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Abstract

This paper suggests a new nonparametric testing procedure for determining the rank of nonstationary multivariate cointegrated systems. The asymptotic properties of the procedure are determined and a Monte Carlo study is carried out.

Keywords: Cointegration rank, Nonparametric Analysis *JEL Codes:* C32, C14

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1 Introduction

Interest in cointegration analysis has led to the development of a large number of testing procedures to determine the presence and rank of cointegration in nonstationary multivariate systems. Recently interest has partly moved away from the standard vector error correction model towards other possible generating mechanisms. An example is the threshold error correction model whose theoretical properties are still largely unknown but which has been used in a number of empirical applications (see e.g. Balke and Fomby (1997)).

This paper suggests a new nonparametric testing procedure for determining the rank of cointegrated systems to complement procedures by Shintani (2001), Poskitt (2000), Saikkonen and Luukkonen (1997) and Bierens (1997). The procedure relies on two steps. The first step involves estimation of the long run coefficient matrix of the infinite MA representation of the differences of the nonstationary process. The second step which forms the main part of the innovation of the paper is the use of a test of rank on the estimate of the long run coefficient matrix. We use the Cragg and Donald (1996) test of rank. This test requires a consistent and asymptotically normally distributed estimate of the matrix in whose rank we are interested and an estimate of its covariance matrix. One major advantage of the new procedure is that unlike any other nonparametric testing method it tests for r against an alternative of $r - 1$ rather than $r + 1$ cointegrating vectors. As argued by Snell (1999) such a test is preferable to the standard tests for economic data as the null of more cointegration usually follows from a priori economic reasoning.

2 Theory

In general terms we follow the setup of Bierens (1997). Consider the m -variate unit root process with drift $x_t = \mu + x_{t-1} + \epsilon_t$, $t = 1, \dots, T$, where ϵ_t is a zero mean stationary process and μ is a vector of drift parameters. The

Wold decomposition can be used to write the process ϵ_t as

$$\epsilon_t = \sum_{i=0}^{\infty} C_i \xi_{t-i} = C(L)\xi_t$$

where ξ_t is an m -variate white noise process and $C(L)$ is a square q -dimensional matrix of lag polynomials.

We make the following assumptions:

Assumption 1 ϵ_t is a stationary and ergodic process with finite fourth moments.

Assumption 2 $\|C_i\| \sim c^i$ as $i \rightarrow \infty$, for some $c < 1$.

The first assumption guarantees that the Wold decomposition holds for the process u_t . The second assumption describes the limit behaviour of the sequence C_i . Cointegration is assumed to imply that the matrix $C(1)$ is of reduced rank. If r^* is the rank of $C(1)$, the cointegration rank of the system is $r = m - r^*$. Our aim is to obtain a \sqrt{T} -consistent and asymptotically normal estimate of the matrix $C(1)$ and apply a test of rank on this matrix to determine its rank. We achieve this as follows: We firstly estimate a long (to be defined below) error correction model of the form

$$\Delta x_t = \mu + D_0 x_{t-1} + \sum_{i=1}^k D_i \Delta x_{t-i} + \eta_t \quad (1)$$

An infinite ECM representation of the nonstationary system is guaranteed to exist under assumption 2 (see Banerjee, Dolado, Galbraith, and Hendry (1993, Chapter 8)). The behaviour of D_i as $i \rightarrow \infty$ mirrors that of C_i . Below we will give conditions for k that guarantee consistent estimation of ξ_t via $\hat{\eta}_t$ at appropriate rates. We then use $\hat{\eta}_t$ in

$$\Delta x_t = \mu + \sum_{i=1}^k C_i \hat{\eta}_{t-i} + \zeta_t \quad (2)$$

to obtain consistent estimates of C_i , $i = 1, \dots, k$ and therefore of $C(1) = \sum_{i=1}^{\infty} C_i$. We then apply the Cragg and Donald test of rank to the estimate of $C(1)$, $\hat{C}(1)$. By the arguments of Berk (1974), if the following two assumptions on the permitted rate of increase of k apply then the parameter estimates of the long autoregression are \sqrt{T} consistent.

