 - L)^d] \varepsilon_t^2 \quad (5)$$

where  $TM$  represents the time to maturity on the contract used to construct the futures return for period  $t$ , and  $\gamma$  is the associated parameter.

The above model (1), (2), and (5) is estimated for futures returns on our six commodities of interest by maximizing the Gaussian log likelihood function,

$$\ln(L; \Theta) = -(0.5T) \ln(2\pi) - 0.5 \sum_{t=1}^T [\ln(\sigma_t^2) + \varepsilon_t^2 \sigma_t^{-2}], \quad (6)$$

where  $\Theta' = (\mu, \theta_1, \dots, \theta_n, \omega, \beta_1, \dots, \beta_p, \phi_1, \dots, \phi_l)$  is the vector of unknown parameters. However, it has long been recognized that most asset returns are not well represented by assuming  $z_t$  in equation (2) is normally distributed; for example see McFarland, Pettit and Sung (1982), and Booth (1987). Consequently, inference is usually based on the quasi maximum likelihood estimator (QMLE) of Bollerslev and Wooldridge (1992), which is valid when  $z_t$  is non-Gaussian. Denoting the vector of parameter estimates obtained from maximizing (6) using a sample of  $T$  observations on equations (1), (2) and (5) with  $z_t$  being non-normal by  $\hat{\Theta}_T$ , then the limiting distribution of  $\hat{\Theta}_T$ , is then

$$T^{1/2}(\hat{\Theta}_T - \Theta_0) \rightarrow N[0, A(\Theta_0)^{-1} B(\Theta_0) A(\Theta_0)^{-1}], \quad (7)$$

where  $A(\cdot)$  and  $B(\cdot)$  represent the Hessian and outer product gradient respectively, and  $\Theta_0$  denotes the vector of true parameter values. Equation (7) is used to calculate the robust standard errors that are reported in the subsequent results in this paper, with the Hessian and outer product gradient

matrices being evaluated at the point  $\hat{\Theta}_T$  for practical implementation.

[Table 2 about here]

Table 2 presents the results of applying the above model (1), (2), and (5) to daily futures returns for the six commodities discussed earlier. The exact parametric specification of the model which best represents the degree of autocorrelation in the conditional mean and conditional variance of daily commodity returns, varies by commodity. The exact model specification for each commodity is indicated by the number of non-zero estimates provided for the polynomial in the lag operator terms in Table 2. For corn and soybean futures returns, we apply FIGARCH estimation to the FFF filtered returns (see the Appendix). At the bottom of the table there are results from Box-Pierce portmanteau statistics on the standardized residuals. The

standard portmanteau test statistic,  $Q(m) = T(T+2) \sum_{j=1}^m r_j^2 / (T-j)$ , where  $r_j$  is the  $j$ 'th order

sample autocorrelation from the residuals, is known to have an asymptotic  $\chi_{m-k}^2$  distribution, where  $k$  is the number of parameters estimated in the conditional mean. Similar degrees of freedom adjustments are used for the portmanteau test statistic based on the squared standardized residuals when testing for omitted conditional heteroscedasticity. This adjustment is in the spirit of the suggestions by Diebold (1988) and others. The sample skewness and kurtosis of the standardized residuals ( $m_3$  and  $m_4$ ), are also provided at the bottom of Table 2.

The Box-Pierce portmanteau statistics show that the models specified for each commodity do a good job of capturing the autocorrelations in the mean and volatility of the commodity return series. In each case there is no evidence of additional autocorrelation in the standardized residuals or squared standardized residuals, indicating that the chosen model specification provides an adequate fit. It is interesting that autocorrelation in the mean tends to persist more for the livestock commodities of live cattle and hogs than for the other commodities (i.e. more MA terms in the mean required for an adequate fit). Furthermore, these commodities also seem to require more flexible models to capture their autocorrelation in

volatility as well (i.e. more GARCH terms required for an adequate fit). The standardized residuals from all commodities, except perhaps live cattle and hogs, exhibit the usual features of excess kurtosis of daily asset returns. However, this is accommodated through use of the QMLE standard errors for inference.

The estimated MA-FIGARCH models reported in Table 2 seem to fit the data well. For each commodity there is weak evidence of small moving average effects in the mean returns. As stated earlier, this may be attributed to a combination of a small time-varying risk premium, bid-ask bounce, and/or non-synchronous trading phenomena. The volatility autocorrelation parameters in  $\beta(L)$  and  $\phi(L)$  indicate strong evidence of significant serial correlation in volatilities, which is consistent with previous findings of autocorrelated volatility in commodity returns; see Baillie and Myers (1991), Jin and Frechette (2004); and Yang and Brorsen (1992). Furthermore, the time to maturity parameter is statistically significant for all commodities except gold. Gold may not experience a time to maturity effect in volatility because its special role as a store of wealth means that cash and futures prices move very closely together, irrespective of the time to maturity on the futures contract. It is interesting that the time to maturity effect is negative for corn, soybeans and gasoline, but positive for cattle and hogs. This indicates that the upward jumps in time to maturity that occur at contract switching points reduce the volatility of returns for corn, soybeans, and gasoline, but increase volatility in live cattle and hogs. Apparently, live cattle and hogs are relatively more volatile further away from the maturity date, while corn, soybeans and gasoline are relatively more stable.

