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## Algorithm for Identifying Systemically Important Banks in Payment Systems

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*Financial Network Analytics*

**Abstract** The ability to accurately estimate the extent to which the failure of a bank disrupts the financial system is very valuable for regulators of the financial system. One important part of the financial system is the interbank payment system. This paper develops a robust measure, SinkRank, that accurately predicts the magnitude of disruption caused by the failure of a bank in a payment system and identifies banks most affected by the failure. SinkRank is based on absorbing Markov chains, which are well-suited to model liquidity dynamics in payment systems. Because actual bank failures are rare and the data is not generally publicly available, the authors test the metric by simulating payment networks and inducing failures in them. The authors use two metrics to evaluate the magnitude of the disruption: the duration of delays in the system (Congestion) aggregated over all banks and the average reduction in available funds of the other banks due to the failing bank (Liquidity dislocation). The authors test SinkRank on Barabasi–Albert types of scale-free networks modeled on the Fedwire system and find that the failing bank’s SinkRank is highly correlated with the resulting disruption in the system overall; moreover, the SinkRank technology can identify which individual banks would be most disrupted by a given failure.

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**Keywords** Systemic risk; interbank payment system; liquidity; Markov chains; simulation

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# 1 Introduction

The ability to accurately estimate the extent to which the failure of a bank disrupts the financial system is very valuable for regulators of the financial system. This paper develops a robust measure based on absorbing Markov chains, SinkRank, that accurately predicts the magnitude of disruption caused by the failure of a bank in an interbank payment system and identifies banks most affected by the failure.

Interbank payment systems provide the backbone for all financial transactions. Virtually all economic activity is facilitated by transfers of claims by financial institutions. In turn, these claim transfers generate payments between banks whenever they are not settled across the books of a single bank. These payments are settled in interbank payment systems. In 2010, the annual value of interbank payments made e.g. in the Pan-European system TARGET2 was \$839 trillion. In the corresponding US system Fedwire, the amount was \$608 trillion - over 40 times its annual GDP ([BIS, 2010]). Due to the sheer size of the transfers, and their pivotal role in the functioning of financial markets and the implementation of monetary policy, payment systems are central for policymakers and regulators.

Systemic risk in payment systems has been studied since [Humphrey, 1986] who found significant risk in the U.S. Fedwire system in the mid 1980s. Subsequent studies by [Angelini et al., 1996], [Bech and Soramäki, 2002] and [Galos and Soramäki, 2005] found the risks to be limited. Since then, however, most payment systems have switched from net settlement to real-time gross settlement ([Bech et al., 2008]) – transforming credit risk into liquidity risk as gross settlement eliminates the former but at the cost of the latter. Various works have since used simulations to study risks and liquidity needs in real-time gross settlement systems, either by creating entirely simulated systems or introducing changes in data from real payment systems. A growing body of work ([Schulz, 2011]; [Grat-Osinka and Pawliszyn, 2007]; [Arjani, 2006]) uses simulation to study the relationship between liquidity requirements and delays in payment systems. Simulations of failures in payment systems generally focus on system-wide risks and liquidity effects ([Glaser and Haene, 2009]; [McAndrews and Wasilyew, 2005]; [Ledrut, 2007]; [Ball and Engert, 2007]; [Docherty and Wang, 2010]). [Schmitz and Puhr, 2009] studied network structure in payment systems with induced shocks, but

found that network properties were of limited use for stability analysis. Here we use network methods to develop a metric that not only identifies systematically important banks but can also predict the banks most affected by a failure, and validate the metric using simulated payment systems.

The paper is organized as follows. In the next section we discuss existing measures of centrality in the network theory and introduce SinkRank. Section 3 describes the model that is used to simulate bank failures for testing SinkRank and Section 4 presents simulation results that evaluate the accuracy of SinkRank for forecasting the impact of failures and the banks most affected. Section 5 concludes.

Technical details and computer code for reproducing all calculations presented in this paper are given in the Annexes. Interactive versions of the charts are available in [www.fna.fi/sinkrank](http://www.fna.fi/sinkrank).

## 2 Centrality in Network Theory

In the past decade, significant progress towards understanding the structure and functioning of complex networks has been made within the fields of statistical mechanics and social network analysis (SNA).

A multitude of centrality measures has been developed – each with an explicit or implicit network process in mind. [Borgatti, 2005] identifies several stylized processes. According to his typology, a process can progress in the network through geodesic paths, paths, trails or walks. Processes that travel via geodesic (shortest) paths are, for example, problems of the type “traveling salesman”, i.e. they always take the shortest route between two nodes. Processes that travel via paths need not necessarily use the shortest one, but do not visit any node more than once. These can be, for example, viral infections (a person becomes immune once infected) or the routing of internet traffic.

