

De Clippel, Geoffroy

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# Comment on “The Veil of Public Ignorance”\*

Geoffroy de Clippel<sup>†</sup>

February 2010

Nehring (2004) proposes an interesting methodology to extend the utilitarian criterion defined under complete information to an interim social welfare ordering allowing to compare acts. The first axiom defining his approach, called “State Independence,” requires the interim social welfare ordering to be consistent with ex-post utilitarian comparisons: if it is commonly known that the act  $f$  selects in each state an outcome that is socially preferred according to the utilitarian criterion to the lottery selected by an alternative act  $g$ , then  $f$  should be interim socially preferred to  $g$ . The second axiom is a classical condition of consistency with respect to interim Pareto comparisons: if an act  $f$  interim Pareto dominates and act  $g$ , then  $f$  should be interim socially preferred to  $g$ . Nehring proves that 1) these two axioms are incompatible if there is no common prior, and 2) that the unique interim social welfare ordering that satisfies these two axioms when there is a common prior is the ex-ante utilitarian criterion.

The purpose of this comment is to show that Nehring’s methodology does not prove helpful in finding ways of extending other classical social welfare orderings. I show indeed that the corresponding state-independence property becomes incompatible with the interim Pareto criterion for a very large class of common priors, as soon as the social welfare ordering satisfies the strict Pigou-Dalton transfer principle (strict PD for short). I also show that his impossibility result in the absence of a common prior extends to any social welfare ordering that satisfies PD. The Pigou-Dalton principle states that transferring utility so as to reduce inequality should never hurt from a social perspective. Strict PD requires that the resulting utility profile is socially strictly preferred. PD is often viewed as a very appealing axiom in social choice theory, and indeed all the classical social welfare orderings (e.g. utilitarian sum, egalitarian minimum, and Nash product)

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\*K. Nehring, 2004, *Journal of Economic Theory* 119, 247-270

<sup>†</sup>Department of Economics, Brown University, Providence, Rhode Island - declippel@brown.edu. Financial support from the National Science Foundation (grant SES-0851210) is gratefully acknowledged.

satisfy it. The utilitarian criterion has the distinctive property of satisfying PD, but not its strict variant.

I start the formal analysis by quickly reminding the content of Nehring's (2004) model. Let  $I$  be the finite set of individuals, let  $X$  be the set of deterministic *social alternatives*, and let  $\mathcal{L}$  be the set of probability distribution over  $X$  (with finite support). Individuals are expected utility maximizers. Let  $u_i : X \rightarrow \mathbb{R}$  be individual  $i$ 's von Neumann-Morgenstern utility function. There is a finite set  $\Omega$  of *states* that determine the agents' *beliefs*  $p_i : \Omega \rightarrow \Delta(\Omega)$ , and hence their preferences over *acts* :

$$f \succeq_i^\omega g \text{ if and only if } \sum_{\alpha \in \Omega} \sum_{x \in X} p_i^\omega(\alpha) f_x(\alpha) u_i(x) \geq \sum_{\alpha \in \Omega} \sum_{x \in X} p_i^\omega(\alpha) g_x(\alpha) u_i(x),$$

for each  $i \in I$ , each  $f : \Omega \rightarrow \mathcal{L}$ , and each  $g : \Omega \rightarrow \mathcal{L}$ . The belief functions are assumed to satisfy the following assumptions:

**Assumption 1** (Introspection)  $p_i^\omega(\{\alpha \in \Omega | p_i^\alpha = p_i^\omega\}) = 1$ , for all  $\omega \in \Omega$  and all  $i \in I$ .

**Assumption 2** (Truth)  $p_i^\omega(\{\omega\}) > 0$ , for all  $\omega \in \Omega$  and all  $i \in I$ .

Introspection says that agents are always (at any state  $\omega$ ) certain of their own belief  $p_i^\omega$ . The Truth assumption requires that individuals put positive probability on the true state; agents therefore can never be wrong in their probability-one beliefs. Nehring finally assumes that the individuals' utility functions and the set of social alternatives are such that any real number can be derived as the utility of some lottery over  $X$ :

**Assumption 3** (Rich Domain) For each  $\nu \in \mathbb{R}^I$ , there exists  $l \in \mathcal{L}$  such that  $\nu = \sum_{x \in X} l(x) u_i(x)$ , for each  $i \in I$ .