Assumption 3 $k^3/T \rightarrow 0$, and $k \rightarrow \infty$ as $T \rightarrow \infty$.

Assumption 4 $\ni c > 0, r > 0$ such that $ck > T^{1/r}$

Assumption 3 ensures that the rate at which regressors are added to the regression is low enough not to introduce excessive variation in the parameter estimates. Assumption 4 ensures that the number of regressors left out of the regression is of negligible significance for the \sqrt{T} -consistency of the parameter estimates. Lower rates than those postulated in Assumption 4 such as a logarithmic rate ensure consistency of the parameter estimates as long as $k \rightarrow \infty$ but at a lower rate of convergence¹. Then the following result may be shown

Theorem 1 *Under assumptions 1-4, the estimate of $C(1)$ obtained from (2) is consistent at rate \sqrt{T} and asymptotically normally distributed under the \sqrt{T} normalisation.*

The proof of this theorem may be found in the appendix.

Next we use the Cragg and Donald test of rank to determine the rank of $C(1)$. The procedure is based on the transformation of the matrix $C(1)$ using Gaussian elimination with complete pivoting². r^* steps of Gaussian elimination with full pivoting on matrix $C(1)$ amounts to the following operations:

$$Q_{r^*} R_{r^*} Q_{r^*-1} R_{r^*-1} \dots Q_1 R_1 C(1) L_1 \dots L_{r^*-1} L_{r^*} = \begin{bmatrix} \Sigma_{11}(r^*) & \Sigma_{12}(r^*) \\ 0 & \Sigma_{22}(r^*) \end{bmatrix}$$

¹It is interesting to note that a power logarithmic rate for k , i.e. $k \sim [\ln(T)]^\alpha$, $\alpha > 1$ is enough for the result

²For details on Gaussian elimination with complete pivoting see Cragg and Donald (1996).

where R_i and L_i are pivoting matrices for step i and Q_i are Gauss transformation matrices. The pivoting matrices used to perform the first r^* steps of Gaussian elimination are applied to $C(1)$ to obtain the following relation

$$R_{r^*}R_{r^*-1} \dots R_1C(1)C_1 \dots L_{r^*-1}L_{r^*} = RC(1)L = F = \begin{bmatrix} F_{11}(r^*) & F_{12}(r^*) \\ F_{21}(r^*) & F_{22}(r^*) \end{bmatrix}$$

where F is partitioned accordingly, i.e. $F_{11}(r^*)$ is of dimension $r^* \times r^*$. Note that in this case $F_{11}(r^*)$ has full rank, under the null hypothesis that r^* is equal to the true rank. It then follows, see Cragg and Donald (1996), that $F_{22}(r^*) - F_{21}(r^*)F_{11}^{-1}(r^*)F_{12}(r^*) = 0$. The estimated counterpart of the above relation, i.e. $\hat{F}_{22} - \hat{F}_{21}\hat{F}_{11}^{-1}\hat{F}_{12} = \hat{\Lambda}_{22}(r^*)$, may be used as a test statistic of the hypothesis that the rank of $C(1)$ is r^* . Under regularity conditions, including the requirement that the covariance matrix of the asymptotically normally distributed matrix $\sqrt{T}vec(\hat{C}(1) - C(1))$, V , has full rank, we can show, that, under H_0 , $\sqrt{T}vec(\hat{\Lambda}_{22}(r^*)) \xrightarrow{d} N(0, \Gamma V \Gamma')$ where $\Gamma = \Phi_2 \otimes \Phi_1$ and $\Phi_1 = \begin{bmatrix} -\hat{F}_{21}\hat{F}_{11}^{-1} & I_{m-r^*} \end{bmatrix} R$, $\Phi_2 = \begin{bmatrix} -\hat{F}'_{12}\hat{F}_{11}^{-1'} & I_{m-r^*} \end{bmatrix} C'$ and \xrightarrow{d} denotes convergence in distribution. Then, $\hat{\xi} = Tvec \hat{\Lambda}_{22}(r^*)'(\hat{\Gamma}\hat{V}\hat{\Gamma}')^{-1}vec \hat{\Lambda}_{22}(r^*) \xrightarrow{d} \chi_{(m-r^*)^2}^2$ where $\hat{\Gamma}$ and \hat{V} are the sample estimates of Γ and V and χ_l^2 denotes the χ^2 distribution with l degrees of freedom.

