

Department of Economics

Inflation Forecast Targeting in an Overlapping Generations Model

Gerhard Sorger

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Queen Mary
University of London

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GERHARD SORGER

Department of Economics

Queen Mary, University of London

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Abstract: In the framework of a standard overlapping generations model, it is shown that active inflation forecast targeting reinforces mechanisms that lead to indeterminacy of the monetary steady state and to countercyclical behavior of young-age consumption. The inflation forecast targeting rule which minimizes the volatility of inflation can be active or passive, depending on the characteristics of shocks and the risk aversion of households. Inflation forecast errors are always greater under active inflation forecast targeting than under passive inflation forecast targeting or strict money growth targeting. The monetary steady state is more likely to be indeterminate under an active rule of inflation forecast targeting than under the corresponding backward-looking rule (inflation targeting), but backward-looking rules can render the monetary steady state unstable.

Journal of Economic Literature **classification codes:** E32, E52

1 Introduction

In recent years many prominent central banks have adopted inflation targeting or inflation forecast targeting as their preferred framework for the conduct of monetary policy. In a parallel development in the academic literature, instrument rules with varying degrees of responsiveness to inflation or inflation forecasts have been explored across a number of different models; see, e.g., Taylor [8] or Benhabib et al. [1]. Following this line of research, the present paper studies a standard overlapping generations model under the assumption that monetary policy is implemented through an instrument rule of the form

$$\ln \mu_t - \ln n = \ln g - \frac{1 - \gamma}{\gamma} (\ln \hat{\pi}_t - \ln g). \quad (1)$$

Here μ_t and $\hat{\pi}_t$ are the gross rate of nominal money growth and the expected gross rate of inflation, respectively, from period t to period $t + 1$, and n is the gross rate of population growth. The target for the adjusted money growth is given by g and the elasticity of the money growth rate with respect to deviations of inflation forecasts from target is $-(1 - \gamma)/\gamma$, where γ is a positive parameter. The main goal of the paper is to study how the policy parameter γ affects the stability and determinacy of the monetary steady state in a deterministic version of the model and the characteristics of business cycles in a stochastic setting. We also analyse the economy under an alternative instrument rule, which is obtained from (1) by replacing the inflation forecast $\hat{\pi}_t$ by actual past inflation π_{t-1} . We call (1) an inflation forecast targeting rule or a forward-looking rule, and we refer to the modified version as an inflation targeting rule or a backward-looking rule.¹

If $\gamma = 1$, then equation (1) requires the nominal money supply to grow at the constant gross rate $\mu_t = gn$. In other words, $\gamma = 0$ corresponds to a regime of strict money growth targeting, in which the adjusted nominal money growth rate is fixed at g . For $\gamma \in (0, 1)$, the nominal money growth rate μ_t is proportional to the population growth rate n and decreasing with respect to the inflation forecast $\hat{\pi}_t$. We therefore call a rule with $\gamma \in (0, 1)$ an active rule. The limiting case $\gamma = 0$ describes a regime of strict inflation forecast targeting, in which the inflation forecast $\hat{\pi}_t$

¹Part of the literature reserves the term ‘target variable’ to a variable occurring in a loss function of the central bank. We adopt an alternative definition according to which the expression ‘targeting variable x ’ means ‘using a feedback rule that depends directly on x ’. See Svensson [7] for a discussion of this terminology.

must coincide with g . If $\gamma > 1$, then it is still the case that the money growth rate is proportional to n , but now it increases with the inflation forecast. Note, however, that $-(1 - \gamma)/\gamma < 1$ holds for all $\gamma > 1$, such that the money growth rate reacts less than one-for-one to deviations of the inflation forecast from target. Thus, we call a rule with $\gamma > 1$ a passive rule.²

We conduct our analysis in the framework of a standard overlapping generations model. The determinacy and stability of the monetary steady state in this model have been thoroughly investigated by Grandmont [5] under the assumption of a constant money supply. The present paper generalizes some of these results to an economy in which the central bank uses the rule (1) and distributes newly created money in the form of lump-sum transfers to old agents. Furthermore, we also consider a stochastic version of Grandmont's model and study how the volatility of inflation and the accuracy of inflation forecasts depends on γ .

In section 2 we formulate the model and show that by making the instrument rule more active (i.e., by decreasing γ towards 0) the set of economies for which the monetary steady state is indeterminate unambiguously increases. Because of the well-known relation between indeterminacy and the existence of periodic equilibria and sunspot equilibria, this shows that active inflation forecast targeting renders the economy more susceptible to endogenous business cycles than strict money growth targeting or passive inflation forecast targeting. The intuition for this result is quite simple. Suppose that $\hat{\pi}_t$, the forecast for inflation from period t to period $t + 1$, is high. Under active inflation forecast targeting, this implies that monetary policy will be tightened. Young households in period t therefore rationally expect to receive low transfers during their old age, which increases their incentive to transfer wealth from period t to period $t + 1$. This obviously increases their demand for money (the only store of value) while it reduces aggregate demand for goods in period t . Consequently, the price level in period t goes down and this helps to validate the high expected inflation rate $\hat{\pi}_t$. We therefore conclude that active inflation forecast targeting reinforces the mechanism which generates periodic equilibria in the overlapping generations model in the first place, and which results from the interplay between income and substitution effects.

In section 3 we introduce exogenous stochastic shocks into the model. If the monetary steady state

²The terminology of active and passive rules is chosen in analogy to Leeper [6].

is a locally unique (determinate) equilibrium of the underlying deterministic model, then there exists a unique stationary Markov equilibrium of the stochastic model close to the steady state. We investigate how the properties of this equilibrium depend on the variance and autocorrelation of the endowment shocks and on the policy parameter γ . It is shown that young-age consumption is procyclical and less volatile than the stochastic endowment process whenever shocks are non-negatively autocorrelated. If shocks are negatively autocorrelated and γ is sufficiently small, on the other hand, then young-age consumption may become countercyclical. The potential for countercyclical young-age consumption already exists in the model with constant money growth, but it is amplified by active inflation forecast targeting for reasons similar to those mentioned above in connection with indeterminacy.

We then determine the standard deviation of inflation as a function of γ . This allows us to compute the policy rule which minimizes the effect of exogenous shocks on the volatility of inflation. The optimal policy rule may be active or passive, depending on the characteristics of the shocks and the risk aversion of households. Active inflation forecast targeting minimizes the volatility of inflation, if shocks are non-negatively autocorrelated and households are sufficiently risk averse in either of their two periods of life. Our last result for the stochastic model deals with the accuracy of inflation forecasts under inflation forecast targeting. Due to the rationality of expectations, inflation forecasts are always unbiased. We show that the standard variation of inflation forecasts normally increases with γ , but that the standard variation of inflation forecast *errors* always decreases with γ . The conclusion from the latter result is that inflation forecasts are less reliable under an active policy than under a passive rule. In other words, the harder the central bank tries to keep inflation forecasts at their target, the less accurate these forecasts become. This finding is related to the those made by Woodford [9] and Bernanke and Woodford [4], who pointed out that it is impossible to extract accurate information about exogenous shocks from inflation forecasts and, at the same time, to offset the effect of these shocks on the economy.

Section 4 studies again the deterministic model, but now under the instrument rule (1) with the inflation forecast $\hat{\pi}_t$ replaced by the actual past inflation rate π_{t-1} . We study this case because several authors have claimed that backward-looking instrument rules perform better than

forward-looking ones; see Benhabib et al. [2, 3] and references therein. We can confirm within our framework that, by switching from an active forward-looking rule to the corresponding backward-looking rule, the set of economies for which indeterminacy occurs is reduced. If we also allow passive rules, however, this is not true. Furthermore, we find that the monetary steady state can become unstable if an active backward-looking instrument rule is used. This is a consequence of the fact that a backward-looking rule introduces a pre-determined variable into the equilibrium dynamics, namely last period's price level. If the monetary steady state is unstable in this sense, then the economy cannot converge to the monetary steady state unless the pre-determined variable has its steady state value.

The paper concludes with section 5, where we summarize our findings and discuss possible caveats and extensions to our analysis.

2 Indeterminacy

The present section starts by developing the framework for our study of the properties of inflation forecast targeting. Except for the description of monetary policy, this framework is essentially identical to the one used by Grandmont [5] in his study of endogenous competitive business cycles.³ We then study how the sensitivity of the monetary policy rule with respect to inflation forecasts affects the local uniqueness of monetary equilibria under perfect foresight. The lack of local uniqueness, also referred to as indeterminacy, is closely related to the existence of periodic equilibria and to the susceptibility of the economy to the influence of extrinsic uncertainty (sunspots). The results of the present section have therefore immediate implications for the existence of deterministic business cycles and self-fulfilling beliefs.

We consider an economy populated by a sequence of overlapping generations of two-period lived households (agents). Each generation consists of a continuum of identical households. The size of the generation born in period t is denoted by N_t . We assume constant population growth at the gross rate $n > 0$, that is, $N_{t+1} = nN_t$ for all t . In both periods of their life, households are endowed

³For the most part of his paper, Grandmont [5] considers the case of a constant nominal money supply.

with positive quantities of a homogeneous consumption good that can neither be produced nor stored.⁴ Every young agent living in period t is endowed with $y_t > 0$ units of the good, whereas every old agent is endowed with 1 unit of the good. The sequence (\dots, y_t, \dots) will either be assumed to be constant (sections 2 and 4) or to be a stationary Markov process (section 3). In either case it follows that the endowment patterns of all agents are ex ante identical and that there is neither exogenous nor endogenous growth of per-capita endowments. As for the preferences, we assume that all households have the common, additively separable utility function $u(c_1) + v(c_2)$, where c_1 and c_2 denote consumption in the first and second period of life, respectively. The functions u and v are assumed to have standard smoothness and curvature properties, i.e., both of these functions are twice continuously differentiable on the interval $(0, +\infty)$, and they satisfy $u'(c) > 0$, $v'(c) > 0$, $u''(c) < 0$, and $v''(c) < 0$ for all $c > 0$. Moreover, $\lim_{c \rightarrow 0} u'(c) = +\infty$ and $\lim_{c \rightarrow 0} v'(c) = +\infty$ are assumed to hold.

We denote by $M_t > 0$ the nominal money supply in period t and by $\mu_t > 0$ the gross rate of money growth from period t to period $t + 1$, that is, $\mu_t = M_{t+1}/M_t$. The central bank controls the money supply by using the money growth rate μ_t as its instrument variable. Newly created money is supplied to the economy in the form of lump-sum transfers to old agents.⁵ All agents of a given generation receive the same transfer. We denote by τ_{t+1} the nominal lump-sum transfer paid out in period $t + 1$ to the old agents of generation t . Consequently, we have $M_{t+1} = M_t + N_t \tau_{t+1}$.

We assume that the central bank uses the policy rule (1), which can also be written in the more compact form

$$(\mu_t/n)^\gamma \hat{\pi}_t^{1-\gamma} = g. \quad (2)$$

The variable $\hat{\pi}_t$ denotes the inflation rate that is rationally expected to prevail between periods t and $t + 1$. We shall therefore refer to $\hat{\pi}_t$ as the inflation forecast for period t . Since the rational expectation of current inflation is neither exogenous nor pre-determined at the time when the current growth rate of the money supply is set, the instrument rule (2) is an *implicit* rule in the sense of Svensson [7].