Processes that travel along trails do not visit any given link more than once. Such a process is for example the spread of gossip where a person may forward it to several other people or hear the same news from several different people - but a normal person will not learn about new information twice. Processes that are characterized as walks are not restricted in their behavior. An

example of such are the money flows studied here, where everyone can pay everyone multiple times.

Further on, Borgatti characterizes the process in the dimensions of parallel duplication, serial duplication and transfers. In parallel duplication the process spreads at the same time from a node to all its neighbors. In serial duplication it duplicates one link at a time. An example of the former is an e-mail broadcast and of the latter viral infection. Instead, in transfer the process moves something in the network. When it is moved, it leaves the originating node and is now possessed by the receiving node. This is the case with payments.

The most commonly used centrality measures are Degree, Closeness and Betweenness proposed by [Freeman, 1978] and different variations of Eigenvector centrality which was pioneered by [Katz, 1953] and [Bonacich, 1972], [Bonacich, 1987].

*Degree centrality* (or simply Degree) counts the number of neighbors of each node. It is a local measure that only takes the immediate neighborhood of the node into account. It can count neighbors with incoming links, outgoing links or either, and can be weighted by link properties; for example, the weighted out-degree is referred to as out-strength.

The insight underlying *Closeness centrality* is that nodes which have shorter geodesic paths to other nodes are more central. It is generally calculated as the average length of geodesic paths from a node to each other node in the network. *Betweenness centrality* defines nodes through which a high share of geodesic paths pass as central.

What is known today as *Eigenvector centrality* encapsulates the idea that the centrality of a node depends directly on the centrality of the nodes that link to it (or that it links to). Eigenvector centrality measures assume parallel duplication along walks. A famous commercialization of Eigenvector centrality is Google's PageRank algorithm ([Page et al., 1999]), which adds a random jump probability for 'dangling' nodes and thus allows the measure to be calculated for all types of networks. PageRank and Eigenvector centrality can be thought of as the proportion of time spent visiting each node in an infinite random walk through the network. For calculating Eigenvector centrality, the network must be strongly connected (i.e. the underlying transition matrix must be nonsingular).

In payment networks banks (nodes) transfer payments related to customer requests or their own trading along directed links of the network. When a payment is made the money is no longer available to the sender, and the receiver of the funds can make a payment to any other bank in the system. Using terminology in [Borgatti, 2005], the *transfer* process takes place along *walks* in the network as any bank can pay other multiple times without constraints (assuming the paying bank has sufficient funds or credit).

Payment networks are accompanied with liquidity and risk-management constraints and exhibit feedback loops. Banks may not have enough liquidity to settle a payment or may decide to postpone a payment due to liquidity and risk management concerns. These decisions again depend on the state of the system at that time, and also influence the state of the system. Traditional measures of centrality that have been developed with other types of processes in mind (e.g. processes transmitted along geodesic paths or trails or processes based on duplications instead of transfer) and may not be able to accurately identify central nodes in payment systems.

Liquidity constraints may make banks unable to make payments and may alter the unconstrained process significantly. When the constraints are hard, the system may become very unpredictable and be governed by a congestion and cascades process. When liquidity is scarce, the settlement process loses correlation with the process of payments that would need to be settled. These dynamics are described in [Beyeler et al., 2007].

The new centrality measure proposed here, SinkRank, is based on absorbing Markov chains which are well-suited to model transfers along walks. A Markov system is a system that can be in one of several states, and can pass from one state to another at each time step according to fixed probabilities. If a Markov system is in state  $i$ , there is a fixed probability,  $p_{ij}$ , of its going into state  $j$  at the next time step;  $p_{ij}$  is called a transition probability.

An absorbing random walk is a random walk that starts from a node and terminates at an absorbing node. In terms of centrality our interest is the expected number of steps that are taken before termination when the walk starts from a random other node. Absorbing nodes that require a smaller expected number of steps are considered more central than absorbing nodes that require a large number of steps.

Any network can be represented as a matrix and such a matrix can be turned

into a transition matrix. The transition matrix for  $M = [s_{ij}]_{n \times n}$  is defined by dividing each element by the row sum,  $P = \left[ \frac{s_{ij}}{\sum_j s_{ij}} \right]_{n \times n}$ , where the transition probabilities for a random walk are defined by the link weights  $s_{ij}$ .