For any  $\alpha \in \Omega$ ,  $T_i(\alpha) = \{\omega \in \Omega | p_i^\alpha(\omega) > 0\}$  is the set of states that individual  $i$  thinks possible. Introspection and Truth implies that  $\{T_i(\omega) | \omega \in \Omega\}$  forms a partition of  $\Omega$ . Individual  $i$  *knows* an event  $E \subseteq \Omega$  at  $\alpha$  if  $T_i(\alpha) \subseteq E$ .  $E$  is *common knowledge* if everybody knows  $E$ , everybody knows that everybody knows  $E$ , and so forth. Formally, if  $T_I$  is the finest common coarsening of the individuals' knowledge partitions, then  $E$  is common knowledge at  $\alpha$  if  $T_I(\alpha) \subseteq E$ . A probability distribution  $\mu \in \Delta(\Omega)$  is a *common prior* if  $p_i^\omega(A) = \mu(A | T_i(\omega))$ , for all  $i \in I$ ,  $A \subseteq \Omega$ , and  $\omega \in \Omega$  such that  $\mu(\omega) > 0$ .

A *social welfare ordering* (under complete information) is a complete and transitive binary relation  $R$  defined on  $\mathbb{R}^I$ . Classical examples include the utilitarian criterion,  $R^U$  with  $uR^U v$  if and only if  $\sum_{i \in I} u_i \geq \sum_{i \in I} v_i$ , the egalitarian criterion,  $R^E$  with  $uR^E v$  if

and only if  $\min_{i \in I} u_i \geq \min_{i \in I} v_i$ , and the Nash criterion,<sup>1</sup>  $R^N$  with  $uR^Nv$  if and only if  $\prod_{i \in I} u_i \geq \prod_{i \in I} v_i$ .  $P$  will denote the strict component of  $R$ , i.e.  $uPv$  if  $uRv$  and not  $vRu$ .  $R$  satisfies the *Pigou-Dalton transfer principle* (PD) if  $vRu$  for any  $u, v$  such that there exist two individuals  $i$  and  $j$  such that  $u_i < u_j$ ,  $v_i \leq v_j$ ,  $u_i + u_j = v_i + v_j$ ,  $u_i < v_i$ , and  $u_{-ij} = v_{-ij}$ . Inequality is thus reduced when moving from  $u$  to  $v$ , since  $v$  is obtained from  $u$  by “transferring” some utility from  $j$  to  $i$ , while  $i$  was and remains after the transfer with less utility compared to  $j$ .  $R$  satisfies the *strict Pigou-Dalton transfer principle* (strict PD) if  $vRu$  is replaced by  $vPu$ .

An *interim social welfare ordering* is a transitive ordering  $\succeq_I$  (even possibly incomplete) defined on the set of acts. Two axioms will be imposed. The first is directly reproduced from Nehring (2004).

**Interim Pareto Dominance (IPD)**  $f \succeq_I g$  (resp.  $f \succ_i g$ ) whenever it is commonly known that  $f \succeq_i^\alpha g$  (resp.  $f \succ_i^\alpha g$ ) for all  $i \in I$ .

The second axiom is the direct analogue of Nehring’s second axiom, where the ex-post utilitarian criterion is replaced by a generic social welfare ordering  $R$ .

**State Independence Given  $R$  (SI- $R$ )**  $f \succeq_I g$  whenever it is commonly known that  $f(\omega)Rg(\omega)$ .

Following Nehring’s terminology, say that a function  $\phi : \Omega \rightarrow \mathbb{R}$  is *acceptable* if there exists a collection  $(\phi_i)_{i \in I}$  from  $\Omega$  to  $\mathbb{R}$  such that  $\phi = \sum_{i \in I} \phi_i$  and such that it is common knowledge that  $E_i^\alpha \phi_i > 0$ , for all  $i \in I$ . Thinking of  $\phi$  as determining an aggregate level of transferable utility in every state, being acceptable then means that there is a way to share this total amount of utility in each state so that it is common knowledge that the resulting contingent allocation of utilities is strictly interim individually rational.