In this paper we are primarily interested in the long memory parameter  $d$ . The estimated long memory parameters reported in Table 2 are strongly statistically significant for all six futures return series, and the hypotheses that  $d = 0$  (stationary GARCH) and also  $d = 1$  (integrated GARCH) are consistently rejected for all commodities using standard significance levels. Table 2 also reports robust Wald test statistics, denoted by  $W$ , for testing the null hypothesis of GARCH versus a FIGARCH data generating process. Under the null,  $W$  will have an asymptotic  $\chi_1^2$  distribution and, from Table 2, the GARCH model is rejected for every commodity at standard significance levels. This formal statistical test supports the conclusion obtained both here and in Jin and Frechette (2004) that FIGARCH is superior to GARCH for

modeling the conditional variances of commodity returns. Evidently, long memory is a characteristic feature of daily commodity futures returns, and FIGARCH represents a significant improvement over GARCH.

### **3. Analysis of High Frequency Commodity Returns**

Considerable previous work has examined the properties of high frequency returns in equity and currency markets, but to date very little analysis has been done on high frequency commodity returns. The only study we are aware of is Cai, Cheung and Wong (2001) who studied high frequency gold futures prices. Their study analyzed 5 minute gold futures returns between 1994 and 1997, and they discovered slow hyperbolic decay associated with the autocorrelation function of the returns. However, they used an informal method for approximating the long memory parameter and did not estimate formal FIGARCH models. This section of the paper represents a first attempt at extensive analysis of the volatility properties of high frequency commodity futures returns using FIGARCH models.

The raw futures tick data for the analysis were obtained from the Futures Industry Institute data center along with the daily data (see footnote 2), and correspond to the same six commodities studied in the previous section. The prices are for real-time transaction records, which we initially convert to 5-minute price intervals by using the last price quoted before the end of every 5-minute interval over the trading day. For 5-minute intervals that have no price recorded we linearly interpolate between surrounding intervals to fill in the missing data. As with all high frequency asset price analyses, there are potential problems with data unreliability due to the sheer amount of data being used and the fact that there is considerable noise in the series because of little trade occurring at some of the recorded prices. However, we minimize these problems by running the data through a filter to identify and adjust anomalous observations. This was done by locating return observations greater than 3 standard deviations and evaluating these as possible data errors. A careful check and evaluation of these observations revealed a small number of what appeared to be data errors in the high frequency gold returns. These were then eliminated and replaced with a linearly interpolated value using the two contiguous observations. No errors were detected in high frequency commodity returns

other than gold. Furthermore, instead of analyzing the 5-minute interval data (which will be the most susceptible to data errors and noise) we convert the data to lower frequencies (10-minute for corn and soybeans, and 15-minute for live cattle and hogs, gasoline, and gold) to undertake the analysis. Different intervals were chosen for different commodities because they are traded on markets that have different trading day lengths. Hence, in order to make sure interval returns could be computed that exhausted the recorded daily price change, but did not use consecutive intervals that stretched over two different trading days, it was convenient to use 10-minute intervals for corn and soybeans but 15-minute intervals for live cattle, live hogs, gasoline, and gold.

An interval return during day  $t$  is defined as  $y_{t,n} = 100 [\ln(P_{t,n}) - \ln(P_{t,n-1})]$  where  $P_{t,n}$  is futures price for the  $n$ -th intraday interval during trading day  $t$ . As with many analyses of high frequency asset price returns, it was found that the high frequency commodity returns display considerable intra-day periodicity, which is usually attributed to institutional trading features. This periodicity was removed using the FFF filtering method, which is explained in detail in the Appendix.

Figure 4 plots the sample autocorrelations for lags of up to 5 trading days in 15-minute intervals displayed in the horizontal axis for the absolute returns of the unadjusted (raw) and the filtered 15-minute gasoline futures returns series. The dotted line represents sample autocorrelations for the filtered absolute 15-minute returns while the solid line indicates the autocorrelations for the unfiltered absolute 15-minute returns. The FFF filter seems to remove much of intraday periodicity present in the raw absolute returns. As usual, there is a small negative but significant first-order autocorrelation in returns, which may be due to the non-synchronous trading phenomenon while higher order autocorrelations are not significant at conventional levels. The autocorrelation functions of the absolute returns also exhibit a pronounced U shape, suggesting substantial intra-day periodicity. Similar U-shaped patterns are found in the equity markets (Harris, 1986; Wood et al., 1985; Chang et al., 1995; and Andersen and Bollerslev, 1997a). Unless otherwise indicated, all remaining analyses were done on the filtered series.

The MA-FIGARCH model (1) through (3) was estimated on the filtered high frequency filtered returns. As with the daily data, the orders of the MA and GARCH polynomials in the lag

operator were chosen to be as parsimonious as possible but still provide an adequate representation of the autocorrelation structure of the high frequency data. For the high frequency data MA(1)-FIGARCH(1, $d$ ,1) models proved adequate for all commodities. Long high frequency series were constructed by splicing several nearby futures contracts together, in the same way as described for the daily data. A time to maturity effect in volatility was tested, similar to that found in the daily return series. For the high frequency return data, however, the time to maturity effect was not statistically significant and so the time to maturity effect was restricted to zero. One possible reason is that there are many fewer contract switches in the high frequency series, which combines a smaller number of futures contracts than the daily futures return series. The number of trading days and the number of intra day periods are different across the different commodities and this information is provided in Table 3.

