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**Working Paper**

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IZA Discussion Papers, No. 6781

**Provided in Cooperation with:**

IZA – Institute of Labor Economics

*Suggested Citation:* Henderson, Daniel J.; Kumbhakar, Subal C.; Parmeter, Christopher F. (2012) : A simple method to visualize results in nonlinear regression models, IZA Discussion Papers, No. 6781, Institute for the Study of Labor (IZA), Bonn

This Version is available at:

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August 2012

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## ABSTRACT

### **A Simple Method to Visualize Results in Nonlinear Regression Models<sup>\*</sup>**

A simple graphical approach to presenting results from nonlinear regression models is described. In the face of multiple covariates, 'partial mean' plots may be unattractive. The approach here is portable to a variety of settings and can be tailored to the specific application at hand. A simple four variable nonparametric regression example is provided to illustrate the technique.

JEL Classification: C1, C13, C14

Keywords: gradient estimation, dimensionality, kernel smoothing, nonlinear, mean plots, least squares cross validation

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<sup>\*</sup> All code in this paper is available upon request. We thank an anonymous referee for valuable comments that lead to an improved version of this paper.

## 1. INTRODUCTION

A variety of information is produced when one estimates nonlinear models. A key empirical issue is how one chooses to relate this information to readers. Calculation of either the estimated surface or the respective gradients (marginal effects) offers an array of choices when presenting results to readers, something that is important for every applied researcher. In a univariate context it is relatively straightforward to present a plot of the estimated relationship (both the function and its slope) against the data to highlight nonlinearities. However, in multivariate settings this is not typically possible, especially for the estimated slopes since these slopes are observation-specific and more than likely will depend on the values of other covariates. For example, the marginal change in the probability of success for a given covariate within a logit model is observation specific and depends on the value of every covariate.

The presence of interactions among covariates typically makes univariate plotting of estimated partial effects cumbersome since these covariate values need to be fixed prior to presentation of the estimates. Further, what values they should be fixed at is not clear. Given that in applied econometric settings the key object of interest is typically not the estimated function itself, but the gradient of the estimated function with respect to various covariates, this issue is important to practitioners.

The common practice in multivariate settings is to select a variable of interest, hold the remaining variables fixed at some value (usually the mean) and present either partial mean or partial slope functions. The key issue with this presentation is that the results are dependent upon where the remaining variables are held fixed. While holding variables fixed at the mean may appear innocuous, it is well known that as the dimensionality of the data increases, the distance of any data point from the overall mean is increasing. Thus, it is quite likely that even in settings with two or three covariates, many of the actual observations will not be close to the mean. This draws into question how ‘appropriate’ it is to discuss features of a partial mean or partial slope plot if it is not ‘near’ any of the observed data.

An alternative is to estimate models which allow for nonlinearities in a univariate context, such as with additive models or partially linear models. For example Kottaridi & Stengos (2010) construct an economic model of growth whereby three variables enter the model in an additive form. This allows the presentation of both partial mean and slope plots for each variable individually. However, their additive setup does not allow for any form of interactions between these three variables. Kottaridi & Stengos (2010) remedy this lack of interactions by estimating a partially linear model including these three variables nonparametrically. To present their results they provide partial mean plots where the other two variables are each held constant at their respective means.

Here we will present a novel approach to visualize slope estimates from any nonlinear model which are valid for any dimension, easily constructed and presented. This approach allows for two-dimensional figures which do not require that we fix covariates at specific values (say, the mean).

This new approach should prove to be an invaluable component to the applied econometrician's toolkit.

To properly frame our discussion, consider a typical nonlinear (in  $x$ ) regression model

$$y_i = g(x_i; \beta) + \varepsilon_i, \quad i = 1, \dots, n,$$

where  $y_i$  is the univariate response,  $x_i \in \mathbb{R}^d$  and  $\varepsilon_i$  is the error. For nonlinear models rarely does  $\beta$  have a direct interpretation.<sup>1</sup> More practically, interest lies in  $\partial y / \partial x$ . The issue with calculating this partial derivative is that given the nonlinear nature of  $g(\cdot; \beta)$ , unless the components of  $x$  are given specific values, the derivative cannot be calculated; that is for variables  $w$  and  $z$  (included in  $x$ ), it is usually the case that  $\partial^2 y / \partial w \partial z \neq 0$ . Thus, the key aspect is “which values does one give to the covariates when calculating a partial derivative estimate of a nonlinear function?” In applied work, it is common to allow the variable of interest to vary and to hold the remaining variables fixed at their mean, a counterfactual analysis.

