

Department of Economics

Reason-Based Choice Correspondences

Michele Lombardi

Working Paper No. 607

September 2007

ISSN 1473-0278



Queen Mary
University of London

Reason-Based Choice Correspondences

Michele Lombardi¹

Department of Economics
Queen Mary, University of London
E-mail: M.Lombardi@qmul.ac.uk

August, 2007

¹Helpful comments and suggestions by Paola Manzini and Marco Mariotti are gratefully acknowledged. The usual caveat applies.

Abstract

A reason-based choice correspondence rationalizes choice behaviour in terms of a two-stage choice procedure. Given a feasible set S , the individual eliminates from it all of the dominated alternatives according to her fixed (not necessarily complete) strict preference relation, in the first step. In the second step, first she constructs for each maximal alternative identified in the first step its *lower contour set* (i.e., the set of alternatives which are dominated by it in S), and then she eliminates from the maximal set all of those alternatives so that the following justification holds: there exists another maximal alternative whose lower contour set strictly contains that of another maximal alternative. This procedural model captures the basic idea behind the experimental finding known as "attraction effect". We study the rationalizability of reason-based choice correspondences axiomatically. We relate our choice-consistency conditions to standard consistency properties. Our characterization result offers testable restrictions on this 'choice anomaly' for large (but finite) set of alternatives.

J.E.L. codes: D0.

Keywords: Reason-Based Choice, Revealed Preferences.

1 Introduction

Rationality of choice behaviour cannot be assessed without seeing it in the context in which a choice is made (Sen, (1993) and (1997)). This view is confirmed by a sizeable amount of experimental findings which show that when added to a choice set a new relatively inferior alternative can increase the attractiveness of one of the alternatives obtainable from the original set (see, Rieskamp, Busemeyer, and Mellers, (2006)).

This systematic observed choice behaviour, known as “asymmetric dominance effect” or “attraction effect”,¹ is explained in terms of bounded rationality. In a difficult and conflict-filled decision, where there is no escape from choosing, individuals choose by tallying defensible reasons for one alternative versus the other, rather than by trading off costs and benefits. Furthermore, in this respect, the dominant structure of alternatives in the choice set provides the decision-maker with good reasons for her choice (see, Simonson (1989), Tversky and Simonson (1993), and Shafir, Simonson, and Tversky (1993), and the references cited therein).²

Let us give an example. Suppose that an individual wishes to buy herself a digital camera for next holiday in Rome, and she has a choice among three competing models, say, x , y , and x' , where each model is characterized by exactly two equally important dimensions, say, price and quality. She may find the choice between x (resp., x') and y hard because x (resp., x') is better than y on one dimension (say, price) while y is better than x (resp., x') on the other dimension (say, quality). She would find the choice between

¹Strictly speaking these two effects are slightly different, and the difference refers to the attributed levels of the new alternative that is added to the choice set. In this paper we will refer only to the attraction effect since the asymmetric dominance effect is a special case.

²A first formalization of how reasons affect the individual’s decisions in a game theoretical framework appears in Spiegler (2002).

x and x' an easy one because the former dominates the latter with respect to both dimensions. Thus, while she has a clear and indisputable reason for choosing x over x' , she cannot hold any compelling reason for choosing only x (resp., x') from $\{x, y\}$ (resp., $\{x', y\}$) or only y from $\{x, y\}$ and $\{x', y\}$. However, the fact that x' is obtainable from $\{x, y, x'\}$ and x is better priced and of higher quality than x' , whilst y is only of higher quality, may provide her with a reason for choosing only x from $\{x, y, x'\}$.

This pattern of observed choices - which is not confined to consumer products, but also extends to choices among gambles, job applicants, political candidates (Rieskamp, Busemeyer, and Mellers, (2006)) - is partially consistent with the standard economic interpretation of rationality which is preference maximization.

In our example, the individual has an incomplete preference relation on $\{x, y, x'\}$ because she deems x and y choosable from $\{x, y\}$, x' and y from $\{x', y\}$, and only x from $\{x, x'\}$ and $\{x, y, x'\}$. For any feasible set she faces, she chooses undominated alternatives relative to her preferences in that set. However, contrary to what is envisaged from the standard preference maximization hypothesis, she discards y from her choice. This suggests that our individual may have refined her choice by using the information available from the entire choice set (given her preferences) as a tie-breaking rule: As x dominates x' , but y does not, the set of alternatives dominated by x strictly contains that dominated by y , providing the individual with a convincing reason for choosing only x from $\{x, y, x'\}$ (see, e.g., Tversky and Simonson, (1993), p. 1185).

The idea of rationalizing choice correspondences in terms of a two-stage choice procedure whereby the individual arrives at a choice by using the information obtainable from the entire set in the second round of elimination appears in Ok (2004), who identifies in these terms all the choice corre-

spondences satisfying the canonical Property α (also known as Chernoff choice-consistency condition or basic contraction consistency). Property α requires that an alternative that is deemed choosable from a feasible set T and belongs to a subset S of T must be deemed choosable from S (Sen, (1971)).

Indeed many contexts of choice which lead individuals to violate the normatively appealing Property α , and so the weak axiom of revealed preference (WARP) proposed by Samuelson (1938),³ and the ways in which they interact, await further investigation.