A sequential application of the Cragg and Donald test of rank can provide a consistent estimate of the rank of $C(1)$ if the significance level used in the test converges to zero as the number of observations tends to infinity (See Hosoya (1989)). For other applications see Camba-Mendez, Kapetanios, Smith, and Weale (2000) and Camba-Mendez and Kapetanios (2001). We also note in passing that we can use our results to get nonparametric estimates of the cointegrating vectors β since $\beta'C(1) = 0$ and we have a consistent estimate of $C(1)$.

3 Monte Carlo

We consider a number of experiments. We assume that the m dimensional I(1) process, y_t may be represented by the following nonstationary VAR(p)

model

$$y_t = \Phi_1 y_{t-1} + \dots + \Phi_p y_{t-p} + \epsilon_t$$

Then, the following error correction representation may be obtained.

$$\Delta y_t = \Pi y_{t-1} + \Psi_1 \Delta y_{t-1} + \dots + \Psi_{p-1} \Delta y_{t-p+1} + \epsilon_t \quad (3)$$

where $\Pi = \Phi_1 + \dots + \Phi_p - I_m$ and $\Psi_s = -[\Phi_{s+1} + \dots + \Phi_p]$. Cointegration implies that the rank, r , of Π is greater than zero but less than m . We concentrate on a multivariate model with 5 variables. We control the rank of the coefficient matrix, Π in the error correction representation by specifying the vector of its eigenvalues. 5 different vectors are considered. They are given in Table 1.

We construct a standard normal random matrix of eigenvectors, E which are almost surely linearly independent. These are transformed into an orthonormal basis, \tilde{E} , using the Gram-Schmidt process. The coefficient matrix is then given by $\tilde{E}\Lambda\tilde{E}'$ where Λ is a diagonal matrix containing the eigenvalues of the required coefficient matrix. We consider two different lag structures. In experiments labelled A0-A4 the lag order is set to one and the matrix Π is constructed as above. In experiments B0-B4, the lag order is set to 2 and the matrix Ψ_1 is constructed as above but with full rank and all eigenvalues equal to 0.2. Using these matrices and random normal disturbances generated by the GAUSS random number generator with identity covariance matrix a sample from a process following the error correction representation in (3) is obtained. However, the class of finite order VECM models is not the most appropriate class to consider for nonparametric procedures. We therefore also consider the following model

$$\Delta y_t = F(\Delta y_{t-1})\Pi y_{t-1} + \epsilon_t \quad (4)$$

where we allow two forms for $F(\cdot)$. We set $F(\Delta y_{t-1}) = 1 - e^{-(\sum_{i=1}^m \Delta y_{i,t-1})^2}$ and $F(\Delta y_{t-1}) = 1\{|\sum_{i=1}^m \Delta y_{i,t-1}| > r\}$, $r = 2$. The rest of the specification of the model is as before. These modifications lead to nonlinear VECM

models where the speed of convergence to equilibrium depends on Δy_{t-1} . The first model is inspired by univariate smooth transition autoregressive (STAR) models whereas the other comes from self-exciting threshold autoregressive (SETAR) models. We carry out exactly the same experiments as in A0-A4 for the testing procedures using these models leading to experiment C0-C4 and D0-D4. The sample sizes considered are: 200, 400 and 1000. For each Monte Carlo experiment, 500 replications have been carried out. The significance level for all the tests has been set to 95%. We choose to set k (the order of the long autoregression) quite high to obtain a good approximation of the matrix $C(1)$. We set it to $\alpha_T T^{1/3-0.01}$ where $\alpha_{200} = 4$, $\alpha_{400} = \alpha_{1000} = 2$. The motivation of the rate is obvious from the theoretical arguments. The choice of α follows experimentation and was chosen to be as high as possible without causing collinearity problems.