Nehring’s impossibility result follows from a classical characterization of the non-existence of a common prior (see Nehring (2004, Theorem A.1.(i)) who traces the result back to Morris (1994)): a common prior exists if and only if  $\phi = 0$  is not acceptable. It is then straightforward to adapt Nehring’s (2004, Theorem 2) argument to show that his impossibility result in the absence of a common prior extends to any social welfare ordering that satisfies PD.

**Proposition 1** *Let  $R$  be a social welfare ordering that satisfies PD. If there is no common prior, then there is no interim social welfare ordering that satisfies both IPD and SI- $R$ .*

Proof:  $\phi = 0$  is acceptable, since there is no common prior, and hence there exists  $(\phi_i)_{i \in I}$

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<sup>1</sup>The Nash criterion is defined only over  $\mathbb{R}_{++}^I$ , and all the results presented in this comment apply also to that more restrictive domain.

such that (a)  $\sum_{i \in I} \phi_i(\omega) = 0$ , for each  $\omega \in \Omega$ , and (b) it is common knowledge that  $E_i^\alpha \phi_i > 0$ , for all  $i \in I$ . Let now  $f$  and  $g$  be two acts such that  $u_i(f(\omega)) = \phi_i(\omega)$  and  $u_i(g(\omega)) = 0$ , for all  $i \in I$  and  $\omega \in \Omega$  (existence guaranteed under the assumption of Rich Domain). Since  $R$  satisfies PD, (a) implies that  $g(\omega) R f(\omega)$ , for each  $\omega$ , and hence  $g \succeq_I f$ , by SI- $R$ . On the other hand, (b) implies that it is common knowledge that  $f \succ_i^\omega g$ , and hence  $f \succ_I g$ , by IPD, which contradicts the previous comparison. ■

Nehring proves that the converse is true when  $R$  is the utilitarian criterion: if there is a common prior, then there exists an interim social welfare ordering that satisfies both IPD and SI- $R^U$  (in addition, it is unique, and it coincides with the ex-ante utilitarian criterion). I will show that this possibility result does not extend to other classical social welfare orderings. I will say that a function  $\phi : \Omega \rightarrow \mathbb{R}$  is *weakly acceptable* if there exists a collection  $(\phi_i)_{i \in I}$  from  $\Omega$  to  $\mathbb{R}$  such that  $\phi = \sum_{i \in I} \phi_i$ , and such that it is common knowledge that  $E_i^\alpha \phi_i \geq 0$ , for all  $i \in I$ . If there is no common prior, then  $\phi = 0$  is acceptable, and hence a fortiori weakly acceptable. If there is a common prior  $\mu$ , then  $\phi = 0$  is not acceptable, but it might be weakly acceptable in a non trivial sense meaning that there exists  $\omega$  in the support of  $\mu$  and  $i \in I$  such that  $\phi_i(\omega) \neq 0$ . The next lemma offers a characterization of those common priors.

**Lemma 1** *Suppose that there exists a common prior  $\mu$ . Then  $\phi = 0$  is weakly acceptable in a non trivial sense if and only if there exist a sequence  $(\omega_k)_{k=1}^K$  and a sequence  $(i_k)_{k=1}^K$  of individuals such that  $\omega_{k+1} \neq \omega_k$ ,  $i_{k+1} \neq i_k$ , and  $\omega_{k-1} \in T_{i_k}(\omega_k)$ , for each  $k \in \{1, \dots, K\}$ , with the convention  $0 = K$ .*