⁴Although we present the model as an endowment economy, it is possible to interpret it also as a production economy; see Grandmont [5, page 1000].

⁵The transfers can also be negative, in which case they are interpreted as lump-sum taxes.

Both policy parameters g and γ are strictly positive numbers. We shall refer to g as the target growth rate of the central bank and to γ as the type of the policy rule. As explained in the introduction, we say that the rule is active if $\gamma \in (0, 1)$ and that it is passive if $\gamma > 1$. The special cases $\gamma = 1$ and $\gamma = 0$ correspond to strict money growth targeting and strict inflation forecast targeting, respectively. Throughout the paper we assume that the policy parameters g and γ are chosen once and for all, and that the central bank makes a credible commitment to follow the rule determined by these parameters. Because of this commitment, deviations from the announced policy are ruled out and the problem of dynamic inconsistency does not arise.

We conclude the description of the model by explaining the sequence of events in period t . At the beginning of the period, the central bank makes transfers τ_t to all old agents of generation $t - 1$ and the endowments y_t of young agents of generation t are realized. Then, young households of generation t make their consumption/saving decision and the central bank chooses the money growth rate μ_t according to (2). Finally, markets are cleared.

For the rest of this section we assume that the endowments are deterministic and constant, that is, $y_t = \bar{y}$ for all t , where \bar{y} is a positive number. Because of this assumption, the model description does not involve any form of intrinsic uncertainty and it is therefore feasible to make the assumption of perfect foresight. In the present case, this means that the inflation forecast used by the central bank, $\hat{\pi}_t$, coincides with the actual inflation rate. Denoting by p_t the price level that prevails in period t , this condition can be written as

$$\hat{\pi}_t = p_{t+1}/p_t. \quad (3)$$

Under the above assumptions, the utility maximization problem of a household of generation t can be written as

$$\begin{aligned} & \text{maximize} && \{u(c_1) + v(c_2)\} \\ & \text{subject to} && c_1 + m/p_t \leq \bar{y}, \\ & && c_2 \leq 1 + (m + \tau_{t+1})/p_{t+1}, \\ & && c_1 \geq 0, c_2 \geq 0, m \geq 0, \end{aligned}$$

where m denotes the nominal money balance carried from period t to period $t + 1$. The first-order

optimality condition for this problem is $u'(c_1)/p_t = v'(c_2)/p_{t+1}$, market clearing in period t implies $m = M_t/N_t$, and our assumptions about population growth and transfers (spelled out above) imply $m + \tau_{t+1} = M_{t+1}/N_t = nM_{t+1}/N_{t+1}$. Taking all these conditions together and using the budget constraints, we obtain the equilibrium condition

$$\frac{1}{p_t} u' \left(\bar{y} - \frac{M_t}{N_t p_t} \right) = \frac{1}{p_{t+1}} v' \left(1 + \frac{nM_{t+1}}{N_{t+1} p_{t+1}} \right). \quad (4)$$

In order to determine the equilibrium dynamics under the policy rule (2), we denote the per-capita real money balances carried from period t to $t + 1$ by x_t , i.e., $x_t = M_t/(N_t p_t)$. It follows that

$$\frac{x_t}{x_{t+1}} = \frac{np_{t+1}}{\mu_t p_t}. \quad (5)$$

Combining this with equations (2) and (3), we obtain $x_t/x_{t+1} = \hat{\pi}_t(n/\mu_t) = (\hat{\pi}_t/g)^{1/\gamma}$, which can be solved for $\hat{\pi}_t$ to yield $\hat{\pi}_t = g(x_t/x_{t+1})^\gamma$. Finally, by combining this equation with (3) and (4) we obtain the equilibrium dynamics

$$gx_t^\gamma u'(\bar{y} - x_t) = x_{t+1}^\gamma v'(1 + nx_{t+1}). \quad (6)$$

One obvious solution to the difference equation (6) is $x_t = 0$ for all t . This is the non-monetary steady state which exists under all parameter constellations. If the target growth rate g is sufficiently low or the first-period endowment \bar{y} is sufficiently large, then there exists also a monetary steady state. This is stated in a formal way in the following lemma.

Lemma 1 *If $gu'(\bar{y}) \geq v'(1)$, then the only constant solution to equation (6) is the non-monetary steady state $x_t = 0$ for all t . If $gu'(\bar{y}) < v'(1)$, then there exists a unique monetary steady state equilibrium. This equilibrium is given by $x_t = \bar{x}$ for all t , where \bar{x} is the unique value in the interval $(0, \bar{y})$ satisfying*

$$gu'(\bar{y} - \bar{x}) = v'(1 + n\bar{x}). \quad (7)$$

PROOF: If $x_t = x$ is a constant solution to equation (6), then it follows that either $x = 0$ or $gu'(\bar{y} - x) = v'(1 + nx)$ must hold. The solution $x_t = 0$ is the non-monetary steady state. The function $x \mapsto gu'(\bar{y} - x)$ is strictly increasing on $(0, \bar{y})$ whereas the function $x \mapsto v'(1 + nx)$ is strictly decreasing on $(0, \bar{y})$. Moreover, we have $\lim_{x \rightarrow \bar{y}} gu'(\bar{y} - x) = +\infty > v'(1 + n\bar{y})$. Taking

these two properties together, it follows that the equation $gu'(\bar{y} - x) = v'(1 + nx)$ has a positive solution if and only if $gu'(\bar{y}) < v'(1)$. Moreover, if a positive solution exists it must be equal to $x = \bar{x}$. \triangleleft

A few remarks are in order with regard to the above lemma. First, in the monetary steady state it holds that $x_t = x_{t+1} = \bar{x}$ and, therefore, equations (3) and (5) imply that the adjusted money growth rate μ_t/n equals the inflation forecast $\hat{\pi}_t$. Because of (2), this implies that both of these rates are equal to g . Second, note that both the condition ensuring the existence of a monetary steady state and the value \bar{x} itself are independent of the type of the policy rule γ . The corresponding consumption rates $\bar{c}_1 = \bar{y} - \bar{x}$ and $\bar{c}_2 = 1 + n\bar{x}$ are therefore also independent of γ . For the remainder of the paper we assume that a monetary steady state exists. Because of lemma 1, this assumption can be formulated as follows.

Assumption 1 It holds that $gu'(\bar{y}) < v'(1)$.

Since the left-hand side of (6) is a strictly increasing function of x_t with domain $[0, \bar{y})$ and range $[0, +\infty)$, one can solve (6) for x_t as a function of x_{t+1} , say, $x_t = f(x_{t+1})$. The function f is usually called the backward perfect foresight dynamics and it is known to be continuously differentiable under our assumptions (see, e.g., Grandmont [5]). Of course, we have $f(\bar{x}) = \bar{x}$. The monetary steady state $x_t = \bar{x}$ is called indeterminate if, for every $\eta > 0$, there exist infinitely many solutions of the difference equation (6) satisfying $|x_t - \bar{x}| < \eta$ for all t . It is obvious that this is the case if $|f'(\bar{x})| > 1$, whereas it is ruled out if $|f'(\bar{x})| < 1$. Theorem 1 below characterizes the conditions under which the monetary steady state is determinate or indeterminate, respectively. It will be convenient to define $\bar{A}_u = -u''(\bar{c}_1)/u'(\bar{c}_1)$ and $\bar{A}_v = -v''(\bar{c}_2)/v'(\bar{c}_2)$. The numbers \bar{A}_u and \bar{A}_v are positive and measure the degrees of absolute risk aversion of u and v , respectively, both evaluated at the monetary steady state.⁶ Furthermore, we define

$$\gamma_1 = \bar{x}(n\bar{A}_v - \bar{A}_u)/2.$$

⁶Grandmont [5] discusses his results in terms of the degrees of *relative* risk aversion $\bar{A}_u\bar{c}_1$ and $\bar{A}_v\bar{c}_2$, respectively. We find it easier to formulate our results in terms of the degrees of *absolute* risk aversion.

Theorem 1 *Let assumption 1 be satisfied.*

(a) *If $n\bar{A}_v - \bar{A}_u \leq 0$, then $\gamma_1 < 0$ and the monetary steady state is determinate for all policy types $\gamma > 0$.*

(b) *If $n\bar{A}_v - \bar{A}_u > 0$, then it follows that $\gamma_1 > 0$. In this case, the monetary steady state is determinate for all policy types $\gamma \in (\gamma_1, +\infty)$ and it is indeterminate for all policy types $\gamma \in (0, \gamma_1)$.*

PROOF: Substituting $x_t = f(x_{t+1})$ into (6), differentiating the resulting identity with respect to x_{t+1} , and evaluating the result at $x_{t+1} = \bar{x}$ one gets $f'(\bar{x}) = B(\gamma)$, where $B(\gamma) = (\gamma - n\bar{x}\bar{A}_v)/(\gamma + \bar{x}\bar{A}_u)$. It is obvious that $B(\gamma) < 1$ holds for all $\gamma > 0$. Thus, indeterminacy occurs if $B(\gamma) < -1$ and determinacy occurs if $B(\gamma) > -1$. Because of $B'(\gamma) = \bar{x}(\bar{A}_u + n\bar{A}_v)/(\gamma + \bar{x}\bar{A}_u)^2 > 0$ it follows that B is a strictly increasing function. The borderline policy type between determinacy and indeterminacy is therefore uniquely characterized by the equation $B(\gamma) = -1$. It is straightforward to see that the solution of this equation is $\gamma = \gamma_1$. Determinacy occurs for policy types $\gamma > \gamma_1$ and indeterminacy occurs for policy types $\gamma < \gamma_1$. If $n\bar{A}_v - \bar{A}_u \leq 0$, then it follows that $\gamma_1 \leq 0$ so that all feasible policy types give rise to determinacy. ◁

The above theorem demonstrates that the occurrence of indeterminacy depends on the type γ of the policy rule and on the difference between the degrees of absolute risk aversion of old and young agents, $n\bar{A}_v - \bar{A}_u$. Note that the latter is independent of γ . To understand this result intuitively, it is instructive to recall why indeterminacy can occur in the model with constant money growth, i.e., the model with $\gamma = 1$. The crucial property of that model is that income effects may dominate substitution effects, in which case first-period consumption becomes a decreasing function of expected inflation. As shown by Grandmont [5, lemma 1.3], this is the case if old agents are sufficiently risk averse. Suppose now that, for some reason, agents expect high inflation from period t to $t + 1$, i.e., the inflation forecast $\hat{\pi}_t$ is large. If the income effect dominates the substitution effect, this means that desired first-period consumption is relatively low and households wish to transfer wealth from period t to period $t + 1$. The low aggregate demand for goods in period t implies that the price level p_t is also low. Analogously, if expected inflation in period $t + 1$, $\hat{\pi}_{t+1}$, is low, then p_{t+1} will be high. Thus, an oscillating pattern of price levels validates the beliefs about inflation. These oscillations will not die out if the risk aversion of old

agents is large not only in absolute terms but also relative to the risk aversion of young agents; see [5, equation (4.10)]. If the central bank uses strict money growth targeting, this is the end of the story, but if active inflation forecast targeting is applied, there exists another effect acting in the same direction. If the inflation forecast $\hat{\pi}_t$ is high, active inflation forecast targeting results in a tight money supply in period $t + 1$. Households who are born in period t will therefore rationally expect to receive a low transfer in their old age. This reinforces their desire to transfer wealth from period t to $t + 1$ and, therefore, makes the expectations-driven oscillations more likely to occur. Passive monetary policy, on the other hand, reduces the desire to transfer wealth from young age to old age. Thus, in this case, indeterminacy and self-fulfilling expectations are less likely to occur.