We also consider the rank determination procedures discussed by Bierens (1997) (λ_{min}), Saikkonen and Luukkonen (1997) (no deterministic components are considered), Poskitt (2000) and Shintani (2001) (we use the test $P^*(n, s)$, set $K = M = 4$, and use the Parzen kernel) denoted by *SL*, *POS* and *SH* respectively. In Tables 2-5 below we provide the mean and mean square error of the rank estimate obtained by the tests.

As we can see from the results no single test dominates all others in all the experiments considered. Our test does very well for high cointegration ranks as expected, dominating the other procedures. It can compete on equal terms with the other procedures overall. In a few cases the MA test does worse for 400 observations compared to 200 observations. Nevertheless, its performance is best for 1000 observations as expected by the asymptotic results.

4 Conclusion

This paper provides a new nonparametric procedure for determining the cointegration rank in nonstationary multivariate series. This procedure requires

only mild regularity assumptions for the stochastic process under investigation and is otherwise nonparametric. We establish the asymptotic properties of the new testing procedure and carry out a relatively extensive Monte Carlo analysis which shows that the new test can compete with extant nonparametric procedures. One major advantage of the new procedure is that unlike any other nonparametric testing method it tests for r against an alternative of $r - 1$ rather than $r + 1$ cointegrating vectors. As argued by Snell (1999) such a test is preferable to the standard tests for economic data as the null of more cointegration usually follows from a priori economic reasoning.

Appendix

Lemma 1 *The estimate of ξ_t given by $\hat{\eta}_t$ from (1) is $T^{1/2-1/r'}$ -consistent.*

Proof of Lemma 1 The estimate of ξ_t given by $\hat{\eta}_t$ is $T^{1/2-1/r'}$ -consistent in the sense $\xi_t - \hat{\eta}_t = o_p(T^{-1/2+1/r'})$, $\forall t, \forall r' < r$ as we now show.

$$\hat{\eta}_t - \xi_t = \Delta x_t - \hat{\mu} - \hat{D}_0 x_{t-1} - \sum_{i=1}^k \hat{D}_i \Delta x_{t-i} - \Delta x_t + \mu + D_0 x_{t-1} + \sum_{i=1}^{\infty} D_i \Delta x_{t-1}$$

or

$$\hat{\eta}_t - \xi_t = (\hat{\mu} - \mu) + (\hat{D}_0 - D_0)x_{t-1} + \sum_{i=1}^k (\hat{D}_i - D_i)\Delta x_{t-i} + \sum_{i=k+1}^{\infty} D_i \Delta x_{t-i}$$

By results of Saikkonen and Luukkonen (1997) the first two terms are $O_p(T^{-1/2})$. The third term can be bounded in absolute value by $\max_i \|\hat{D}_i - D_i\| k \max_i \|\Delta x_{t-i}\|$ which by the properties of maxima of infinite $O_p(T^{-1/2})$ and $O_p(1)$ random sequences may be seen to be equal to a $O_p(T^{-1/2+1/r'} 1/r^2 (\ln(T))^2) = o_p(T^{-1/2+1/r'})$ term where $r' < r$. The above holds in L_2 norm as well as in probability. The fourth term is $o(T^{-1/2})$ by assumption 2. QED