An absorbing state is a state from which there is a zero probability of exiting. An absorbing Markov system is a Markov system that contains at least one absorbing state, and is such that it is possible to get from each non-absorbing state to each other non-absorbing state and to some absorbing state in one or more time-steps (i.e. the network is strongly connected except for the absorbing states). An absorbing Markov system reflects the process taking place when a bank fails in a payment system: Any payments sent to the failing bank remain in the failing bank's account and don't exit until recovery.

In analyzing an absorbing system, we first number the states so that the absorbing states come last in the matrix. The transition matrix  $P$  of an absorbing system is:

$$P = \begin{bmatrix} S & T \\ 0 & I \end{bmatrix}$$

where  $I$  is an  $m \times m$  identity matrix ( $m =$  the number of absorbing states),  $S$  is a square  $(n - m) \times (n - m)$  matrix ( $n =$  total number of states, so  $n - m =$  the number of non-absorbing states),  $0$  is a zero matrix and  $T$  is an  $(n - m) \times m$  matrix. We consider only single failures, i.e.  $m = 1$ , but as detailed above, the measure can easily be extended to analyze multiple simultaneous failures as well.

The matrix  $S$  gives the transition probabilities for movement among the non-absorbing states. To obtain information about the time to absorption in an absorbing Markov system, we first calculate the fundamental matrix  $Q$ .

$$Q = (I - S)^{-1}$$

The  $i, j$ th entry of  $Q$  defines the number of times, starting in state  $i$ , a process is expected to visit state  $j$  before absorption. The total number of steps expected before absorption equals the total number of visits a process expects to make to all the non-absorbing states. This is the sum of all the entries in the  $i$ th row of  $Q$ . We call this the '*Distance to Sink*' of the node.

In calculating SinkRank, we calculate the Distance to Sink of each non-absorbing node and take an average. The measure is analogous to distances

along paths except that the process is based on number of steps in walks defined by the transition matrix and ending at the absorbing node. Thus, our measure of SinkRank is defined by:

$$\text{SinkRank} = \frac{\sum_i \sum_j q_{ij}}{n - m}.$$

SinkRank is an intuitively meaningful metric in a payment system as it can measure how close a failing bank is to the other banks in the system via payment flows. We expect failures to be more disruptive when they occur in banks that are more central, i.e. have lower SinkRank. The SinkRank of a node denotes the average number of payments that need to be made for a unit of liquidity anywhere in the network to reach the node. The minimum possible SinkRank is 1, in the case of the center node of a star network. Payments made by the spokes of the star reach the center in one step. There is no theoretical upper bound for the SinkRank of a node.

### 3 Simulation model of payment system

Because bank failures are rare and the data is not generally publicly available, we test the SinkRank metric by simulating payment networks and inducing failures in them. The simulation model incorporates both liquidity constraints and a queuing mechanism for payments that cannot be settled due to the liquidity constraint. For the simulations we use the FNA payment simulator<sup>1</sup> which has previously been used inter alia in [Berge and Christophersen, 2012] and [McLafferty and Denbee, 2012]. Properties of the BA network used in the simulations are summarized in Table 1<sup>2</sup>.

The payment data used in the simulation is randomly generated as detailed in the Annexes. The generating process is able to produce payment flows with close resemblance to the Fedwire payment network in the United States.

Visualizations of the BA network are shown in Figures 1 and 2; each node (circle) represents a bank and the arcs between them represent payments.

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<sup>1</sup>See [www.fna.fi/interbank](http://www.fna.fi/interbank)

<sup>2</sup>Fedwire network as described in [Soramäki et al., 2007]. Model network is one realization.



Property	Value in simulated BA network
Number of nodes	100
Number of links	1220
Connectivity	0.123
Reciprocity	0.317
Degree ( $k$ )	24.4
Max ( $k$ -in)	57
Max( $k$ -out)	56
Number of payments	5000
Value paid ('1000)	604

Table 1: Properties of BA network topology

Figure 1 shows the entire network. Node sizes are scaled by Out-strength (that is, larger nodes represent banks that make more total payments), and arc width is scaled by the number of payments (that is, thicker arcs represent more payments made). The network is characterized by a few large well-connected banks with high centrality and many more small banks, as is typical in scale-free networks.

Figure 2 shows the maximally connected subgraph of the BA network; that is, the largest subgraph that contains a link between each pair of nodes. The maximally connected subgraph has 17 nodes, which represent the core of the network.

In the simulation model, banks start the day with a given amount of opening balances. Payments are tested for settlement as they are fetched from the file of generated payments. If the value of the payment is larger than the available balance of the sending bank, the payment is put in the sending bank's queue of pending payments. If the value is smaller than the available balance, the payment is settled and the account of the sending bank is debited and the account of the receiving bank is credited.