We illustrate theorem 1 by means of the simple, but important example in which the utility functions exhibit constant relative risk aversion and the target growth rate g is equal to 1. More specifically, suppose that the utility functions are given by $u(c) = c^{1-\alpha}/(1-\alpha)$ and $v(c) = c^{1-\beta}/(1-\beta)$, where α and β are positive numbers.⁷ The economy is therefore fully described by the environmental parameters n and \bar{y} , the preference parameters α and β , and the policy parameter γ . We shall denote this economy by $E(n, \bar{y}, \alpha, \beta, \gamma)$. Assumption 1 is satisfied if and only if $\bar{y} > 1$. The equilibrium dynamics (6) for this economy can be written as

$$x_t^\gamma (\bar{y} - x_t)^{-\alpha} = x_{t+1}^\gamma (1 + nx_{t+1})^{-\beta}.$$

It is obvious from this equation that the equilibrium set of the economy $E(n, \bar{y}, \alpha, \beta, \gamma)$ is identical to the equilibrium set of the economy $E(n, \bar{y}, \alpha', \beta', \gamma')$ where $\alpha' = \alpha/\gamma$, $\beta' = \beta/\gamma$, and $\gamma' = 1$. This shows that equilibrium phenomena that occur under strict money growth targeting ($\gamma = 1$) only for unrealistically high values of α or β , occur under active inflation forecast targeting for smaller values of α and β . It is well-known from Grandmont [5] that perfect foresight equilibria can exhibit complicated dynamics (chaos) if $\gamma = 1$ and β is sufficiently large. Our results imply therefore that complicated dynamics can occur under active inflation forecast targeting for much smaller and, hence, more realistic values of β than under strict money growth targeting.

⁷For $\alpha = 1$ the expression for $u(c)$ has to be replaced by $\ln(c)$. Analogously, if $\beta = 1$, then $v(c) = \ln c$.

3 Volatility

In the present section we consider the model under the assumption that first-period endowments form a stochastic process (\dots, y_t, \dots) . Households maximize the expected utility of life-time consumption. Our main goal is to study under which type of policy rule the introduction of intrinsic uncertainty in the form of endowment shocks generates the least volatile inflation rate. We also analyse how the type of the policy rule affects the accuracy of inflation forecasts.

Let (\dots, y_t, \dots) be a stochastic process with values $y_t \in Y$, where the set Y is called the state space of the economy. Moreover, denote by \mathcal{F}_t the σ -algebra generated by $\{y_s \mid s \leq t\}$. Now recall from the model formulation in section 2 that the uncertainty about the first-period endowment y_t is realized before households of generation t make their consumption decisions and before the central bank determines the money growth rate μ_t . This implies that households of generation t solve the stochastic optimization problem

$$\begin{aligned} & \text{maximize} && \{u(c_1) + \mathbb{E}_t[v(c_2)]\} \\ & \text{subject to} && c_1 + m/p_t \leq y_t, \\ & && c_2 \leq 1 + (m + \tau_{t+1})/p_{t+1}, \\ & && c_1 \geq 0, c_2 \geq 0, m \geq 0, \end{aligned}$$

where \mathbb{E}_t denotes the expectation conditional on information \mathcal{F}_t . Using the same steps that led to equation (4), one can derive the equilibrium condition

$$\frac{1}{p_t} u' \left(y_t - \frac{M_t}{N_t p_t} \right) = \mathbb{E}_t \left[\frac{1}{p_{t+1}} v' \left(1 + \frac{n M_{t+1}}{N_{t+1} p_{t+1}} \right) \right]. \quad (8)$$

Because the price level in period $t+1$ is not known at the time when the central bank determines the money growth rate μ_t , the inflation forecast used in the policy rule (2) is assumed to be

$$\hat{\pi}_t = \mathbb{E}_t \left(\frac{p_{t+1}}{p_t} \right). \quad (9)$$

As in section 2 we denote the per-capita real money balances by x_t , i.e., $x_t = M_t/(N_t p_t)$. Obviously, condition (5) remains to be true. Solving (2) for n/μ_t and substituting the result into (5), one gets

$$\frac{x_t}{x_{t+1}} = g^{-1/\gamma} \hat{\pi}_t^{(1-\gamma)/\gamma} \frac{p_{t+1}}{p_t}. \quad (10)$$

Taking conditional expectations on both sides of this equation and using (9) we obtain

$$\hat{\pi}_t = gx_t^\gamma \left[\mathbb{E}_t \left(\frac{1}{x_{t+1}} \right) \right]^\gamma. \quad (11)$$

Substituting this back into (10) and rearranging terms yields the following expression for the actual inflation rate π_t :

$$\pi_t = \frac{p_{t+1}}{p_t} = \frac{gx_t^\gamma}{x_{t+1}} \left[\mathbb{E}_t \left(\frac{1}{x_{t+1}} \right) \right]^{\gamma-1}. \quad (12)$$

Finally, by multiplying both sides of (8) by p_t and using (12) we obtain the stochastic counterpart to equation (6), namely

$$gx_t^\gamma u'(y_t - x_t) = \left[\mathbb{E}_t \left(\frac{1}{x_{t+1}} \right) \right]^{1-\gamma} \mathbb{E}_t [x_{t+1} v'(1 + nx_{t+1})]. \quad (13)$$

Every stochastic process (\dots, x_t, \dots) satisfying equation (13) is a rational expectations equilibrium of the model. In the rest of this section, however, we restrict ourselves to stationary Markov equilibria defined in the following way.

Definition 1 A stochastic process (\dots, x_t, \dots) is called a stationary Markov equilibrium, if it satisfies (13) and if there exists a function $X : Y \mapsto \mathbb{R}$ such that $x_t = X(y_t)$ holds for all t .

The restriction to stationary Markov equilibria has two facets. First, stationarity rules out time-dependence. In the deterministic framework of the previous section, a stationary equilibrium would have to be a steady state.⁸ Second, the Markov property implies that real money balances in period t , x_t , depend only on the current state of the economy, y_t . In particular, x_t must not depend directly on any past realizations of the state nor can it depend on extrinsic uncertainty (sunspots).

To make the following analysis as transparent as possible, we make two further simplifying assumptions. First, we assume that the endowment process is a two-state Markov chain. More specifically, we assume that the state space of the economy is given by $Y = \{Y_H, Y_L\}$, where Y_H and Y_L are real numbers satisfying $Y_H \geq Y_L > 0$, and that the transition probabilities are given

⁸The deterministic framework of section 2 is a special case of the stochastic model discussed here, which is obtained when $y_t = \bar{y}$ holds with probability 1 for all t .

by $\mathbb{P}(y_{t+1} = Y_L | y_t = Y_H) = \rho_L$ and $\mathbb{P}(y_{t+1} = Y_H | y_t = Y_L) = \rho_H$, where $\rho_H > 0$, $\rho_L > 0$, and $\rho_H + \rho_L < 2$. From this assumption it follows that (\dots, y_t, \dots) is an ergodic Markov chain and that the unconditional probabilities of the economy being in a given state are

$$\mathbb{P}(y_t = Y_H) = \rho_H / (\rho_H + \rho_L) \quad \text{and} \quad \mathbb{P}(y_t = Y_L) = \rho_L / (\rho_H + \rho_L). \quad (14)$$

Second, we want to rule out that the stochastic perturbation of the model changes the long-run average endowment of the economy. To this end we assume that the unconditional mean of (\dots, y_t, \dots) is equal to \bar{y} . Denoting the unconditional standard deviation of the process by σ , it follows that

$$Y_H = \bar{y} + \sigma \sqrt{\rho_L / \rho_H} \quad \text{and} \quad Y_L = \bar{y} - \sigma \sqrt{\rho_H / \rho_L}. \quad (15)$$

It is easy to verify that, under these assumptions, the first-order autocorrelation coefficient of the endowment process (\dots, y_t, \dots) is given by $1 - \rho_H - \rho_L$. For later reference we denote this coefficient by ρ and note that $\rho \in (-1, 1)$ holds.

The limiting case $\sigma = 0$ corresponds to the deterministic economy considered in section 2. Our goal in the present section is to see how the monetary steady state (i.e., a stationary Markov equilibrium in the case $\sigma = 0$) is transformed as σ increases, that is, as intrinsic uncertainty in the form of endowment shocks becomes stronger. To make this analysis meaningful we must of course assume that the monetary steady state exists for the deterministic model and that it is locally unique (determinate). Otherwise it would be impossible to carry out a local comparative dynamics analysis. From lemma 1 we know that the monetary steady state exists if assumption 1 is satisfied, and from theorem 1 we know that the monetary steady state is a locally unique equilibrium if $\gamma > \gamma_1$. For easier reference we formulate the latter condition as a separate assumption.

Assumption 2 It holds that $\gamma > \gamma_1$, where $\gamma_1 = \bar{x}(n\bar{A}_v - \bar{A}_u)/2$.

Our focus on intrinsic uncertainty justifies to rule out any influence of extrinsic uncertainty. As mentioned above, this is achieved by our restriction to Markov equilibria. The following lemma shows that, for sufficiently small σ , there exists a unique stationary Markov equilibrium close to the monetary steady state \bar{x} of the deterministic model.