Proof of Theorem 1 We consider the following regression

$$\Delta x_t = \mu + \sum_{i=1}^k C_i \hat{\eta}_{t-i} + \psi_t$$

First of all we note that $\|C(1) - I - \sum_{i=1}^k C_i\| = O(c^k) = o(T^{-\alpha})$, $\forall \alpha > 0$. Let $C = (C_1, \dots, C_k)$. Then, $\hat{C} = \Delta X' \hat{H} (\hat{H} \hat{H}')^{-1}$, where $\hat{H}_t = (1, \hat{\eta}'_{t-1}, \dots, \hat{\eta}'_{t-k})'$, $\hat{H} = (\hat{H}_1, \dots, \hat{H}_T)'$ and $\Delta X = (\Delta x_1, \dots, \Delta x_T)'$. Define also $\hat{\hat{C}} = \Delta X' H (H H')^{-1}$,

where $H_t = (1, \xi'_{t-1}, \dots, \xi'_{t-k})'$ and $H = (H_1, \dots, H_T)'$. The asymptotic distribution of $\sqrt{T}(\hat{C} - C)$ is straightforward to obtain, under assumptions 1, 3 and 4 and from this the asymptotic distribution of $\hat{C}(1) = I_m + F\hat{C}'$ follows, where $F = (O_{(1 \times m)}, I_m \otimes \iota')$, $\iota = (1, \dots, 1)'$. We need to show that $\hat{C} - \hat{C} = o_p(1)$. But in fact, it is easy to show the stronger result that $\sqrt{T}(\hat{C} - C) - \sqrt{T}(\hat{C} - C) = o_p(T^{-1/r})$ since (i) $\|1/\sqrt{T}(\Delta X' \hat{H} - \Delta X' H)\| = o_p(T^{-1/r})$ and (ii) $\|(1/T \hat{H}' \hat{H})^{-1} - (1/T H' H)^{-1}\| = o_p(T^{-1/r})$. To see (i) we have

$$\begin{aligned} \|1/\sqrt{T}(\Delta X' \hat{H} - \Delta X' H)\| &= \left\| 1/\sqrt{T} \sum_{t=1}^T (\Delta x_t \hat{H}'_t - \Delta x_t H'_t) \right\| = \\ &= \left\| 1/\sqrt{T} \sum_{t=1}^T \left\{ \Delta x_t \left[\sum_{j=1}^k \Delta x'_{t-j} (\hat{D}_j - D_j)' + \sum_{j=k+1}^{\infty} \Delta x'_{t-j} D'_j \right] \right\} \right\| = \\ &= \left\| \sum_{j=1}^k \left\{ (\hat{D}_j - D_j)' \left[1/\sqrt{T} \sum_{t=1}^T \Delta x_t \Delta x'_{t-j} \right] \right\} + \sum_{j=k+1}^{\infty} D'_j \left[1/\sqrt{T} \sum_{t=1}^T \Delta x_t \Delta x'_{t-j} \right] \right\| = \\ &= \left\| \sum_{j=1}^k \sqrt{T} (\hat{D}_j - D_j)' 1/T \sum_{t=1}^T \Delta x_t \Delta x'_{t-j} \right\| + o_p(T^{-\alpha}) \leq \\ &= \left\| \sum_{j=1}^k \sqrt{T} (\hat{D}_j - D_j)' \right\| \left\| 1/T \sum_{t=1}^T \Delta x_t \Delta x'_{t-j} \right\| + o_p(T^{-\alpha}), \quad \forall \alpha > 0 \end{aligned}$$

$1/T \sum_{t=1}^T \Delta x_t \Delta x'_{t-j}$ converges to its non zero expectation, each of $\sqrt{T}(\hat{D}_i - D_i)$ is $O_p(1)$ and therefore the whole term is $o_p(T^{-1/r})$. (ii) follows from similar arguments as above and the facts that, firstly for positive definite matrices A and B , if $A - B = o_p(T^a)$ then $A^{-1} - B^{-1} = o_p(T^a)$ for all finite a , and secondly, if $A - \hat{A} = o_p(T^a)$ and $B - \hat{B} = o_p(T^a)$ then $AB - \hat{A}\hat{B} = o_p(T^a)$.