Returning to our consumer, suppose that another camera model y' - which is dominated by y with respect to both of the dimensions, whilst it is of higher quality than x and x' and worse priced than them - is added to the set $\{x, y, x'\}$. In this new choice-context, the individual loses the compelling reason which led her to choose x from $\{x, y, x'\}$ because the set of alternatives dominated by x does not contain that dominated by y , and vice versa. The presence of y' (which indeed should be irrelevant for her choice) makes x and y reasonably choosable from the grand set, whereas its absence makes only x choosable from $\{x, y, x'\}$. The combination of these choices violates Property α even though there is nothing particularly "unreasonable" in this pair of choices.

What is more, the described tie-breaking rule may lead an individual to suffer from certain framing manipulations. Let us give another example. Suppose an employee spends her lunch vouchers in one of her local restaurants. Assuming that her preferences may be incomplete and that her vouchers are enough to get any kind of luncheon served at any chosen local restaurant, on day 1 she steps into one of the local restaurants finding three

³Analyses of these and related choice-consistency conditions can be found, among others, in Moulin (1985), Sen (1971), Suzumura (1983). For a recent study of what kind of preference maximization WARP characterizes, see Mariotti (2007).

Day	Menu	Choice
1	{luncheon 1, luncheon 1', luncheon 2}	{luncheon 1}
2	{luncheon 2, luncheon 2', luncheon 3}	{luncheon 2}
3	{luncheon 3, luncheon 3', luncheon 1}	{luncheon 3}

Figure 1: Table 1.

kinds of luncheon on the menu (say, *luncheon 1*, *luncheon 1'*, and *luncheon 2*). Our employee strictly prefers *luncheon 1* to *luncheon 1'*, whereas she cannot make up her mind between *luncheon 1* (resp., *luncheon 1'*) and *luncheon 2*. To satisfy one's hunger she goes for *luncheon 1* as it dominates *luncheon 1'*, but *luncheon 2* does not. The day after (day 2) she steps into another available local restaurant to explore her range of choices, and it is serving three luncheons (say, *luncheon 2*, *luncheon 2'* and *luncheon 3*, where *luncheon 2* is the same luncheon served from the restaurant of day 1). Because she dithers between *luncheon 2* (resp., *luncheon 2'*) and *luncheon 3*, whereas she strictly prefers *luncheon 2* to *luncheon 2'*, she goes for *luncheon 2* as it seems the most "attractive" according to the menu of the day (i.e., *luncheon 2* dominates *luncheon 2'*, but *luncheon 3* does not). On day 3, she decides to return to the restaurant of day 1 which is serving only *luncheon 3*, *luncheon 3'* and *luncheon 1* (*luncheon 1* is the same luncheon served on day 1, and *luncheon 3* is the same luncheon served from the restaurant of day 2). Since she strictly prefers *luncheon 3* to *luncheon 3'*, while she cannot make up her mind between *luncheon 3* (resp., *luncheon 3'*) and *luncheon 1*, she goes for *luncheon 3* because it dominates *luncheon 3'*, but *luncheon 1* does not. Her choices are displayed in Table 1.

The choices made over the three days may appear wierd from an eco-

nomic perspective, but they are not as irrational in any minimal significant sense. The reason for this is that the employee’s preferences are insufficient to solve the decision problem that she faces, and so she constructs a reason on the basis of the problem that she faces by using her known preferences. Since each day there is a maximal luncheon (i.e., *luncheon 1* on day 1, *luncheon 2* on day 2, and *luncheon 3* on day 3) which outperforms the other maximal one (i.e., *luncheon 2*, *luncheon 3*, and *luncheon 1*, respectively), this allows her to complete her preferences by knocking the latter off.

Motivated by these observations, we believe that there is a need to shed more light on the phenomenon of how individuals use the set under consideration to identify the most “reasonable” alternatives following the revealed preference approach introduced by Samuelson (1938), the importance of which has been recently emphasized by Rubinstein and Salant (2006, 2007).⁴

With this aim we provide a full characterization of a choice correspondence as exemplified above in terms of a two-stage choice procedure. Given a feasible finite set, the individual eliminates from the decision all of the dominated alternatives according to her fixed (not necessarily complete) strict preference relation, in the first step. In the second step, she eliminates from the maximal set, identified in the first step, those alternatives which have the set of dominated alternatives strictly contained in that of another undominated alternative. Whenever a choice correspondence can be rationalized with the described two-stage rationalization, we say that the choice correspondence is a reason-based choice correspondence.

The rest of the paper is organized as follows. We begin by outlining our axiomatic framework, delineating the behavioural consistency properties used in our characterization result. Next, we provide our characterization of

⁴For a set theoretical foundation of the revealed preference theory see Richter (1966). For a recent survey on revealed preference see Varian (2005).

reason-based choice correspondences. We conclude with a brief discussion of our result in relation to the literature.

2 Preliminaries

Let X be a universal finite set of conceivable alternatives that is fixed from now on. Let \mathcal{S} be a collection of all nonempty subsets of X . By a *choice correspondence* C on \mathcal{S} we mean a map C which assigns a nonempty subset $C(S)$ of S to every $S \in \mathcal{S}$. Following Sen (1993), we read $x \in C(S)$ as x is choosable from S . Moreover, given $x, y \in X$, with $x \neq y$, $x, y \in C(S)$ for some $S \in \mathcal{S}$ does not necessarily mean that x is indifferent to y , but we interpret it as both of them are choosable from S .