Lemma 2 *Let assumptions 1 and 2 be satisfied.*

(a) *There exist positive numbers $\bar{\sigma}$ and η and a function $X : Y \times [0, \bar{\sigma}] \mapsto \mathbb{R}$ such that for all $\sigma \in [0, \bar{\sigma}]$ the following is true. The stochastic process (\dots, x_t, \dots) defined by $x_t = X(y_t, \sigma)$ is the unique stationary Markov equilibrium satisfying $|x_t - \bar{x}| < \eta$ for all t .*

(b) *The function X is continuously differentiable with respect to σ and satisfies⁹*

$$X(Y_H, \sigma) = \bar{x} + \sigma K(\gamma) \sqrt{\rho_L / \rho_H} + O(\sigma^2), \quad (16)$$

$$X(Y_L, \sigma) = \bar{x} - \sigma K(\gamma) \sqrt{\rho_H / \rho_L} + O(\sigma^2),$$

where

$$K(\gamma) = \bar{x} \bar{A}_u / [\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n \bar{A}_v)]. \quad (17)$$

(c) *The function $K : (\gamma_1, +\infty) \mapsto \mathbb{R}$ defined in (17) is strictly decreasing and has strictly positive values. Moreover, $\lim_{\gamma \rightarrow +\infty} K(\gamma) = 0$.*

PROOF: Suppose that (\dots, x_t, \dots) is a stationary Markov equilibrium. By the definition of such an equilibrium, there must exist numbers X_H and X_L such that for all t and all $i \in \{H, L\}$ it holds that $x_t = X_i$ whenever $y_t = Y_i$. Because of (15), the equilibrium condition (13) is equivalent to

$$\begin{aligned} gX_H^\gamma u' \left(\bar{y} + \sigma \sqrt{\frac{\rho_L}{\rho_H}} - X_H \right) &= \left[\frac{1 - \rho_L}{X_H} + \frac{\rho_L}{X_L} \right]^{1-\gamma} [(1 - \rho_L)X_H v'(1 + nX_H) + \rho_L X_L v'(1 + nX_L)], \\ gX_L^\gamma u' \left(\bar{y} - \sigma \sqrt{\frac{\rho_H}{\rho_L}} - X_L \right) &= \left[\frac{\rho_H}{X_H} + \frac{1 - \rho_H}{X_L} \right]^{1-\gamma} [\rho_H X_H v'(1 + nX_H) + (1 - \rho_H)X_L v'(1 + nX_L)]. \end{aligned} \quad (18)$$

Existence of a stationary Markov equilibrium is therefore equivalent to the existence of a solution (X_H, X_L) of system (18). The results stated in parts (a) and (b) of the lemma can now be derived by a straightforward application of the implicit function theorem to system (18). First note that, for $\sigma = 0$, system (18) has the solution $X_H = X_L = \bar{x}$, where \bar{x} is defined by (7). Substituting $X_H = X(Y_H, \sigma)$ and $X_L = X(Y_L, \sigma)$ into (18), differentiating the two equations with respect to

⁹Here and in the remainder of this section, the notation $O(\sigma^2)$ indicates terms of second or higher order with respect to σ .

σ , evaluating at $\sigma = 0$, and dividing by $v'(1 + n\bar{x})\bar{x}^{\gamma-1}$ yields

$$\begin{aligned} (\gamma + \bar{x}\bar{A}_u)X'(Y_H, 0) - \bar{x}\bar{A}_u\sqrt{\rho_L/\rho_H} &= (\gamma - n\bar{x}\bar{A}_v)[(1 - \rho_L)X'(Y_H, 0) + \rho_L X'(Y_L, 0)], \\ (\gamma + \bar{x}\bar{A}_u)X'(Y_L, 0) + \bar{x}\bar{A}_u\sqrt{\rho_H/\rho_L} &= (\gamma - n\bar{x}\bar{A}_v)[\rho_H X'(Y_H, 0) + (1 - \rho_H)X'(Y_L, 0)], \end{aligned} \quad (19)$$

where $X'(y, \sigma)$ is the derivative of $X(y, \sigma)$ with respect to σ . Condition (19) is a system of two linear equations for $X'(Y_H, 0)$ and $X'(Y_L, 0)$. The determinant of this system is

$$\Delta = \bar{x}(\bar{A}_u + n\bar{A}_v)[\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n\bar{A}_v)].$$

Because of assumption 2 and $-1 < \rho < 1$ it follows that

$$\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n\bar{A}_v) > \gamma_1(1 - \rho) + \bar{x}(\bar{A}_u + \rho n\bar{A}_v) = \bar{x}(1 + \rho)(\bar{A}_u + n\bar{A}_v)/2 > 0.$$

This shows that Δ is non-zero. The implicit function theorem can therefore be applied, which proves part (a) of the lemma. The fact that $\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n\bar{A}_v) > 0$ also proves statement (c) of the lemma. Finally, system (19) has a unique solution which is given by $X'(Y_H, 0) = K(\gamma)\sqrt{\rho_L/\rho_H}$ and $X'(Y_L, 0) = -K(\gamma)\sqrt{\rho_H/\rho_L}$. This proves statement (b) of the lemma. \triangleleft

Equipped with the results in lemma 2 we can now study how the parameter γ affects the way in which exogenous stochastic shocks influence endogenous variables. To get started let us discuss a few immediate consequences of lemma 2. First, because $K(\gamma) > 0$, we can see from (15) and (16) that (\dots, x_t, \dots) and (\dots, y_t, \dots) are procyclical processes (at least as long as σ is sufficiently small). Second, from equation (16) and the transition probabilities of the process (\dots, x_t, \dots) , it follows that the unconditional mean of x_t is $\bar{x} + O(\sigma^2)$ and that the unconditional standard deviation of x_t is $\sigma K(\gamma) + O(\sigma^2)$. The result about the mean is of course a consequence of the assumption that the stochastic perturbation does not change the unconditional mean of endowments. The result about the standard deviation, on the other hand, is non-trivial and shows that the first-order effect of σ on the volatility of x_t becomes greater as the policy becomes more active and that it vanishes as γ approaches $+\infty$. In other words, by expanding and contracting the money supply one-for-one with inflation forecasts, the central bank can completely eliminate the first-order effect of exogenous shocks.

Because there is neither storage nor production, aggregate consumption in any given period equals the aggregate endowment. The volatility of aggregate consumption is therefore independent of the type of the policy. Per-capita consumption of old households in period t is given by $1 + nx_t$. Together with the above observations, this implies trivially that, for sufficiently small σ , old-age consumption is procyclical and that its unconditional standard deviation is given by $\sigma nK(\gamma) + O(\sigma^2)$. Thus, the first-order effect of σ on the volatility of per-capita consumption of old agents is decreasing with respect to γ . As for per-capita consumption of young households, the situation is more complicated as shown in the following theorem. We denote by $C_1(y, \sigma)$ the per-capita consumption of young households conditional on the state $y \in \{Y_H, Y_L\}$.

Theorem 2 *Let assumptions 1 and 2 be satisfied. The following results hold in the stationary Markov equilibrium characterized in lemma 2.*

(a) *Per-capita consumption of young households satisfies*

$$C_1(Y_H, \sigma) = \bar{y} - \bar{x} + \sigma[1 - K(\gamma)]\sqrt{\rho_L/\rho_H} + O(\sigma^2),$$

$$C_1(Y_L, \sigma) = \bar{y} - \bar{x} - \sigma[1 - K(\gamma)]\sqrt{\rho_H/\rho_L} + O(\sigma^2),$$

and the unconditional standard deviation of $C_1(y_t, \sigma)$ is equal to $\sigma|1 - K(\gamma)| + O(\sigma^2)$.

(b) *If $\rho \geq \min\{0, (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)\}$, then it follows that $K(\gamma) < 1$ for all $\gamma > \max\{0, \gamma_1\}$. For sufficiently small σ , per-capita consumption of young households is procyclical and less volatile than endowments.*

(c) *If $\rho < \min\{0, (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)\}$ and γ is sufficiently close to $\max\{0, \gamma_1\}$, then it follows that $K(\gamma) > 1$. In this case, per-capita consumption of young households is countercyclical when σ is sufficiently small.*

PROOF: (a) From the budget constraints it follows that consumption by young agents in state $y \in \{Y_H, Y_L\}$ is given by $y - X(y, \sigma)$. Part (a) of the theorem follows immediately from this observation and from (15) and (16).

(b) First note that $K(\gamma_1) \leq 1$ holds if and only if $\rho \geq (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)$. Thus, if $\bar{A}_u - n\bar{A}_v < 0$, then the assumption $\rho \geq \min\{0, (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)\}$ implies $K(\gamma_1) \leq 1$. Because K is a

strictly decreasing function, we have $K(\gamma) < 1$ for all $\gamma > \gamma_1$. On the other hand, if $\bar{A}_u - n\bar{A}_v \geq 0$, then $\rho \geq \min\{0, (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)\}$ implies $\rho \geq 0$ and therefore $K(0) = \bar{A}_u/(\bar{A}_u + \rho n\bar{A}_v) \leq 1$. Again, because K is strictly decreasing we obtain $K(\gamma) < 1$ for all $\gamma > 0$. We have therefore shown that $K(\gamma) < 1$ holds for all $\gamma > \max\{0, \gamma_1\}$. The remaining statements in part (b) are now a simple consequence of the results in part (a).

(c) Using the same arguments as in the proof of part (b), one can easily see that $K(\max\{0, \gamma_1\}) > 1$ holds whenever $\rho < \min\{0, (\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)\}$. By continuity, it follows that $K(\gamma) > 1$ for all γ sufficiently close to $\max\{0, \gamma_1\}$. Together with the results from part (a) this implies that $C_1(y_t, \sigma)$ is countercyclical for those γ . \triangleleft

The above theorem identifies three determinants of the cyclical properties of young-age consumption: the autocorrelation coefficient of shocks, ρ , the type of the policy, γ , and the expression $(\bar{A}_u - n\bar{A}_v)/(\bar{A}_u + n\bar{A}_v)$. In order to understand the intuition of the theorem let us first consider the case where $\bar{A}_u - n\bar{A}_v \geq 0$. In this case, theorem 2(b) deals with non-negatively autocorrelated shocks while theorem 2(c) covers the case of negatively autocorrelated shocks. If $\rho \geq 0$ and if young agents observe a high realization of their first-period endowment, they rationally believe that the total endowment of the economy during their second period of life is likely to be high. Thus, they expect a relatively low price level in their second period of life and there is no strong incentive for them to transfer much wealth from their youth into old age. This, in turn, implies that they consume at a high rate during their young age or, in other words, that young-age consumption is procyclical.

Now assume $\rho < 0$ and consider again a young household who receives a high endowment. Because shocks are negatively autocorrelated, the household knows that the aggregate endowment of the economy during its old age is likely to be low and, hence, the price level during old age is likely to be high. Thus, in order to smooth the life-time consumption stream, the household has to carry high real money balances from young age to old age. According to theorem 2(c), this precautionary saving motive leads to countercyclical young-age consumption if γ is sufficiently low. To understand why γ has to be small, we argue as follows. If $\rho < 0$ and young households in period t receive a high endowment, then, as shown before, the price level in period $t+1$ is expected

to be high. In addition, the low aggregate demand in period t implies a relatively low price level in period t . Consequently, expected inflation in period t , $\hat{\pi}_t$, is relatively high. Under active inflation forecast targeting this triggers tight monetary policy and young agents in period t expect to receive low transfers in period $t + 1$. This obviously reinforces their desire to hold high real money balances and to consume little during young age. Consequently, young-age consumption is more likely to be countercyclical.

Finally, let us consider the case where $\bar{A}_u - n\bar{A}_v < 0$. The difference to the previously discussed case $\bar{A}_u - n\bar{A}_v \geq 0$ is that young-age consumption is now procyclical even for (mildly) negative values of ρ . This is the case simply because for $\bar{A}_u - n\bar{A}_v < 0$ we have $\gamma_1 > 0$ and, hence, assumption 2 rules out those policy types which are most conducive to countercyclical young-age consumption, i.e., the types corresponding to small values of γ .

