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Extracting implicit density functions from short term interest rate options*

Hannah Nielsen[†]

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Abstract

Using option prices the expectations of the market participants concerning the underlying asset can be extracted as well as the uncertainty surrounding these expectations. In this paper a mixture of lognormal density functions will be assumed to analyze options on three-month Euribor futures for the period between August and November 2000. During this period the ECB raised the interest rates and intervened in the exchange markets, both actions that could have an effect on the expectations of a short term interest rate. As will be shown the expected mean as well as the higher moments of the distribution show quite large movements, which can in part be associated directly with these interventions.

Keywords: implicit density function, interest rate options,
market expectations, monetary policy

JEL classification: G13, E44, E52, C13

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1 Introduction

The use of financial market instruments has gained increased attention during the last years. For example the European Central Bank, which has the stability of prices as target, declares the information obtained from financial markets as important indicator within the second pillar of its "Two-Pillar-Strategy". Financial market instruments should not be used as main target or indicator, but can give useful additional information concerning the expectations of the economic agents (ECB (2000)).

There exist various financial market instruments. SÖDERLIND AND SVENSSON (1997) provide an overview over several different approaches to extract information from them. In this paper the focus will lie on derivatives, especially options. The main feature of derivatives is their forward-looking character. The development of the underlying asset is uncertain in the future, therefore the holder of a derivative contract has to have some expectation of the future development of the value of the contract. These expectations, implied in the prices of the derivatives, can now be extracted. Considering option prices the expected mean of the underlying asset, the uncertainty surrounding it and even the whole implicit density function can be derived.

The extraction of the implied density function can be accomplished following several methods; for an overview see e.g. BAHRA (1997) or MCMANUS (1999). In this paper a mixture of lognormal density functions will be assumed as appropriate specification of the implicit density function, which is also employed e.g. by MELICK AND THOMAS (1997), CAMPA, CHANG AND REIDER (1998) or BLISS AND PANIGIRTZOGLU (1999) for different types of options. The distributional parameters of the function can be derived by minimizing the distance between the theoretically determined values and the actually observed option prices. This method will be applied here to options on interest rate derivatives, more precisely to options on three-month Euribor (Euro Interbank Offered Rate) futures. By taking the three-month Euribor future as underlying asset information on market expectation concerning the development of the short term interest rate can be provided and the uncertainty belonging to these expectations. The market for options on three-month Euribor futures is the most liquid market for interest rates in the Euro area (ECB (2000)), therefore their prices can be regarded as a good

measure of the implied expectations. The time span from the end of August to the beginning of November 2000 will be taken into consideration, in which the ECB raised the minimum bid rate on the main refinancing operations, as well as the interest rates on the marginal lending facility and the deposit facility twice. During this time the ECB also intervened alone and together with other major central banks to stabilize the Euro. As will be shown the expectations change according to the interest rate changes or interventions, which will be analyzed in more detail.

The paper is organized as follows. Section 2 starts with an overview of the techniques that can be used to extract expectations from option prices. As the mixture of lognormals is used here, this approach will be explained in detail. At the end of this section a method to compute standard errors for the derived moments of the distribution is introduced. The following section 3, contains the empirical application. At the end the mixed lognormal approach is compared to the benchmark model of just one lognormal density function, followed by a summary and concluding comments.

2 Extraction of expectations from option prices via implied density functions

2.1 Overview of different techniques

A whole set of different techniques has been developed concerning the extraction of expectations from option prices using implied risk neutral probability density functions. The various techniques can be broadly grouped into three categories.

The first group contains methods that assume a specific process which generates the evolution of the underlying asset, such as jump diffusion processes or binomial trees.¹

The second approach proposes the interpolation of the call-pricing-function or the implied volatilities. This technique bases on the result of BREEDEN AND LITZENBERGER (1978) that the second derivative of the call pricing

¹See eg. McMANUS (1999), MALZ (1996), BATES (1996), RUBINSTEIN (1994), JACKWERTH AND RUBINSTEIN (1996), CAMPA, CHANG AND REIDER (1998) or HÖRDAHL (2000).

function with respect to the strike price delivers the probability density for this explicit strike price. Various parametric and nonparametric techniques have been applied to obtain a continuous call pricing function or to smooth the volatility smile.²

At last the parametric form of the density function itself can be assumed, for example a mixture of two or three lognormal density functions, e.g. done by BAHRA (1996, 1997) and MELICK AND THOMAS (1997),³ a mixture of multivariate normal distributions (SÖDERLIND AND SVENSSON (1997) and SÖDERLIND (2000)) or a Burr III-distribution, used by SHERRICK, GARCIA AND TIRUPATTUR (1996). As the mixture of lognormal density function is the method used in this paper, the procedure underlying this approach will be explained in more detail in the following section.

2.2 Mixture of lognormal density functions

2.2.1 The lognormal distribution

As pointed out above, the implicit density function can be specified parametrically through different functional forms. The most popular and the one also chosen here is the assumption of a mixture of lognormal distributions. It is a widely accepted fact in financial theory that stock prices are distributed lognormally.⁴ The lognormal distribution has some desirable properties especially for data from financial markets, such as it is merely defined for positive values, has a positive skewness and a positive excess kurtosis. The lognormal distribution is defined as follows:

Definition 2.1 (Lognormal distribution) ⁵ *Consider two random variables X and Y . If $Y=\log(X)$ is normally distributed, then X is called lognormally distributed with the parameters μ_L and σ_L^2 . The density function of X*

²See e.g. AÏT-SAHALIA AND LO (1995), BATES (1991), SHIMKO (1993), MALZ (1997), COOPER AND TALBOT (1999) BLISS AND PANIGIRTZOGLU (1999), COOPER (1999), CLEWS, PANIRGITZOGLU AND PROUDMAN (2000), NEUHAUS (1995) or NAKAMURA AND SHIRATSUKA (1999).