The dimensionality of the data plays a large role regarding how appropriate plots and estimates are when calculated holding the remainder of the data fixed at a given point. It is natural to think that, regardless of the dimensionality, draws from a multivariate density should be clustered around the ‘center’ of the distribution; for the normal, say, we would expect many of our observations to be near the mean vector. However, this is misleading. For exposition only, assume that our data have mean  $\mu$  and are symmetrically distributed. Suppose we wish to draw a hypersphere around this ‘center’ of the data with radius  $r$ . The volume of such a hypersphere in  $d$  dimensions is  $V_s(d) = C_d r^d$ , where  $C_d = \pi^{d/2} / \Gamma(d/2 + 1)$ . Alternatively, if we were to draw a hypercube of length  $2r$  around  $\mu$ , it would have volume  $V_c(d) = (2r)^d$ . If we look at the ratio of these two volumes we have

$$\frac{V_s(d)}{V_c(d)} = \frac{\pi^{d/2} r^d}{\Gamma(d/2 + 1)(2r)^d} = \frac{\pi^{d/2}}{2^d \Gamma(d/2 + 1)} \rightarrow 0, \quad \text{as } d \uparrow \infty.$$

The area close to the center of the data has no volume relative to the rest of the space. If we think of points in the hypersphere as close and those that are not as far from the mean, then as the dimensionality increases, all points are far from the mean.

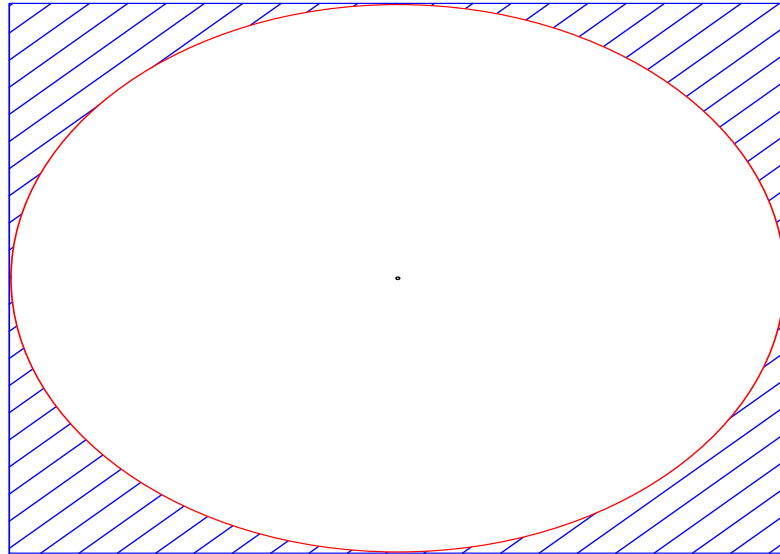
Figure 1 illustrates this phenomena in 2 dimensions. Here the center of the data is  $(0.5, 0.5)$  and we draw a circle of radius 0.5 around this point and a cube with sides of length 1 such that  $(0.5, 0.5)$  is at the center of this cube. The shaded area represents the points not near the center. The previous result tells us that as  $d$  increases, this shaded ‘volume’ increases relative to the ‘volume’ of the points ‘near’ the center. From a data perspective, it suggests that as one goes into higher dimensions, the draws from the distribution will reside in the tails instead of the middle.

What this figure illustrates more generally is that fixing covariates at a given level to evaluate a gradient may not be representative of the data. There is nothing inherently wrong with this process in general. Instead, it is worth suggesting that when an applied researcher fixes the data at a point,

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<sup>1</sup>Here our use of the term nonlinear includes models that are linear in parameters, for example a translog production or cost function.

FIGURE 1. Nonoverlapping ‘volume’ of a hypercube and a hypersphere.



they should think of how realistic the resulting estimates are if they are not representative of the data broadly.

## 2. VISUALIZING GRADIENT ESTIMATES – THE 45° PLOT

To visualize multivariate gradient estimates we introduce the 45° degree plot. This figure plots the estimated gradients against themselves, along with either bootstrap or asymptotic confidence bands. Next, the horizontal and vertical axes are plotted. These plots will easily allow the user to distinguish where a bulk of the effects lie, which effects are significant and which effects are insignificant. In sum, these plots can help to clarify the nature of the heterogeneity that exists in estimates stemming from nonlinear models. The elegance of this type of presentation for the results is that it no longer requires selection of where to fix the remaining variables when attention is placed on a specific variable.

Given this, we argue that the marginal effect of a covariate for each observation is to be calculated at the observed values of all the covariates for the given observation. This means that while calculating, for example the marginal effect of crime on housing prices in a locality, we evaluate the impact at the observed values of the covariates for the given locality. This is more intuitive and meaningful. It does not make much sense to compute the marginal effect of crime on housing prices in high crime areas, holding the values of the other covariates at the mean if areas with high crime also have other covariate values far from the mean. It is easy to acknowledge areas with higher

crime rates also have a lower proportion of high income individuals and may be near industrial areas with poor air quality.

To illustrate this concept we provide a simple example in the statistical environment R. We use the `Hedonic` dataset of Harrison & Rubinfeld (1978).<sup>2</sup> Our dependent variable is `log(mv)`, the logarithm of the median value of owner-occupied homes within a census tract in the Boston metropolitan area, while our explanatory variables are `lstat`, the proportion of people who are of low income status within the tract; `nox`, the annual average nitrogen oxide concentration in each census tract; `ptratio`, the pupil to teacher ratio within each tract; and `crim`, the crime rate within a tract. We use local-constant kernel regression with 2nd-order Gaussian kernels and bandwidths selected using least-squares cross-validation. Standard errors are calculated using a wild bootstrap with 399 replications.

One unattractive component of presenting results from nonparametric models is the lack of a predefined form that allow users/readers to gauge the shape of the estimated function. For example, it is common to present a density or histogram of the estimated effects. However, these density plots cannot adequately convey information regarding the statistical significance of a given point since the height of the density is composed of more than one data point for which the confidence intervals are not the same. Moreover, there is the issue of bandwidth (bin width) selection for the density plot that the user must also grapple with. To remedy this it is common to present partial slope plots which hinge on ‘fixing’ the other covariates in the model. Thus, the results can be entirely dependent upon the level the other variables are set at. Here the 45° plot allows graphical depiction of the results while relieving the user of selecting fixed levels for the additional covariates. In the example above, if each of the other three variables are set at their means to construct a partial slope plot, then at best 50% of the actual data lies within a standard deviation of the mean (for `ptratio`), while for `crim`, only 36.5% of the observations for the fixed observations are within a standard deviation of the mean.

The 45° plots for the example are provided in Figure 2. First, each  $\circ$  represents a single point estimate of the associated gradient, one for each observation. Second, each upper and lower  $\triangle$  represents the upper and lower bounds of a 95% confidence interval (calculated here via the bootstrap) around this specific point estimate. When the upper and lower confidence bounds for an estimate are both in the upper right quadrant then the estimated gradient for that observation is positive and significant. When the upper and lower confidence bounds for a given estimate are both in the lower left quadrant the gradient estimate for that observation is negative and significant. When the upper or lower confidence bounds straddle the horizontal axis, the gradient estimate is not statistically different from zero.

There are several noticeable features from these plots. First, all four of the covariates have estimated positive and negative effects. However, the majority of the estimated gradients for `lstat` are negative. Consistent with a quadratic type specification, the estimated gradients for both `crim`

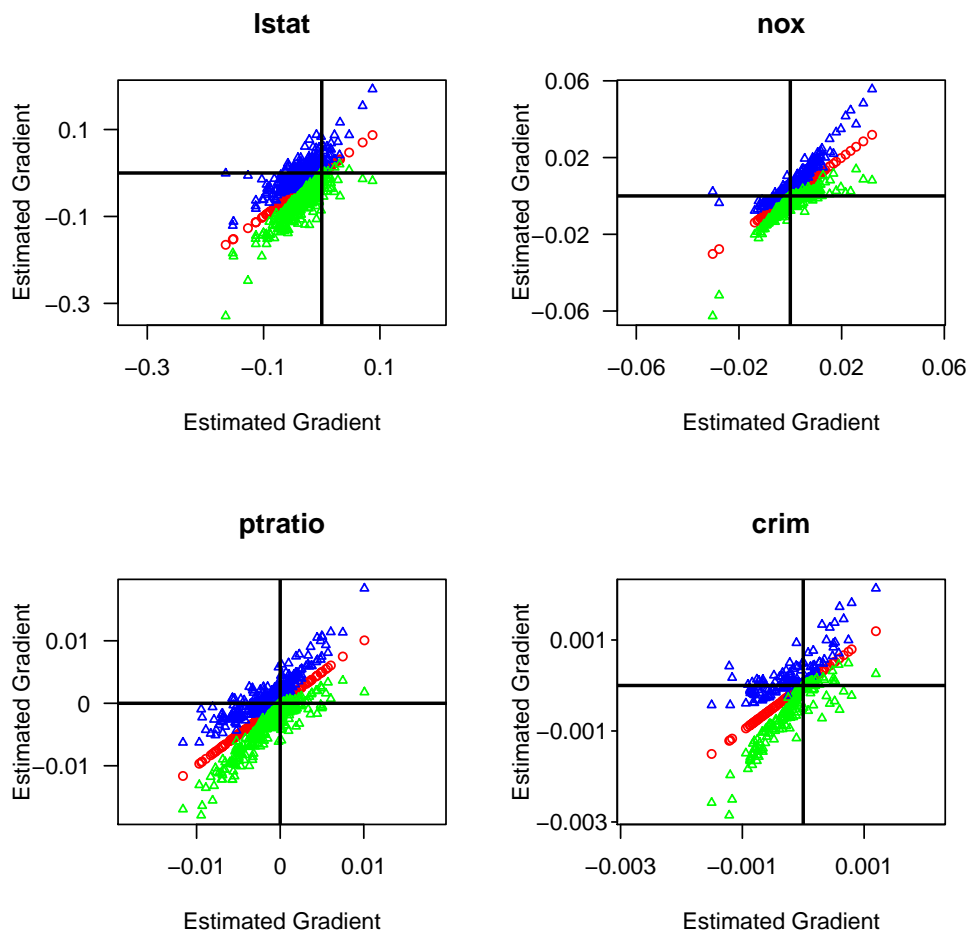
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<sup>2</sup>This dataset is available in the `Ecdat` package (Croissant 2011) in the R statistical environment.