Two distinct alternatives x and y in X are said to be *indistinguishable* on a set $S \in \mathcal{S}$, $x, y \notin S$, if, for all $z \in S$, one of the following holds:

1. $\{x\} = C(\{x, z\}) \Rightarrow \{y\} = C(\{y, z\})$;
2. $\{z\} = C(\{x, z\}) \Rightarrow \{z\} = C(\{y, z\})$;
3. $\{x, z\} = C(\{x, z\}) \Rightarrow \{y, z\} = C(\{y, z\})$.

Then x and y are indistinguishable one another if they behave in the same way with respect to direct choice comparisons with other alternatives. Observe that if x and y are not indistinguishable it does not necessarily mean that they are C -incomparable as we are silent on $C(\{x, y\})$.⁵

The set of positive integers is denoted by $\mathbb{N} = \{1, 2, \dots\}$. Let $\succ \subseteq X \times X$ be a binary relation on X which represents the individual preference relation. As usual we write $x \succ y$ for $(x, y) \in \succ$, and $x \not\succ y$ for $(x, y) \notin \succ$. A relation $\succ \subseteq X \times X$ is *acyclical* if, for all $t \in \mathbb{N}$ and for all $x^1, \dots, x^t \in X$, $x^\tau \succ x^{\tau+1}$ for all $\tau \in \{1, 2, \dots, t-1\}$ implies $x^t \not\succ x^1$. For any $S \in \mathcal{S}$, $\prec(x, S)$ denotes

⁵For a choice theoretical study of incomplete preferences, see Eliaz and Ok (2006).

the lower section of \succ restricted to S at x , i.e., $\prec(x, S) = \{y \in S \mid x \in S \setminus \{y\}, y \prec x\}$. $S \subseteq T$ means that every alternative in S is in T , whilst $S \subset T$ means that $S \subseteq T$ and $S \neq T$.

For $S \in \mathcal{S}$ and a binary relation \succ on X , the set of \succ -maximal alternatives in S is $M(S, \succ) = \{x \in S \mid y \not\succeq x \text{ for all } y \in S \setminus \{x\}\}$. Whenever a choice correspondence C on \mathcal{S} has an acyclical relation \succ on X such that, for all $S \in \mathcal{S}$,

$$C(S) = \{x \in M(S, \succ) \mid \prec(x, S) \subset \prec(y, S) \text{ for no } y \in M(S, \succ) \setminus \{x\}\},$$

we say that C is a *reason-based choice correspondence*.

Now we define some choice-consistency conditions of interest. The first is borrowed by Sen (1977) which is much weaker than Property α .

Property $\alpha 2$ ($\alpha 2$). For all $S \in \mathcal{S}$: $x \in C(S) \Rightarrow x \in C(\{x, y\})$ for all $y \in S$.

The second property is a weakening of Sen's (1971) Property β . Property β demands that for all pair of feasible sets, say S and T , and for all pair of alternatives, say x and y , if x and y are choosable from S , a subset of T , then y is choosable from T if and only if x is choosable from T . Our Weak Property β on the other hand requires Sen's Property β to hold if x and y are indistinguishable one another on $T \setminus \{x, y\}$.

Weak Property β ($\mathbf{W}\beta$). For all $S, T \in \mathcal{S}$: $S \subseteq T$, $x, y \in C(S)$, and x, y indistinguishable on $T \setminus \{x, y\} \Rightarrow [y \in C(T) \Leftrightarrow x \in C(T)]$.

The third property is a weakening of Samuelson's (1938) WARP, according to which if $x \in C(S)$ and $y \in S \setminus C(S)$, then there is no feasible set T , with $x \in T$, such that $y \in C(T)$. Our Weak WARP on the other hand demands Samuelson's WARP to hold if x is uniquely chosen from S , and we

add to the set S a feasible set T such that x and y are indistinguishable one another on T , with $x, y \notin T$.

Weak WARP (WWARP). For all $S, T \in \mathcal{S} : \{x\} = C(S)$ and $y \in S \setminus C(S) \Rightarrow [x, y \text{ indistinguishable on } T, x, y \notin T \Rightarrow y \notin C(S \cup T)]$.

The following property is a straightforward strengthening of the choice formulation of the so called "*Condorcet Winner Principle*" - labeled Binary Dominance Consistency (BDC) by Ehlers and Sprumont (2006) -, according to which for a feasible set, say T , with $x \in T$, if x is uniquely chosen over every other alternative obtainable from T , then x must be the only choice from T . Our Strong BDC demands that for all pairs of feasible sets, say S and T , if x is the only choice from S and it is uniquely chosen over every other alternative obtainable from T , then x must be the only choice from $S \cup T$. Obviously, if our property holds, BDC follows.

Strong BDC (SBDC). For all $S, T \in \mathcal{S} : \{x\} = C(S)$ and $\{x\} = C(\{x, y\})$ for all $y \in T \Rightarrow \{x\} = C(S \cup T)$.

Our next property is a particular weakening of Weak Axiom of Revealed Non-Inferiority (WARNI) of Eliaz and Ok (2006), according to which for any feasible set, say S , if for every $y \in C(S)$ there exists a feasible set, say T , such that $x \in C(T)$ and $y \in T$, then $x \in C(S)$. Our Weak WARNI on the other hand demands WARNI to hold if there exists a T , with $\{x, y\} \subset T \subset S$, such that x is the only choice from T , and x is choosable over every other $y \in S$.