Therefore, \hat{C} is consistent for C and therefore if we develop a central limit theorem for $1/\sqrt{T}(\Delta X' \hat{H})$ or equivalently for $1/\sqrt{T} \sum_{t=1}^T \Delta x_{i,t} \hat{\eta}_{i,t-j}$ $i = 1, \dots, m$, $j = 1, \dots, k$ we have proven that $\text{vec}(\hat{C})$, is asymptotically normal. A straightforward way is to prove that $\hat{\eta}_{it}$ is a near-epoque dependent (NED) process on ξ_t and then use a central limit theorem for NED processes on mixing processes.

A stochastic process y_t is near epoque dependent in L_v -norm (L_v -NED) of size $-\alpha$ on $\{z_t\}_{-\infty}^{\infty}$, where $\|\cdot\|_v$ denotes L_v -norm if, for a stochastic process $\{z_t\}_{-\infty}^{\infty}$ defined on a probability space (Ω, \mathcal{F}, P) , a sequence of σ -fields $\mathcal{F}_{t-m}^{t+m} = \sigma(z_{t-m}, \dots, z_{t+m})$ and for $v > 0$, y_t satisfies $\sup_t \|y_t - E(y_t | \mathcal{F}_{t-m}^{t+m})\|_v \equiv v_m$ and $v_m = O(m^{-\alpha})$. The NED property focuses on the relationship between the process $\{y_t\}$ and the underlying process $\{z_t\}$. When the underlying process, $\{z_t\}$, is mixing, the NED property is used to extend results on limit laws which hold for mixing

processes to the process $\{y_t\}$ which may not be mixing. The fact which permits this extension is that NED processes on mixing processes are, under regularity conditions, mixingales.

The main condition to be satisfied for the NED central limit Theorem 24.7 of Davidson (1994) is that $\hat{\eta}_{it}$ is NED on ξ_t of size -1 . However, closer examination of the proof of that theorem via Lemma 24.8 and Corollary 17.6 and the fact that the error sequence ξ_t is mixing of any size due to independence suggests that the results of the theorem hold for NED processes of size smaller than $-1/2$. To show that $\hat{\eta}_t$ is L_2 -NED on ξ_t we need to show that $(\hat{\eta}_{it} - \xi_{it})^2$ is of stochastic order $T^{-\alpha}$ for all $-d < \alpha$, for some $d < -0.5$. But this follows from the above since $\hat{\eta}_{it} - \xi_{it} = o_p(T^{-1/2+1/r})$ for all $r > 0$ and any $r > 0$ is allowed by Assumption 4. Further, we know that Δx_t is NED of arbitrary size on ξ_t because of assumption 2. The product of two NED processes is NED with size the minimum of the sizes of the two NED processes as shown in Theorem 17.9 of Davidson (1994). We therefore see that the estimate of the matrix $C(1)$ has, suitably normalised, an asymptotic normal distribution. It is easy to see that the asymptotic covariance matrix may be obtained from standard regression results.

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Table 1: Eigenvalues used to generate coefficient matrices

	Experiment				
	1	2	3	4	5
1	0.0	-0.5	-0.6	-0.7	-0.6
2	0.0	0.0	-0.3	-0.7	-0.4
3	0.0	0.0	0.0	-0.7	-0.2
4	0.0	0.0	0.0	0.0	-0.1
5	0.0	0.0	0.0	0.0	0.0
r	0	1	2	3	4