Proof:  $\Rightarrow$ ) Let  $(\phi_i)_{i \in I}$  be a non trivial decomposition of  $\phi = 0$ . Notice that  $E_i^{\omega'} \phi_i = E_i^\omega \phi_i$  if  $\omega' \in T_i(\omega)$ . So, for any  $T_i \in \{T_i(\omega) | \omega \in \Omega\}$ ,  $E_i^{T_i} \phi_i$  will denote  $E_i^\omega \phi_i$ , for some (or all)  $\omega \in T_i$ . Notice then that

$$\sum_{i \in I} \sum_{T_i \in \{T_i(\omega) | \omega \in \Omega\}} \mu(T_i) E_i^{T_i} \phi_i = \sum_{\omega \in \Omega} \mu(\omega) \sum_{i \in I} \phi_i(\omega) = \sum_{\omega \in \Omega} \mu(\omega) \phi(\omega) = 0.$$

Hence it must be that  $E_i^\omega \phi_i = 0$ , for all  $i \in I$  and all  $\omega \in \Omega$ . Since the decomposition of  $\phi$  is non trivial, one can find an  $i$  and an  $\omega$  for which  $\phi_i(\omega) \neq 0$ . Call him  $i_2$ , call it  $\omega_1$ , and let's say to fix our ideas that  $\phi_{i_2}(\omega_1) < 0$  (a similar reasoning applies if the inequality is reversed). Since  $E_{i_2}^{\omega_1} \phi_{i_2} = 0$ , there must exist  $\omega \in T_{i_2}(\omega_1)$  such that  $\phi_{i_2}(\omega) > 0$ . Call it  $\omega_2$ . Since  $\omega_2 \in T_{i_2}(\omega_1)$ , we also have that  $\omega_1 \in T_{i_2}(\omega_2)$ . Since  $\sum_{i \in I} \phi_i(\omega_2) = 0$ , there must exist another individual, call him  $i_3$ , for whom  $\phi_{i_3}(\omega_2) < 0$ . Since  $E_{i_3}^{\omega_2} \phi_{i_3} = 0$ , there must exist  $\omega_3 \in T_{i_3}(\omega_2)$  such that  $\phi_{i_3}(\omega_3) > 0$ . Since  $\omega_3 \in T_{i_3}(\omega_2)$ , we also have

that  $\omega_2 \in T_{i_3}(\omega_3)$ . Iterating the argument, one of the new states, let's say  $\bar{\omega}$ , will have already appeared previously, since  $\Omega$  is finite. The subsequence starting at  $\bar{\omega}$ , and ending at the state right before its reappearance, combined with the associated individuals (take  $i_1$  as the individual that led to the reappearance of  $\bar{\omega}$  - it must be that this individual  $i_1$  is different from  $i_2$ , since  $\phi_{i_2}(\bar{\omega}) < 0 < \phi_{i_1}(\bar{\omega})$ ), satisfies the necessary condition, as desired.

$\Leftrightarrow$ ) Let  $(\omega_k)_{k=1}^K$  be a sequence of states as in the statement, with the additional property that there is no shorter sequence of states with that property. It implies that

$$(\forall k \in \{1, \dots, K\}) : T_{i_k}(\omega_k) \cap \{\omega_l | 1 \leq l \leq K\} = \{\omega_{k-1}, \omega_k\}. \quad (1)$$

I prove this by contradiction. Suppose thus, on the contrary, that there exist  $k \in \{1, \dots, K\}$  and  $l \in \{1, \dots, K\} \setminus \{k-1, k\}$  such that  $\omega_l \in T_{i_k}(\omega_k)$ . Hence  $\omega_{k-1} \in T_{i_k}(\omega_l)$ . Suppose first that  $l < k-1$ . If  $i_{l+1} = i_k$ , then one reaches a contradiction since the subsequence that starts with  $l+1$  and ends with  $k-1$  is shorter than the original sequence and satisfies all the properties of the statement ( $\omega_l \in T_{i_k}(\omega_{l+1})$  and  $\omega_{k-1} \in T_{i_k}(\omega_l)$  imply that  $\omega_{k-1} \in T_{i_k}(\omega_{l+1})$ ). If  $i_{l+1} \neq i_k$ , then again one reaches a contradiction, since the subsequence that starts with  $l$  and ends with  $k-1$ , changing only  $i_l$  into  $i_k$ , is shorter than the original sequence while satisfying all the properties of the statement. Suppose now that  $l > k$ . If  $i_l = i_k$ , then the subsequence that starts with  $k$  and ends with  $l-1$  is shorter than the original sequence while satisfying the properties of the statement ( $\omega_{l-1} \in T_{i_k}(\omega_l)$  and  $\omega_l \in T_{i_k}(\omega_k)$  imply that  $\omega_k \in T_{i_k}(\omega_{l-1})$ ). This is not possible. If  $i_l \neq i_k$ , then the subsequence that starts with  $k$  and ends with  $l$  is shorter than the original sequence (notice that it cannot be that  $k=1$  and  $l=K$ , since  $l \neq k-1$ ), while satisfying all the properties of the statement. Again, this is impossible, and we can conclude that (1) is indeed correct.