Details of the estimated MA(1)-FIGARCH(1, $d$ ,1) high frequency models for the six commodities are reported in Table 4. All the models have small but significant MA(1) parameter estimates, which is usually attributed to the non-synchronous trading phenomenon. Similar features for high frequency exchange rate returns have been noted by Andersen and Bollerslev (1997a), Goodhart and Figliuoli (1992), Goodhart and O'Hara (1997), and Zhou (1996). The estimated long memory volatility parameter,  $d$ , is in the range between 0.2 and 0.3 for most of the commodities considered and are generally statistically significant.

Similar to the daily return results, we found significant long memory volatility in the high frequency returns data as well. In general, the long memory estimates for intra day return volatilities are slightly lower than those for daily returns. Furthermore, as in the daily return models, the robust Wald statistics in Table 3 show strong evidences in favor of the FIGARCH specification against the GARCH specifications in the high frequency model.

#### **4. Local Whittle Estimation and Self Similarity**

An alternative to the parametric long memory models used so far in this paper, is the application of the semi-parametric, local Whittle estimator for estimation of long memory parameters. The advantage of this estimator is that it allows for quite general forms of short run



dynamics; see Kunch (1987) and Robinson (1995); while ARFIMA and FIGARCH models are potentially sensitive to the specification used to represent the short-run dynamics. Of course, semi-parametric estimation has its own problems as it is very data intensive and often exhibits poor performance in terms of bias and mean square error. The local Whittle estimator is used as a robustness check on the estimation of long memory models derived from the estimation of the FIGARCH models. The long memory parameter inherent in absolute returns is related to, but generally not expected to be identical to the long memory component of the FIGARCH model.

A characteristic of long memory that is independent of parametric model specification is that the spectrum of the series will be given by  $f(\omega) \sim G\omega^{-2d}$ , as  $\omega \rightarrow 0+$  and  $G$  is a constant. This suggests a useful objective function for estimating  $d$  would be (see Robinson, 1995)

$$Q = \ln \left[ (1/m) \sum_{j=1}^m \omega_j^{2d} I(\omega_j) \right] - (2d/m) \sum_{j=1}^m \ln(\omega_j)$$

where  $I(\omega_j)$  is the periodogram of the series at frequency  $\omega_j$ . Solving this objective function numerically gives the Local Whittle estimator of  $d$ . Note that it is not necessary to specify the short run dynamics of the process in order to estimate  $d$  in this framework. As shown by Robinson (1995) and others, the main decision variable is  $m$ , the choice of the number of ordinates of the periodogram. For consistency it is necessary that  $[(1/m) + (m/T)] \rightarrow 0$  as  $T \rightarrow \infty$ . While for asymptotic normality, it is required that  $(1/m) + m^{1+2\beta} [\ln(m)]^2 T^{-2\beta} \rightarrow 0$ , as  $T \rightarrow \infty$ . In the empirical results reported in this paper,  $m$  is chosen as  $T^{0.80}$ . Note that the asymptotic variance of the local Whittle estimator is given by  $(1/4m)$ .

Local Whittle estimation of the long memory volatility parameter  $d$  was applied to both the daily and high frequency returns for all six commodities studied earlier. Furthermore, both MA-FIGARCH and local Whittle estimation of  $d$  were undertaken for a range of alternative frequencies (1 day, 2 day, 3 day, 4 day and 5 day using the daily data and various return frequencies between 10 minutes and 2 hours using the high frequency data). Estimation was undertaken over multiple

frequencies to check for the self similarity feature. Self similarity occurs when the magnitude of the long memory parameter does not change across sampling frequencies; e.g. see Beran (1994). If the long memory parameter is invariant across frequencies then it suggests that the long memory property is an intrinsic feature of the data and does not result from regime shifts or exogenous external shocks. The self similarity property is technically extremely difficult to test empirically. However, one can subjectively evaluate changes in long memory parameter estimates across frequencies to see whether the self similarity feature seems to hold in general.

Results of both FIGARCH and local Whittle estimation of the long memory parameter  $d$  are shown for a range of daily return frequencies in Table 5 and a range of intraday return frequencies in Table 6. Numbers in parentheses below the estimates are the estimated standard errors. The first thing to notice is that FIGARCH and local Whittle estimates of  $d$  appear quite consistent with one another, with  $d$  estimated in the range supporting long memory in commodity return volatilities. Hence, previous conclusions about the existence of the long memory property in commodity return volatilities using FIGARCH appear robust to specification of alternative representations of short-run dynamics. The second point of particular interest in Tables 5 and 6 is that the long memory parameter estimates are generally quite consistent across different return frequencies, irrespective of whether we look at daily returns or intra day returns. This result is consistent with the notion of self similarity and suggests that long memory and hyperbolic decay are intrinsic features of commodity return data.

## **5. Conclusions**

This paper has examined the long memory volatility properties of both daily and high frequency intra day futures returns for six important commodities. The absolute and squared returns all possess very significant long memory features and their volatility processes are found to be well described as FIGARCH fractionally integrated volatility processes. We also find small departures from the martingale in mean property. The long memory property in absolute returns was also undertaken by semi-parametric Local Whittle estimation of the long memory parameter. The estimation of MA-FIGARCH models and the application of the Local Whittle estimators to absolute returns were also computed for a range of different sample frequencies using both the daily

and intra day high frequency returns. The long memory parameter estimates are found to be quite robust both across estimators and across sample frequencies. This is consistent with a finding of self-similarity, which implies that long memory in volatility is a pervasive and consistent feature of commodity returns, and is not just being caused by shocks or regime shifts to the underlying price processes.