Table 2: Mean and MSE of Estimated Rank for Experiments A

Test	Exp	MA Test					
		Mean			MSE		
		200	400	1000	200	400	1000
MA	A0	1.906	1.628	0.000	4.424	3.606	0.000
	A1	1.198	1.280	0.004	2.244	2.273	0.004
	A2	0.452	1.010	0.010	0.970	1.475	0.010
	A3	0.086	0.760	0.008	0.695	0.760	0.008
	A4	-1.088	-0.004	-0.002	1.938	0.004	0.002
SL	A0	1.058	0.402	0.124	1.900	0.583	0.136
	A1	0.230	0.012	0.130	0.747	0.341	0.154
	A2	-0.604	-0.248	0.074	1.118	0.545	0.106
	A3	-0.928	-0.078	0.062	1.690	0.166	0.066
	A4	-2.010	-0.366	0.056	4.976	0.479	0.056
POS	A0	0.114	0.006	0.000	0.114	0.006	0.000
	A1	0.060	0.004	0.000	0.060	0.004	0.000
	A2	0.016	0.000	0.000	0.016	0.000	0.000
	A3	0.012	0.000	0.000	0.012	0.000	0.000
	A4	-0.928	-0.386	0.000	1.241	0.403	0.000
BI	A0	0.282	0.284	0.262	0.286	0.288	0.266
	A1	-0.724	-0.714	-0.628	0.728	0.722	0.649
	A2	-1.670	-1.658	-1.548	3.014	2.978	2.665
	A3	-2.594	-2.508	-2.288	6.995	6.577	5.513
	A4	-3.624	-3.558	-3.414	13.380	12.919	11.943
SH	A0	0.000	0.000	0.000	0.000	0.000	0.000
	A1	-0.998	-0.920	0.000	0.998	0.920	0.000
	A2	-1.946	-1.006	0.000	3.838	1.106	0.000
	A3	-1.786	-0.142	0.000	3.611	0.142	0.000
	A4	-3.354	-0.952	0.000	11.647	1.152	0.000

Table 3: Mean and MSE of Estimated Rank for Experiments B

Test	Exp	MA Test					
		Mean			MSE		
		200	400	1000	200	400	1000
MA	B0	1.648	0.990	0.000	3.349	1.551	0.000
	B1	0.966	0.680	0.000	1.692	0.913	0.000
	B2	0.344	0.512	0.000	0.845	0.593	0.000
	B3	-0.158	0.376	0.000	0.792	0.376	0.000
	B4	-1.244	-0.002	0.000	2.354	0.002	0.000
SL	B0	1.172	0.348	0.174	2.246	0.561	0.222
	B1	0.308	0.062	0.098	0.898	0.371	0.102
	B2	-0.476	-0.118	0.076	1.118	0.371	0.092
	B3	-0.814	-0.010	0.064	1.496	0.150	0.072
	B4	-1.670	-0.168	0.052	3.936	0.276	0.052
POS	B0	0.014	0.002	0.000	0.014	0.002	0.000
	B1	0.004	0.000	0.000	0.004	0.000	0.000
	B2	0.002	0.000	0.000	0.002	0.000	0.000
	B3	0.000	0.000	0.000	0.000	0.000	0.000
	B4	-1.402	-0.720	-0.012	2.271	0.817	0.012
BI	B0	0.242	0.296	0.242	0.242	0.296	0.246
	B1	-0.716	-0.680	-0.608	0.716	0.684	0.612
	B2	-1.694	-1.640	-1.580	3.086	2.920	2.757
	B3	-2.604	-2.466	-2.292	7.028	6.359	5.521
	B4	-3.620	-3.546	-3.448	13.344	12.835	12.185
SH	B0	0.000	0.000	0.000	0.000	0.000	0.000
	B1	-1.000	-0.870	0.000	1.000	0.870	0.000
	B2	-1.870	-0.824	0.000	3.610	0.840	0.000
	B3	-1.542	-0.068	0.000	2.767	0.068	0.000
	B4	-2.872	-0.624	0.000	8.853	0.665	0.000