³See also CAMPA, CHANG AND REIDER (1998), LEVIN, MCMANUS AND WATT (1998), COOPER (1999), BLISS AND PANIGIRTZOGLU (1999), GEMMILL AND SAFLEKOS (1999), MCMANUS (1999) and CLEWS, PANIGIRTZOGLU AND PROUDMAN (2000).

⁴See e.g. BAXTER AND RENNIE (1996) or CAMPBELL, LO AND MACKINLAY (1997).

⁵See e.g. AITCHINSON AND BROWN (1957) or SCHLITTGEN (1996).

is therefore

$$f(x; \mu_L, \sigma_L^2) = \frac{1}{\sqrt{2\pi}\sigma_L x} \cdot e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu_L}{\sigma_L}\right)^2} \quad (1)$$

for $x > 0$. The distribution is completely defined by these two parameters. What has to be regarded is that the parameters are not equal to the mean and the variance of the distribution. The mean, the variance, the skewness and the excess kurtosis of the lognormal distribution are given by:

$$E[X] = e^{\mu_L + \frac{1}{2}\sigma_L^2} \quad (2)$$

$$Var(X) = e^{2\mu_L + \sigma_L^2} \cdot (e^{\sigma_L^2} - 1) \quad (3)$$

$$Skew(X) = \sqrt{e^{\sigma_L^2} - 1} \cdot (e^{\sigma_L^2} + 2) \quad (4)$$

$$Kurt(X) = (e^{\sigma_L^2} - 1)[(e^{\sigma_L^2} - 1)[e^{\sigma_L^2} + 14 + 6\sqrt{e^{\sigma_L^2} - 1}] + 16]. \quad (5)$$

2.2.2 The mixture of lognormals

Although the lognormal distribution captures some of the properties of the observed option prices it is not adequate enough, as will be shown in section 3.4. Because of that, a mixture of lognormal distributions is considered, that fits the properties of the option prices better than just one lognormal. With a mixture of distributions e.g. a multimodal density function could be produced, which can be possible if the market participants expect different potential scenarios. In most cases it is sufficient to use a mixture of two lognormals. Accordingly the mixed density function can be written as:

$$g(x; \theta) = \phi f_1(x; \mu_{1L}, \sigma_{1L}^2) + (1 - \phi) f_2(x; \mu_{2L}, \sigma_{2L}^2) \quad (6)$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are the individual lognormal density functions as defined in equation (1), μ_{iL} and σ_{iL}^2 ($i = 1, 2$) the parameters of the individual functions and ϕ the mixing parameter. The mixed density function is completely characterized through the five distributional parameters $\mu_{1L}, \mu_{2L}, \sigma_{1L}^2, \sigma_{2L}^2$ and ϕ , which are combined within the vector θ .

According to this distributional assumption and the general valuation formula of a call option, following COX AND ROSS (1976),

$$C(K, r, \tau; \theta) = e^{-r\tau} \int_K^\infty (F_T - K) g(F_T; \theta) dF_T, \quad (7)$$

where r is the risk free interest rate, τ the time till expiration, i.e. $e^{-r\tau}$ is the discount factor, F the value of the underlying asset, T the maturity and

K the exercise price, the value of the option can be rewritten as:

$$C(K, r, \tau; \theta) = e^{-r\tau} \int_K^\infty (F_T - K) [\phi f_1(F_T; \mu_{1L}, \sigma_{1L}^2) + (1 - \phi) f_2(F_T; \mu_{2L}, \sigma_{2L}^2)] dF_T. \quad (8)$$

The distributional parameters can now be obtained by minimizing the distance between the theoretical call prices $C(K, r, \tau; \theta)$ and the actually observed call prices \hat{C} . This procedure can also be applied to the put prices of the option, which can be calculated in an analogous manner as:

$$P(K, r, \tau; \theta) = e^{-r\tau} \int_0^K (K - F_T) [\phi f_1(F_T; \mu_{1L}, \sigma_{1L}^2) + (1 - \phi) f_2(F_T; \mu_{2L}, \sigma_{2L}^2)] dF_T. \quad (9)$$

By considering the call as well as the put prices more observations can be included into the minimization procedure, which can be made more precise through that.

The overall minimization procedure can be written as:

$$\min_{\theta} M(\theta) = \sum_{i=1}^{n_1} [C(K_i, r, \tau; \theta) - \hat{C}_i]^2 + \sum_{i=1}^{n_2} [P(K_i, r, \tau; \theta) - \hat{P}_i]^2. \quad (10)$$

The variable n_i , $i = 1, 2$, is the number of strike prices at which calls and puts are available.

After having estimated the five parameters, the moments of the mixed distribution of the option prices can be computed. Because of the mixing this is not straightforward.⁶ Especially the quantiles can not be computed exactly, but have to be simulated instead. The moments of the distribution can be calculated as follows:⁷

$$E[X] = \tilde{\mu} = \tilde{\phi}\tilde{\mu}_1 + (1 - \tilde{\phi})\tilde{\mu}_2 \quad (11)$$

$$Var(X) = \tilde{\sigma}^2 = \tilde{\phi}\tilde{\sigma}_1^2 + (1 - \tilde{\phi})\tilde{\sigma}_2^2 + \tilde{\phi}(1 - \tilde{\phi})(\tilde{\mu}_1 - \tilde{\mu}_2)^2 \quad (12)$$

$$Skew(X) = \tilde{\beta}_1(X) = \frac{1}{\tilde{\sigma}^3} [\tilde{\phi}(\tilde{\beta}_{11}\tilde{\sigma}_1^3 + 3\tilde{\mu}_1\tilde{\sigma}_1^2 + \tilde{\mu}_1^3) + (1 - \tilde{\phi})(\tilde{\beta}_{12}\tilde{\sigma}_2^3 + 3\tilde{\mu}_2\tilde{\sigma}_2^2 + \tilde{\mu}_2^3) - 3\tilde{\mu}\tilde{\sigma}^2 - \tilde{\mu}^3] \quad (13)$$

⁶See e.g. BÜNING (1991).