Given any  $\alpha > 0$ , construct the collection  $(\phi_i)_{i \in I}$  by the following recursive formula:

$$\begin{aligned} \phi_{i_1}(\omega_1) &= \alpha, \phi_{i_2}(\omega_1) = -\alpha, \text{ and } (\forall i \in I \setminus \{i_1, i_2\}) : \phi_i(\omega_1) = 0 \\ (\forall 2 \leq k \leq K) : &\begin{cases} \phi_{i_k}(\omega_k) = -\phi_{i_k}(\omega_{k-1}) \frac{\mu(\omega_{k-1})}{\mu(\omega_k)}, \\ \phi_{i_{k+1}}(\omega_k) = \phi_{i_k}(\omega_{k-1}) \frac{\mu(\omega_{k-1})}{\mu(\omega_k)}, \text{ and} \\ (\forall i \in I \setminus \{i_k, i_{k+1}\}) : \phi_i(\omega_k) = 0 \end{cases} \\ (\forall \omega \in \Omega \setminus \{\omega_k | 1 \leq k \leq K\}) &(\forall i \in I) : \phi_i(\omega) = 0. \end{aligned}$$

Notice that, by construction,  $\sum_{i \in I} \phi_i(\omega) = 0$ , for all  $\omega \in \Omega$ , and  $E_i^\omega \phi_i = 0$ , for all

$(\omega, i) \in \Omega \times I$  for which there does not exist  $1 \leq k \leq K$  such that  $i = i_k$  and  $\omega \in T_{i_k}(\omega)$ . Consider now a pair  $(\omega, i)$  and a  $k$  such that  $i = i_k$  and  $\omega \in T_{i_k}(\omega)$ . The property proved in the previous paragraph implies that  $\mu(T_i(\omega))E_i^\omega \phi_i = \mu(\omega_{k-1})\phi_{i_k}(\omega_{k-1}) + \mu(\omega_k)\phi_{i_k}(\omega_k)$ . If  $k \neq 1$ , then it is straightforward to check that  $E_i^\omega \phi_i = 0$ , by definition of  $\phi_{i_k}(\omega_k)$ . If  $k = 1$ , then

$$\mu(T_i(\omega))E_i^\omega \phi_i = -\mu(\omega_K)\frac{\mu(\omega_1)\mu(\omega_2)}{\mu(\omega_2)\mu(\omega_3)} \cdots \frac{\mu(\omega_{K-1})}{\mu(\omega_K)}\alpha + \mu(\omega_1)\alpha = 0.$$

Hence  $E_i^\omega \phi_i = 0$ , for all  $i \in I$  and all  $\omega \in \Omega$ , and it is thus also common knowledge that  $E_i^\omega \phi_i = 0$ , for all  $i \in I$ , as desired. ■