The bank whose account was credited may now be able to settle some of its previously queued payments. Queued payments are released on a 'First-in-First-Out' (FIFO) basis. If a payment from the queue can be settled, the recipient of the newly released payment may now be able to release its first

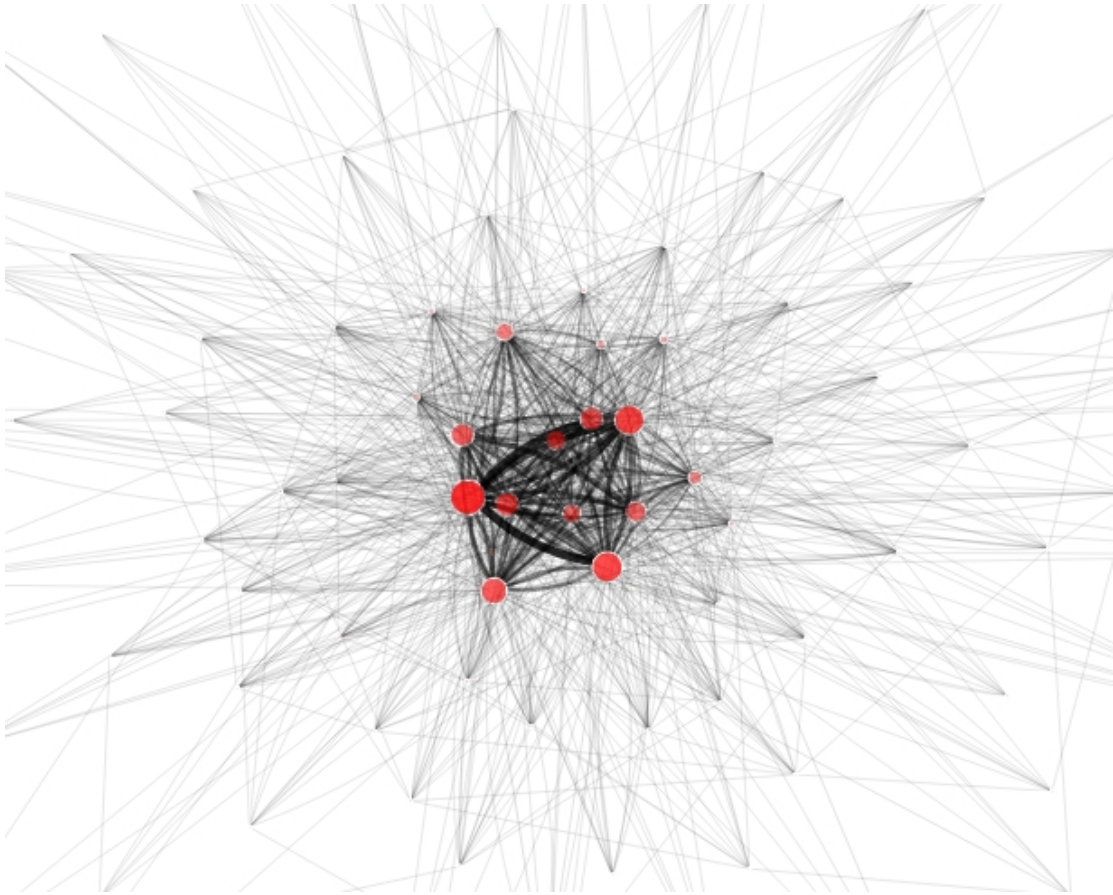


Figure 1: BA network visualization, entire network

queued payment - i.e. a single payment can cause the release of many queued payments in a cascade. At the aggregate level this creates a process where the system gets randomly congested, manifesting as an increase in queued payments, and occasionally queued payments are settled in cascades when payments that can be settled due to incoming funds from previously settled payments are released to others. The behavior of such a system is described in detail in [Beyeler et al., 2007].

In the failure simulations we set each bank in turn to be unable to send any payments during the day; that is, we set each bank in turn to be an absorbing state. The failing bank continues, however, to receive payments and will therefore trap some of the total liquidity on its account. As a consequence