⁷These results are taken from BÜNING (1991) or LEVIN, McMANUS AND WATT (1998) and own computations.

$$\begin{aligned}
Kurt(X) = \tilde{\beta}_2(X) &= \frac{1}{\tilde{\sigma}^4} [\tilde{\phi}((\tilde{\beta}_{21} + 3)\tilde{\sigma}_1^4 + 4\tilde{\beta}_{11}\tilde{\sigma}_1^3\tilde{\mu}_1 \\
&+ 6\tilde{\sigma}_1^2\tilde{\mu}_1^2 + \tilde{\mu}_1^4) + (1 - \tilde{\phi})((\tilde{\beta}_{22} + 3)\tilde{\sigma}_2^4 + 4\tilde{\beta}_{12}\tilde{\sigma}_2^3\tilde{\mu}_2 \\
&+ 6\tilde{\sigma}_2^2\tilde{\mu}_2^2 + \tilde{\mu}_2^4) - 4\tilde{\beta}_{11}\tilde{\sigma}_1^3\tilde{\mu}_1 - 6\tilde{\sigma}_1^2\tilde{\mu}_1^2 - \tilde{\mu}_1^4] - 3.
\end{aligned} \tag{14}$$

The parameters $\tilde{\mu}_i$, $\tilde{\sigma}_i^2$, $\tilde{\beta}_{1i}$ and $\tilde{\beta}_{2i}$ ($i = 1, 2$) are the mean, variance, skewness and excess kurtosis of the individual lognormal density function as defined in equations (2)-(5).⁸

BAHRA (1997) provides the closed form solution for equations (8) and (9), so the theoretical option prices can be computed more easily:⁹

$$\begin{aligned}
C(K, r, \tau; \theta) &= e^{-r\tau} [\phi(e^{\mu_{1L} + 0.5\sigma_{1L}^2} N(d_1) - KN(d_2)) \\
&+ (1 - \phi)(e^{\mu_{2L} + 0.5\sigma_{2L}^2} N(d_3) - KN(d_4))], \tag{15}
\end{aligned}$$

$$\begin{aligned}
P(K, r, \tau; \theta) &= e^{-r\tau} [\phi(-e^{\mu_{1L} + 0.5\sigma_{1L}^2} N(-d_1) + KN(-d_2)) \\
&+ (1 - \phi)(-e^{\mu_{2L} + 0.5\sigma_{2L}^2} N(-d_3) + KN(-d_4))], \tag{16}
\end{aligned}$$

with $d_1 = \frac{-\ln K + \mu_{1L} + \sigma_{1L}^2}{\sigma_{1L}}$, $d_3 = \frac{-\ln K + \mu_{2L} + \sigma_{2L}^2}{\sigma_{2L}}$, $d_2 = d_1 - \sigma_{1L}$ and $d_4 = d_3 - \sigma_{2L}$.

These are the general option valuation formulae. Depending on the underlying asset (stock index, currency, future) and its specific characteristics these valuation formulae have to be modified.

2.3 Computation of standard errors

To evaluate the movements of the descriptive statistics and their significance the standard errors have to be computed. As a result of the nonlinear transformations of the variables, this is not trivial. Other authors have constructed confidence intervals for the estimated density functions using Monte Carlo and bootstrap methods (MELICK AND THOMAS (1998)). Here a more analytical method is applied to provide standard errors for the parameters and moments.

To compute the moments of the single and mixed lognormal distributions the five estimated parameters μ_{1L} , μ_{2L} , σ_{1L} , σ_{2L} and ϕ have to be transformed

⁸The parameters denoted with $\tilde{}$ are the empirically determined parameters.

⁹The derivation of the solution can be found in the appendix of BAHRA (1997).

in a nonlinear way. Due to that the derivation of the standard errors of these moments is not straightforward. The standard errors of the estimated parameters can be obtained quite easily by using the final Hessian matrix delivered by the optimization procedure. The eigenvalues of this matrix have been constrained to ensure that it is positive definite. The Variance-Covariance-Matrix therefore is

$$Cov(\theta) = \hat{\sigma}^2 \left[\frac{\partial^2 M(\theta^0)}{\partial \theta \partial \theta'} \right]^{-1} \quad (17)$$

with

$$\hat{\sigma}^2 = \frac{1}{n} (\hat{u} - \bar{\hat{u}})' (\hat{u} - \bar{\hat{u}}), \quad (18)$$

as consistent estimator of the residual variance σ^2 , where \hat{u} is the vector of the residuals and $\bar{\hat{u}}$ the mean of the residual series. θ^0 is the parameter vector of the estimated parameter values. The square root of the diagonal elements are the standard errors of the estimated parameters, that will be denoted as $\hat{s}_{\mu_{1L}}$, $\hat{s}_{\mu_{2L}}$, $\hat{s}_{\sigma_{1L}}$, $\hat{s}_{\sigma_{2L}}$ and \hat{s}_{ϕ} .

The derivation of the standard errors of the moments of the distribution is more complex. Due to the nonlinearity of the necessary transformations, as explained above, the standard error of the transformed variable will only be approximated. If $h(x_1, \dots, x_k)$ is a function of the variables x_1, \dots, x_k and the variances $(\hat{s}_{x_1}^2, \dots, \hat{s}_{x_k}^2)$ and covariances $\widehat{Cov}(x_i, x_j) \forall i, j = 1, \dots, k$ are known, the variance of the function can be approximated by¹⁰

$$\hat{s}_h^2 \approx \sum_{i=1}^k \sum_{j=1}^k \frac{\partial h}{\partial x_i} \cdot \frac{\partial h}{\partial x_j} \widehat{Cov}(x_i, x_j), \quad (19)$$

evaluated at the particular parameter estimates, and the standard error accordingly as the square root of this expression. In the context of this paper the variables x_1, \dots, x_k are $\mu_{1L}, \mu_{2L}, \sigma_{1L}, \sigma_{2L}$ and ϕ .

This method can now be applied to the moments of the single distribution as well as to the moments of the mixed lognormal distribution in an analogous manner. For the parameters of the mixed distributions this is rather complex due to the fact that five variables have to be taken into account and that the formulae are more complicated.

¹⁰See GRAF ET AL. (1987).

3 Empirical application

3.1 Data

The data used has been obtained from the London International Financial Futures and Options Exchange (LIFFE). Options on three-month Euribor futures will be examined. The Euribor (Euro Interbank Offered Rate) is available since December 30th, 1998. It is calculated daily from the offered rates for one- to twelve-month-deposits of up to 57 banks following the method act/360 and reported from Bridge Telerate.¹¹ The expiry date of the options analyzed here is March 2002. There are 37 strike prices at which calls and puts on this future can be purchased.

Since these are options on short term interest rates, the option valuation formulae, given by equation (15) and (16), have a slightly different appearance. The quotation of the underlying future is 100 minus the interest rate, in percent, implied by the contract.¹² Therefore the closed form solution of the value of the call option on an interest rate future can be written as put option on the implied interest rate. This is applicable in the same way for the put option. Responsible for that is the linear inverse relationship between the futures price and the implied interest rate: When the futures price decreases the implied interest rate increases. Hence, equation (15) and (16) change to:

$$C(K, r, \tau; \theta) = e^{-r\tau} [\phi(-e^{\mu_{1L} + 0.5\sigma_{1L}^2} N(-d_1) + (100 - K)N(-d_2)) \\ + (1 - \phi)(-e^{\mu_{2L} + 0.5\sigma_{2L}^2} N(-d_3) + (100 - K)N(-d_4))] \quad (20)$$

and

$$P(K, r, \tau; \theta) = e^{-r\tau} [\phi(e^{\mu_{1L} + 0.5\sigma_{1L}^2} N(d_1) - (100 - K)N(d_2)) \\ + (1 - \phi)(e^{\mu_{2L} + 0.5\sigma_{2L}^2} N(d_3) - (100 - K)N(d_4))], \quad (21)$$

with d_i ($i = 1, \dots, 4$) defined as in equations (15) and (16).

A further important feature of options traded at the LIFFE is the delayed payment of the option premium, which works like the margining system for futures. The buyer of an option does not have to pay the premium up front,

¹¹For more details see e.g. DIWALD (1999).

¹²For this and the other described features of the considered options see LIFFE (1999) or GEMMILL (1993).

but is only required to deposit a certain amount, that will still remain his property. The position of the option will then be corrected for the gains and losses associated with it each day, a procedure known as marking-to-market. At the time of exercise the price of the option has to be paid. Due to that the price of the contract does not have to be discounted and can be written as:

$$C(K, \tau) = \phi(-e^{\mu_{1L} + 0.5\sigma_{1L}^2} N(-d_1) + (100 - K)N(-d_2)) \\ + (1 - \phi)(-e^{\mu_{2L} + 0.5\sigma_{2L}^2} N(-d_3) + (100 - K)N(-d_4)), \quad (22)$$

and analogously for the put option, d_1 to d_4 being again the same as in (15) and (16).

Another very important property resulting from this peculiar trading rule is that the contracts can be priced as European options. The options investigated here are actually all American style, for which the pricing is more complicated than for European style options. The advantage of American style options is that they can be exercised at any point in time. Because of this extra right, American options must be worth at least as much as European ones and could even be worth more. In the absence of dividend payments and regarding the futures-style margining with the delayed payment of the premium the right to exercise early is no additional value. This is the reason, why these options can be valued as in equation (22) at all.

The market for options on three-month Euribor futures is the most liquid market for interest rates in the Euro area (ECB (2000)), therefore their prices can be regarded as a good measure of the implied expectations. The average daily traded volume of that future at the LIFFE on an annual basis came to 139,834 contracts in 1999 and 228,413 in 2000, the total annual number was 35,657,690 in 1999 and 58,016,852 in 2000. The average daily number of options on that future amounted (again on an annual basis) to 19,433 in 1999 and 31,103 in 2000, the total annual number summed up to 4,819,366 in 1999 and 7,900,121 contracts for the year 2000. With that the option on the three-month Euribor future was the option with the highest total annual volume of all traded options at the LIFFE in these years.

3.2 Date of interest

For this study the time span from end of August to mid November 2000 will be taken into consideration, in which the ECB raised the minimum bid rate on the main refinancing operations, as well as the interest rates on the marginal lending facility and the deposit facility twice. During this time the ECB also intervened alone and together with other major central banks to stabilize the Euro.

In Table 1 the dates and the interventions during the considered period are summarized. In addition to these events the Danish population voted against the joining of Denmark of the Eurosystem on September, 28th.

Date	Action	Effective at
8/31/2000	Raise of interest rates on marginal lending facility (5.25% \rightarrow 5.5%) and on deposit facility (3.25% \rightarrow 3.5%)	9/1/2000
	Raise of minimum bid rate on the main refinancing operations of the Eurosystem (4.25% \rightarrow 4.5%)	9/6/2000
9/22/2000	Joint intervention in the exchange markets of the ECB and the monetary authorities of the United States and Japan	
10/5/2000	Raise of interest rates on marginal lending facility (5.5% \rightarrow 5.75%) and on deposit facility (3.5% \rightarrow 3.75%)	10/6/2000
	Raise of minimum bid rate on the main refinancing operations of the Eurosystem (4.5% \rightarrow 4.75%)	10/11/2000
11/3/2000	Intervention of the ECB in the exchange markets	

Table 1: Summary of actions of the ECB

The actions of the ECB had become necessary to stabilize the Euro, because its exchange rate to the USDollar fell constantly since its introduction with only small phases of recovery.

3.3 Results and interpretation

The mixture of lognormal distributions, described in section 2.2, is applied to extract the implied risk neutral density functions.¹³ The analyzed time period starts with the 24th of August and ends with the 8th of November. For every day the computation has been carried out. Mean, variance, skewness and excess kurtosis have been calculated according to the formulae given by equation (11), (12), (13) and (14), while the upper and lower quartiles have been simulated, due to the fact, that they cannot be determined exactly.¹⁴ As representative example the shape of the two single lognormal and the resulting mixed lognormal density function from one date are presented in Figure 1. For this example the 16th of October has been chosen, but this choice is arbitrary.

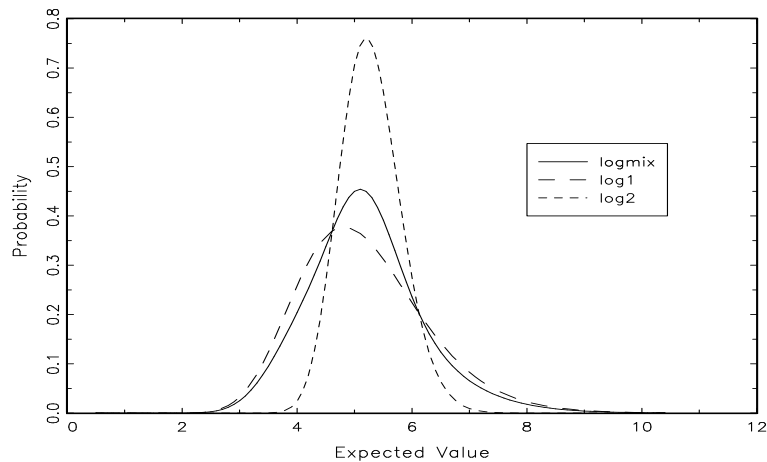


Figure 1: Density functions of three-month Euribor Future for October, 16th

In the following the descriptive statistics will be analyzed with respect to the dates of the considered events, which are summarized in Table 1, i.e. (i) the double raise of the interest rates by the ECB and (ii) the two interventions in the exchange markets. The moments of the distribution are depicted together with the parameter \pm twice the standard error, that has

¹³The computation has been done using the CONSTRAINED OPTIMIZATION-Modul for the software program GAUSS.

¹⁴The simulation has been fulfilled by a GAUSS-program, which has been supplied to me by BÜNING.

been calculated according to section 2.3. The shaded regions in the following figures cover the dates of the interventions of the ECB as summarized in Table 1.

3.3.1 Mean

At first the resulting mean for the whole period is depicted in Figure 2. As can be seen the expected value of the underlying futures contract shows a negative trend over the whole period, interrupted by some short upward movements, which did not last very long. Surrounding the considered events, the following observations can be made.

(i) The first break of this series coincides with the first raise of the interest rates at the beginning of September. First the mean moves down on September, 1st, the day after the announcement, the same day the raise of the interest rates on marginal lending facility and the deposit facility became effective, but then it gets back to its old level by the 6th of September, the day the raise of the main refinancing operations becomes effective. Around the date of the second raise of the interest rates by the ECB no large movement is detectable. The explanation for these findings could be that the market participants were surprised by the first raise, but suspected the second one and integrated it therefore into their calculations in advance.

(ii) The second sharp upward shift matches the date after the joint intervention in the exchange market on September 22nd. The interpretation of the timing of this shift could again be that the economic agents did not anticipate the intervention and did therefore not incorporate it into their expectations beforehand. Surrounding the date of the second intervention of the ECB in the exchange markets at the 3rd of November, no change can be seen. Probably the market participants expected this event.

The standard error bands indicate that the optimization leads to quite precise results. Taking the mean \pm twice the standard error, the bands would not be visible, because they are too close to the coefficient itself. Due to this the mean is shown \pm twenty times the standard error.

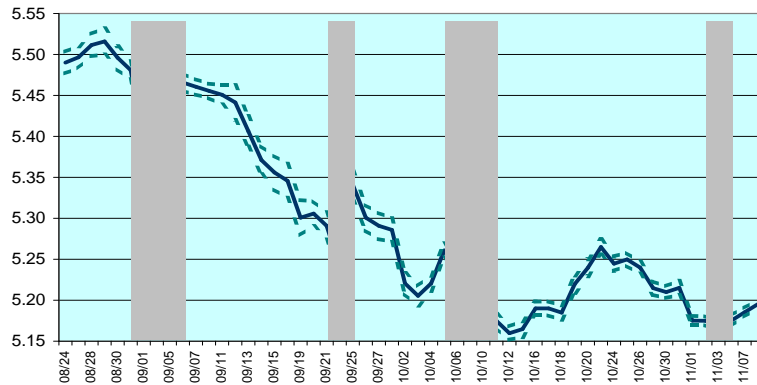


Figure 2: Mean of three-month Euribor Future

Further disclosure can be achieved by examining the components of the double lognormal mixture separately. In Figure 3 the means of the two single lognormal density functions (left scale) are depicted together with the mixing parameter ϕ , that can take values between zero and one (right scale). A value of ϕ , which is greater than 0.5 therefore means, that the first lognormal density has been given more weight than the second one. As can be seen from Figure 3 the two means move in general in the same direction, but differ at some dates up to 0.2 percentage points.¹⁵

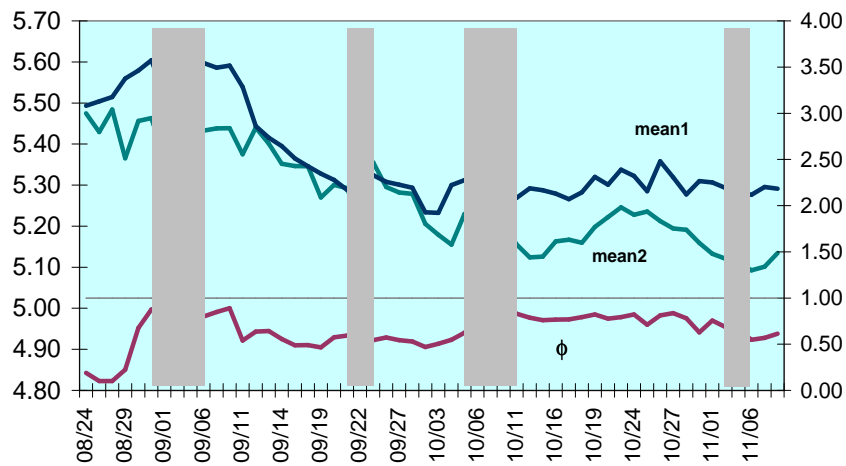


Figure 3: Mean 1, Mean 2 and ϕ of three-month Euribor Future

The analysis of the dates of interest leads to the following observations:

¹⁵For reasons of clearness the standard error bands have been omitted.

(i) The same break that is detectable within the mean of the mixed distribution can be found considering the single means around the first raise of the interest rates by the ECB. The difference between the two means could imply that the economic agents had a bimodal expectation as they could imagine two different scenarios. Following this period the two means move rather similar for about a month, interrupted only by a sharp upward shift of the second mean at the day following the joint intervention. Following the second raise of the interest rates within the observed period at the 5th of October the two series drift apart again. During this time span, the mixing parameter ϕ is rather high, i.e. the first distribution has a higher weight. A lower mean, implied by the second distribution therefore seems not to be very likely for the market participants, though not impossible.

(ii) As already stated, the first intervention in the exchange markets on September, 22nd leads to a sharp upward shift of the second mean on the day after. The occurrence of this event in only one of the mean-series leads again to a bimodal distribution. The second intervention of the ECB does not contribute to a change of the single means, which is the same concerning the joint mean.

3.3.2 Variance and difference of quartiles

In Figure 4 the variance of the joint distribution as well as the standard error bands can be seen. In general the variance behaves similar to the joint mean. Overall it shows a downward trend with only small upward or downward shifts, which means that the uncertainty in the market was smaller at the end of the considered time span. The standard error bands are again rather close to the series itself, except again at the beginning of the sample. Concerning the specific dates, the following statements can be made:

(i) During the dates of first interest rate raise the same break as in the series of the first moments can be observed, but the second raise did not lead to a shift. These findings lead to the interpretation that the first raise was not expected and changed the uncertainty, whereas the second raise was already anticipated.

(ii) Looking at the dates of the interventions of the ECB alone or together with other major central banks, the series reveals no systematic movement. This is contrary to the behaviour of the mean, where the first intervention

lead to a sharp upward shift.

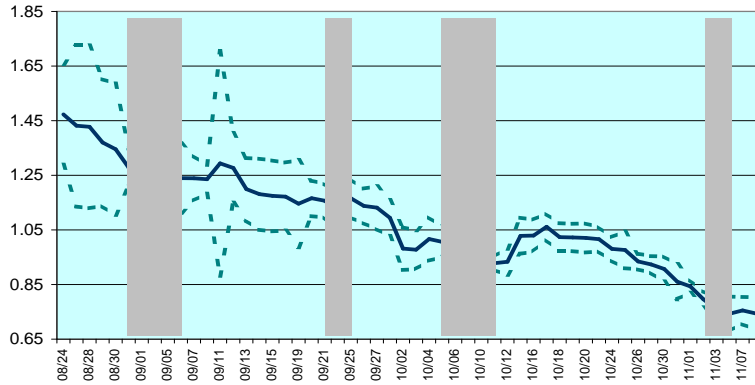


Figure 4: Variance of three-month Euribor Future

As the variance reveals not as much usable information about the uncertainty in the market surrounding these expectations as expected the difference between the upper and the lower quartile as an alternative measure has been computed. For reasons of standardization these differences have been divided by the mean. The resulting series is shown in Figure 5. A general observation is that the uncertainty in the market is more volatile at the beginning of the regarded period. In contrast to the development of the mean, some larger movements of this time series cannot be associated that easily with the considered events, especially at the beginning of the sample. Regarding the events separately leads to the following conclusions:

(i) Following the first raise of the interest rates by the ECB the difference between the upper and the lower quartile is at a lower level than before, starting to move upwards again at the 4th of October, one day before the second raise was announced. That means that the first raise lowered the market uncertainty, but not substantially, and the second raise did not contribute to that evolution. Because the change in the market takes place on the day before the announcement the information extracted from the mean series is confirmed, i.e. the economic agents anticipated this action by the ECB.

(ii) In contrast to the development of the mean, no changes due to the joint intervention of the ECB and other major central banks can be found in these data, while during the days before and after the second intervention a down-

ward movement can be recognized, which can be interpreted as success of the ECB concerning the decrease of uncertainty within the market regarding the further development.

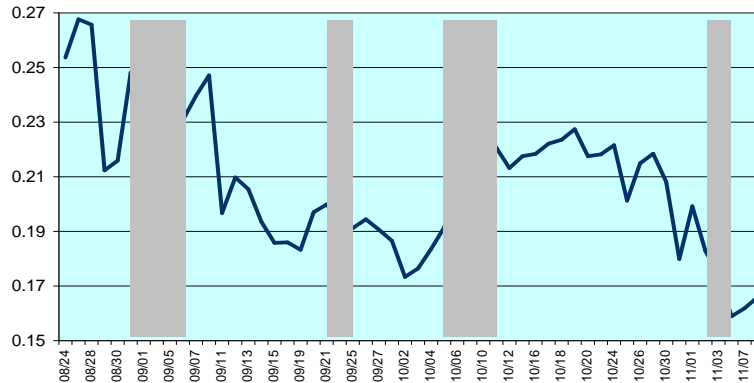


Figure 5: Normalized difference of quartiles for three-month Euribor Future

3.3.3 Skewness and excess kurtosis

As further descriptive devices the skewness and the excess kurtosis of the joint distribution, shown in Figure 6 and 7, can be referred to. The general movement of the two parameters looks very similar. As expected the skewness is positive for the whole period. The parameter is not large, that means the distribution is rather symmetric, what can be confirmed by a look at the density function in Figure 1.¹⁶ The excess kurtosis, as well, is positive for the entire time span, i.e. the distribution is leptokurtic, as expected by theory. In more detail:

(i) Before the day of the first raise, skewness and excess kurtosis show a downward trend, but shift upwards a few days later, which does not necessarily coincide with the action of the ECB. Until the second raise both parameters are higher than compared to the rest of the period, i.e. the underlying distribution is less symmetric. Some days before the second raise skewness and excess kurtosis move down to the same level as before, leading again to the conclusion that this event was already incorporated into the expectations of the market participants at the day of the announcement.

¹⁶This is merely the density function for one day, but the result is similar for most of the other dates.

(ii) One day after the joint intervention in the exchange markets the skewness parameter shifts upwards, which corresponds to a rather large shift in the mean series. It seems therefore that this action has not been anticipated and increased the asymmetry in the market temporarily. After the second, single, intervention the skewness is very low. The downward trend at the end coincides with the development of the quartiles difference as measure of uncertainty. This can be interpreted as calming down of the market after a relatively turbulent period. In contrast, no large movement can be detected within the series of the excess kurtosis before and after both interventions into the exchange market. Either the actions have been anticipated or they did not change the excess kurtosis.

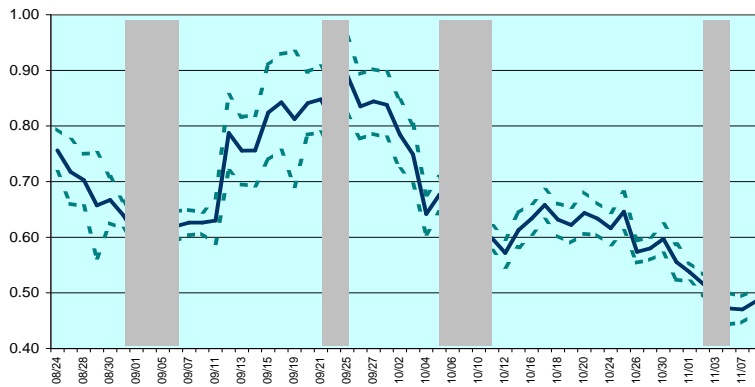


Figure 6: Skewness of three-month Euribor Future

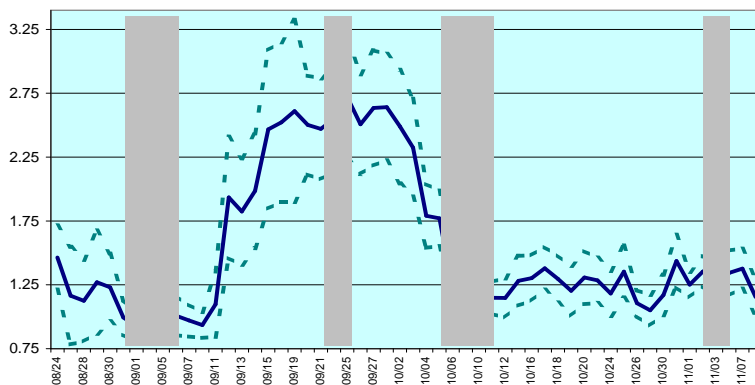


Figure 7: Excess kurtosis of three-month Euribor Future

With the exception of the middle of the sample, where the skewness and the excess kurtosis itself also shows the most movements, the standard errors are not very large, which confirms the goodness of the optimization.

3.3.4 Concluding remarks

Concluding it can be stated that some of the actions undertaken by the ECB had an effect on the expectations of the economic agents, but these effects did not last. The first raise of the interest rates at the beginning of September was obviously not expected by the market participants, because mean and difference of quartiles show quite large movements and these after the intervention. The shift in the skewness and excess kurtosis parameter occurs some days later and can therefore not be associated with that action that easily. A different behaviour can be observed concerning the second raise of the interest rates at the beginning of October. The joint mean shows no change, while the difference of quartiles, the single means, the skewness and the excess kurtosis shift already before the announcement of the raise, which means that this action was anticipated by the market participants. The joint intervention of the ECB and other major central banks in the exchange markets on September 22nd changed the mean and the skewness, while the quartiles difference and the excess kurtosis reveal no larger movements. As the shift occurs one day after the event the conclusion is again that it was not expected. The second, single, intervention at November 3rd can not be detected considering the mean and the excess kurtosis, whereas the two other parameters move downwards surrounding this date, leading to the conclusion that the uncertainty and asymmetry within the market has decreased. This calming down of the market at the end of the considered time period is a general observation that can be made by analyzing the higher moments of the extracted distribution.

The voting of the Danish population against the joining of the Eurosystem obviously did not effect the expectations considerably, because none of the descriptive devices show a large movement around that date.

3.4 Comparison with one lognormal distribution

To illustrate that a mixture of lognormal distributions fits the data better than just one, the optimization procedure has been applied also to that approach.

As first device to evaluate the two methods the residual variance is compared. This comparison is shown in Figure 8.

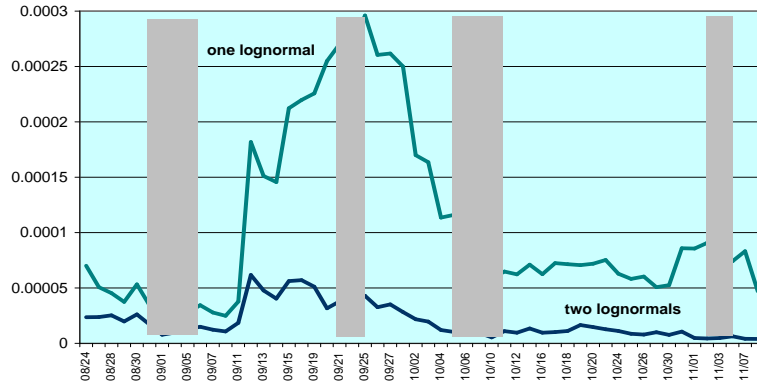


Figure 8: Residual variance of one and two lognormal density functions

It is obvious that the residual variance of one lognormal density function is higher than that of a mixture of two density functions at any date. What is also striking is the large difference of the two series during the middle of the observed sample. This observation coincides with the lack of ability of one lognormal function to capture the development of the higher moments adequately. This can be seen in Figure 9 and Figure 10.

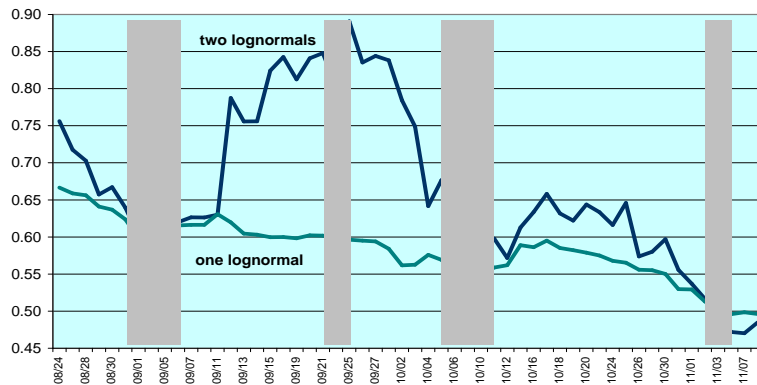


Figure 9: Skewness of one and two lognormal density functions

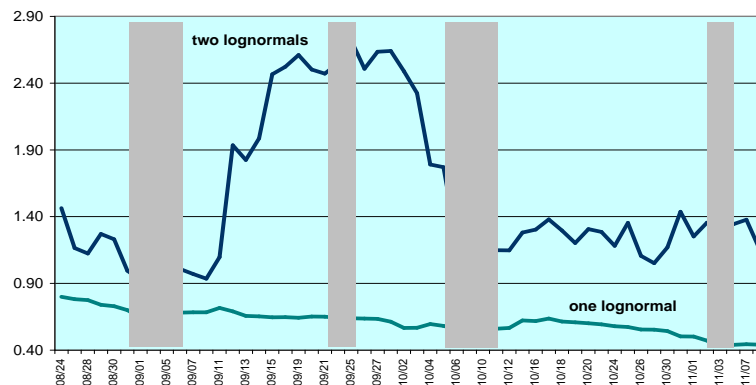


Figure 10: Excess kurtosis of one and two lognormal density functions

While the mean and the variance move rather similar and are therefore not depicted here, the skewness and excess kurtosis behave contrarily, especially in the middle of the time period.

Considering the series obtained by assuming just one lognormal density function almost no structure can be detected within the series of the higher moments.

Concluding it can be stated that the approach using only one lognormal function is not appropriate to capture the characteristics of the data and especially the behaviour of the higher moments.

4 Conclusion

Options on three-month Euribor futures were used to extract the expectations, uncertainty, asymmetry and concentration in the market concerning the further development of this short term interest rate. By applying an approach that assumes a mixture of lognormal distributions as parametric form of the implicit density function, this density has been extracted from the options prices for a period of about 2.5 months, covering two decisions of the ECB to raise the interest rates and two interventions in the exchange markets. By analyzing the moments of this distribution, statements about changes of these moments following or preceding these interventions can be made.

As pointed out in the last section, some of the large movements of the se-

ries can be associated directly with the considered events, while other changes cannot be explained directly with the method used here. To make statements about the significance of the movements of the series, standard errors have to be computed, which is not straightforward due to the nonlinearity of the optimization problem. This has been done by using a procedure to approximate the standard errors. The computed standard errors confirm the precision of the parameter values obtained by the optimization process.

What has to be mentioned is that these expectations cannot deliver all the information you would need to evaluate monetary policy actions, because these are only the expectations concerning March 2002. What would be necessary are different maturities, so the expectations regarding shorter periods can be extracted. Having obtained these, this method can be a useful tool to evaluate monetary policy actions by analyzing the beliefs of the market and especially the uncertainty in the market.

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