We now move on to analyze the statistical properties of inflation. These properties are summarized in the following theorem.

Theorem 3 *Let assumptions 1 and 2 be satisfied. The following results hold in the stationary Markov equilibrium characterized in lemma 2. The unconditional mean of π_t is given by $g + O(\sigma^2)$ and the unconditional standard deviation of π_t is*

$$\sigma \frac{gK(\gamma)\sqrt{2(1-\rho) - (1-\rho)^2(1-\gamma^2)}}{\bar{x}} + O(\sigma^2). \quad (20)$$

PROOF: Fix any $\sigma \in [0, \bar{\sigma})$. As in the proof of lemma 2 we write X_H and X_L instead of $X(Y_H, \sigma)$ and $X(Y_L, \sigma)$, respectively. From (12) one can see that actual inflation in period t , π_t , is a function of x_t , x_{t+1} , and $\mathbb{E}_t(1/x_{t+1})$. In a stationary Markov equilibrium it must be the case that both x_t and $\mathbb{E}_t(1/x_{t+1})$ are functions of the state y_t and that x_{t+1} is a function of y_{t+1} . It follows that in such an equilibrium π_t is a function of y_t and y_{t+1} . It is therefore convenient to define the augmented state variable $z_t = (y_t, y_{t+1})$. From the properties imposed on (\dots, y_t, \dots) it follows immediately that (\dots, z_t, \dots) is a Markov chain with the four states (Y_H, Y_H) , (Y_H, Y_L) , (Y_L, Y_H) ,

and (Y_L, Y_L) . The matrix of transition probabilities between these states is given by

$$\begin{pmatrix} 1 - \rho_L & \rho_L & 0 & 0 \\ 0 & 0 & \rho_H & 1 - \rho_H \\ 1 - \rho_L & \rho_L & 0 & 0 \\ 0 & 0 & \rho_H & 1 - \rho_H \end{pmatrix}$$

and the unconditional probabilities of the states are

$$\begin{aligned} \mathbb{P}(z_t = (Y_H, Y_H)) &= \rho_H(1 - \rho_L)/(\rho_H + \rho_L), \\ \mathbb{P}(z_t = (Y_H, Y_L)) &= \mathbb{P}(z_t = (Y_L, Y_H)) = \rho_H\rho_L/(\rho_H + \rho_L), \\ \mathbb{P}(z_t = (Y_L, Y_L)) &= (1 - \rho_H)\rho_L/(\rho_H + \rho_L). \end{aligned} \tag{21}$$

For every $(i, j) \in \{(H, H), (H, L), (L, H), (L, L)\}$ we define $\pi_{(i,j)} = \mathbb{E}(\pi_t | z_t = (Y_i, Y_j))$. Using (12) it is straightforward to derive

$$\begin{aligned} \pi_{(H,H)} &= g \left(1 - \rho_L + \rho_L \frac{X_H}{X_L} \right)^{\gamma-1}, \\ \pi_{(H,L)} &= g \frac{X_H}{X_L} \left(1 - \rho_L + \rho_L \frac{X_H}{X_L} \right)^{\gamma-1}, \\ \pi_{(L,H)} &= g \frac{X_L}{X_H} \left(1 - \rho_H + \rho_H \frac{X_L}{X_H} \right)^{\gamma-1}, \\ \pi_{(L,L)} &= g \left(1 - \rho_H + \rho_H \frac{X_L}{X_H} \right)^{\gamma-1}. \end{aligned} \tag{22}$$

Using (21) and (22), the unconditional mean of π_t is seen to be

$$\frac{g}{\rho_H + \rho_L} \left[\rho_H \left(1 - \rho_L + \rho_L \frac{X_H}{X_L} \right)^\gamma + \rho_L \left(1 - \rho_H + \rho_H \frac{X_L}{X_H} \right)^\gamma \right]. \tag{23}$$

From lemma 2 we have

$$\begin{aligned} \frac{X_H}{X_L} &= \frac{X(Y_H, \sigma)}{X(Y_L, \sigma)} = 1 + \sigma \frac{K(\gamma)Q}{\bar{x}} + O(\sigma^2), \\ \frac{X_L}{X_H} &= \frac{X(Y_L, \sigma)}{X(Y_H, \sigma)} = 1 - \sigma \frac{K(\gamma)Q}{\bar{x}} + O(\sigma^2), \end{aligned}$$

where $Q = \sqrt{\rho_H/\rho_L} + \sqrt{\rho_L/\rho_H}$. Substituting these first-order approximations into (23) it follows that the unconditional mean of inflation is equal to $g + O(\sigma^2)$. Similarly, by substituting the first-order approximations into (22) one can derive

$$\begin{aligned}\pi_{(H,H)} &= g - \sigma \frac{g\rho_L(1-\gamma)K(\gamma)Q}{\bar{x}} + O(\sigma^2), \\ \pi_{(H,L)} &= g + \sigma \frac{g[1-\rho_L(1-\gamma)]K(\gamma)Q}{\bar{x}} + O(\sigma^2), \\ \pi_{(L,H)} &= g - \sigma \frac{g[1-\rho_H(1-\gamma)]K(\gamma)Q}{\bar{x}} + O(\sigma^2), \\ \pi_{(L,L)} &= g + \sigma \frac{g\rho_H(1-\gamma)K(\gamma)Q}{\bar{x}} + O(\sigma^2).\end{aligned}$$

Using these equations together with $\mathbb{E}(\pi_t) = g + O(\sigma^2)$ and (21) it is straightforward to verify that the unconditional standard deviation of π_t is given by (20). \triangleleft

The first result of the above theorem, namely that the unconditional mean of inflation is equal to $g + O(\sigma^2)$, is not very surprising, because both the money growth target and the inflation forecast target are equal to g . It can be shown that the second and higher-order terms in the expression $g + O(\sigma^2)$ are unambiguously positive, that is, that the unconditional mean of inflation is strictly greater than g for all $\sigma > 0$ and all $\gamma > 0$. However, since this finding is merely a consequence of the non-linearity involved in measuring average inflation, which has nothing to do with the specific details of our model, we prove and discuss it in the appendix.¹⁰ The second result from theorem 3, however, is more interesting. We know from lemma 2 that $K(\gamma)$ is decreasing with respect to γ . The square root in equation (20), however, is a strictly increasing function of γ . There exist therefore two opposing effects of the policy type γ on the way in which exogenous shocks influence the volatility of inflation. The following theorem determines which type of policy minimizes this influence. To state the theorem, let us define

$$\gamma^* = (1 + \rho) / [\bar{x}(\bar{A}_u + \rho n \bar{A}_v)].$$

Theorem 4 *Let assumptions 1 and 2 be satisfied.*

(a) *If $\bar{A}_u + \rho n \bar{A}_v > 0$, then it follows that $\gamma^* > 0$ and the first-order term in (20) is minimized at $\gamma = \gamma^*$.*

¹⁰The exact expression for the unconditional mean of inflation is stated in equation (23) in the proof of theorem 3.

(b) If $\bar{A}_u + \rho n \bar{A}_v \leq 0$, then it follows that the first-order term in (20) is strictly decreasing with respect to γ .

PROOF: Using (17) it follows that the first-order term in (20) is proportional to

$$H(\gamma) = \frac{\sqrt{2 - (1 - \rho)(1 - \gamma^2)}}{\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n \bar{A}_v)},$$

where the factor of proportionality is positive and independent of γ . Minimizing the first-order term in (20) with respect to γ is therefore equivalent to minimizing $H(\gamma)$. It is straightforward to verify that

$$H'(\gamma) = -\frac{(1 - \rho) [1 + \rho - \gamma \bar{x}(\bar{A}_u + \rho n \bar{A}_v)]}{[\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n \bar{A}_v)]^2 \sqrt{2 - (1 - \rho)(1 - \gamma^2)}}.$$

If $\bar{A}_u + \rho n \bar{A}_v \leq 0$, then it follows that $H'(\gamma) < 0$ for all $\gamma > 0$ which proves part (b) of the theorem. On the other hand, if $\bar{A}_u + \rho n \bar{A}_v > 0$, then $\gamma^* > 0$ and $H'(\gamma)$ is negative for all $\gamma \in (0, \gamma^*)$ and positive for all $\gamma \in (\gamma^*, +\infty)$. Thus, in this case $H(\gamma)$ attains its minimum at $\gamma = \gamma^* > 0$. \triangleleft

Theorem 4 determines the type of the policy rule which minimizes the first-order effect of σ on the volatility of inflation. If $\bar{A}_u + \rho n \bar{A}_v > 0$, then there exists a unique optimal type of rule, namely $\gamma = \gamma^*$. If $\bar{A}_u + \rho n \bar{A}_v \leq 0$, however, there is no optimal type because the first-order effect of shocks on the volatility of inflation is globally decreasing with respect to γ . We shall henceforth refer to the case $\bar{A}_u + \rho n \bar{A}_v > 0$ as the normal case. Note that normality holds whenever $\rho \geq 0$ or $n \bar{A}_v - \bar{A}_u \leq 0$. The latter condition has been identified in Theorem 1 as the one which renders the monetary steady state determinate under all policy rules. It is interesting to note that the optimal type γ^* is always strictly decreasing with respect to the degree of absolute risk aversion of young households, \bar{A}_u , and that it is strictly decreasing with respect to the degree of absolute risk aversion of old households, \bar{A}_v , if and only if $\rho > 0$. Thus, if shocks are positively autocorrelated, then active inflation forecast targeting is optimal provided that households are sufficiently risk averse in at least one of their two periods of life.

We conclude this section by calculating the first-order effects of σ on the volatility of inflation forecasts and on the average size of forecast errors. The inflation forecast error made in period t

is $\varepsilon_t = \hat{\pi}_t - \pi_t$. From (11) and (12) we obtain

$$\varepsilon_t = g x_t^\gamma \left[\mathbb{E}_t \left(\frac{1}{x_{t+1}} \right) \right]^{\gamma-1} \left[\mathbb{E}_t \left(\frac{1}{x_{t+1}} \right) - \frac{1}{x_{t+1}} \right].$$

The statistical properties of ε_t and $\hat{\pi}_t$ can now be derived.

Theorem 5 *Let assumptions 1 and 2 be satisfied. The following results hold in the stationary Markov equilibrium characterized in lemma 2.*

(a) *The unconditional mean of ε_t is equal to 0 and the unconditional standard variation of ε_t is*

$$\sigma \frac{g \sqrt{1 - \rho^2} K(\gamma)}{\bar{x}} + O(\sigma^2). \quad (24)$$

(b) *The unconditional mean of $\hat{\pi}_t$ is equal to $g + O(\sigma^2)$ and the unconditional standard variation of $\hat{\pi}_t$ is*

$$\sigma \frac{g(1 - \rho)\gamma K(\gamma)}{\bar{x}} + O(\sigma^2). \quad (25)$$

If $\bar{A}_u + \rho n \bar{A}_v > 0$, then the first-order term in (25) is strictly increasing with respect to γ , otherwise it is decreasing with respect to γ .