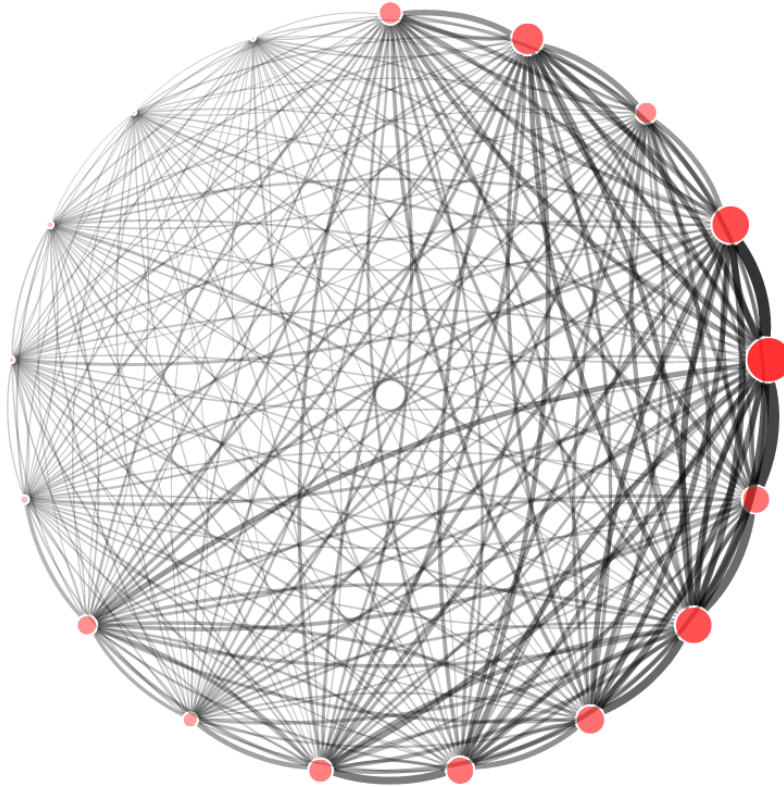


Figure 2: BA network visualization, maximally connected subgraph

other banks will run short of liquidity and queues will build, first causing existing liquidity buffers to be used more and eventually causing payments to be delayed. The FNA code to replicate these simulations can be found in Annex II.

We calculate duration of delays in the system aggregated over all banks ('Congestion') and the average reduction in available funds of the other banks due to the failing bank, ('Liquidity Dislocation'); their duration-weighted sum is used as a measure of the extent of the disruption caused by the failing bank ('Disruption').

The magnitude of the disruption is dependent on the level of liquidity in the system. If other banks have enough liquidity to offset the funds that did not arrive from the failing bank, no delays will occur. In the trivial

case of unlimited liquidity, no Congestion would ever occur and each bank's Liquidity Dislocation would be equal to the amount of payments not received from the failing bank.

In the simulations we set the initial balance of each bank at the minimum level that allows all banks to process all payments immediately when no bank failure is present. Thus, when a failure occurs, Congestion will be caused by lack of sufficient liquidity in at least some banks - which in turn will cause Liquidity Dislocation and/or Congestion at other banks.

Figure 3 summarizes the two disruption measures considered and the relationship between them. Each point represents a single failed bank and shows the Liquidity Dislocation and Congestion calculated for all other banks in the network. All bank failures cause at least some Liquidity Dislocation, whereas Congestion only occurs in about half (62 / 100) of the bank failures; if all banks affected by a failure have enough liquidity to make their payments, no delay will occur. The relationship between Liquidity Dislocation and Congestion is convex as theoretically shown in [Galbiati and Soramäki, 2011]. As more liquidity is dislocated, more delays occur that dislocate more liquidity.

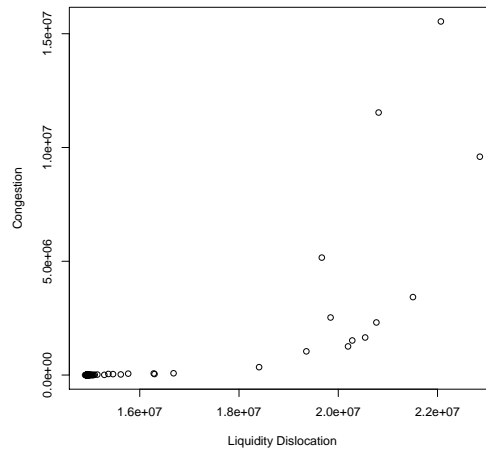


Figure 3: Network-level disruption measures

## 4 SinkRank and Failure Distance

We calculate for each bank in the network its SinkRank (as described in Section 2), Out-Strength (that is, the sum of all its outgoing payments a measure of the size of the bank), and PageRank. These centrality measures are related to the disruption experienced in the simulation. Figure 4 shows that SinkRank, Out-strength, and PageRank are all very strongly related ( $r > 0.99$ ) to disruption in BA networks. Note that banks with high centrality have low SinkRank and high Out-strength and PageRank, so the correlations have opposite signs.