Weak WARNI (WWARNI). For all $S \in \mathcal{S}, x \in S : \text{for all } y \in C(S)$ there exists $T \subset S : \{x\} = C(T)$ and $\{x, y\} \subset T$, and $x \in C(\{x, y\})$ for all $y \in S \Rightarrow x \in C(S)$.

The final property that we will consider here for reason-based choice correspondences plays a key role in the development of this paper. It posits

that for three distinct alternatives obtainable from a universal set, say x , y , and z , if x is strictly better than y and not worse than z , and y is not worse than z , then x must be the only choice from $\{x, y, z\}$. This property is motivated by the empirical research which established the importance of the attraction effect in decision making. Our property captures this phenomenon requiring a bias toward the most defensible alternative in term of reasons.

Reason-Based Bias (RBB). For all distinct $x, y, z \in X : \{x\} = C(\{x, y\})$, $x \in C(\{x, z\})$, and $y \in C(\{y, z\}) \Rightarrow \{x\} = C(\{x, y, z\})$.

3 Reason-Based Choice Correspondences

The following theorem shows that whenever X is a universal finite set of alternatives, the axioms above characterize completely a reason-based choice correspondence.

Theorem 1 *A choice correspondence C on \mathcal{S} is a reason-based choice correspondence if and only if it satisfies Property $\alpha 2$ ($\alpha 2$), Weak Property β ($W\beta$), Weak WARP ($WWARP$), Strong BDC ($SBDC$), Weak WARNI ($WWARNI$), and Reason-Based Bias (RBB).*

Proof. Suppose that C is a reason-based choice correspondence on \mathcal{S} . That C satisfies $\alpha 2$ is straightforward, thus omitted. We show that C satisfies the remaining choice-consistency conditions listed above.

To prove that C satisfies $W\beta$, take any $S, T \in \mathcal{S}$, such that $S \subseteq T$, and assume that $x, y \in C(S)$ and x and y are indistinguishable on $T \setminus \{x, y\}$. Let $y \in C(T)$. We show that $x \in C(T)$. Because $x, y \in M(S, \succ)$, neither $x \succ y$ nor $y \succ x$. Since $y \in M(T, \succ)$, then $z \succ y$ for no $z \in T \setminus \{x, y\}$. As x and y are indistinguishable on $T \setminus \{x, y\}$, $z \not\succeq x$ for all $z \in T \setminus \{x, y\}$. It follows that $x \in M(T, \succ)$. Moreover, there does not exist $z \in M(T, \succ) \setminus \{y\}$

such that $\prec (y, T) \subset \prec (z, T)$, by our supposition. Because x and y are indistinguishable on $T \setminus \{x, y\} \supseteq S \setminus \{x, y\}$, and neither $x \succ y$ nor $y \succ x$, we have that $\prec (y, T) = \prec (x, T)$. It follows from our supposition that $x \in C(T)$. Suppose that $y \notin C(T)$. We show that $x \notin C(T)$. Assume, to the contrary, that $x \in C(T)$. By an argument similar to the case above, we have that $y \in C(T)$, a contradiction.

To show that C satisfies WWARP, let $x, y \in X$ be two distinct alternatives, and take any $S, T \in \mathcal{S}$ such that $x, y \in S$ and $x, y \notin T$. Suppose that $\{x\} = C(S)$, and x and y are indistinguishable on T . Then $y \notin M(S, \succ)$ or $\prec (y, S) \subset \prec (x, S)$. If $y \notin M(S, \succ)$, then $y \notin M(S \cup T, \succ)$. As C is a reason-based choice correspondence it follows that $y \notin C(S \cup T)$. Otherwise, let consider $\prec (y, S) \subset \prec (x, S)$. If $\{z\} = C(\{y, z\})$ for some $z \in T \cup S$, then $y \notin M(S \cup T, \succ)$, and so $y \notin C(S \cup T)$, by our supposition. Otherwise, suppose $\{z\} \neq C(\{y, z\})$ for all $z \in T \cup S$. It follows that $y \in M(S \cup T, \succ)$. As x and y are indistinguishable on T , and $\prec (y, S) \subset \prec (x, S)$, it follows that $\prec (y, S \cup T) \subset \prec (x, S \cup T)$. Therefore, we have that $y \notin C(S \cup T)$, as desired.

To show that C satisfies SBDC, take any $S, T \in \mathcal{S}$, and suppose that $\{x\} = C(S)$ and $\{x\} = C(\{x, y\})$ for all $y \in T$. Because $\{x\} = C(S)$, it follows that either $\{x\} = M(S, \succ)$ or $\prec (z, S) \subset \prec (x, S)$ for all $z \in M(S, \succ) \setminus \{x\}$. As $x \succ y$ for all $y \in T \setminus \{x\}$ it follows that $x \in M(S \cup T, \succ)$. Suppose that $M(S \cup T, \succ) \neq \{x\}$. Then, the only possible case is that $z \in M(S \cup T, \succ) \setminus \{x\}$ for some $z \in M(S, \succ) \setminus \{x\}$. Because $\prec (z, S) \subset \prec (x, S)$ and $x \succ y$ for all $y \in T \setminus \{x\}$, it follows that $\prec (z, S \cup T) \subset \prec (x, S \cup T)$. Because this holds for any $z \in M(S \cup T, \succ) \setminus \{x\}$, with $z \in M(S, \succ) \setminus \{x\}$, our supposition implies that $\{x\} = C(S \cup T)$. Otherwise, let $M(S \cup T, \succ) = \{x\}$. It follows from our supposition that $\{x\} = C(S \cup T)$.