PROOF: (a) Using arguments analogous to those used in the proof of theorem 3 and defining $\varepsilon_{(i,j)} = \mathbb{E}(\varepsilon_t | z_t = (Y_i, Y_j))$ one can derive

$$\begin{aligned} \varepsilon_{(H,H)} &= g \rho_L \left(\frac{x_H}{x_L} - 1 \right) \left(1 - \rho_L + \rho_L \frac{x_H}{x_L} \right)^{\gamma-1} = \sigma \frac{g \rho_L K(\gamma) Q}{\bar{x}} + O(\sigma^2), \\ \varepsilon_{(H,L)} &= -g(1 - \rho_L) \left(\frac{x_H}{x_L} - 1 \right) \left(1 - \rho_L + \rho_L \frac{x_H}{x_L} \right)^{\gamma-1} = -\sigma \frac{g(1 - \rho_L) K(\gamma) Q}{\bar{x}} + O(\sigma^2), \\ \varepsilon_{(L,H)} &= g(1 - \rho_H) \left(1 - \frac{x_L}{x_H} \right) \left(1 - \rho_H + \rho_H \frac{x_L}{x_H} \right)^{\gamma-1} = \sigma \frac{g(1 - \rho_H) K(\gamma) Q}{\bar{x}} + O(\sigma^2), \\ \varepsilon_{(L,L)} &= -g \rho_H \left(1 - \frac{x_L}{x_H} \right) \left(1 - \rho_H + \rho_H \frac{x_L}{x_H} \right)^{\gamma-1} = -\sigma \frac{g \rho_H K(\gamma) Q}{\bar{x}} + O(\sigma^2). \end{aligned}$$

Together with (21) it is now straightforward to show that the unconditional mean of forecast errors is 0 and that the unconditional standard deviation of ε_t is given by (24).

(b) From (11) it follows that the inflation forecast in state Y_i is equal to

$$g \left(1 - \rho_j + \rho_j \frac{X_i}{X_j} \right)^\gamma,$$

where $\{i, j\} = \{H, L\}$. Together with the state probabilities (14) this shows that the unconditional mean of $\hat{\pi}_t$ coincides with the unconditional mean of inflation as given by (23). Using the first-order approximations to X_H/X_L and X_L/X_H derived in the proof of theorem 3 it is furthermore easy to verify that the unconditional standard deviation of $\hat{\pi}_t$ is given by (25). The first-order term in (25) is proportional to $\gamma K(\gamma)$. We have

$$\frac{d[\gamma K(\gamma)]}{d\gamma} = \frac{\bar{x}^2 \bar{A}_u (\bar{A}_u + \rho n \bar{A}_v)}{[\gamma(1 - \rho) + \bar{x}(\bar{A}_u + \rho n \bar{A}_v)]^2}.$$

This proves immediately the last part of statement (b) of the theorem. \triangleleft

The unconditional mean of inflation forecast errors ε_t is equal to 0. This is of course a consequence of the rationality of expectations. Similarly, the unconditional mean of the inflation forecasts $\hat{\pi}_t$ is equal to the unconditional mean of inflation itself. The non-trivial parts of theorem 5 regard the unconditional standard deviations of inflation forecasts and forecast errors.

Theorem 5(a) shows that the first-order effect of σ on the volatility of forecast errors is proportional to $K(\gamma)$ which, according to lemma 2, is a strictly decreasing function of γ . Furthermore, this first-order effect converges to 0 as γ approaches $+\infty$. It follows that inflation forecast errors are high (in the sense of high variance) if the central bank applies active inflation forecast targeting whereas they are low for passive policy rules. In other words, the harder the central bank tries to keep inflation forecasts at target, the less reliable these forecasts become. This finding is closely related to results derived by Woodford [9] and Bernanke and Woodford [4] and follows essentially from the fact that the information content of inflation forecasts is reduced by the central bank's efforts to meet the inflation forecast target.

As for the volatility of inflation forecasts, the situation is not so clear cut. In the normal case $\bar{A}_u + \rho n \bar{A}_v > 0$, theorem 5(b) shows that the first-order effect of σ on the volatility of inflation forecasts is increasing with respect to γ . Thus, as one would expect, active policy rules lead to smaller variances of inflation forecasts. In the abnormal case $\bar{A}_u + \rho n \bar{A}_v \leq 0$, however, the opposite is the case.

4 Inflation targeting

So far we have assumed that the policy rule prescribes the money growth rate during period t as a function of the inflation forecast for that period. It is often argued that backward-looking monetary policy rules tend to avoid the possible indeterminacy of equilibrium, see, e.g., Benhabib et al. [2, 3]. In the present section, we therefore consider equilibria under the rule

$$(\mu_t/n)^\gamma \pi_{t-1}^{1-\gamma} = g, \quad (26)$$

where $\pi_{t-1} = p_t/p_{t-1}$ is actual inflation during period $t - 1$. Because actual past inflation is used as an argument of this rule (instead of a forecast of current inflation) we refer to the rule as one of inflation targeting. An important difference between the forward-looking rule (2) and the backward-looking rule (26) is that, according to the latter, the money growth rate from period t to period $t + 1$ depends on the past price level p_{t-1} , which is pre-determined when the money growth rate is set. As we shall see below, the existence of this pre-determined variable may render the equilibrium dynamics under the instrument rule (26) unstable. If this is the case, the monetary steady state cannot be reached from a generically chosen initial price level.

As in section 2, we assume that first-period endowments are constant, i.e., $y_t = \bar{y}$ for all t . To determine the equilibrium dynamics for the model under the backward-looking monetary policy rule (26), we note that conditions (4) and (5) remain valid. Combining (26) with (5), we obtain

$$\pi_t^\gamma = \left(\frac{p_{t+1}}{p_t}\right)^\gamma = \left(\frac{\mu_t}{n}\right)^\gamma \left(\frac{x_t}{x_{t+1}}\right)^\gamma = g \left(\frac{x_t}{x_{t+1}}\right)^\gamma \pi_{t-1}^{\gamma-1}. \quad (27)$$

Furthermore, condition (4) can be written as $\pi_t = v'(1 + nx_{t+1})/u'(\bar{y} - x_t)$. Substituting the latter equation as well as its lagged version into (27), one gets

$$\left[\frac{v'(1 + nx_{t+1})}{u'(\bar{y} - x_t)}\right]^\gamma = g \left(\frac{x_t}{x_{t+1}}\right)^\gamma \left[\frac{v'(1 + nx_t)}{u'(\bar{y} - x_{t-1})}\right]^{\gamma-1}. \quad (28)$$

Under assumption 1 there exists a unique monetary steady state, i.e., a unique constant solution of equation (28). This is formally stated in the following lemma, the proof of which is analogous to that of lemma 1 and therefore omitted.

Lemma 3 *Let assumption 1 be satisfied. There exists a unique monetary steady state equilibrium. This equilibrium is given by $x_t = \bar{x}$ for all t , where \bar{x} is the unique value in the interval $(0, \bar{y})$ satisfying (7).*

The main goal of the present section is to characterize the determinacy and stability properties of the monetary steady state. Note that the equilibrium dynamics (28) are described by a second-order difference equation. Since there is one pre-determined variable in period t (the past price p_{t-1} or, equivalently, the past per-capita real money balance x_{t-1}), there are three generic cases to distinguish. First, if all eigenvalues of the linearization of (28) are unstable (absolute value greater than 1), then the monetary steady state is locally unique (determinate) but unstable. Instability means that, if the central bank starts using the policy rule (26) in period t , then the monetary steady state will not be reached unless the past real money balance x_{t-1} coincides with the steady state value \bar{x} . Second, if one eigenvalue of the linearization of (28) is stable (absolute value smaller than 1) whereas the other one is unstable, then the monetary steady state is determinate and stable. This means that starting from any past value x_{t-1} close to \bar{x} , there exists a unique equilibrium that remains close to \bar{x} and converges to the monetary steady state. In particular, if $x_{t-1} = \bar{x}$, then the monetary steady state is a locally unique equilibrium. Finally, if both eigenvalues of the linearization of (28) are stable, then, from any initial value x_{t-1} close to \bar{x} , there exist infinitely many equilibria that converge to the monetary steady state. In particular, if $x_{t-1} = \bar{x}$ there exist infinitely many equilibria close to, but different from the monetary steady state. The monetary steady state is therefore indeterminate. The following theorem characterizes the occurrence of these three generic cases. It will be useful to define

$$\gamma_2 = \frac{\bar{x}(n\bar{A}_v - \bar{A}_u)}{2[\bar{x}(n\bar{A}_v - \bar{A}_u) - 1]}.$$

Theorem 6 *Consider the policy rule (26) and let assumption 1 be satisfied. Furthermore, assume $n\bar{x}\bar{A}_v \neq 1$.*

- (a) *If $n\bar{A}_v - \bar{A}_u < 0$, then it holds that $\gamma_2 \in (0, 1/2)$. Furthermore, the monetary steady state is determinate and stable if $\gamma > \gamma_2$ and it is determinate and unstable if $\gamma < \gamma_2$.*
- (b) *If $0 \leq n\bar{A}_v - \bar{A}_u < 1/\bar{x}$, then the monetary steady state is determinate and stable for all policy types $\gamma > 0$.*

(c) If $n\bar{A}_v - \bar{A}_u > 1/\bar{x}$, then it holds that $\gamma_2 > 1/2$. Furthermore, the monetary steady state is determinate and stable if $\gamma < \gamma_2$ and it is indeterminate if $\gamma > \gamma_2$.

PROOF: We start by linearizing equation (28) around the monetary steady state \bar{x} . Defining $dx_t = x_t - \bar{x}$, the linearized equation is

$$\gamma(1 - n\bar{x}\bar{A}_v)dx_{t+1} - [\gamma(1 + \bar{x}\bar{A}_u) + (1 - \gamma)n\bar{x}\bar{A}_v]dx_t - (1 - \gamma)\bar{x}\bar{A}_u dx_{t-1} = 0.$$

If $n\bar{A}_v \neq 1/\bar{x}$, then the characteristic polynomial of this second-order linear difference equation is given by $P(\lambda) = \lambda^2 + a\lambda + b$, where the real numbers a and b are given by

$$a = \frac{\gamma(1 + \bar{x}\bar{A}_u) + (1 - \gamma)n\bar{x}\bar{A}_v}{\gamma(n\bar{x}\bar{A}_v - 1)} \quad \text{and} \quad b = \frac{(1 - \gamma)\bar{x}\bar{A}_u}{\gamma(n\bar{x}\bar{A}_v - 1)}.$$

Figure 1 depicts the (a, b) -parameter space. This space is partitioned into 5 areas which are separated by the loci $\{(a, b) \mid 1 + a + b = 0\}$, $\{(a, b) \mid 1 - a + b = 0\}$, and $\{(a, b) \mid -2 < a < 2, b = 1\}$. The 5 areas are denoted by $C_0^1, C_0^2, C_1^1, C_1^2$, and C_2 . Define $C_0 = C_0^1 \cup C_0^2$ and $C_1 = C_1^1 \cup C_1^2$. It is well-known that the characteristic equation $P(\lambda) = 0$ has i roots with absolute value smaller than 1 provided that $(a, b) \in C_i$.

Insert figure 1 approximately here.

We have

$$1 + a + b = \frac{\bar{x}(\bar{A}_u + n\bar{A}_v)}{\gamma(n\bar{x}\bar{A}_v - 1)}$$

and

$$1 - a + b = \frac{2\gamma[\bar{x}(n\bar{A}_v - \bar{A}_u) - 1] - \bar{x}(n\bar{A}_v - \bar{A}_u)}{\gamma(n\bar{x}\bar{A}_v - 1)}.$$

(a) It is easy to see that $n\bar{A}_v - \bar{A}_u < 0$ implies that $\gamma_2 \in (0, 1/2)$. Furthermore, if $\gamma > \gamma_2$, then we have $\text{sgn}(1 + a + b) = -\text{sgn}(1 - a + b) = \text{sgn}(n\bar{x}\bar{A}_v - 1)$. If $1 + a + b < 0 < 1 - a + b$, then $(a, b) \in C_1^1$, whereas if $1 - a + b < 0 < 1 + a + b$, then $(a, b) \in C_1^2$. Thus, in both cases there is exactly one stable eigenvalue and the monetary steady state is determinate and stable.

Now assume $\gamma < \gamma_2$ and $n\bar{x}\bar{A}_v - 1 < 0$. In this case it holds that $1 + a + b < 0$ and $1 - a + b < 0$ such that $(a, b) \in C_0^2$. Thus, if $\gamma < \gamma_2$ and $n\bar{x}\bar{A}_v - 1 < 0$, then there are two unstable eigenvalues. On

the other hand, if $\gamma < \gamma_2$ and $n\bar{x}\bar{A}_v - 1 > 0$, then it holds that $1 + a + b > 0$ and $1 - a + b > 0$ such that $(a, b) \in C_0^1 \cup C_2$. To decide whether $(a, b) \in C_0^1$ or $(a, b) \in C_2$, we have to check whether $b > 1$ or $b < 1$. Under the present assumptions $n\bar{A}_v - \bar{A}_u < 0$ and $n\bar{x}\bar{A}_v - 1 > 0$, it is straightforward to verify that $\gamma_2 < \bar{x}\bar{A}_u/[\bar{x}(n\bar{A}_v + \bar{A}_u) - 1]$ holds and that the condition $b > 1$ is equivalent to $\gamma < \bar{x}\bar{A}_u/[\bar{x}(n\bar{A}_v + \bar{A}_u) - 1]$. Thus, if $\gamma < \gamma_2$, then it must be the case that $b > 1$ and therefore $(a, b) \in C_0^1$. This implies that for $n\bar{x}\bar{A}_v - 1 > 0$ and $\gamma < \gamma_2$ there are also two unstable eigenvalues. Thus, if $n\bar{A}_v - \bar{A}_u < 0$ and $\gamma < \gamma_2$, the monetary steady state is determinate and unstable.

(b) If $0 \leq n\bar{A}_v - \bar{A}_u < 1/\bar{x}$, then $\text{sgn}(1 + a + b) = -\text{sgn}(1 - a + b) = \text{sgn}(n\bar{x}\bar{A}_v - 1)$ holds for all $\gamma > 0$. As in the first part of the proof of statement (a) one can see that this implies that the monetary steady state is determinate and stable.

(c) It is easy to see that $n\bar{A}_v - \bar{A}_u > 1/\bar{x}$ implies $n\bar{x}\bar{A}_v - 1 > 0$ and $\gamma_2 > 1/2$. Furthermore, if $\gamma < \gamma_2$, then we have $1 + a + b > 0$ and $1 - a + b < 0$. This shows that $(a, b) \in C_1^2$ which implies that the monetary steady state is determinate and stable.

On the other hand, if $\gamma > \gamma_2$, then we have $1 + a + b > 0$ and $1 - a + b > 0$. This shows that $(a, b) \in C_0^1 \cup C_2$. To decide whether $(a, b) \in C_0^1$ or $(a, b) \in C_2$, we have to check whether $b > 1$ or $b < 1$. Under the present assumption $n\bar{A}_v - \bar{A}_u \geq 1/\bar{x}$, it is straightforward to verify that $\gamma_2 > \bar{x}\bar{A}_u/[\bar{x}(n\bar{A}_v + \bar{A}_u) - 1]$ holds and that the condition $b < 1$ is equivalent to $\gamma > \bar{x}\bar{A}_u/[\bar{x}(n\bar{A}_v + \bar{A}_u) - 1]$. Thus, if $\gamma > \gamma_2$, then it must be the case that $b < 1$ and therefore $(a, b) \in C_2$. This implies that, for $n\bar{x}\bar{A}_v - 1 > 1/\bar{x}$ and $\gamma > \gamma_2$, the monetary steady state is indeterminate. \triangleleft

Let us compare the results of theorems 1 and 6. We illustrate this comparison by means of figure 2. On the horizontal axis we draw $n\bar{A}_v - \bar{A}_u$ and on the vertical axis we draw γ . The upward sloping line AA represents γ_1 . The two hyperbolas BB and CC together represent γ_2 . Theorem 1 states that, under the forward-looking rule, all policy types above the line AA lead to determinacy, whereas all policy types below AA lead to indeterminacy. Theorem 6 states that, for the backward-looking rule, all policy types below BB lead to determinacy and instability, all policy types above CC lead to indeterminacy, and all policy types between BB and CC lead to determinacy and stability.

Insert figure 2 approximately here.

Figure 2 highlights the following interesting observation. Whereas in the case of the forward-looking rule (2) high values of γ are conducive to determinacy and low values of γ generate indeterminacy, the opposite is true under the backward-looking rule (26). As a matter of fact, as shown by theorem 6 and figure 2, decreasing γ in the case of a backward-looking rule reduces the number of stable eigenvalues and renders the monetary steady state determinate or even unstable. One can also use theorems 1 and 6 to check whether it is true that forward-looking policy rules lead to indeterminacy of the monetary steady state for a larger set of parameters than backward-looking rules. If we allow for all policy types $\gamma > 0$, then the answer to this question is ambiguous. As can be seen from figure 2, there exist forward-looking rules that lead to determinacy while the corresponding backward-looking rules lead to indeterminacy (for example, all points above both AA and CC in figure 2). Conversely, there are also backward-looking rules that lead to determinacy while the corresponding forward-looking rules lead to indeterminacy (for example, all points below both AA and CC in figure 2). However, if we restrict ourselves to active policy types $\gamma \in (0, 1]$, then one can see from figure 2 that the range of parameter values for which indeterminacy occurs under the backward-looking rule is a proper subset of the range of parameter values for which indeterminacy occurs under the forward-looking rule. Thus, assuming $\gamma \leq 1$, the application of backward-looking instrument rules makes indeterminacy less likely to occur in the present model. This is in line with the consensus view that has emerged from the study of alternative models; see the references given at the beginning of this section.

To conclude this section, consider the model with the backward-looking instrument rule (26) and stochastic endowments (\dots, y_t, \dots) . The equilibrium condition (8), which obviously remains valid in this setting, can be written as

$$u'(y_t - x_t) = \mathbb{E}_t \left[\frac{v'(1 + nx_{t+1})}{\pi_t} \right]. \quad (29)$$

Condition (27), which must also hold, implies that for all $\gamma \neq 1$ the inflation rate in period t depends in a non-trivial way on the inflation rate from the previous period. This introduces memory into the model and thereby rules out that the price level in period t depends only on the current shock, as it was the case in the model studied in section 3. Consequently, the definition of stationary Markov equilibria (definition 1) has to be amended in order for such equilibria to

exist in the present model. More specifically, it seems to be reasonable to call a pair of stochastic process (\dots, x_t, \dots) and (\dots, π_t, \dots) a stationary Markov equilibrium if conditions (27) and (29) are satisfied and if there exists a function $X : Y \times \mathbb{R} \mapsto \mathbb{R}$ such that $x_t = X(y_t, \pi_{t-1})$ holds for all t . Analyzing stationary Markov equilibria in this sense is much more complicated than the corresponding analysis for the model with the forward-looking instrument rule (2) and is therefore left for future research.

5 Concluding remarks

The purpose of the present paper was to study the business cycle effects of monetary policy rules in the standard overlapping generations model used by Grandmont [5]. We have focused on inflation forecast targeting rules, which prescribe the money growth rate as a function of the rational forecast of current inflation, but we have also briefly looked at an inflation targeting rule, in which the money growth rate depends on the actual past inflation rate.

Our main findings are that active inflation forecast targeting reinforces those mechanisms that lead to indeterminacy of equilibrium and to countercyclical behavior of young-age consumption. Although active inflation forecast targeting normally reduces the volatility of inflation forecasts, it does not necessarily reduce the volatility of inflation itself. Active inflation forecast targeting minimizes the volatility of inflation if shocks are non-negatively autocorrelated and if agents are sufficiently risk averse in at least one of their two periods of life. We have also shown that inflation forecast errors are always higher under active inflation forecast targeting than under passive inflation forecast targeting or strict money growth targeting. This is a consequence of the fact that the central bank's efforts to stabilize inflation forecasts reduces the information content of these forecasts, an observation already exploited in the papers by Woodford [9] and Bernanke and Woodford [4]. Finally, we have investigated whether a backward-looking instrument rule (inflation targeting) can avoid the problem of indeterminacy, as often claimed in the literature. This is indeed the case in the overlapping generations model for active rules but not for passive ones. We have also found that the application of an active backward-looking rule can make the

monetary steady state unstable.

There are several restrictions of our analysis that we want to point out. First and foremost, the model we have used is a very stylised one and it does not include many features that figure prominently in other studies of monetary policy rules. In particular, the present model does not include any form of imperfect competition or nominal rigidity. We have deliberately chosen this simple framework in order to emphasize the way in which monetary policy rules can reinforce some of the mechanisms that are known to generate endogenous business cycles in such an idealized world. Second, we have restricted ourselves to the equilibrium dynamics locally around the monetary steady state. We are aware that a local analysis of this kind is not sufficient to fully evaluate the properties of monetary policy rules, but we think that it is a necessary and important first step towards a full understanding of the equilibrium dynamics under various rules. A global analysis would be particularly important for the case in which the monetary steady state is unstable; see theorem 6(a). We leave a more complete investigation of the global dynamics for another paper. Finally, we have focussed entirely on equilibria under rational expectations (perfect foresight) and have not considered any learning dynamics. It is well known (see, e.g., Grandmont [5]) that the stability properties of equilibria are usually reversed if one considers learning dynamics instead of rational expectations equilibria. Since the literature still has not come to a unanimous view on the question of which of the two approaches (learning or rational expectations) is the more relevant one, we feel that our restriction to one of them is justified. We do, however, caution the reader that the assumption of rational expectations is an important one for our analysis and that it should be born in mind when the results of the present paper are interpreted.

Appendix

In this appendix we show that the unconditional mean of inflation in the stationary Markov equilibria characterized by lemma 2 is strictly greater than g . The result is formally stated in the following lemma.

Lemma 4 *The expression in (23) is strictly greater than g for all $\sigma \in (0, \bar{\sigma})$ and all $\gamma > 0$.*

PROOF: Define

$$h_1(\xi) = \frac{\rho_H}{\rho_H + \rho_L} (1 - \rho_L + \rho_L \xi)^\gamma + \frac{\rho_L}{\rho_H + \rho_L} \left(1 - \rho_H + \frac{\rho_H}{\xi}\right)^\gamma.$$

Obviously, the lemma is proven if we can show that $h_1(\xi) > 1$ holds for all positive numbers ξ satisfying $\xi \neq 1$. We have $h_1(1) = 1$ and

$$h_1'(\xi) = \frac{\gamma \rho_H \rho_L}{\rho_H + \rho_L} h_2(\xi; \rho_H, \rho_L),$$

where

$$h_2(\xi; \rho_H, \rho_L) = (1 - \rho_L + \rho_L \xi)^{\gamma-1} - \frac{1}{\xi^2} \left(1 - \rho_H + \frac{\rho_H}{\xi}\right)^{\gamma-1}.$$

It is therefore sufficient to prove that $h_2(\xi; \rho_H, \rho_L) > 0$ for all $\xi > 1$ and $h_2(\xi; \rho_H, \rho_L) < 0$ for all $\xi \in (0, 1)$. Let us start by assuming $\gamma \leq 1$ and $\xi > 1$. In this case it is easy to see that $h_2(\xi; \rho_H, \rho_L)$ is non-increasing with respect to ρ_H and ρ_L . Therefore, $h_2(\xi; \rho_H, \rho_L) \geq h_2(\xi; 1, 1) = (\xi^{2\gamma} - 1)/\xi^{\gamma+1} > 0$, whereby the last inequality follows from $\gamma > 0$ and $\xi > 1$. Now assume $\gamma > 1$ and $\xi > 1$. In this case $h_2(\xi; \rho_H, \rho_L)$ is strictly increasing with respect to ρ_H and ρ_L . Therefore, $h_2(\xi; \rho_H, \rho_L) \geq h_2(\xi; 0, 0) = 1 - 1/\xi^2 > 0$, whereby the last inequality follows from $\xi > 1$. Analogous arguments can be applied in the case $\xi \in (0, 1)$. This completes the proof of the lemma. \triangleleft

The above result is essentially a consequence of the non-linearity that is involved in the measurement of average inflation and has nothing to do with the specific details of our model. To see why, consider the following example. Suppose that the price level p_t jumps randomly between two values, say, P_H and P_L with $P_H > P_L$. Assume furthermore that the distribution of p_t is stationary, that is, there is no long-run trend in the price level. In this case, the inflation rate π_t takes values in the set $\{P_H/P_L, P_L/P_H, 1\}$ and, on average, the price level must switch from P_H to P_L as many times as it switches from P_L to P_H . The long-run average inflation is therefore a weighted average of the form $w(P_H/P_L) + w(P_L/P_H) + (1 - 2w)$, where $w \in (0, 1/2)$. It is easy to see that any such average is greater than 1. Lemma 4 is a generalization of this example to the more general setting of our model.

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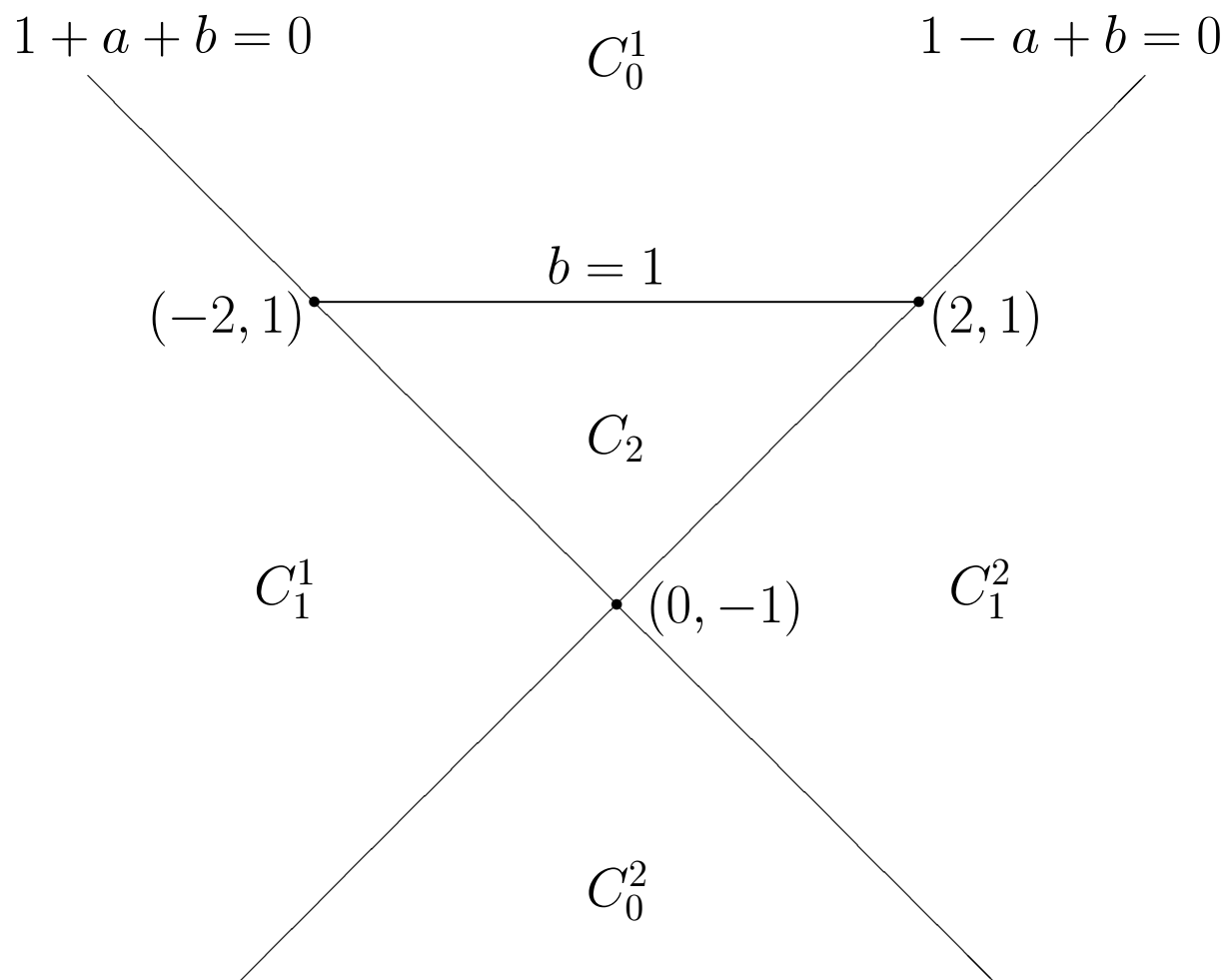


Figure 1: The (a, b) -parameter space.

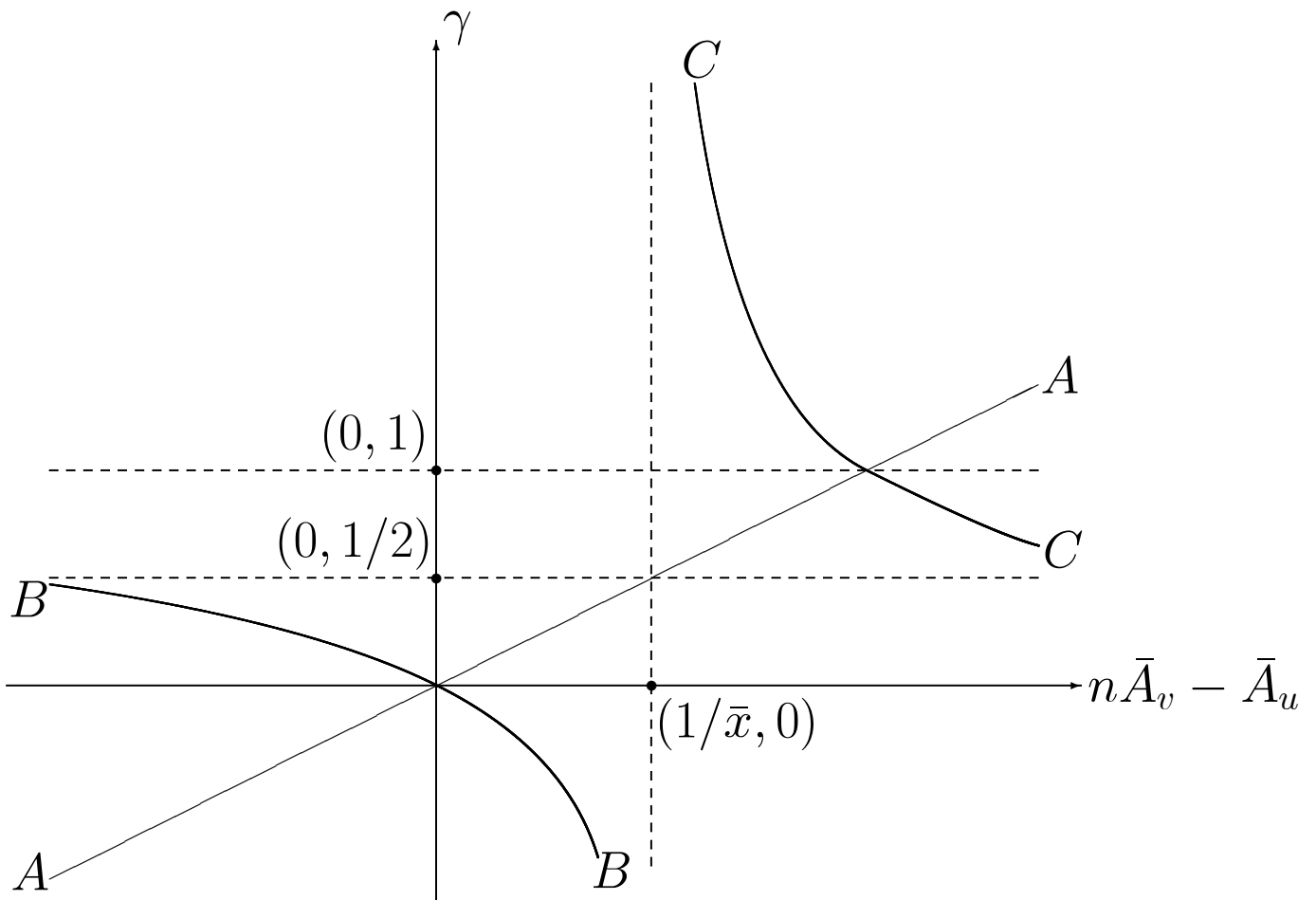


Figure 2: Comparison of forward-looking and backward-looking rules.

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**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096 or Fax: +44 (0)20 8983 3580
Email: j.conner@qmul.ac.uk
Website: www.econ.qmul.ac.uk/papers/wp.htm**