Table 4: Mean and MSE of Estimated Rank for Experiments C

Test	Exp	MA Test					
		Mean			MSE		
		200	400	1000	200	400	1000
MA	C0	1.882	1.604	0.000	4.267	3.530	0.000
	C1	1.148	1.256	0.004	2.082	2.245	0.004
	C2	0.354	1.068	0.002	0.960	1.589	0.006
	C3	-0.142	0.744	0.012	0.808	0.748	0.012
	C4	-1.298	-0.022	-0.108	2.387	0.022	0.108
SL	C0	1.114	0.390	0.156	2.112	0.591	0.192
	C1	0.158	-0.016	0.102	0.671	0.465	0.130
	C2	-0.662	-0.416	0.060	1.292	0.657	0.072
	C3	-1.096	-0.110	0.046	2.058	0.299	0.054
	C4	-2.216	-0.658	0.058	6.086	1.031	0.058
POS	C0	0.116	0.010	0.000	0.116	0.010	0.000
	C1	0.058	0.012	0.000	0.058	0.012	0.000
	C2	-0.002	0.000	0.000	0.058	0.000	0.000
	C3	0.004	0.000	0.000	0.004	0.000	0.000
	C4	-1.470	-1.074	-0.160	2.507	1.579	0.160
BI	C0	0.244	0.258	0.284	0.248	0.262	0.288
	C1	-0.726	-0.704	-0.692	0.726	0.704	0.696
	C2	-1.688	-1.694	-1.630	3.068	3.082	2.898
	C3	-2.606	-2.544	-2.432	7.030	6.741	6.185
	C4	-3.652	-3.608	-3.504	13.564	13.260	12.573
SH	C0	0.000	0.000	0.000	0.000	0.000	0.000
	C1	-1.000	-0.986	-0.006	1.000	0.986	0.006
	C2	-1.992	-1.326	-0.008	3.976	1.998	0.008
	C3	-2.252	-0.448	0.000	5.481	0.453	0.000
	C4	-3.832	-1.668	-0.030	14.836	3.153	0.038

Table 5: Mean and MSE of Estimated Rank for Experiments D

Test	Exp	MA Test					
		Mean			MSE		
		200	400	1000	200	400	1000
MA	D0	1.888	1.596	0.000	4.297	3.526	0.000
	D1	1.028	1.164	-0.340	1.773	1.993	0.340
	D2	0.046	0.742	-0.898	0.808	1.263	1.203
	D3	-0.748	0.638	-0.004	1.386	0.671	0.028
	D4	-1.892	-0.634	-2.180	4.386	0.915	5.205
SL	D0	1.114	0.406	0.156	2.084	0.679	0.180
	D1	0.044	-0.456	-0.102	0.766	0.665	0.230
	D2	-0.990	-1.108	-0.350	1.732	1.961	0.499
	D3	-1.736	-0.994	0.068	3.858	1.727	0.080
	D4	-3.034	-2.666	-0.600	9.863	8.016	0.877
POS	D0	0.150	0.014	0.000	0.154	0.014	0.000
	D1	-0.638	-0.606	-0.104	0.663	0.606	0.104
	D2	-1.448	-1.434	-0.848	2.441	2.359	0.940
	D3	-1.660	-0.860	-0.012	3.726	1.810	0.012
	D4	-3.528	-3.452	-2.910	12.809	12.297	8.907
BI	D0	0.292	0.250	0.270	0.292	0.254	0.274
	D1	-0.744	-0.738	-0.746	0.744	0.738	0.746
	D2	-1.714	-1.720	-1.692	3.146	3.160	3.076
	D3	-2.674	-2.652	-2.654	7.378	7.260	7.274
	D4	-3.714	-3.684	-3.610	13.998	13.792	13.274
SH	D0	0.000	0.000	0.000	0.000	0.000	0.000
	D1	-1.000	-1.000	-0.982	1.000	1.000	0.982
	D2	-2.000	-2.000	-1.302	4.000	4.000	1.926
	D3	-2.998	-2.566	-0.084	8.990	6.959	0.084
	D4	-4.000	-3.986	-1.866	16.000	15.902	3.751

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