To prove that C satisfies WWARNI, take any $S \in \mathcal{S}$, with $x \in S$, and suppose that for every $y \in C(S)$ there exists $T \subset S$ such that $\{x\} = C(T)$ and $\{x, y\} \subset T$, and $x \in C(\{x, y\})$ for all $y \in S$. We show that $x \in C(S)$. By the way of contradiction, let $x \notin C(S)$. Thus $x \notin M(S, \succ)$ or $\prec(x, S) \subset \prec(z, S)$ for some $z \in M(S, \succ) \setminus \{x\}$. As $x \in C(\{x, y\})$ for all $y \in S$, it follows that $x \in M(S, \succ)$. Thus, it must be the case that $\prec(x, S) \subset \prec(z, S)$ for some $z \in M(S, \succ) \setminus \{x\}$. If $z \in C(S)$, it follows from our supposition that for no $T \subset S$ it can be that $\{x\} = C(T)$ and $\{x, z\} \subset T$, a contradiction. Otherwise, let $z \notin C(S)$. As S is finite and C is a reason-based choice correspondence, there exists $y \in C(S)$ such that $y \in M(S, \succ)$ and $\prec(z, S) \subset \prec(y, S)$. By the transitivity of set inclusion, $\prec(x, S) \subset \prec(y, S)$. Therefore, by our supposition, we have that for no $T \subset S$ it can be that $\{x\} = C(T)$ and $\{x, y\} \subset T$, a contradiction.

To prove that C meets RBB, let $x, y, z \in X$ be three distinct alternatives such that $\{x\} = C(\{x, y\})$, $x \in C(\{x, z\})$, and $y \in C(\{y, z\})$. We show that $\{x\} = C(\{x, y, z\})$. Assume, to the contrary, that $\{x\} \neq C(\{x, y, z\})$. As C is a reason-based choice rule and $y \neq x$ and $z \neq x$, we have that $x \in M(\{x, y, z\}, \succ)$. If $x \succ z$, then $z \notin M(\{x, y, z\}, \succ)$, and so that $\{x\} = M(\{x, y, z\}, \succ)$. It follows from our supposition that $\{x\} = C(\{x, y, z\})$, a contradiction. Otherwise, consider $x, z \in C(\{x, z\})$. If $\{y\} = C(\{y, z\})$, then $z \notin M(\{x, y, z\}, \succ)$. Because $\{x\} = M(\{x, y, z\}, \succ)$, it follows that $\{x\} = C(\{x, y, z\})$, a contradiction. Therefore, let $y, z \in C(\{y, z\})$. So, we have that $x, z \in M(\{x, y, z\}, \succ)$. Because $\prec(z, \{x, y, z\}) \subset \prec(x, \{x, y, z\})$, it follows from our supposition that $\{x\} = C(\{x, y, z\})$, a contradiction.

For the converse, assume that C satisfies $\alpha 2$, $W\beta$, WWARNI, SBDC, WWARP, and RBB. Given X , define the relation \succ on X as follows:

$$\text{for } x, y \in X, \text{ with } x \neq y: x \succ y \Leftrightarrow C(\{x, y\}) = \{x\}.$$

We have to prove that, for all $S \in \mathcal{S}$,

$$C(S) = \{x \in M(S, \succ) \mid \prec(x, S) \subset \prec(y, S) \text{ for no } y \in M(S, \succ) \setminus \{x\}\}$$

holds true and that \succ is acyclic.

To show acyclicity of \succ , suppose $x^1, x^2, \dots, x^t \in X$ are such that $x^{\tau-1} \succ x^\tau$ for $\tau \in \{2, \dots, t\}$, that is, $C(\{x^{\tau-1}, x^\tau\}) = \{x^{\tau-1}\}$ for $\tau \in \{2, \dots, t\}$. Let $S = \{x^1, x^2, \dots, x^t\} \subseteq \mathcal{S}$. Suppose that $x^\tau \in C(S)$ for $\tau \in \{2, \dots, t\}$. As $\alpha 2$ holds, we have $x^\tau \in C(\{x^{\tau-1}, x^\tau\})$, and so $x^{\tau-1} \not\succeq x^\tau$, a contradiction. Then $x^\tau \notin C(S)$ for $\tau \in \{2, \dots, t\}$. It follows from the nonemptiness of C that $\{x^1\} = C(S)$. Because $x^t \in S \setminus \{x^1\}$ and $\alpha 2$ holds, we have $x^1 \in C(\{x^1, x^t\})$. This implies $x^t \not\succeq x^1$, as desired.

Take any $S \in \mathcal{S}$, and let $x \in C(S)$. We show that $x \in M(S, \succ)$ and $\prec(x, S) \subset \prec(y, S)$ for no $y \in M(S, \succ) \setminus \{x\}$. Assume, to the contrary, that $x \notin M(S, \succ)$ or there exists $y \in M(S, \succ) \setminus \{x\}$ such that $\prec(x, S) \subset \prec(y, S)$. As $x \in C(\{x, y\})$ for all $y \in S$, by $\alpha 2$, the case $x \notin M(S, \succ)$ is not possible. Thus, let $x \in M(S, \succ)$ and $\prec(x, S) \subset \prec(y, S)$ for some $y \in M(S, \succ) \setminus \{x\}$. Take any $z \in \prec(y, S) \setminus \prec(x, S)$. Because $\{y\} = C(\{y, z\})$, $y \in C(\{x, y\})$, and $z \in C(\{x, z\})$, RBB implies $\{y\} = C(\{x, y, z\})$. It follows from SBDC that $\{y\} = C(\prec(y, S) \cup \{x, y\})$. If $S \setminus (\prec(y, S) \cup \{x, y\})$ is empty, then $x \notin C(S)$, a contradiction. Otherwise, let $S \setminus (\prec(y, S) \cup \{x, y\})$ be a nonempty set. Because x and y are indistinguishable on $S \setminus (\prec(y, S) \cup \{x, y\})$ and $\{y\} = C(\prec(y, S) \cup \{x, y\})$, WWARP implies $x \notin C(S)$, a contradiction.