Our findings suggest that any future empirical application using daily or intra day commodity futures returns, for example optimal hedge ratio estimation, tests for futures market efficiency, tests for the announcement effect of market news, option valuation, farm risk portfolio management, etc., will need to account for the long memory property in commodity return volatilities.

## Appendix

The regular opening and closing of commodity markets and the institutionalized features of lunch hours, etc gives rise to strong intra day periodicity that is readily observable from the recurrent U shape patterns in the correlograms of the squared and absolute returns data. This is similar but different to the currency markets where world-wide trading occurs. Following Andersen and Bollerslev (1998), we first remove these deterministic intra-day periodicities by applying Gallant's Flexible Fourier Form (FFF) filter-see Gallant (1981) and (1982). The estimated model becomes

$$y_{t,n} = E(y_{t,n}) + (\sigma_t s_{t,n} z_{t,n} N^{-1/2}) \quad (\text{A1})$$

where  $E(y_{t,n})$  is the unconditional mean of returns,  $\sigma_t$  is the conditional variance of daily returns,  $s_{t,n}$  is a deterministic function to represent intra day seasonality,  $z_{t,n}$  is an i.i.d.(0,1) process, which is independent of the daily volatility process  $\sigma_t$ , and  $N$  is the number of return intervals per day. From equation (A1),

$$x_{t,n} = 2 \ln |y_{t,n} - E(y_{t,n})| - \ln(\sigma_t^2) + \ln(N) = \ln(s_{t,n}^2) + \ln(z_{t,n}^2).$$

The observable variable  $x_{t,n}$  is then a nonlinear regression on the time interval  $n$ , and daily volatility  $\sigma_t$ , or

$$x_{t,n} = f(\theta; t, n) + u_{t,n},$$

where  $u_{t,n} = \ln(z_{t,n}^2) - E[\ln(z_{t,n}^2)]$  is an i.i.d.(0,1) process and the functional form for  $f$  is,

$$f(\theta; t, n) = \sum_{j=0}^J \sigma_t^j \left\{ \mu_{0j} + \mu_{1j} \frac{n}{N_1} + \mu_{2j} \frac{n^2}{N_2} + \sum_{p=1, k} [\delta_{c,p} \cdot \cos(p2\pi n / N) + \delta_{s,p} \cdot \sin(p2\pi n / N)] \right\} \quad (\text{A2})$$

where  $N_1 = (1/N) \sum_{i=1}^N i = (N+1)/2$ ,  $N_2 = (1/N) \sum_{i=1}^N i^2 = (N+1)(2N+1)/6$ . On taking the variable  $x_{t,n}$  as the dependent variable, the parameters in the equation (A2) were estimated by OLS. The intra day periodicity for interval n, on day t is then estimated as

$$\hat{s}_{t,n} = T \cdot \left[ \exp(f_{t,n} / 2) \right] / \left[ \sum_{t=1, (T/N)} \sum_{n=1, N} \exp(f_{t,n} / 2) \right]. \quad (\text{A3})$$

The 10 or 15 minute high frequency returns are then filtered by the estimated intra day periodicity series,  $\hat{s}_{t,n}$  to generate the filtered returns, which are defined as

$$\tilde{y}_{t,n} = y_{t,n} / \hat{s}_{t,n}. \quad (\text{A4})$$

**Table 1: Summary Statistics of Returns**

	Corn	Soybean	Cattle	Hogs	Gasoline	Gold
First Day	1/02/80	1/02/80	1/02/80	1/02/80	1/02/91	1/02/80
Last Day	3/30/01	12/29/00	12/29/00	12/29/00	12/29/00	12/29/00
Sample Size	5362	5300	5306	5306	2509	5283
Mean	-0.016	-0.005	0.037	0.042	0.0406	-0.0298
High	8.606	7.806	2.867	6.307	12.107	9.745
Low	-10.472	-11.665	-2.812	-7.632	-30.987	-9.909
Std. Dev.	1.279	1.341	0.898	1.403	1.9594	1.227

**Key: The above statistics refer to  $100\Delta\ln(P_t)$ , where  $P_t$  is the price of the asset in time period  $t$ .**

**Table 2: Estimated MA-FIGARCH Models for Daily Futures Returns**

	Corn	Soybeans	Cattle	Hog	Gasoline	Gold
$\mu$	-0.0171 (0.0152)	-0.0229 (0.0152)	0.0456 (0.0117)	0.0524 (0.0217)	0.0097 (0.0337)	-0.0367 (0.0102)
$\theta$	0.0618 (0.0151)	-0.0220 (0.0144)	*	*	0.0695 (0.0211)	-0.0247 (0.0163)
$d$	0.3154 (0.0362)	0.3451 (0.0493)	0.3718 (0.0422)	0.3687 (0.0609)	0.3179 (0.0577)	0.2969 (0.0261)
$\omega$	0.2036 (0.0473)	0.2727 (0.0607)	0.0185 (0.0141)	0.0621 (0.0386)	0.7625 (0.2151)	0.0399 (0.0288)
$\beta_1$	0.2542 (0.0442)	0.3313 (0.0597)	0.3603 (0.0466)	0.3420 (0.0639)	0.2852 (0.0650)	0.1923 (0.0438)
$\beta_2$	—	—	0.0819 (0.0212)	0.1206 (0.0202)	—	—
$\gamma$	-0.1820 (0.0890)	-0.4218 (0.1226)	0.0701 (0.0383)	0.1933 (0.0994)	-1.0264 (0.2978)	0.0595 (0.0587)
$m_3$	-0.003	0.016	-0.170	-0.142	-0.166	-0.097
$m_4$	4.218	4.917	3.100	3.079	3.916	8.750
Q(20)	20.232	21.446	29.906	17.147	25.765	22.476
Q <sup>2</sup> (20)	30.887	34.976	16.875	21.426	13.407	19.697
W	76.092	48.950	77.698	36.699	30.406	55.693

Key: Robust standard errors based on QMLE are in parentheses below the corresponding parameter estimates. The diagnostic statistics Q(20) and Q<sup>2</sup>(20) are the Ljung-Box statistics based on the first 20 autocorrelations of the standardized residuals and the autocorrelations of the squared standardized residuals respectively. The statistics  $m_3$  and  $m_4$  are the sample skewness and kurtosis respectively of the standardized residuals.