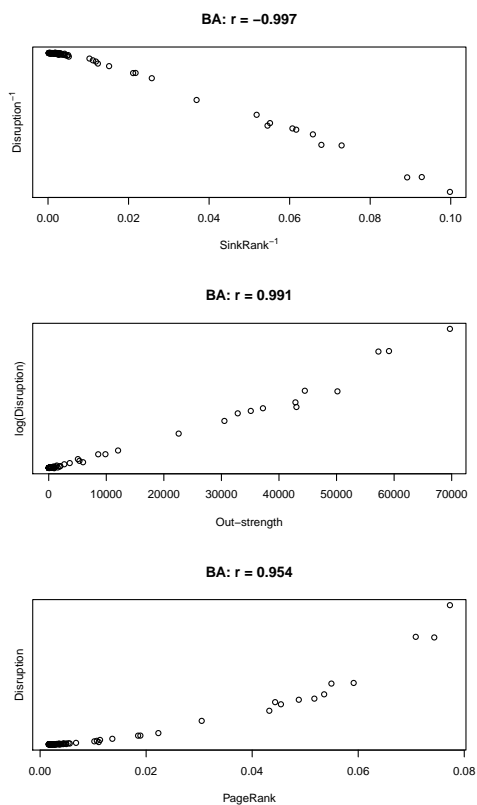


Figure 4: Relationship between disruption and centrality measures in BA networks

Figure 5 shows that the relationship between centrality and disruption in other types of payment network. We consider random and complete networks

of the same size (number of links) as the BA network, with link weights assigned randomly and payments generated as in Annex I, but with linear scaling of payment values to ensure that the total value is approximately the same across networks. SinkRank and PageRank are both strongly related to disruption in all three networks, whereas the relationship between Out-strength and disruption appears to hold only in the BA network which has strong correlations with strength and degree of nodes.

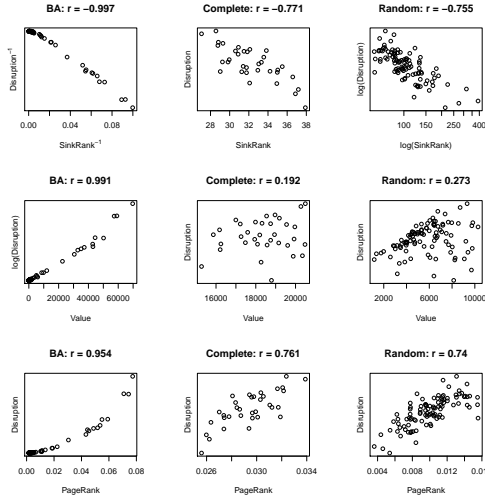


Figure 5: Relationship between disruption and centrality measures in BA, complete, and random networks

The results in Figures 4 and 5 are for aggregate network properties: A bank's centrality is strongly related to the *overall* disruption seen in the system if that bank fails. We can further utilize the SinkRank technology to identify which individual banks are most susceptible to disruption in the case of a bank failure. We define the *Failure Distance* as the Distance to Sink *from* a failing bank *to* any other bank. Banks with small Failure Distances are close to the failing bank and downstream from it in the payment chain, and so should be most disrupted by the failure. Figure 6 shows the Failure Distances and disruptions when the bank with the lowest SinkRank (that is, the most central bank) fails. Banks with smaller Failure Distances indeed exhibit larger disruptions, and the relationship is quite strong ( $r < -0.85$ ).

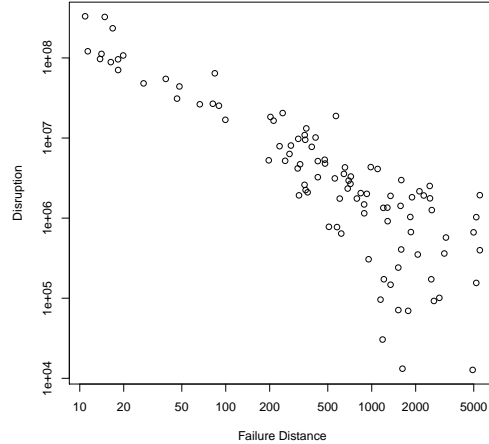


Figure 6: Relationship between Failure Distance and Disruption when the most central bank fails

## 5 Conclusions

This paper developed a new metric (SinkRank) based on absorbing Markov chains and evaluated its accuracy by comparing it with results from simulated failure scenarios in payment systems modeled after the Fedwire system. SinkRank was shown to be predictive of network-level disruption in the case of a bank failure, and the related metric Failure Distance was shown to be predictive of disruption in individual banks.

Several possibilities exist for extending the work. First, a more robust analysis with regression models to investigate the explanatory power of different metrics or combined metrics could be carried out. A longer time series of different realizations of the networks and failure simulations would also make the results more robust.