and `nox` appear equally clustered around zero. These figures could further be enhanced by also plotting a straight line (either vertically or horizontally) where the mean effect lies.

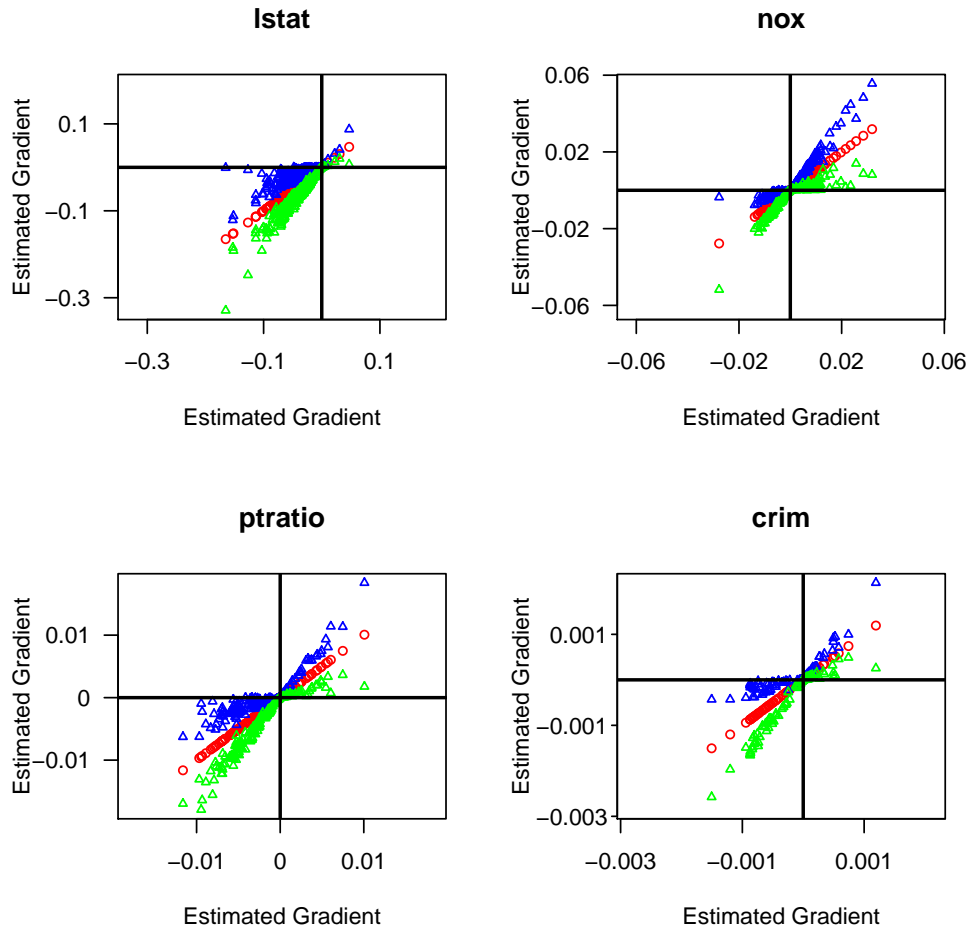
FIGURE 2. 45° plot of the estimated gradients for `lstat`, `nox`, `ptratio` and `crim`.