Assume that $x \in M(S, \succ)$ and $\prec(x, S) \subset \prec(y, S)$ for no $y \in M(S, \succ) \setminus \{x\}$. We show that $x \in C(S)$. Because $x \in M(S, \succ)$, it follows that $x \in C(\{x, y\})$ for all $y \in S$. If $\{x\} = M(S, \succ)$, it is clear, by $\alpha 2$ and the nonemptiness of C , that $\{x\} = C(S)$. Otherwise, consider $\{x\} \neq M(S, \succ)$. By the nonemptiness of C , $\{x\} = C(S)$ whenever $y \notin C(S)$ for all $y \in S \setminus \{x\}$. Thus, let $y \in C(S)$ for some $y \in S \setminus \{x\}$. It follows from the paragraph above that $y \in M(S, \succ)$ and $\prec(y, S) \subset \prec(z, S)$ for no $z \in M(S, \succ) \setminus \{y\}$.

Therefore, $\{x, y\} = C(\{x, y\})$. If $\prec(y, S)$ is empty, then $\prec(x, S)$ must be empty, and so $W\beta$ implies $x \in C(S)$. Thus, let $\prec(y, S)$ be a nonempty set. It follows that $\prec(x, S)$ is nonempty as well. If $\prec(x, S) = \prec(y, S)$ for some $y \in C(S)$, $W\beta$ implies $x \in C(S)$. So, let $\prec(x, S) \neq \prec(y, S)$ for all $y \in C(S)$. Thus, for any $y \in C(S)$, there exists $z \in \prec(x, S) \setminus \prec(y, S)$ and $w \in \prec(y, S) \setminus \prec(x, S)$. Therefore, for all $y \in C(S)$, $\{x, y, z\} \subset S$ for some $z \in \prec(x, S) \setminus \prec(y, S)$. Since $\{x\} = C(\{x, z\})$, $z \in C(\{y, z\})$, and $x \in C(\{x, y\})$ it follows from RBB that $\{x\} = C(\{x, y, z\})$. Because this holds for any $y \in C(S)$, $WWARNI$ implies $x \in C(S)$. ■

The mutual independence of choice-consistency conditions used in Theorem 1 has been relegated to the Appendix A.

Observe that a reason-based choice correspondence does not meet Sen's (1977) Property $\gamma 2$, according to which for any feasible set, say S , if $x \in C(\{x, y\})$ for all $y \in S$, then $x \in C(S)$. To see it, suppose that an individual has the following preferences among three distinct alternatives: $x \succ x'$, $x \not\succeq y$ and $y \not\succeq x$, and $x' \not\succeq y$ and $y \not\succeq x'$. If our individual follows the reason-based procedural choice, then her choices are $y \in C(\{x, y\})$, $y \in C(\{x', y\})$, and $\{x\} = C(\{x, x', y\})$ which contradicts Property $\gamma 2$. It follows that Property $\alpha 2$ is not equivalent to Property α in our framework (that is, Sen's (1977, p.65) Proposition 10 is not fulfilled). Needless to say, Sen's (1971) Property γ is not necessary for a reason-based choice correspondence. It also follows that our reason-based choice correspondences do not necessarily meet the standard 'binariness' property.

4 Concluding Remarks

Motivated by the vast literature on the attraction effect, we provide a characterization of reason-based choice correspondences which captures the basic

idea behind this choice ‘anomaly’.

Our characterization result is obtained by using the standard revealed preference methodology. Thus we suppose that our individual possesses a (not necessarily complete) preference relation which is revealed by her choices. From the normative point of view our choice-consistency conditions are appealing because they never lead an individual to make ‘bad’ choices (i.e., dominated alternatives) even though most of them are weaker than the conventional choice-consistency conditions (i.e., Property α , Property β , Samuelson’s WARP). Nonetheless, a reason-based choice correspondence lends itself to certain framing manipulations that are hard to explain only from the point of view of preference maximization. However, it differs from other rules which allow an individual to reveal a fixed cyclic preference relation (see, Ehlers and Sprumont (2006), Lombardi (2006)).

In this work we have attempted to analyse how offered sets may induce an individual to follow a particular guidance in her decisions, and how this affects her choices across sets. Needless to say, reasons that guide decisions are likely to be diverse. In this respect, for example, Baigent and Gaertner (1996) characterize a choice procedure in which the individual’s choices are guided by a self-imposed constraint of “choosing a non unique largest or otherwise a second largest alternative” from each offered set.