More simulations on alternative network topologies with longer path lengths and different correlations among network topology and link values could provide better information on the relative merits of the different metrics across network topologies. These networks could be artificial (lattice, random, etc.) or constructed from real payment data.

Finally, this initial analysis showed that it is possible to accurately rank banks on the basis of metrics calculated from network topology to estimate the potential disruption their failure would cause in the payment system. The new metric introduced, SinkRank, has conceptually the right underpinnings and did well in identifying banks with capabilities for various magnitudes of disruption. It is possible to further improve it also by taking into account the liquidity distribution at the time of failure.

## **Annex I: Algorithm for generating payment data**

Payment networks exhibit complex properties. We take as a starting point the Fedwire network which consists of almost 8000 banks processing over 411.000 payments on an average day. The network is described in detail in [Soramäki et al., 2007]. Due to the highly confidential nature of the data, it is rarely available for research outside central banks and therefore artificial data needs to be used.

There are three main aspects in describing the payment network: the structure of the links, link weight distributions, and individual payment distributions; in other words, who pays whom, how often, and how much. Both link weights and payment values have also correlations with each other. Generating a mechanism that produces all desired aspects of the data is thus challenging.

The main structural characteristic of the network is a power law degree distribution. This means that a few very large banks connect to a large number of very small banks. The in and out -degrees correlate strongly, i.e. banks that receive payments from many different banks also send payments to many different banks, and vice versa. The largest degree in the Fedwire network on an average day is 1922 for incoming links and 2097 for outgoing links. The network also has a very low connectivity. Only 0.3% of all possible links are present on an average day. In addition, the links have a very high reciprocity of 0.22. This means that 22% of relationships between two nodes are bidirectional if a link exists from A to B, then a link also exists from B to A. Reciprocity in a random network is on average equal to its connectivity,



i.e. over 70 times smaller in this case.

The link weights (number of payments) also follow a power law distribution and have a very high positive correlation with the degree of the node. This means that large banks have both more links and that these links transmit more payments than links of smaller banks. The number of payments in reciprocal links also has a high correlation, denoting strong bi-directional business relationships between banks.

The payment values have a lognormal distribution, and again their value depends on the size of the banks measured as the number of counterparties or the total value sent (i.e. out- or in- strength).

We develop a simple payment generation process extending the BA model by Barabási and Albert (1999) for generating random scale-free networks. The BA model is based on two processes, growth and preferential attachment. Growth in the model means that initially the network only has a few nodes and nodes are gradually added to the system. Preferential attachment means that the more connected a node is, the more likely it is to receive new links. Newly added nodes are therefore more likely to connect to nodes with many existing links.

The model developed here aims at reproducing the main statistical properties described for the Fedwire network above. The model applies growth and preferential attachment as the main drivers of the generation process, but instead of adding links, it adds payments. A link is formed when the first payment is drawn from a bank to another. Additional payments between banks with existing payments add to the weight of the link.

We start with an initial number of nodes  $n_0$ . We then draw new payments one by one,  $m$  payments for each new node, until we have the desired number of nodes,  $n$ . We use a vector  $H[h_i]_{i=1,\dots,n}$  to track the amount of preferential attachment strength that has been allocated to each bank (node)  $i$  and a matrix  $M = [s_{ij}]_{n \times n}$  to track the number of payments created to and from each bank. The matrix  $M$  can also be interpreted as a weighted adjacency matrix of the payment network, where the weight is the number of payments.

The other main difference from the BA model is the addition of a parameter  $\alpha$  that denotes the “strength of preferential attachment”, i.e. how much is added to  $h$  when a payment is sent or received. In the BA model,  $\alpha = 1$ ; because the number of payments here is vastly higher than the number of

links to draw, the addition to the preferential attachment must be smaller so as not to skew the degree distribution too much. The pseudo-code for the algorithm is given below.

```

FOR  $i = 1, \dots, n_0$  (add initial banks/nodes)
  SET  $h_i = 1$ 
FOR  $k = n_0 + 1, \dots, n$  (banks)
  FOR  $l = 1, \dots, m$  (average number of payments per bank)
    SELECT random sender  $i^*$  such that bank  $i$  has the probability  $\frac{h_i}{\sum h_i}$  of being selected as a sender
    SET  $h_i^* = h_i^* + \alpha$  (update preferential attachment strength)
    WHILE  $i^* \neq j^*$  (exclude loops, payments to oneself)
      SELECT random receiver  $j^*$  such that bank  $j$  has the probability of  $\frac{h_j}{\sum h_j}$  of being selected as recipient of the payment
      SET  $h_j^* = h_j^* + \alpha$  (update preferential attachment strength)
      SET  $s_{ij} = s_{ij} + 1$  (create payment/link)
  SET  $h_{m_0+k} = 1$  (create new bank/node)

```

Table 2 summarizes the comparison.<sup>3</sup> The model seems to be able to reproduce the main characteristics of the Fedwire topology very well with  $n_0 = 10$  and  $\alpha = 0.1$ . The parameter  $n_0$  determines the number of core banks, and  $\alpha$  the slope of the power law co-efficient in the degree distribution. Both the real and the generated network are sparse, with power law degree distributions and high clustering and reciprocity. In and out degrees are highly correlated and the degrees of the largest bank are very similar.