It is also possible to plot out only the significant effects. Doing this results in Figure 3. A slightly clearer picture of how these four variables influence property values. Now a majority of the significant gradients estimates for `crim` are negative and the effects for `nox` are equally distributed around 0. The variables also provide estimated effects that are economically significant given that the dependent variable is measured logarithmically while the independent variables all enter the model in levels. Thus, the correct interpretation for  $\partial y/\partial x$  is to multiply these estimated gradients by 100.

As a basis for comparison we provide partial mean plots in Figure 4. This figure presents the estimated gradient for `crim` holding the remaining three variables fixed at either their lower quartile (Q1), their median (Q2), their upper quartile (Q3) and their mean (Mean). One will immediately

FIGURE 3. 45° plot of the statistically significant estimated gradients for `lstat`, `nox`, `ptratio` and `crim`.

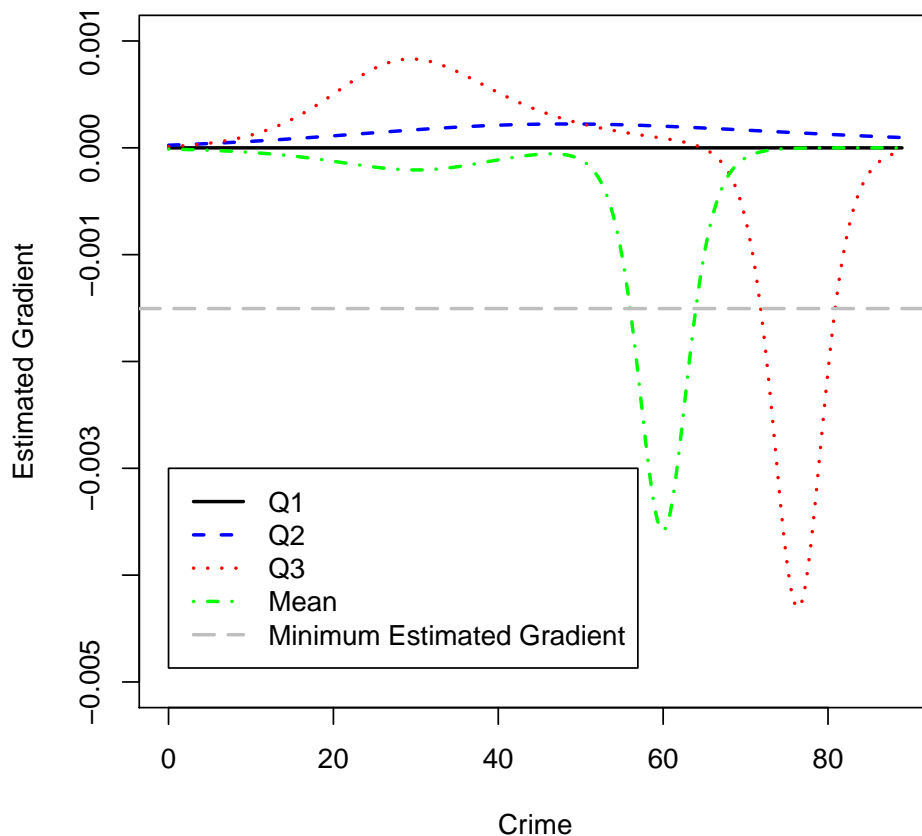


notice that there is no discernible pattern of the estimated curves as we move across the four points of reference. Moreover, the long dashed horizontal line, which represents the smallest estimated gradient from the initial set of point estimates is almost three times smaller in magnitude than a substantial range of the estimates for Q3 and Mean. Again, what Figure 4 highlights is the issue with presenting partial mean plots, the ambiguity over the point to hold the remaining covariates fixed at. We see here that one would derive different insights based on which point was selected. Moreover, if we limited ourselves to only the quartile and mean values, one could construct  $2^4 = 16$  different combinations of points.

### 3. CONCLUSION

While no figure can fully characterize the features of a nonlinear model, the 45° plots introduced here should become a useful component of the applied researcher's toolkit when evaluating and

FIGURE 4. Standard ‘fixed’ partial plots of the estimated gradient for `crim`. The other three variables are held fixed at their quartile or mean values. The long-dashed line is the minimum estimated gradient estimate from the 506 point estimates.



presenting results. Recall that the method is not designed to relay the explicit form of the nonlinearity of the estimated slope, but to provide general features of the extant nonlinearity in a graph that does not hinge on fixing the additional covariates at pre-specified levels. Alternative uses of the 45° plot could be to construct returns to scale from a translog production function, and to plot out estimated returns to scale (along with their standard errors) with horizontal and vertical lines at 1 (as opposed to the axes).

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