The symbol \* indicates that MA(5) and MA(10) models respectively were estimated for live cattle and live hogs respectively. The parameter estimates are not reported to conserve space.

**Table 3: Summary Statistics for Five Minute Futures Returns**

	Number of trading days	Number of intraday intervals	First time period	Last time period
Corn	471	44	9:40	13:15
Soybeans	409	44	9:40	13:15
Gasoline	401	63	10:00	15:00
Live Cattle	405	45	9:20	13:00
Live Hogs	400	45	9:20	13:00
Gold	401	72	8:30	14:25

	<b>Corn</b>	<b>Soybean</b>	<b>Cattle</b>	<b>Hogs</b>	<b>Gasoline</b>	<b>Gold</b>
First Day	50399	50399	50399	50399	50399	50399
Last Day	33001	122800	122800	122800	122800	122800
Sample Size	20724	17996	18225	18000	25263	25842
Mean	-0.003	-0.001	0.013	0.027	0.033	-0.009
High	14.706	14.721	7.231	22.422	33.416	25.168
Low	-15.783	-14.846	-7.160	-23.530	-32.308	-28.664
Standard Dev.	1.658	1.571	0.819	1.982	2.463	1.052



**Table 4: Estimated MA-FIGARCH model for Filtered High Frequency Futures Returns**

	Corn	Soybean	Cattle	Hog	Gasoline	Gold
Sample frequency	10 min.	10 min.	15 min.	15 min.	15 min.	15 min.
$\mu$	-0.0030 (0.0017)	-0.0023 (0.0021)	0.0031 (0.0015)	0.0091 (0.0032)	0.0145 (0.0039)	-0.0037 (0.0011)
$\theta$	-0.1560 (0.0112)	-0.0659 (0.0120)	-0.0525 (0.0144)	-0.0490 (0.0158)	-0.0274 (0.0127)	-0.0750 (0.0134)
$d$	0.2429 (0.0368)	0.2213 (0.0329)	0.2097 (0.0367)	0.3503 (0.0620)	0.1843 (0.0218)	0.2047 (0.0421)
$\omega$	0.0014 (0.0004)	0.0018 (0.0005)	0.0024 (0.0019)	0.0030 (0.0012)	0.0449 (0.0062)	0.0026 (0.0006)
$\beta$	0.8866 (0.0339)	0.8736 (0.0309)	0.4234 (0.3885)	0.7242 (0.0816)	0.0572 (0.0291)	0.2534 (0.1666)
$\phi$	0.8314 (0.0462)	0.8279 (0.0417)	0.3450 (0.3810)	0.5485 (0.0964)	-----	0.3573 (0.1726)
$m_3$	0.366	0.106	-0.111	-0.199	-0.134	0.048
$m_4$	6.882	7.335	4.728	6.138	5.151	8.508
$Q(20)$	28.673	19.973	17.685	23.944	34.336	25.893
$Q^2(20)$	16.592	15.556	7.917	24.340	17.650	11.566
$W$	43.548	45.173	32.716	31.944	71.753	23.619

Key: As for Table 2

**Table 5: Long Memory Parameter Estimation at Different Daily Sample Frequencies.**

	1 day	2 day	3 day	4 day	5 day
<b>Corn</b>					
FIGARCH	0.3154 (0.0362)	0.2734 (0.0460)	0.3096 (0.0670)	0.3312 (0.0833)	0.2510 (0.0671)
Local Whittle	0.4072 (0.0376)	0.3446 (0.0471)	0.3052 (0.0539)	0.3122 (0.0592)	0.2359 (0.0635)
<b>Soybeans</b>					
FIGARCH	0.3451 (0.0493)	0.3403 (0.0780)	0.4052 (0.1385)	0.3096 (0.0783)	0.3294 (0.0921)
Local Whittle	0.3902 (0.0378)	0.3688 (0.0474)	0.3918 (0.0541)	0.3356 (0.0592)	0.3260 (0.0638)
<b>Live Cattle</b>					
FIGARCH	0.3718 (0.0422)	0.4399 (0.0863)	0.4208 (0.0821)	0.4335 (0.0984)	0.4747 (0.1395)
Local Whittle	0.3866 (0.0378)	0.3383 (0.0472)	0.3361 (0.0539)	0.3234 (0.0592)	0.3226 (0.0603)
<b>Live Hogs</b>					
FIGARCH	0.3687 (0.0609)	0.3041 (0.0578)	0.3085 (0.0659)	0.2900 (0.0935)	0.2411 (0.0805)
Local Whittle	0.4061 (0.0378)	0.3416 (0.0472)	0.3609 (0.0539)	0.2987 (0.0592)	0.2455 (0.0603)
<b>Gasoline</b>					
FIGARCH	0.3179 (0.0577)	0.3140 (0.0707)	0.2851 (0.1041)	0.2999 (0.1430)	0.2052 (0.0874)
Local Whittle	0.2935 (0.0481)	0.2548 (0.0603)	0.2967 (0.0689)	0.2722 (0.0760)	0.2400 (0.0818)
<b>Gold</b>					
FIGARCH	0.2969 (0.0261)	0.3435 (0.0520)	0.2754 (0.0400)	0.3357 (0.0470)	0.3108 (0.0857)
Local Whittle	0.4323 (0.0378)	0.3766 (0.0474)	0.3565 (0.0541)	0.3659 (0.0595)	0.3432 (0.0638)