The next step after creating the interaction topology and the number of payments each bank sends to each other is to add time of submission and value to each payment. The time for each payment is drawn from a uniform

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<sup>3</sup>Fedwire network as described in [Soramäki et al., 2007]. Model network is one realization.

	Fedwire	Model
nodes	5066	5066
links	75397	70710
connectivity	0.003	0.003
reciprocity	0.215	0.213
degree ( $k$ )	14.9	14.0
max ( $k$ -in)	2097	2210
max ( $k$ -out)	1922	2215
payments ('1000)	411	411

Table 2: Properties of BA network topology

distribution between 08:00 and 18:00 (opening hours of the simulated payment system) and the value is drawn from a normal distribution with a mean of 1 and standard deviation of 0.2. As the individual payment values were log-normally distributed in the real data, we then exponentiate the drawn values. In addition, larger banks interchange larger payments with each other than do smaller banks. We achieve this by scaling the payment values by  $Min(k_{sender}, k_{receiver})$ .

In the robustness analysis we also consider random networks and complete networks whose link weights (number of payments) are assigned randomly and whose payments are generated as detailed above, but with linear scaling of payment values to ensure that the total value is approximately the same across the different network types. FNA commands for generating the networks are given below.

## Annex II: FNA Commands for Reproducing Results

### Generating Networks

```
# Generate Barabasi-Albert (BA) network with link weights showing
# number of payments from each node to the other
ba -nv 100 -m 50 -v0 10 -alpha 0.1 -preserve false -seed 123
```

```

# Generate Random network and assign each link with a weight
# drawn from a uniform distribution between 1 and 7
random -nv 100 -na 1200 -preserve false -seed 123
calcap -e [?random:uniform:1,7:123?] -saveas number

# Generate Complete network and assign each link with a weight
# drawn from a uniform distribution between 1 and 7
complete -v 34 -directed -preserve false
calcap -e [?random:uniform:1,7:123?] -saveas number

#
# Creating payment files (one for each of the above networks)
#
# Create one day (8h - 17h = 28800 - 61200 in seconds) of payments
# Log payments have mean 1 and sd 0.2
createpayments -number number -open 28800 -close 61200
  -mean 1 -stdev 0.2 -saveas network.csv -seed 123

```

## Calculating Network Metrics

```

# Calculate SinkRank of each node, weighted by
# number of payments
sinkrank -ap number

# Calculate weighted out-degree (Out-strength)
degree -p value -direction out -saveas value

# Calculate number of nodes and links in each network
order -saveas numnodes
size -saveas numlinks

# Calculate connectivity and reciprocity
connectivity
reciprocity

# Calculate average reciprocity of each node
avgvform -p reciprocity -saveas reciprocity

# Calculate degree and average degree in network

```

```

degree -direction both -saveas degree
avgvfor -p degree -saveas degree

# Calculate in-degree and average in-degree in network
degree -direction in -saveas indegree
maxvfor -p indegree -saveas maxindegree

# Calculate out-degree and average out-degree in network
degree -direction out -saveas outdegree
maxvfor -p outdegree -saveas maxoutdegree

# Calculate number of payments in each network
sumafor -p number -saveas numpayments

# Calculate value of payments in each network
sumafor -p value -saveas value

```

## Simulating Payments

```

# Simulate Payment system without failure
rtgs -paymentsfile network_BA.csv[skiplines=1]
    -fundsfile funds_BA.csv[skiplines=1]
    -openingtime 080000 -closingtime 170000
    -outrecords out_records -outbanks out_banks_BA_success
    -dateformat yyyyMMdd

# Simulate Payments system where Bank ID 00000 Fails
# (Repeated for each bank in the network)
rtgs -paymentsfile network_BA.csv[skiplines=1]
    -overdraftsfile overdrafts_BA.csv[skiplines=1]
    -openingtime 080000 -closingtime 170000
    -outrecords out_records -outbanks out_banks_BA00000
    -dateformat yyyyMMdd -strickenbank 00000 -capacity 1e-04

```

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