Our two-stage choice model can be contrasted with other decision-making procedures recently suggested in the literature. Closer to our reason-based choice rules are the two-stage procedural choice models suggested by Houy (2006), Manzini and Mariotti (2006, 2007), Rubinstein and Salant (2006). The first author proposes a choice model in which an individual eliminates maximal alternatives identified in the first-stage according to her conservative mood. In contrast to the history dependent choice model of Houy, in our model the choices of an individual are driven by the set under consider-

ation whenever her preferences are insufficient to solve the decision problem that she deals with. Manzini and Mariotti (2006, 2007) and Rubinstein and Salant characterize slightly different two-stage choice models which have the following property in common: the individual arrives at a choice by eliminating some of the shortlisted alternatives identified in the first stage according to a fixed asymmetric (and not transitive) preference relation.⁶ In contrast to their two-stage choice procedures, in our model an individual constructs in the second-stage a (not necessarily complete) binary relation (i.e., the strict set inclusion) according to her known preferences on the set of alternatives under consideration to knock off some of the alternatives that survive the first round of elimination.

To conclude we observe that Masatlioglu and Ok (2006) axiomatize a reference-dependent procedural choice model in which an individual, endowed with an objective utility function, solves sequentially a two-stage constrained utility maximization problem where the constraints depend on her status quo alternative - if she cannot find an alternative yielding her a higher utility level than that brought to her by the status quo alternative, she keeps the latter. Our model differs from the choice model of Masatlioglu and Ok in many respects. Mainly we have investigated how in the second-stage a convincing reason (i.e., the information inferable from the entire offered set) plays a role in decision making rather than investigating how a reference alternative affects the decisions that an individual makes.

⁶Indeed, Manzini and Mariotti (2007) suggest a choice model in which an individual may use several asymmetric preference relations to arrive at a choice.

References

- [1] Baigent, N., and W. Gaertner (1996): “Never Choose the Uniquely Largest: A characterization”, *Economic Theory*, 8(2): 239-249.
- [2] Ehlers, L., and Y. Sprumont (2006): “Weakened WARP and Top-Cycle Choice Rules”, forthcoming in *Journal of Mathematical Economics*.
- [3] Eliaz, K., and E.A. Ok (2006): “Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences”, *Games and Economic Behavior*, 56(1): 61-86.
- [4] Houy, N. (2006): “Choice Correspondence with States of Mind”, mimeo.
- [5] Lombardi, M. (2006): “Uncovered Set Choice Rules”, mimeo.
- [6] Manzini, P., and M. Mariotti (2006): “Two-Stage Boundedly Rational Choice Procedures: Theory and Experimental Evidence”, Queen Mary Working Paper n. 561.
- [7] Manzini, P., and M. Mariotti (2007): “Sequentially Rationalizable Choice”, forthcoming in *American Economic Review*.
- [8] Mariotti, M. (2007): “What kind of preference maximization does the weak axiom of revealed preference characterize?”, forthcoming in *Economic Theory*.
- [9] Masatlioglu, Y., and E.A. Ok (2006): “Reference-Dependent Procedural Decision Making”, Department of Economics, New York University.
- [10] Moulin, H. (1985): “Choice Functions over a Finite Set: A Summary,” *Social Choice and Welfare*, 2(2): 147-160.

- [11] Ok, E.A. (2004): “Independence of Irrelevant Alternatives and Individual Choice”, mimeo.
- [12] Richter, M.K. (1966): “Revealed Preference Theory”, *Econometrica*, 34(3): 635-645.
- [13] Rieskamp, J., J.R. Busemeyer, and B.A. Mellers (2006): “Extending the Bounds of Rationality: Evidence and Theories of Preferential Choice”, *Journal of Economic Literature*, 44(3): 631-661.
- [14] Rubinstein, A., and Y. Salant (2006): “Two Comments on the Principle of Revealed Preference”, mimeo, available online at: <http://arielrubinstein.tau.ac.il/papers/frames.pdf>.
- [15] Rubinstein, A., and Y. Salant (2007): “Some thoughts on the principle of revealed preferences”, forthcoming in *Foundations of Positive and Normative Economics* (Andrew Caplin and Andrew Schotter, eds).
- [16] Samuelson, P. (1938): “A Note on the Pure Theory of Consumer’s Behaviour”, *Economica*, 5(17): 61-71.
- [17] Sen, A.K. (1971): “Choice Functions and Revealed Preference”, *Review of Economic Studies*, 38(3): 307-317.
- [18] Sen, A.K. (1977): “Social Choice Theory: A Re-Examination”, *Econometrica*, 45(1): 53-88.
- [19] Sen, A.K. (1993): “Internal Consistency of Choice”, *Econometrica*, 61(3): 495-521.
- [20] Sen, A.K. (1997): “Maximization and the Act of Choice” *Econometrica*, 65(4): 745-779.