**Table 6: Long Memory Parameter Estimation at Different Intraday Sample Frequencies.**

<b>Corn</b>	<u>10 min.</u>	<u>20 min.</u>	<u>55 min.</u>	<u>1 hr. 50 min.</u>
FIGARCH	0.2429 (0.0368)	0.1919 (0.0430)	0.2196 (0.0951)	0.0814 (0.1556)
Local Whittle	0.1941 (0.0255)	0.2037 (0.0324)	0.1622 (0.0462)	0.1652 (0.0595)
<b>Soybeans</b>	<u>10 min.</u>	<u>20 min.</u>	<u>55 min.</u>	<u>1 hr. 50 min.</u>
FIGARCH	0.2213 (0.0329)	0.2689 (0.0560)	0.2431 (0.0758)	0.3111 (0.1378)
Local Whittle	0.2533 (0.0268)	0.2365 (0.0340)	0.1706 (0.0486)	0.1448 (0.0625)
<b>Live Cattle</b>	<u>15 min.</u>	<u>25 min.</u>	<u>45 min.</u>	<u>1 hr. 15 min.</u>
FIGARCH	0.2097 (0.0367)	0.2580 (0.0492)	0.2519 (0.0806)	0.2483 (0.0986)
Local Whittle	0.2128 (0.0307)	0.1833 (0.0366)	0.1796 (0.0451)	0.1421 (0.0540)
<b>Live Hogs</b>	<u>15 min.</u>	<u>25 min.</u>	<u>45 min.</u>	<u>1 hr. 15 min.</u>
FIGARCH	0.3503 (0.0620)	0.3987 (0.0835)	0.3936 (0.1127)	0.4045 (0.1405)
Local Whittle	0.2993 (0.0308)	0.3414 (0.0368)	0.2988 (0.0453)	0.2802 (0.0543)
<b>Gasoline</b>	<u>15 min.</u>	<u>35 min.</u>	<u>45 min.</u>	<u>1 hr. 45 min.</u>
FIGARCH	0.1843 (0.0218)	0.2672 (0.0556)	0.2215 (0.0590)	0.2191 (0.0828)
Local Whittle	0.2243 (0.0274)	0.1876 (0.0376)	0.1870 (0.0401)	0.2436 (0.0543)
<b>Gold</b>	<u>15 min.</u>	<u>45 min.</u>	<u>1 hr. 30 min.</u>	<u>2 hr. min.</u>
FIGARCH	0.2047 (0.0421)	0.2870 (0.3087)	0.3167 (0.1180)	0.4403 (0.2742)
Local Whittle	0.3832 (0.0261)	0.3818 (0.0381)	0.2704 (0.0486)	0.2486 (0.0540)

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Figure 1. Autocorrelation of Daily Live Cattle Futures

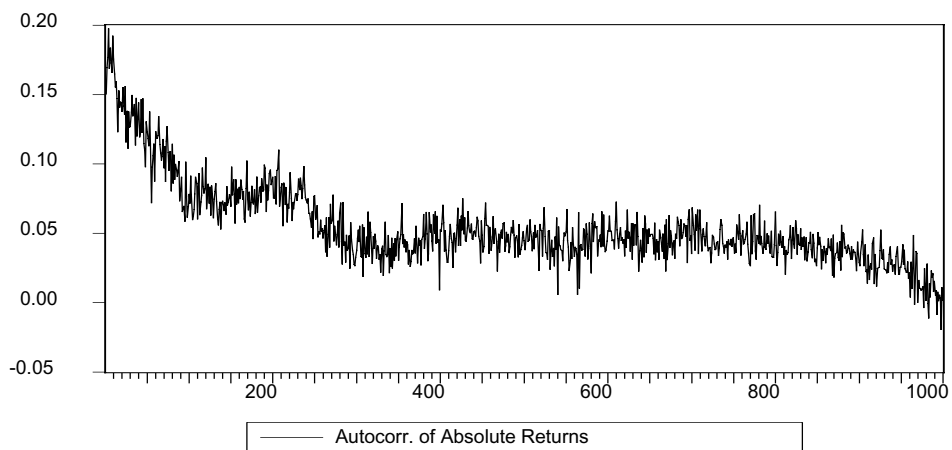
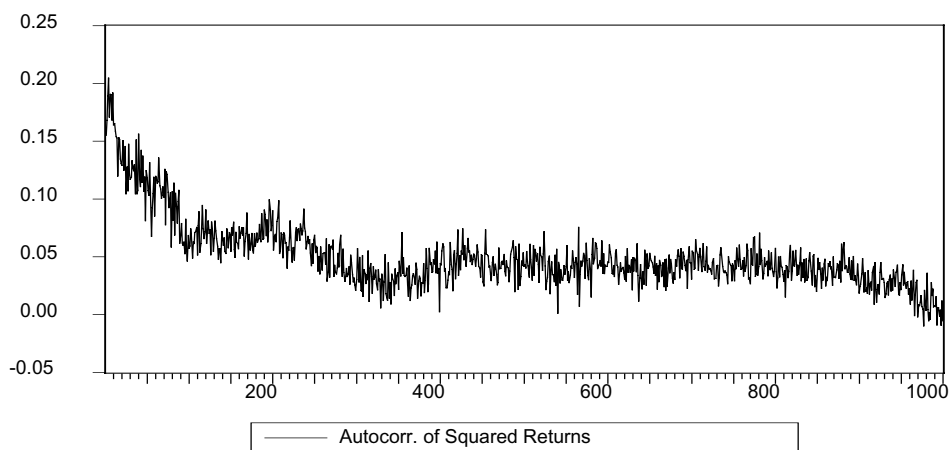
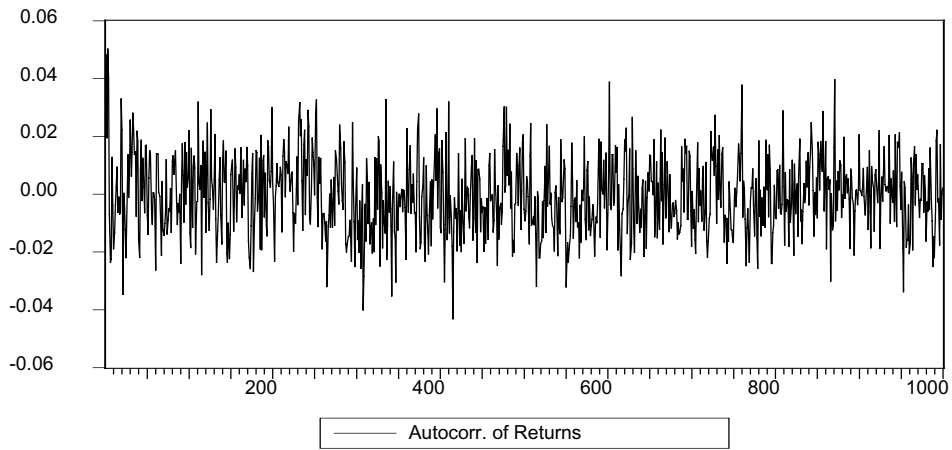


Figure 2. Autocorrelation of Daily Corn Futures

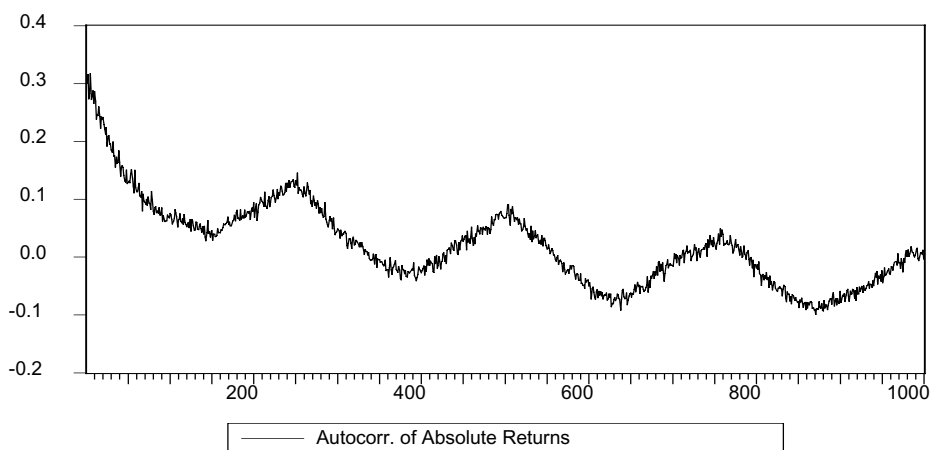
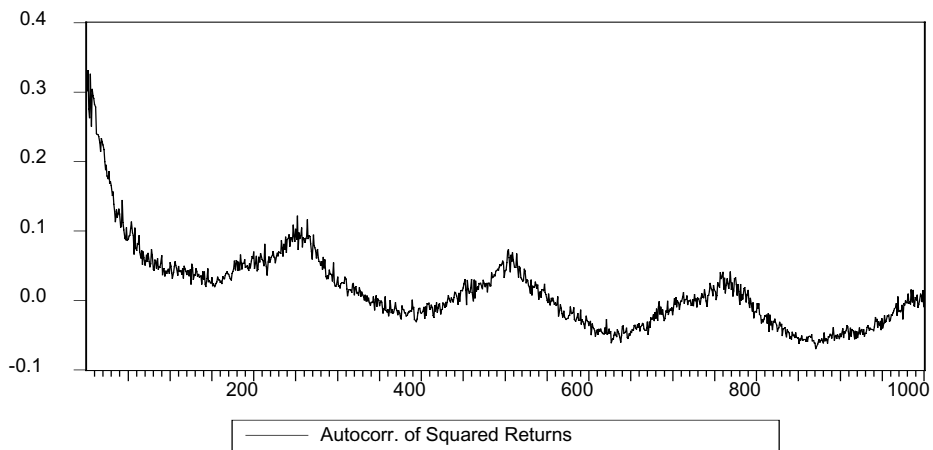
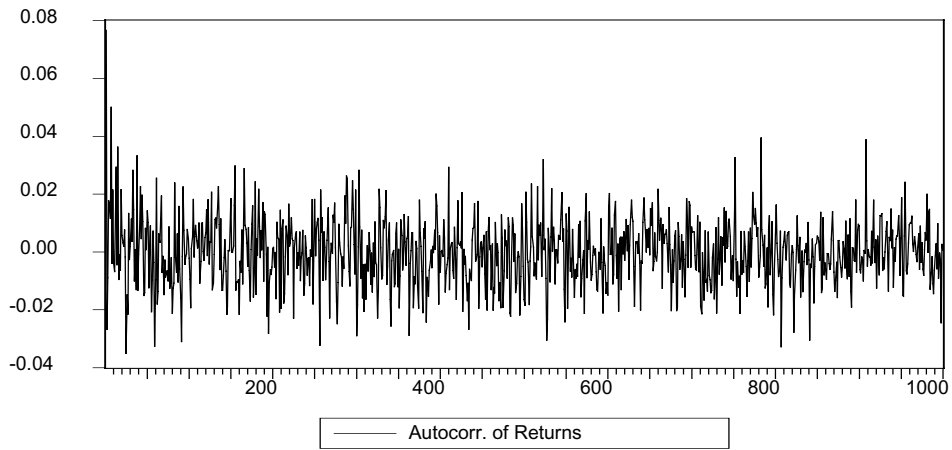
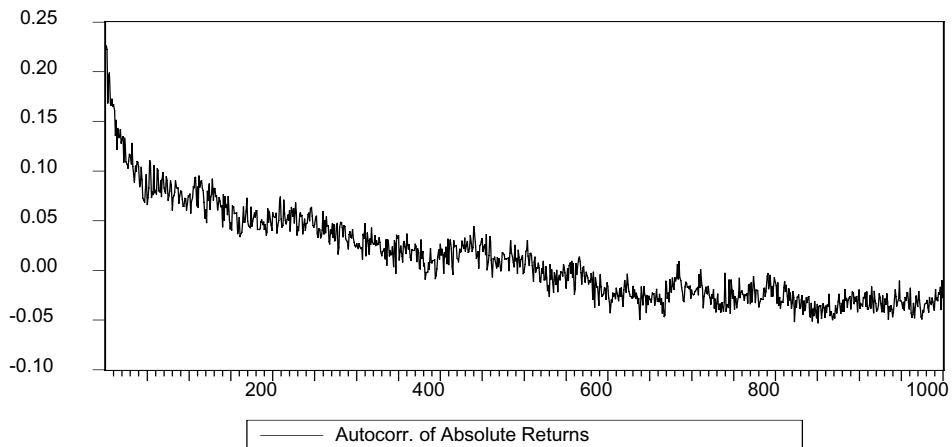
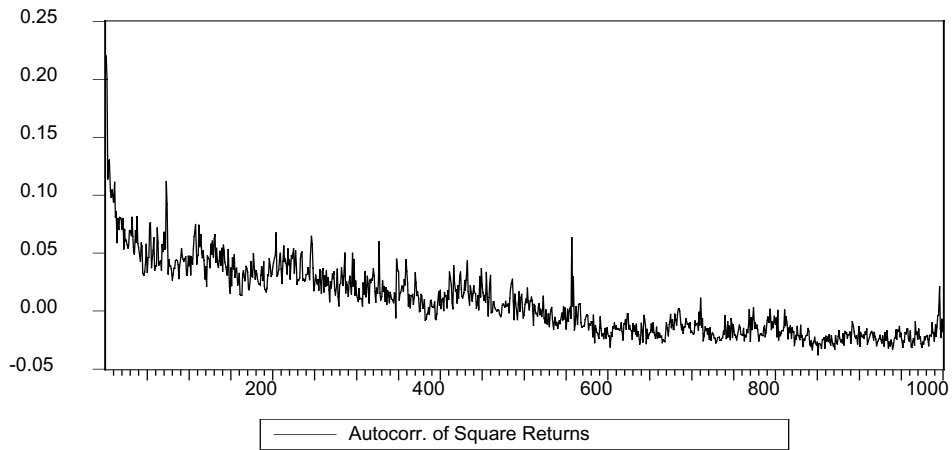
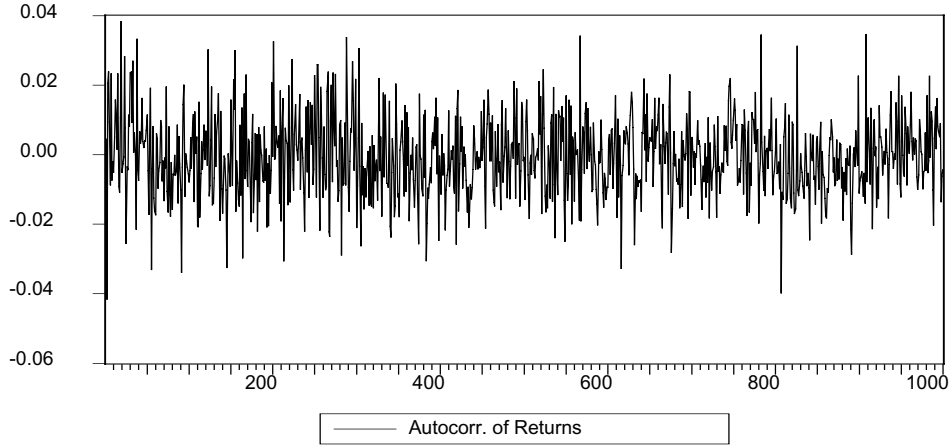




Figure 3. Autocorrelation of Filtered Daily Corn Futures



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