Notice that the condition characterizing priors for which  $\phi = 0$  is weakly acceptable in a non trivial sense, is very weak. For instance, it is satisfied if there exist two individuals  $i$  and  $j$ , and a state  $\omega$  such that  $T_i(\omega) \cap T_j(\omega)$  contains at least two states. Indeed, suppose that the intersection contains  $\omega'$  in addition to  $\omega$ . The condition in the Lemma is satisfied with  $\omega_1 = \omega$ ,  $i_1 = i$ ,  $\omega_2 = \omega'$ , and  $i_2 = j$ . In particular, of course, it is satisfied when, while facing uncertainty, all the individuals have the same information ( $T_i(\omega) = \Omega$ , for all  $i \in I$  and all  $\omega \in \Omega$ ). The condition in the Lemma is also satisfied when the information structure is derived from types with a joint probability distribution that has full support: a set of types  $\mathcal{T}_i$  with at least two elements is associated to each individual  $i$ ,  $\Omega = \times_{i \in I} \mathcal{T}_i$ , and  $\mu$  has full support over  $\Omega$ . Indeed, the condition of the Lemma is satisfied with  $i_1 = 2$ ,  $i_2 = 1$ ,  $i_3 = 2$ ,  $i_4 = 1$ ,  $\omega_1 = (t_1, t_2, t_{-12})$ ,  $\omega_2 = (t_1, t'_2, t_{-12})$ ,  $\omega_3 = (t'_1, t'_2, t_{-12})$ , and  $\omega_4 = (t'_1, t_2, t_{-12})$ . Here is yet another example where the condition of the Lemma is satisfied, while not falling in the two previous cases. Suppose that  $\Omega = \{\omega_1, \omega_2, \omega_3\}$ ,  $I = \{1, 2, 3\}$ , the first individual's information partition is  $\{\{\omega_1, \omega_2\}, \{\omega_3\}\}$ , the second individual's information partition is  $\{\{\omega_1, \omega_3\}, \{\omega_2\}\}$ , and the third individual's information partition is  $\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$ . The condition of the Lemma is satisfied for  $(\omega_k)_{k=1}^3$  by choosing  $i_1 = 2$ ,  $i_2 = 1$ , and  $i_3 = 3$ . So finally here are two examples where the condition does not apply, and hence where a weakly acceptable decomposition of  $\phi = 0$  is necessarily trivial. As a first example, consider the case where all the agents but one are fully informed. As a second example, consider the case where  $I = \{1, 2\}$ ,  $\Omega = \{\omega_1, \omega_2, \omega_3\}$ , the first individual's information partition is  $\{\{\omega_1, \omega_2\}, \{\omega_3\}\}$ , and the second individual's information partition is  $\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$ .

Lemma 1 allows to show that IPD and SI- $R$  are essentially incompatible when  $R$  satisfies strong PD. I will slightly strengthen SI- $R$  by requiring that the resulting interim

social comparison appearing in the axiom is strict if there is some state  $\omega$  in the support of  $\mu$  for which  $f(\omega)Pg(\omega)$ . There are ways to show the incompatibility of IPD and SI- $R$  with some additional technical assumptions on  $R$ , but the modification of SI- $R$  seems so innocuous and natural that I will not pursue that direction.

**Strong State Independence Given  $R$  (SSI- $R$ )**  $f \succeq_I g$  whenever it is commonly known that  $f(\omega)Rg(\omega)$ . If, in addition,  $f(\omega)Pg(\omega)$  for some  $\omega$  in the support of the common prior, then  $f \succ_I g$ .

**Proposition 2** *Suppose that there is a common prior  $\mu$  that satisfies the condition in Lemma 1, and let  $R$  be a social welfare ordering that satisfies strict PD. Then there is no interim social welfare ordering that satisfies both IPD and SSI- $R$ .*

Proof:  $\phi = 0$  is weakly acceptable in a non-trivial sense, by Lemma 1, and hence there exists  $(\phi_i)_{i \in I}$  such that (a)  $\sum_{i \in I} \phi_i(\omega) = 0$ , for each  $\omega \in \Omega$ , (b) it is common knowledge that  $E_i^\alpha \phi_i \geq 0$ , for all  $i \in I$ , and (c)  $\phi_i(\omega) \neq 0$ , for some  $i \in I$  and some  $\omega$  in the support of  $\mu$ . Let now  $f$  and  $g$  be two acts such that  $u_i(f(\omega)) = \phi_i(\omega)$  and  $u_i(g(\omega)) = 0$ , for all  $i \in I$  and  $\omega \in \Omega$  (existence guaranteed under the assumption of Rich Domain). Since  $R$  satisfies strict PD, (a) and (c) imply that  $g(\omega)Rf(\omega)$ , for each  $\omega$ , and  $g(\omega)Pf(\omega)$ , for some  $\omega$  in the support of  $\mu$ . Hence  $g \succ_I f$ , by SSI- $R$ . On the other hand, (b) implies that it is common knowledge that  $f \sim_i^\omega g$ , and hence  $f \sim_I g$ , by IPD, which contradicts the previous strict comparison. ■

$R^N$  is defined only over  $\mathbb{R}_{++}$ , and hence one may wonder whether the reasoning for Lemma 1 and Proposition 2 also apply on that restricted domain. It does. Although the comparison in the proof of Proposition 2 involve  $\phi = 0$ , one could also have started instead with any strictly positive number to be split equally among the individuals in every state. Then the non trivial decomposition of  $\phi = 0$  can be added to this equal-split. One can make sure that this new allocation of utilities remains in  $\mathbb{R}_{++}$ , as the magnitude of the transfers in  $(\phi_i)_{i \in I}$  can be made as small as needed, given that we are free to choose  $\alpha$  in the proof of Lemma 1.

The proof of Proposition 2 proceeds by showing the direct incompatibility of the interim social comparisons imposed by IPD and SSI- $R$  when the common prior satisfies the condition of Lemma 1. Unfortunately, one cannot be sure of the existence of a transitive interim social welfare ordering that satisfies IPD and SSI- $R$  (or SI- $R$ ) even in the rare cases where that condition is not satisfied and these interim comparisons are not directly incompatible. Consider for instance the case where  $I = \{1, 2\}$ ,  $\Omega = \{\omega_1, \omega_2\}$ ,



the common prior is uniform, the first individual's information partition is  $\{\{\omega_1\}, \{\omega_2\}\}$ , and the second individual's information partition is  $\{\{\omega_1, \omega_2\}\}$ . It is easy to check that interim Pareto comparisons are not in contradiction with the comparisons derived from  $SI-R^E$ . In fact, it is not even possible to find a cycle from these comparisons with only three acts, but well with four. Consider the four acts  $f^1$ ,  $f^2$ ,  $f^3$ , and  $f^4$  that generate the following ex-post utilities:

|            | $f^1$  | $f^2$    | $f^3$    | $f^4$    |
|------------|--------|----------|----------|----------|
| $\omega_1$ | (0, 5) | (4, 100) | (3, 50)  | (50, 3)  |
| $\omega_2$ | (0, 5) | (50, 4)  | (49, 50) | (50, 49) |

$SI-R^E$  implies that  $f^1 \succeq_I f^2$  and  $f^3 \succeq_I f^4$ , while IPD implies that  $f^2 \succ_I f^3$  and  $f^4 \succ_I f^1$ , hence the contradiction with transitivity.

Following the strong negative results presented in this note, and yet the importance of equity considerations in social choice, I have developed an alternative methodology to obtain a notion of interim egalitarianism in de Clippel (2010). This new approach differs in many respect from Nehring's work. First, attention is paid to interim social choice functions instead of interim social welfare orderings. Second, incentive constraints are taken into account by modeling the physical description underlying social choice problems. Third, the analysis does not involve any explicit comparison with the ex-post stage. Instead, it is conducted entirely at the interim stage, aiming at characterizing the solution that satisfies natural analogues of Kalai's (1977, Theorem 1) axioms. The resulting criteria may violate ex-post egalitarian comparisons, but are still viewed as an extension of the egalitarian principle to frameworks under asymmetric information since they do coincide with that principle in the special case where information is complete. As an analogy, when the individuals face symmetric uncertainty, applying the social welfare ordering  $R$  to the ex-ante utilities is often considered as a natural criterion, even if the resulting comparisons may conflict with uniform comparisons through  $R$  at the ex-post stage (see Myerson, 1981).

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