- [21] Simonson, I. (1989): “Choice Based on Reasons: The Case of Attraction and Compromise Effects”, *Journal of Consumer Research*, 16(2): 158-174.
- [22] Shafir, E., I. Simonson, and A. Tversky (1993): “Reason-based Choice”, *Cognition*, 49(1-2): 11-36.
- [23] Spiegler, R. (2002): “Equilibrium in Justifiable Strategies: A Model of Reason-Based Choice in Extensive-Form Games”, *The Review of Economic Studies*, 69(3): 691-706.
- [24] Suzumura, K. (1983): *Rational Choice, Collective Decisions, and Social Welfare*, Cambridge University Press, Cambridge U.K..
- [25] Tversky, A., and I. Simonson (1993): “Context-dependent Preferences”, *Management Science*, 13(10): 1179-1189.
- [26] Varian, H.R (2005): “Revealed Preference”, <http://www.ischool.berkeley.edu/~hal/Papers/2005/revpref.pdf>

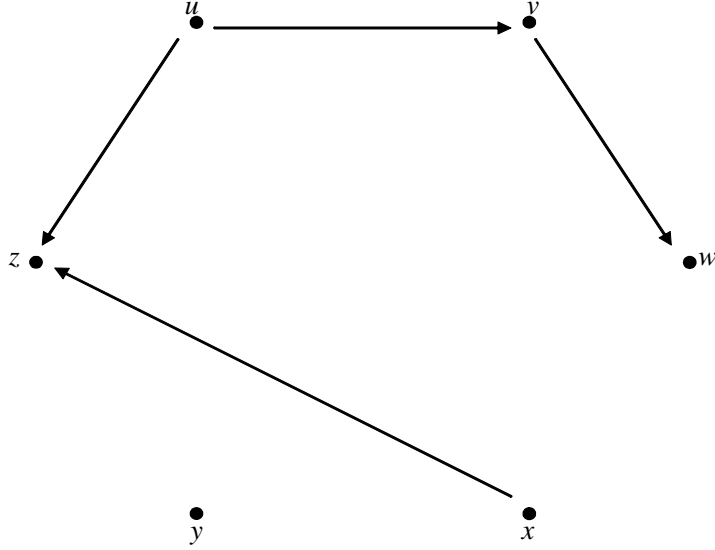


Figure 2: Binary Choices.

A Appendix

To complete the proof of Theorem 1, we show that Property $\alpha 2$ ($\alpha 2$), Weak Property β ($W\beta$), Weak WARNI (WWARNI), Strong BDC (SBDC), Weak WARP (WWARP), and Reason-Based Bias (RBB) are independent.

Suppose that u, v, w, x, y and z are distinct feasible alternatives, and let choice in pairs be as displayed in Figure 2, where $a \rightarrow b$ stands for $\{a\} = C(\{a, b\})$, whilst no arrow between a and b stands for $\{a, b\} = C(\{a, b\})$.

For an example violating only $\alpha 2$, fix $X = \{u, x, z\}$, and suppose $C(X) = X$. C is not a reason-based choice correspondence because $z \notin M(S, \succ)$ but $z \in C(S)$. $\alpha 2$ is violated as $z \notin C(\{x, z\})$. All other choice-consistency conditions are satisfied.

For an example violating only $W\beta$, fix $X = \{u, x, z\}$, and suppose

$C(X) = \{u\}$. C is not a reason-based choice correspondence because $x \in M(S, \succ)$ and there does not exist $a \in M(S, \succ) \setminus \{x\}$ such that $\prec(x, S) \subset \prec(a, S)$ but $x \notin C(X)$. $W\beta$ is violated because $\{u, x\} = C(\{u, x\})$, u and x are indistinguishable on $\{z\}$, and $u \in C(X)$ but $x \notin C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only $WWARNI$, fix $X = \{v, w, x, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \setminus X$, and suppose $C(X) = \{v\}$. C is not a reason-based choice correspondence because $x \in M(X, \succ)$ and there does not exist $a \in M(X, \succ) \setminus \{x\}$ such that $\prec(x, X) \subset \prec(a, X)$, but $x \notin C(X)$. $WWARNI$ is violated because $v \in C(X)$, and there exists $S = \{v, x, z\} \subset X$ such that $\{x\} = C(S)$ and $\{x, v\} \subset S$, $x \in C(\{a, x\})$ for all $a \in X$, but $x \notin C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only $SBDC$, fix $X = \{u, v, y, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \setminus X$, and suppose $C(X) = \{u, y\}$. C is not a reason-based choice correspondence because $u, y \in M(S, \succ)$ and $\prec(y, S) \subset \prec(u, S)$, but $y \in C(X)$. $SBDC$ is violated because $C(\{u, v, y\}) = \{u\}$ and $\{u\} = C(\{u, z\})$ but $C(X) \neq \{u\}$. All other choice-consistency conditions are satisfied.

For an example violating only $WWARP$, fix $X = \{w, x, y, z\}$. Let $C(S)$ be a reason-based choice correspondence for all $S \in \mathcal{S} \setminus X$, and suppose $C(X) = X \setminus \{z\}$. C is not a reason-based choice correspondence because $w, x, y \in M(S, \succ)$, $\prec(w, S), \prec(y, S) \subset \prec(x, S)$, but $w, y \in C(X)$. $WWARP$ is violated because $C(\{x, y, z\}) = \{x\}$, $y \notin C(\{x, y, z\})$, x and y are indistinguishable on $\{w\}$, but $y \in C(X)$. All other choice-consistency conditions are satisfied.

For an example violating only RBB , fix $X = \{x, y, z\}$. Let $C(X) = \{x, y\}$. C is not a reason-based choice correspondence because $x, y \in M(S, \succ)$,

$\prec (y, S) \subset \prec (x, S)$, but $y \in C(X)$. RBB is violated because $\{x\} = C(\{x, z\})$, $x \in C(\{x, y\})$, $z \in C(\{y, z\})$, but $C(\{x, y, z\}) \neq \{x\}$. All other choice-consistency conditions are satisfied.

**This working paper has been produced by
the Department of Economics at
Queen Mary, University of London**

**Copyright © 2007 Michele Lombardi
All rights reserved**

**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm**