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Abstract

The persistence properties of economic time series has been a primary object of investigation in a variety of guises since the early days of econometrics. This paper suggests investigating the persistence of processes conditioning on their history. In particular we suggest that examining the derivatives of the conditional expectation of a variable with respect to its lags maybe a useful indicator of the variation in persistence with respect to its past history. We discuss in detail the implementation of the measure. We present a Monte Carlo investigation of the suggested measure. We further apply the persistence analysis to real exchange rates.

JEL Classification: C22, C14, F31.
Key Words: Persistence, Nonparametric Regression, Nonlinear Models, Real Exchange Rates.

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1 Introduction

The persistence properties of economic time series has been a primary object of investigation in a variety of guises since the early days of econometrics. The majority of the studies has concentrated on linear models and their persistence properties. Traditionally stationary processes have been investigated but following the advent of unit root econometrics, and the implication of the existence of permanent shocks to economic variables, nonstationary processes have been investigated as well.

The persistence properties of nonlinear processes have received increased attention in recent years. Important milestones in this literature include papers by Gallant, Rossi, and Tauchen (1993) and Koop, Pesaran, and Potter (1996) on generalised impulse response analysis. The use of simulation techniques has enabled the investigation of the impulse reponses and persistence properties of nonlinear properties without the need for analytical expressions for the evaluation of the relevant expectations.

The evaluation of persistence in nonlinear econometric models is of paramount importance for a number of economic problems. These include the investigation of the PPP hypothesis and the implications of the Fisher equation for the stationarity of real interest rates. Using a linear model to investigate such problems has led to the rejection of the economic hypotheses in question since data appear nonstationary according to standard unit root tests. Neverthless recent work (see e.g. Kapetanios, Snell, and Shin (2002)) indicates that the use of tests and models designed for nonlinear processes may uncover evidence supporting economic theory. Taking the analysis one step further involves looking at the effect of shocks in different parts of the state space and investigating whether shocks have different effects for different process histories. Generalised impulse responses are useful in this context. These however have usually been considered in the parametric context of particular nonlinear models.

This paper suggests investigating the persistence of processes conditioning on their history. In particular we suggest that examining the derivatives of the conditional expectation of a variable with respect to its lags maybe a useful indicator of the variation in persistence with respect to its past history. This measure is related to the generalised impulse responses proposed by Koop, Pesaran, and Potter (1996) but simplifies the analysis in two respects making it more tractable. Firstly, we do not consider nonlinearity with respect to the size of the shocks but instead take a look at the limit of the response as the shock goes to zero (the definition of the derivative). Secondly, we suggest a nonparametric way of computing the expectation involved in the calculation of the impulse responses thus avoiding the need for either a model or computationally expensive simulation techniques.

The layout of the paper is a follows: Section 2 discusses the measure we suggest and the implementation of the measure. Section 3 presents a Monte Carlo investigation of the suggested measure. Section 4 applies the persistence analysis to real exchange rates. Finally, Section 5 concludes.

2 Theoretical Considerations

2.1 The Persistence Measure

Our focus is a stationary and ergodic series y_t. We do not posit a parametric model for the series but instead suggest that it can be described by

$$y_t = m(y_{t-1}, \ldots, y_{t-p}) + \epsilon_t$$

where the error ϵ_t is independent of $y_{t-1}, \ldots, y_{t-p}, \ldots$. The unknown function $m(.)$ is assumed to be a continuous conditional mean function.

We suggest that a measure of persistence at horizon $h > 0$ conditional on a history $y_1, \ldots, y_p$ be given by

$$\hat{q}(h; y_1, \ldots, y_p) = \frac{\partial \hat{E}(y_{t+h-1}|y_{t-1} = y_1, \ldots, y_{t-p} = y_p)}{\partial y_{t-1}}$$

where $\hat{E}(.|.)$ denotes a nonparametric estimate of the conditional mean function $m^{(h)}(.)$ and $m^{(h)}(.) = m(m(\ldots(.)\ldots))$.

Clearly this measure is related to the generalised impulse response function (GIRF) suggested by Koop, Pesaran, and Potter (1996) and the relationship between the two is given by

$$q(y_1, \ldots, y_p) = \lim_{\delta \to 0} GIRF(h; y_1, \ldots, y_p; \delta)/\delta$$

where

$$GIRF(h; y_1, \ldots, y_p; \delta) = \hat{E}(y_{t+h-1}|y_{t-1} = y_1, \ldots, y_{t-p} = y_p, \epsilon_t = \delta) -$$

$$\hat{E}(y_{t+h-1}|y_{t-1} = y_1, \ldots, y_{t-p} = y_p, \epsilon_t = 0)$$

Koop, Pesaran, and Potter (1996) do not specify an estimator for $\hat{E}(.)$ but allow both for parametric and nonparametric estimators.

An alternative measure may be based on the largest eigenvalue of the coefficient matrix $\boldsymbol{A}^{(h)}$, of the companion form of the linearised nonparametric specification given by

$$\boldsymbol{y}_{t+h-1} = \boldsymbol{A}^{(h)} \boldsymbol{y}_{t-1}$$

where $\boldsymbol{y}_t = (y_t, \ldots, y_{t-p+1})'$

$$\boldsymbol{A}^{(h)} = [q(h - i + 1; y_1, \ldots, y_p; j)]$$

$$q(h - i + 1; y_1, \ldots, y_p; j) = \frac{\partial \hat{E}(y_{t+h-1}|y_{t-1} = y_1, \ldots, y_{t-p} = y_p)}{\partial y_{t-j}}$$

$q(h-i+1; y_1, \ldots, y_p; j) = 1$ for $-h+i = j$ and $q(h-i+1; y_1, \ldots, y_p; j) = 0$ for $h - i < 0$ and $h - i \neq -j$.

An estimate of $\boldsymbol{A}^{(h)}$ may be given by either by

$$\hat{\boldsymbol{A}}^{(h)} = [\hat{q}(h-i; y_1, \ldots, y_p; j)]$$

or

$$\hat{\boldsymbol{A}}^{(h)} = \hat{\boldsymbol{A}}^{h}, \quad \hat{\boldsymbol{A}} = [\hat{q}(1-i; y_1, \ldots, y_p; j)]$$

We will denote this measure of persistence by $\hat{\lambda}(h; y_1, \ldots, y_p)$.

Both persistence measures estimate the effect an infinitesimally small shock on the recent history of the process will have on the evolution of the process at given horizons. The higher the measure, the larger the effect and therefore the more persistent the process is for that particular history. The nonparametric nature of the measure enables model-free evaluation of the persistence of the process and therefore allows for a very wide range of nonlinearities.

The use of the nonparametric persistence measure, when calculated and plotted over relevant values of the history of the process, may reveal a lot of information about the process. For example issues of asymmetry may be addressed as the process may be more persistent (have a higher persistence measure) for large values than for small values or vice versa. Further, as Kapetanios (2003) has discussed a number of macroeconomic processes may have regions in their range where they may exhibit explosive behaviour. As discussed by Kapetanios (2003), an example of such a series is GDP growth for the US where the use of a nonlinear threshold model[1] reveals a corridor regime for the series where the autoregressive polynomial underlying the regime has explosive roots. The process is however globally stable as the outer regimes for the GDP growth process, on either side of the corridor regime, have autoregressive polynomials with stable roots. Testing for nonstationarity using linear models for such a process may lead to concluding

[1]The nonlinear model used belongs to the EDTAR class and was introduced by Pesaran and Potter (1997).

that the process is indeed unit root nonstationary whereas in reality it is globally stationary (geometrically ergodic) and therefore is not affected permanently by shocks. The nonparametric persistence analysis we suggest can reveal such features for the series. On the other hand summary measures such as the half life measure suggested by Shintani (2002) using nonparametric regression will not reveal such features.

An important question arises on the choice of what is a relevant history for the process on which to condition and obtain the conditional persistence measure. Clearly, as the lag order p grows the dimension of the history space over which to compute the persistence measure increases dramatically and therefore high values of p may be problematic. An obvious choice is to restrict p to be equal to one and search over a grid spanning the observed range of the process. Nevertheless, a more appropriate choice may be to choose a higher p and simply search over all available histories in the sample ordering them by the values taken by y_{t-1}. Another possibility is to order the persistence measures according to the one step ahead forecast of the nonparametric regression. Concerning the choice of p, a data-dependent method maybe used along the lines of work by e.g. Gozalo (1993) or Lavergne and Vuong (1996).

Of course, we can carry out inference on the persistence measure using standard results from nonparametric estimation. Standard errors for the persistence measure $\hat{q}$ are readily available and tests on whether conditional persistence is different at different regions of the range of the process are possible. An interesting byproduct of this measure may be a nonparametric test of nonlinearity since a flat peristence measure over a range of the process indicates linearity. So a Kolmogorov-Smirnov type test of equality of the nonparametric persistence measure q with a flat line at the level of the coefficient of an AR(1) model for y_t, or more generally of the λ peristence measure with the maximum eigenvalue of the estimated companion matrix of an $AR(p)$ model for y_t is feasible. The bootstrap may be used as a straightforward way of obtaining the distribution under the null hypothesis for such a test[2]. Of course such tests exist, but the persistence measure may enable nonlinearity testing of particular regions of the range of the process leading to nonparametric characterisations of processes as locally linear mirroring similar characterisations from parametric models such as the piecewise linear threshold model.

2.2 Nonparametric Estimation

Any nonparametric estimator may be used to obtain $\hat{E}(.|.)$ and its derivatives. We consider a kernel based (Nadaraya-Watson) regression estimator in the Monte Carlo section of the paper and so we give details of this estimator here.

The kernel based estimator for $m(\boldsymbol{y})$, where $\boldsymbol{y} = (y_1, \ldots, y_p)'$ is given by

$$\hat{m}(\boldsymbol{y}) = \sum_{t=p+1}^{T} K\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right) y_t \Bigg/ \sum_{t=p+1}^{T} K\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right)$$

where $K(.)$ is a multivariate kernel function and h is the window width. We adopt the standard normal kernel given by

$$K(\boldsymbol{y}) = \frac{1}{(2\pi)^{-p/2}} e^{-1/2\boldsymbol{y}'\boldsymbol{y}}$$

The value of the window width h which minimises the mean integrated squared error for this kernel is given by $h = cT^{-1/(4+p)}$ where $c = [4/(2p + 1)]^{1/(d+4)}$.

[2]More specifically the test statistic may be the maximum distance between the nonparametric persistence measure and the persistence measure under linearity. Bootstrap samples may easily be constructed using an $AR(p)$ as the model under the null and using the resampled residuals from the nonparametric regression as errors terms for the AR model.

The kernel estimate of the derivative of the regression estimator is given by[3]

$$\widehat{\frac{\partial m(\boldsymbol{y})}{\partial y_i}} = \hat{f}(\boldsymbol{y})^{-1}[\hat{g}^{(1)}(\boldsymbol{y}) - \hat{f}^{(1)}(\boldsymbol{y})\hat{m}(\boldsymbol{y})]$$

where

$$\hat{f}(\boldsymbol{y}) = \frac{1}{Th^p}\sum_{t=p+1}^{T} K\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right)$$

$$\hat{g}^{(1)}(\boldsymbol{y}) = -\frac{1}{Th^{p+1}}\sum_{t=p+1}^{T} K_j^{(1)}\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right) y_t$$

$$\hat{f}^{(1)}(\boldsymbol{y}) = -\frac{1}{Th^{p+1}}\sum_{t=p+1}^{T} K_j^{(1)}\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right)$$

and $K_j^{(1)}(\boldsymbol{y}) = \partial K(\boldsymbol{y})/\partial y_j$. An alternative estimator for the derivative may be obtained by taking a numerical derivative of $\hat{m}(\boldsymbol{y})$. Numerical evidence from our experiments suggests that the two estimators are very close to each other. Estimation of $m^{(h)}(\boldsymbol{y})$ may be carried out either via estimation of the regression

$$y_{t+h} = \tilde{m}(y_{t-1}, \ldots, y_{t-p}) + \tilde{\epsilon}_t$$

or by using

$$\widehat{m^{(h)}(\boldsymbol{y})} = \hat{m}(\hat{m}(\ldots(\hat{m}(\boldsymbol{y}))\ldots)$$

We choose the latter method so that the estimates are obtained in a consistent manner across horizons. Estimates of the derivatives of $m^{(h)}(\boldsymbol{y})$, for $h > 1$, may be obtained numerically from $\widehat{m^{(h)}(\boldsymbol{y})}$. Finally, for $h = 1$ the asymptotic variance of $\widehat{\frac{\partial m(\boldsymbol{y})}{\partial y_i}}$ is given by

$$[Th^3]^{-1/2}\frac{\sigma^2}{f(\boldsymbol{y})}\int K^{(1)}(\boldsymbol{y})^2 d\boldsymbol{y}$$

where $f(\boldsymbol{y})$ is the density function of $\boldsymbol{y}_t$ at $\boldsymbol{y}$ and σ^2 is the variance of the error of the regression estimated by

$$\hat{\sigma}^2(\boldsymbol{y}) = \sum_{t=p+1}^{T} K\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right)(y_t - \hat{m}(\boldsymbol{y}_t))^2 \Bigg/ \sum_{t=p+1}^{T} K\left(\frac{\boldsymbol{y}_t - \boldsymbol{y}}{h}\right)$$

[3] For more details see Pagan and Ullah (1999).

This expression enables estimation of standard errors for $h = 1$. For $h > 1$ an estimate of the variance of the derivative may be obtained numerically via the variance of $\hat{m}(.)$ and the Delta method. Alternatively, a heteroscedasticity and autocorrelation consistent (HAC) standard error estimate may be used.

An alternative estimator we have examined for $h = 1$ is the local quadratic estimator which for given $\boldsymbol{y}$ simply involves regressing y_t on a constant, $\boldsymbol{y}_t$ and squares and cross products of the variables in $\boldsymbol{y}_t$ using weighted least squares where the weights are given by $K[(\boldsymbol{y}_t - \boldsymbol{y})/h]/h$ (see e.g. Fan and Gijbels (1996)). Initial investigation has found that this estimator although in theory preferable to the Nadaraya-Watson estimator is not performing well for regions at the edges of the range of the process, i.e. very high and very low values of y_t. So we concentrate on the kernel regression estimator.

3 Monte Carlo Investigation

We carry out an extensive Monte Carlo investigation of the persistence measure we suggest. We consider three classes of nonlinear parametric models, i.e. logistic smooth transition autoregressive (LSTAR) models, exponential smooth transition autoregressive (ESTAR) models and self-exciting threshold autoregressive (SETAR) models. These classes, between themselves, can exhibit a wide variation of conditional persistence properties.

For the class of ESTAR models we generate data from the following specification

$$y_t = \alpha_1 y_{t-1} + \gamma_1 y_{t-1} \left[1 - \exp\left(-\theta_1 y_{t-1}^2\right)\right] + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim N(0,1)$. For the class of LSTAR models we generate data from the following specification

$$y_t = \alpha_2 y_{t-1} + \gamma_2 y_{t-1} \left[\frac{1}{1 - \exp\left(-\theta_2 y_{t-1}\right)}\right] + \varepsilon_t, \tag{2}$$

Finally, for the class of SETAR models we have the specification

$$y_t = \begin{cases} \phi_1 y_{t-1} + u_t & if \ y_{t-1} \le r_1 \\ \phi_0 y_{t-1} + u_t & if \ r_1 < y_{t-1} \le r_2 \\ \phi_2 y_{t-1} + u_t & if \ y_{t-1} > r_2 \end{cases}, \ t = 1, 2, ..., T, \tag{3}$$

We consider 13 experiments in total. More specifically we have 5 ESTAR experiments (experiments 1-5), 4 LSTAR experiments (experiments 6-9) and 4 SETAR experiments (experiments 10-13). The parameter specifications are given below:

- ESTAR
  - Experiment 1: $\alpha_1 = 1$, $\gamma_1 = -0.4$, $\theta_1 = 0.1$
  - Experiment 2: $\alpha_1 = 1$, $\gamma_1 = -0.4$, $\theta_1 = 0.8$
  - Experiment 3: $\alpha_1 = 1$, $\gamma_1 = -0.8$, $\theta_1 = 0.1$
  - Experiment 4: $\alpha_1 = 1$, $\gamma_1 = -0.8$, $\theta_1 = 0.8$
  - Experiment 5: $\alpha_1 = 1.5$, $\gamma_1 = -1$, $\theta_1 = 0.8$
- LSTAR
  - Experiment 6: $\alpha_2 = 0.95$, $\gamma_2 = -0.4$, $\theta_2 = 2$
  - Experiment 7: $\alpha_2 = 0.95$, $\gamma_2 = -0.4$, $\theta_2 = 8$
  - Experiment 8: $\alpha_2 = 0.95$, $\gamma_2 = -0.8$, $\theta_2 = 2$
  - Experiment 9: $\alpha_2 = 0.95$, $\gamma_2 = -0.8$, $\theta_2 = 8$
- SETAR
  - Experiment 10: $\phi_1 = 0.9$, $\phi_0 = 1$, $\phi_2 = 0.9$, $r_1 = -0.9$, $r_2 = 0.9$
  - Experiment 11: $\phi_1 = 0.95$, $\phi_0 = 1$, $\phi_2 = 0.85$, $r_1 = -0.9$, $r_2 = 0.9$
  - Experiment 12: $\phi_1 = 0.9$, $\phi_0 = 1.3$, $\phi_2 = 0.9$, $r_1 = -0.9$, $r_2 = 0.9$
  - Experiment 13: $\phi_1 = 0.95$, $\phi_0 = 1.3$, $\phi_2 = 0.85$, $r_1 = -0.9$, $r_2 = 0.9$

Some experiments have parameter combinations which impose an explosive regime in the centre of the range of the process thus implying a persistence measure above one. These processes are still stationary and geometrically ergodic. This follows from the fact that the outer regimes have AR representations which are stable[4]. A proof of this, using the drift criterion by Tweedie (1975), is given by Kapetanios and Shin (2002b). Further, note that all the processes which have symmetric coefficient structure around zero have mean zero. We consider samples of 150 observations. We report estimates of the persistence measure $\hat{q}$ for a horizon of one to three steps ahead and for a set of 500 points equally spaced in the range -5 to 5 for each process. The measure is obtained using 1000 replications and taking the average over them for each point. The results for $h = 1$ are presented in Figures 1-4 together with the true persistence measure. For this horizon we also report confidence intervals using the average estimated standard error[5]. Results for $h = 2$ and $h = 3$ are reported in Figures 5-8 together with the true persistence measure.

The results make interesting reading. We see that the estimated persistence measure tracks quite closely the true one. Asymmetry as well as increased persistence in the middle of the range of the process is captured relatively well. The estimated standard errors are quite wide thereby including both the true measure and its estimate in the 95% confidence interval most of the time. For $h = 2, 3$ the estimated measure perform less well with considerable wiggles in the average in the tails of the range especially for $h = 3$. We note that in the simulations we drop any replication for which the persistence measure for any point in the range exceeds 10 in absolute value. Also the estimated measure seems in general to underestimate the true persistence. Intuitively, this underestimation is more pronounced for processes which exhibit very high persistence in the middle of the range of the process,

[4]Outer regimes are those that hold for large absolute values of the process.

[5]We do not use a HAC standard error estimate as we know the DGP does not suffer from either heteroscedasticity or serial correlation.

as the nonparametric estimator is essentially a smoother.

4 Empirical Application

In this section we apply our persistence measure to the investigation of Yen real exchange rates. It is well known in the literature that Yen real exchange rates have consistently contradicted the PPP hypothesis since they appear to be unit root nonstationary using standard unit root tests.

However, recent work by Chortareas, Kapetanios, and Shin (2002) and Kapetanios and Shin (2002a) indicates that this apparent rejection of the theory may have more to do with the tools that have been used to test the PPP hypothesis rather than with the hypothesis itself. More specifically using tests of the unit root hypothesis against the alternative of a nonlinear stationary model, these papers manage to reject the unit root hypothesis for a large number of Yen real exchange rates. The model that underlies the alternative hypotheses in these papers is the ESTAR model leading credence to the possibility that these real exchange rates may follow stationary, yet highly and variably conditionally persistent processes. We therefore examine nonparametrically the conditional persistence of these processes.

We construct bilateral real exchange rates against the *i-th* currency at time t ($q_{i,t}$) as $q_{i,t} = s_{i,t} + p_{J,t} - p^*_{i,t}$, where $s_{i,t}$ is the corresponding nominal exchange rate (*i-th* currency per yen), $p_{J,t}$ the price level in the home country, and $p^*_{i,t}$ the price level of the *i-th* country. Thus, a rise in $q_{i,t}$ implies a real yen appreciation against the *i-th* currency. The price levels are consumer price indices for Yen and wholesale price indices for the DM. All variables are in logs. All data are from the International Monetary Fund's *International Financial Statistics* in CD-ROM. The data are not seasonally adjusted. All data are quarterly, spanning from 1960Q1 to 2000Q4 and the bilateral nominal exchange rates against the currencies other than the US dollar are cross-rates computed using the US dollar rates.

We consider a large sample of countries in an attempt to make the empirical analysis more comprehensive. In particular we consider three groups of countries: A) Western countries (US, Germany, France, Italy, UK and Canada, Austria, and Turkey), B) Asian countries (Singapore, Malaysia, Indonesia, Thailand,Phillipines and Sri Lanka) and C) Latin Americal countries (Argentina, Chile, Colombia, and Mexico). We examine the persistence measure, $\hat{q}$, using $p = 1$ and $h = 1, 2$. Results are presented in Figures 9-11.

Most series exhibit a rise in conditional persistence in the center of the range of the process. This is as expected if theoretical considerations such as transaction costs are taken into account. All Western countries exhibit such behaviour apart from Turkey where the real exchange rate process is more persistent for low values of the process than for high values. For all these countries no explosive behaviour is observed apart from Austria. There we see that the persistence profile has two peaks with the highest peak reaching 1.2 for $h = 1$ and 2.2 for $h = 2$.

Moving on to the Asian group of countries we see that higher persistence in the middle of the range is again the norm. The only exception is the Phillipines where the process is more persistent for low values than for high values. There are two case of locally explosive processes. These are Sri Lanka and Indonesia. Finally, for the Latin American countries similar conclusions are reached with Colombia and Mexico exhibiting explosive behaviour. It interesting to note that two countries which have been primarily affected by financial crises (i.e. Indonesia and Mexico) exhibit explosive behaviour in the middle of the range of the real exchange rate process. This points to the possibility that the processes may be modelled by a SETAR or STAR model with explosive corridor regimes.

5 Conclusion

The investigation of persistence in macroeconomic time series is extremely relevant for the analysis of a wide variety of economic phenomena. The incidence or not of permanent shocks leads to radically different economic theories and therefore an accurate measurement of persistence is of interest. The presence of nonlinearity complicates the analysis since it leads to varying levels of persistence depending on the history of the process. Since the state of nonlinear modelling is not fully developed a nonparametric approach is of relevance for the analysis of persistence.

We suggest a new measure of conditional persistence which is based on the derivative of the nonparametric estimate of the expectation of a process conditional on its lags. We use a standard kernel based nonparametric estimator which we find to have reasonably good properties for our purposes through a Monte Carlo study. We apply the new measure to Yen real exchange rates and we find that as expected persistence is higher when the real exchange rate process is near the middle of its range and lower when the process takes more extreme values. In a number of cases we find that real exchange rate processes have regions where persistence exceeds one and therefore the process becomes locally explosive.

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Figure 1: Experiments 1-4 ($h = 1$)

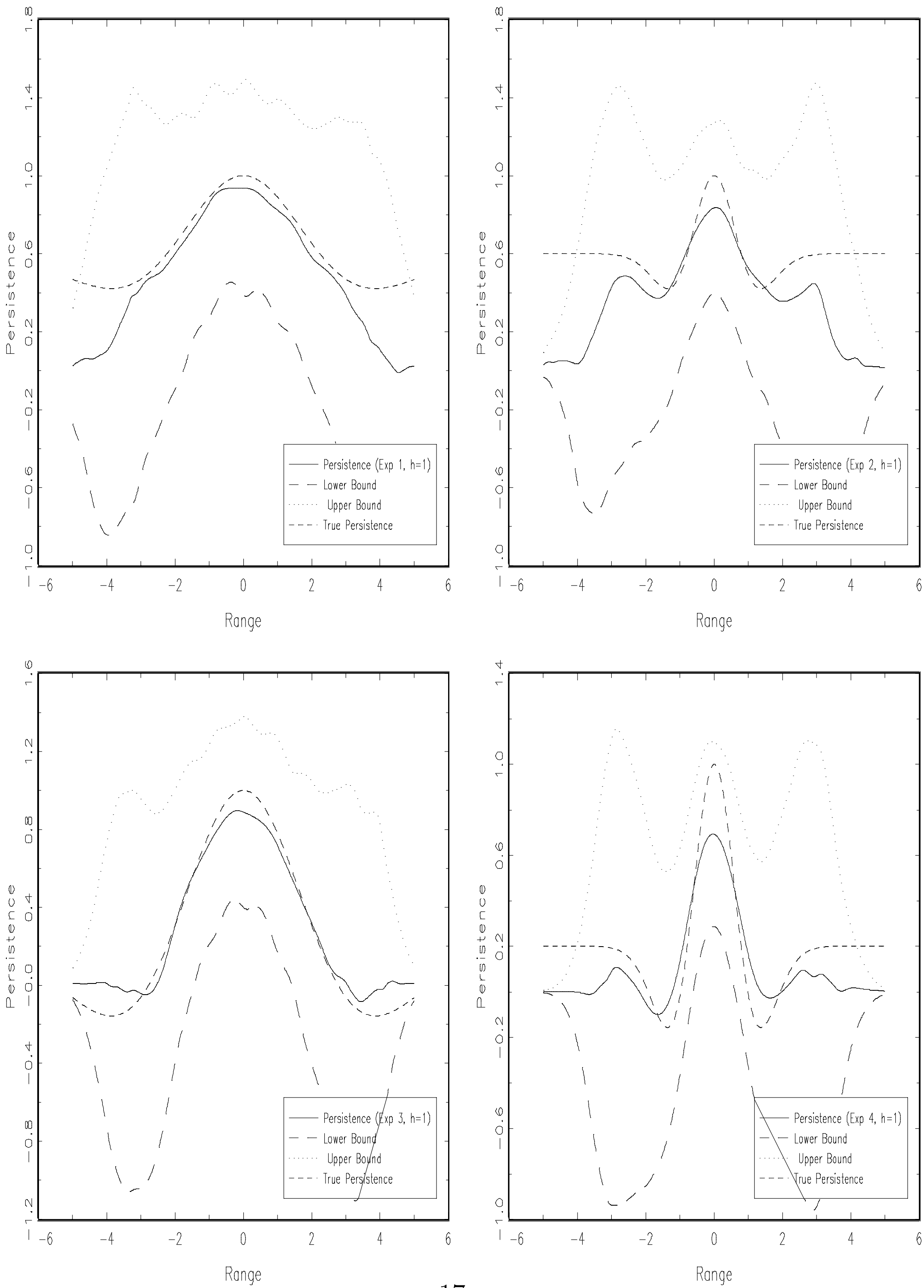


Figure 2: Experiments 5-8 ($h = 1$)

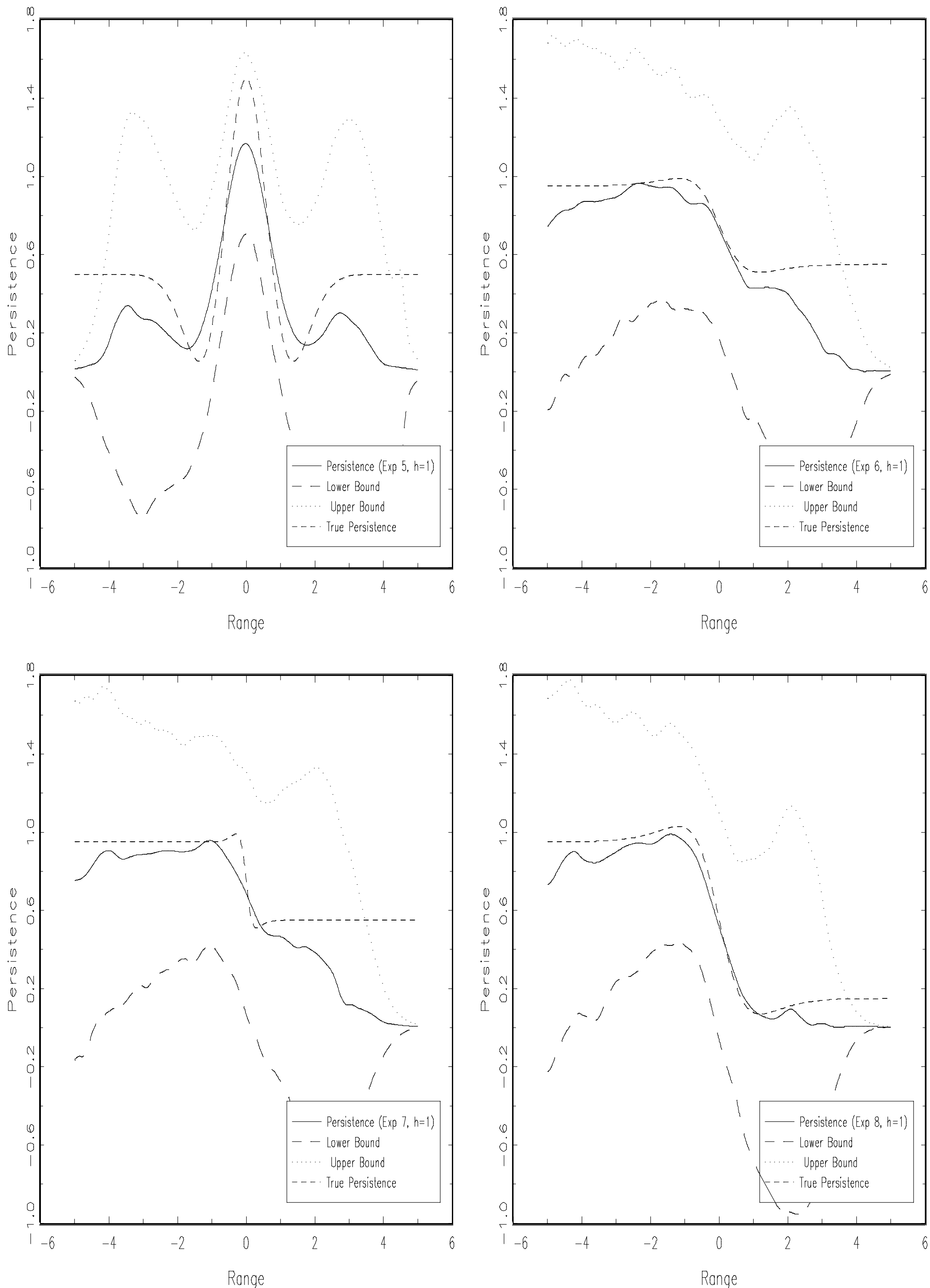


Figure 3: Experiments 9-12 ($h = 1$)

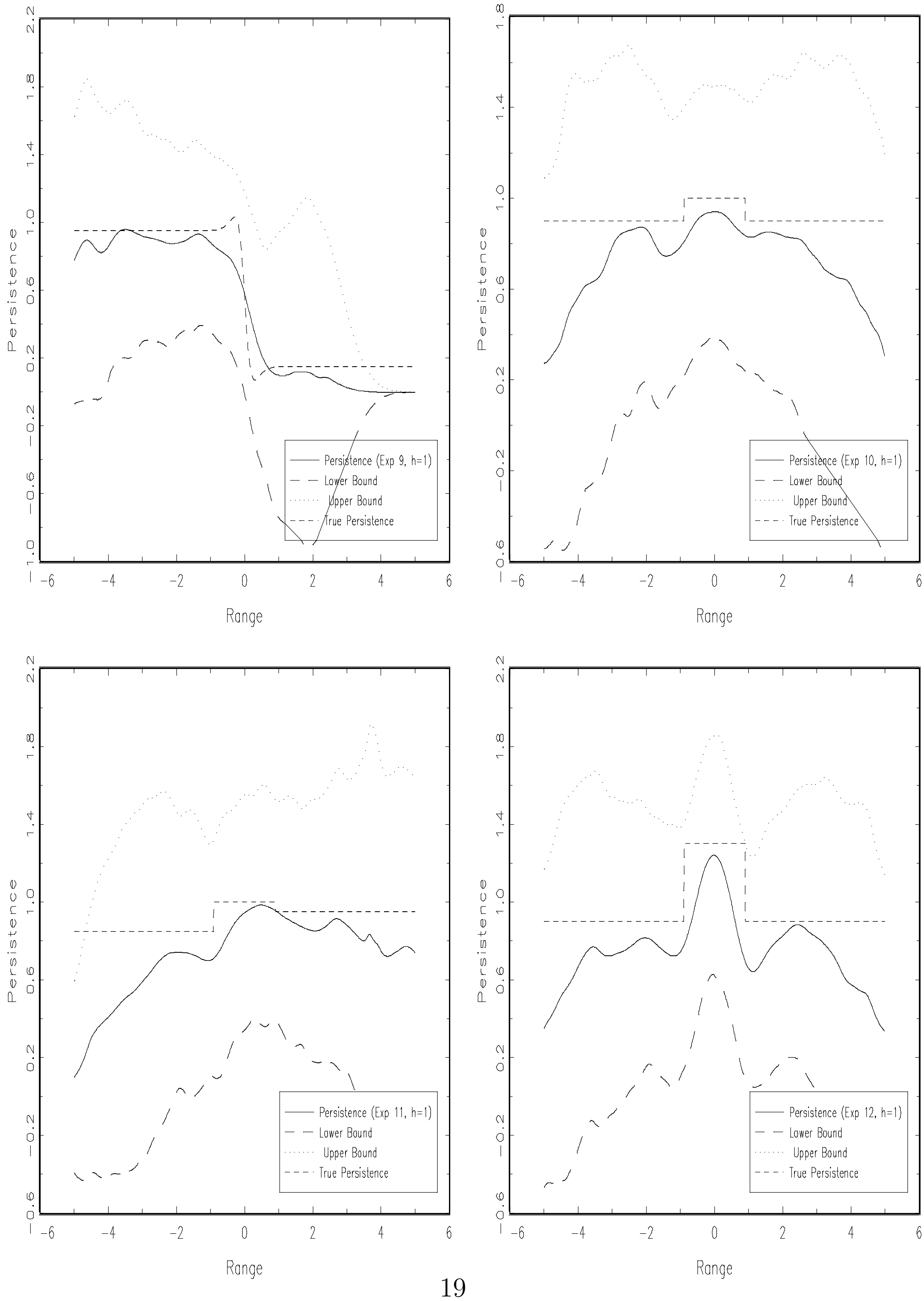


Figure 4: Experiment 13 ($h = 1$)

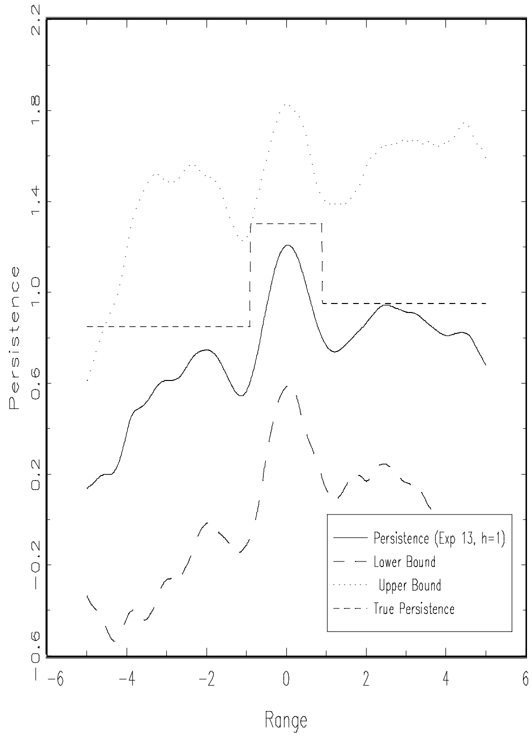


Figure 5: Experiments 1-4 ($h = 2, 3$)

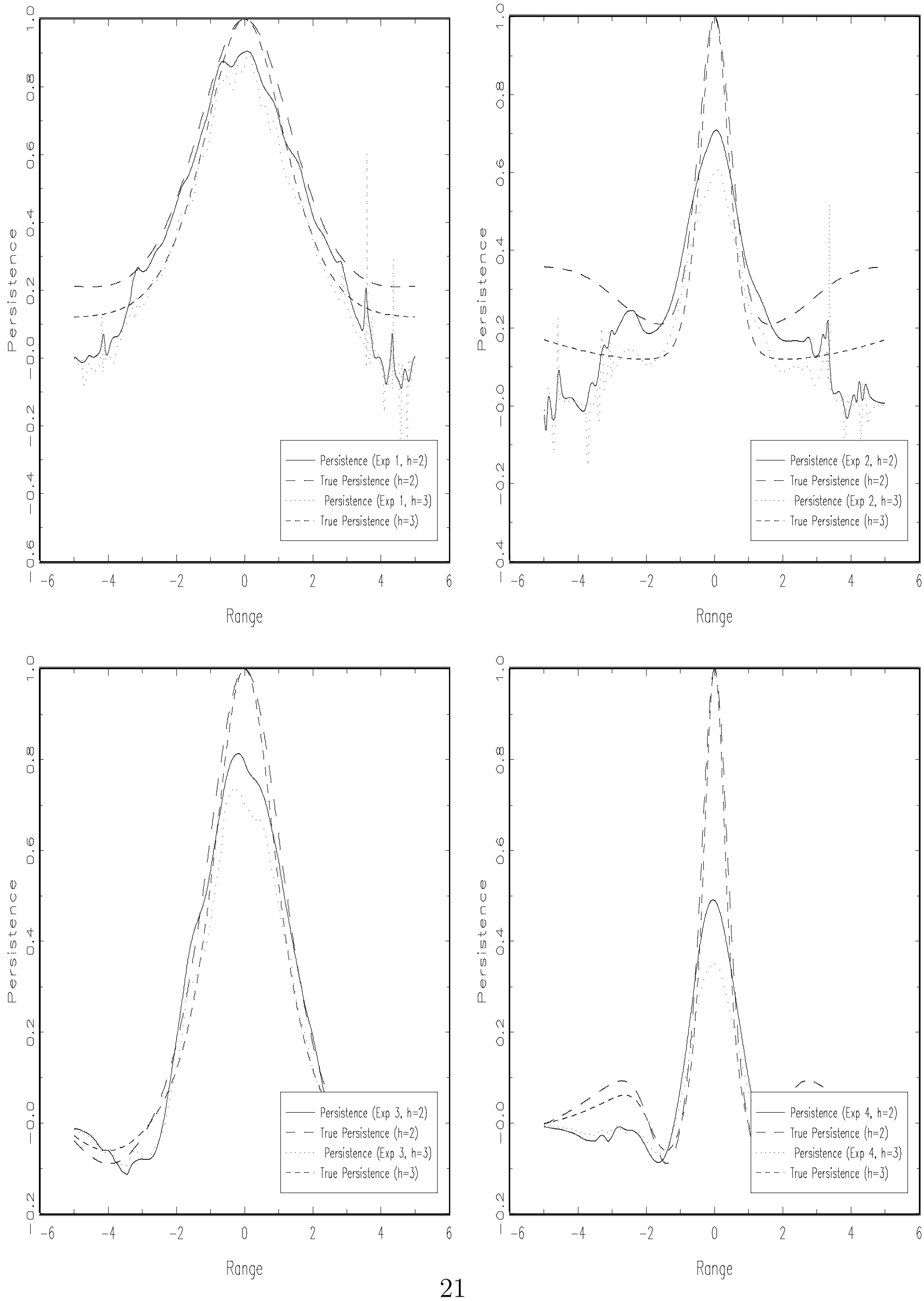


Figure 6: Experiments 5-8 ($h = 2, 3$)

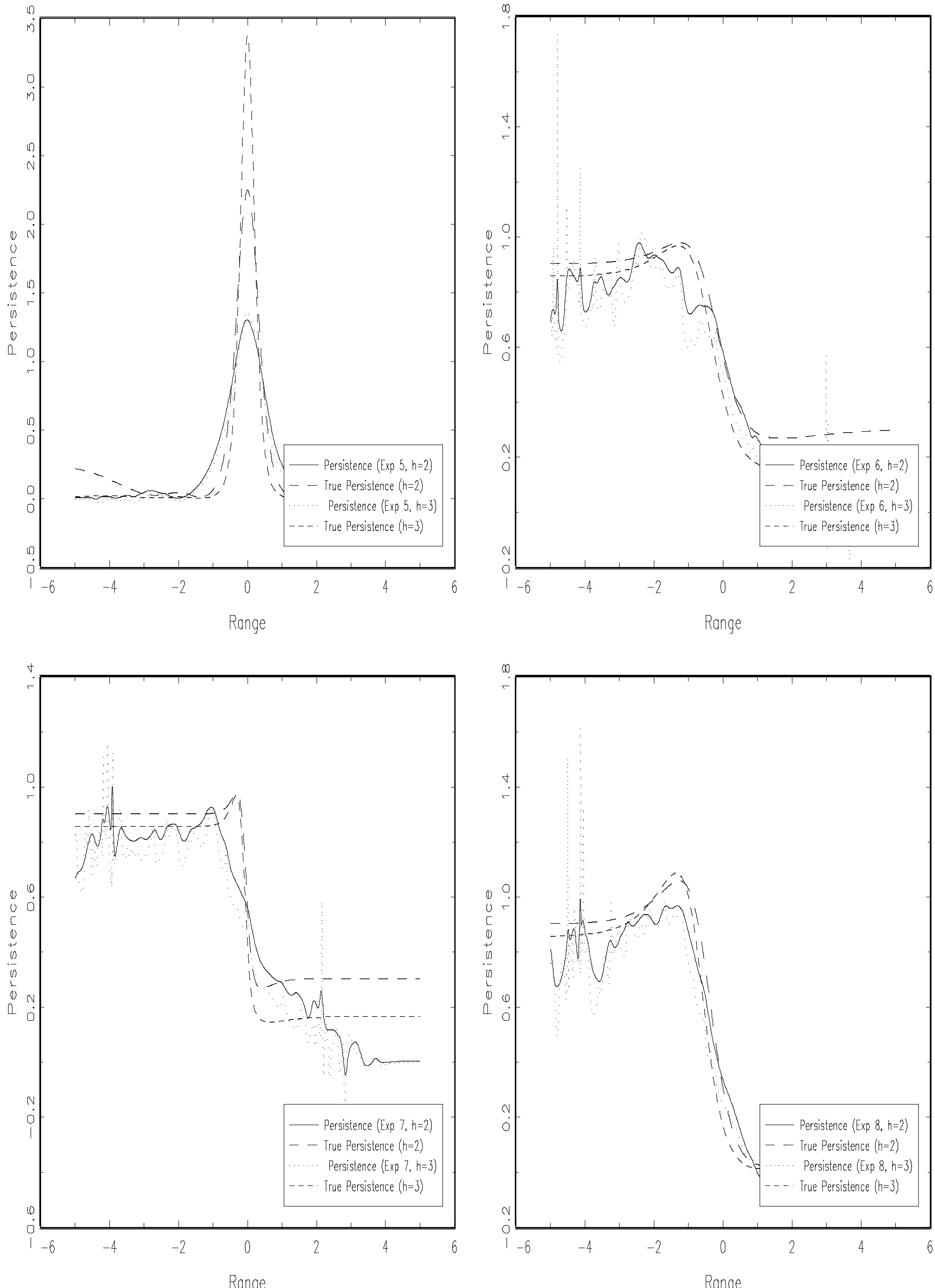


Figure 7: Experiments 9-12 ($h = 2, 3$)

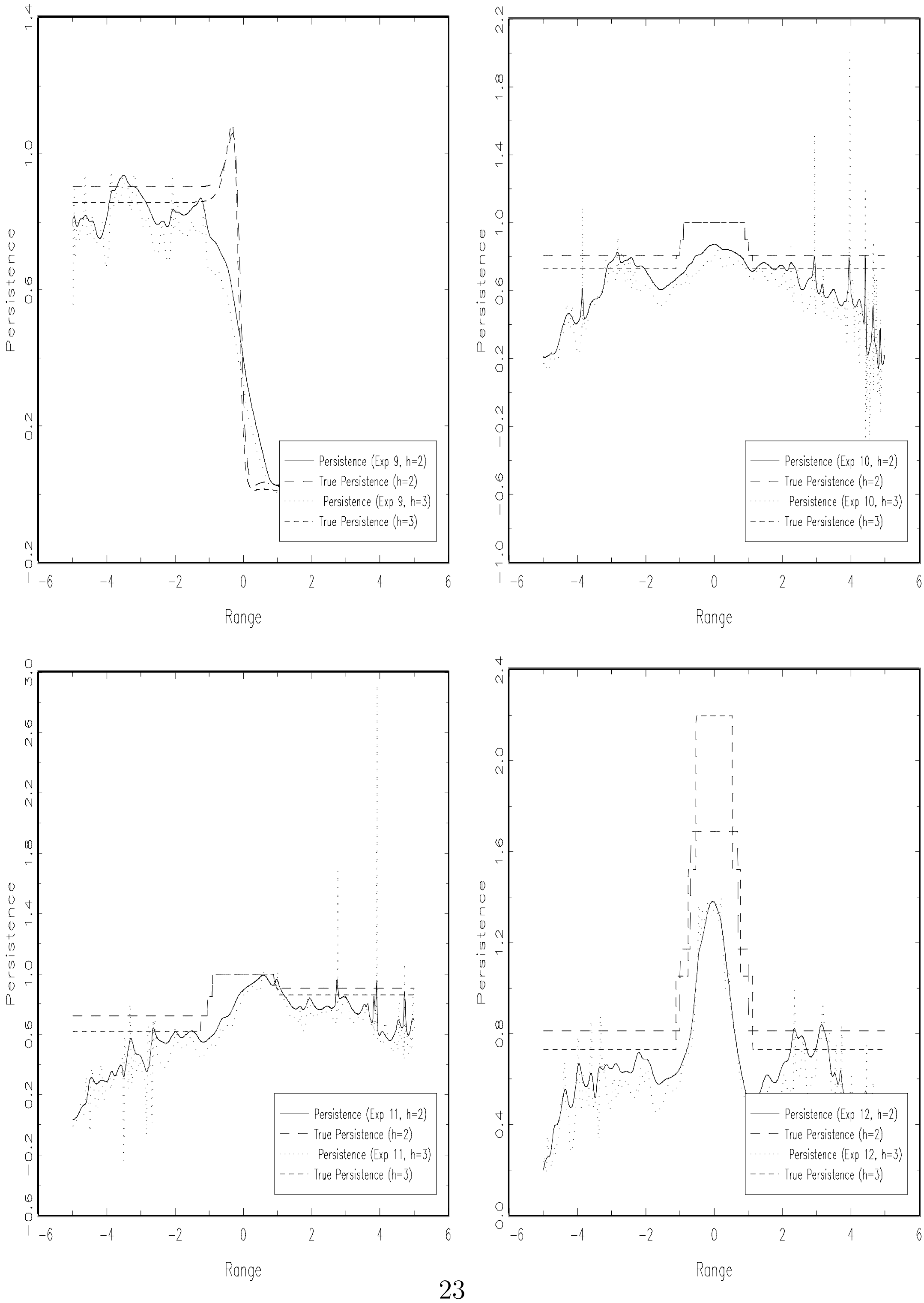


Figure 8: Experiment 13 ($h = 2, 3$)

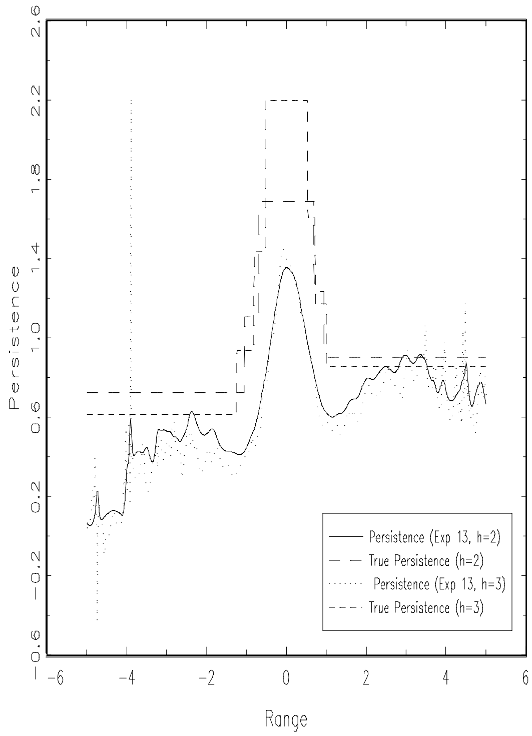


Figure 9: Empirical results for US, Germany, France, Italy, UK and Canada

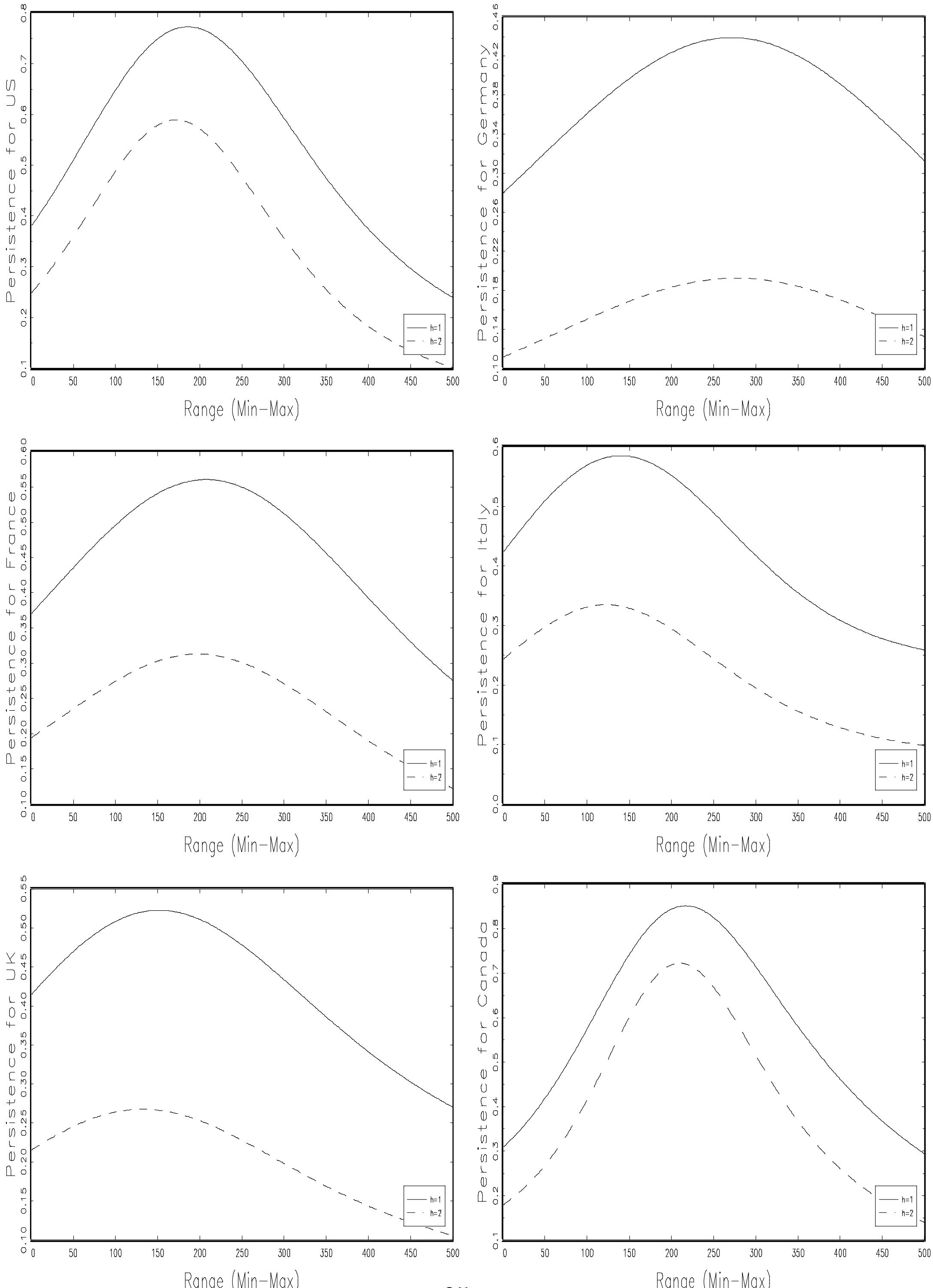


Figure 10: Empirical results for Austria, Turkey, Singapore, Malaysia, Indonesia and Thailand

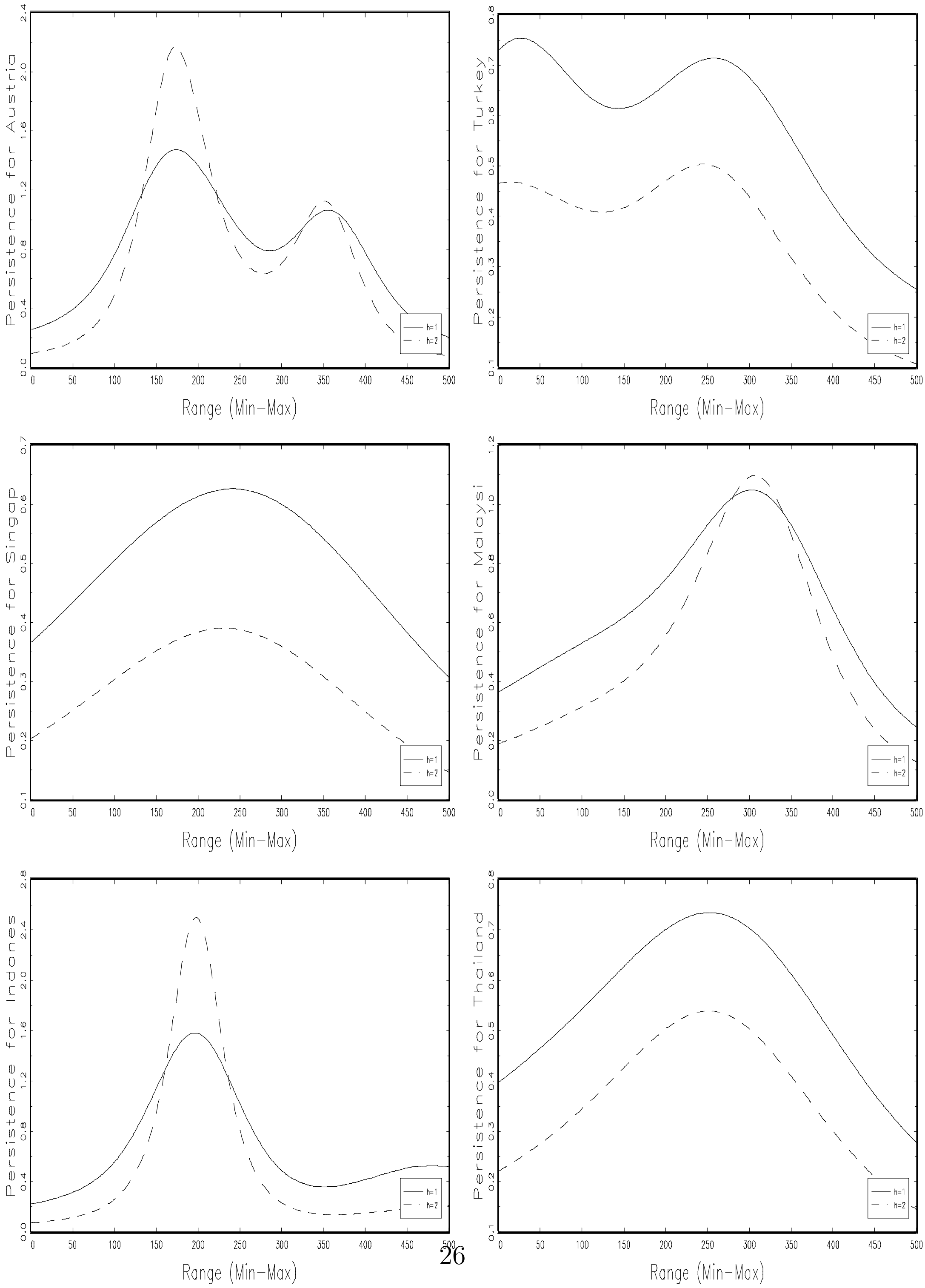
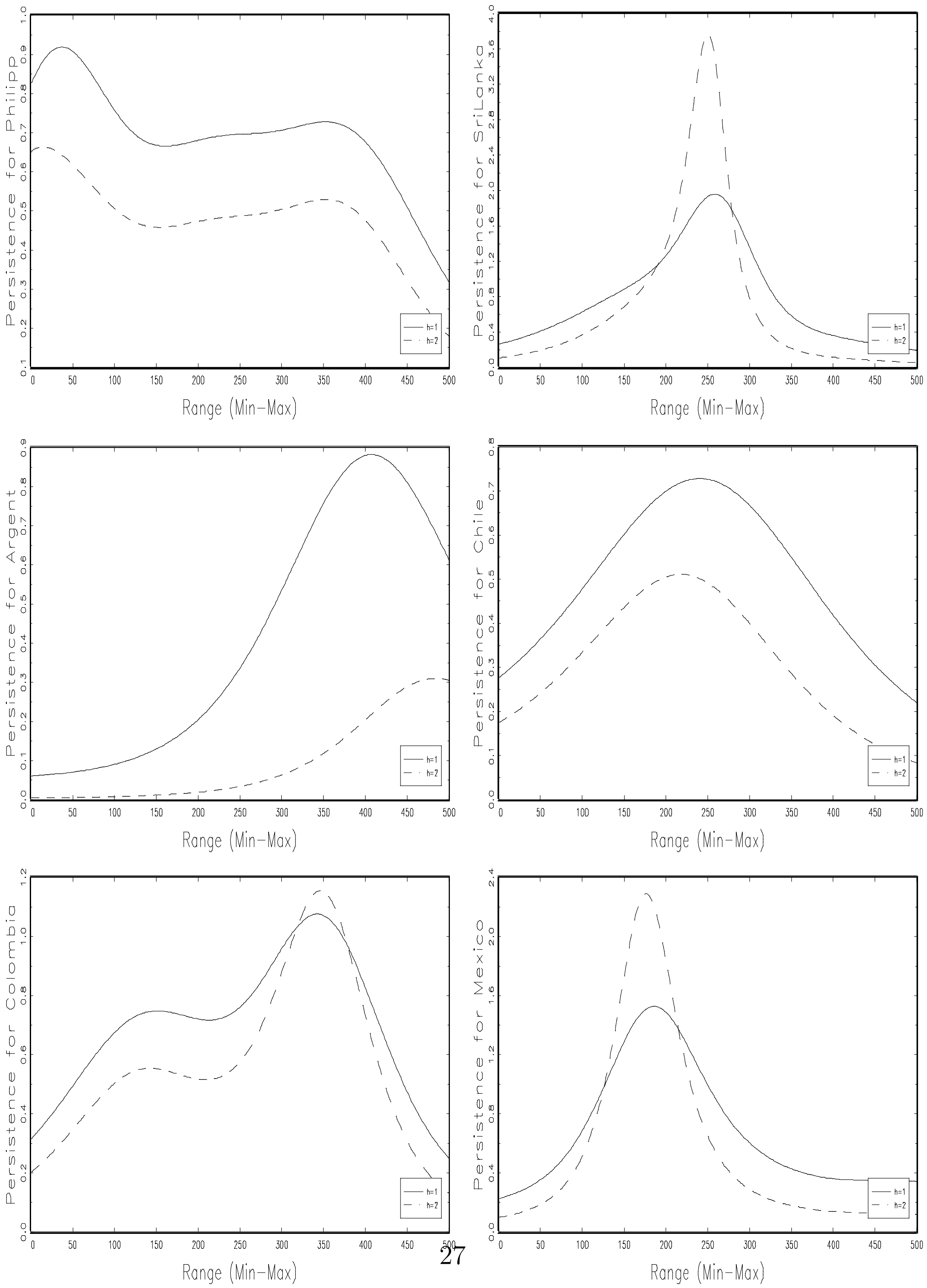


Figure 11: Empirical results for Phillipines, Sri Lanka, Argentina, Chile, Colombia, and Mexico



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