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# Taylor rule cross-checking and selective monetary policy adjustment*

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**Abstract**

We examine whether the robustifying nature of Taylor rule cross-checking under model uncertainty carries over to the case of parameter uncertainty. Adjusting monetary policy based on this kind of cross-checking can improve the outcome for the monetary authority in selected specifications. This, however, depends on the functional form and also on the degree of the misspecification. Increasing the relative weight attached to Taylor rule cross-checking results in a trade-off between a reduction of loss and higher standard deviations of the relative losses. In cases where the monetary authority is not able to commit, it may be beneficial only to selectively adjust its monetary policy.

*JEL-Classification:* E47, E52, E58.

*Keywords:* Optimal monetary policy, parameter uncertainty, Taylor rule.

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# 1 Introduction

The Taylor rule is a widely used concept in monetary macroeconomics. Even though the idea is simple, it has been used in various areas. Taylor (1993) employs a *positive* analysis in the sense that he points out that the rule explains U.S. monetary policy extraordinarily well. Subsequently, this finding has been frequently captured and confirmed. Gerlach and Schnabel (2000) apply the concept to pre-European Monetary Union data and estimate a policy rule for Euro area countries. They show that monetary policy can also be described well by a Taylor rule and obtain similar coefficient estimates as the ones initially assumed by Taylor (1993). Other studies suggest that using real time data and projections for estimating the policy rule parameters might even improve the explanatory power of the Taylor rule (Orphanides and Wieland, 2008).

On the other hand, Taylor rules can be used in order to ex-post evaluate monetary policy and therefore to employ a *normative* analysis. The quality of monetary policy can be assessed by comparing actual developments in the short term interest rate with the interest rate implied by a Taylor rule, in other words policy was too loose when the monetary policy instrument was below the Taylor rule-implied interest rate, whereas it was too tight if it was above the implied rate. Poole (2007) defines monetary policy following the Taylor rule as being "systematic", hence he is able to find periods where U.S. monetary policy is not systematic according to his definition.

Furthermore, the *Taylor principle*, in other words the requirement for the coefficient on inflation in a Taylor rule to exceed unity, plays a key role

when it comes to determinacy in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models. In stylized DSGE models, this condition is equivalent to meeting the Blanchard and Kahn (1980) regularity conditions. This, in some way, very technical condition is interesting in view of the fact that it also provides an economic interpretation. The monetary authority should sufficiently increase its policy instrument in order to be able to influence the evolution of the real interest rate. On the contrary, a reaction less than one-to-one to inflation is not enough to offset the change in inflation which in turn might trigger an increase in inflation expectations, thereby leading to higher future inflation. Empirical studies follow this idea and evaluate the quality of monetary policy by examining the responsiveness of the interest rate to developments in the inflation rate. Judd and Rudebusch (1998) look at monetary policy of different Federal Reserve chairmen in terms of estimated policy reaction functions. The Burns chairmanship, for example, is identified as being less responsive to inflation which is put forward as a reason for high realized inflation during the same time period. Hence, the Taylor rule might also have some value in evaluating the quality of monetary policy, or, put differently, it provides information about what a "good" action for a monetary authority might be.

The fact that interest rates based on the Taylor rule are indeed an indicator for adequate monetary policy is reflected in selected statements from either policy makers or academics. Governor Janet Yellen indicated the Taylor rule as a means of providing her "a rough sense of whether or not the funds rate is at a reasonable level". "I do not disagree with the Greenbook strategy. But the Taylor rule and other rules ... call for a rate in the 5 per-

cent range, which is where we already are. Therefore, I am not imagining another 150 basis points." (FOMC transcripts, January 31 to February 1, 1995). Among others, Taylor and Williams (2010) argue that Taylor rules "are designed to take account of only the most basic principle of monetary policy [and] ... because they are not fine-tuned to specific assumptions, they are more robust to mistaken assumptions." We therefore try to investigate the usefulness of the Taylor rule for monetary policy as a "guideline" in the sense that it provides valuable information for the monetary authority about the adequateness of its monetary policy.

Røisland and Sveen (2011) show that the Taylor rule can robustify monetary policy in case of model uncertainty, in other words in the case of a complete mismatch between the model that the monetary authority uses in order to determine its monetary policy and the true model and therefore the true data generating process of the economy. They find that in such a framework, even putting a small weight on the information resulting from Taylor rule cross-checking in the process of the determination of optimal monetary policy is able to insure against bad outcomes. In an empirical exercise, they argue that actual monetary policy may be described by optimal monetary policy which incorporates cross-checking of this kind. Other approaches on cross-checking are discussed, for instance, in Beck and Wieland (2008) and Christiano and Rostagno (2001). Their approaches can be seen as alternatives to the robust policy proposed by Hansen and Sargent (2008) which is discussed for DSGE models in Giordani and Söderlind (2004) where the monetary authority also has a reference model at hand and considers the possibility of a bad shock hitting the model economy. This approach is fre-

quently referred to as an "evil agent" choosing the worst possible shock for the economy.

The results of Røisland and Sveen (2011), however, crucially depend on the assumption of a complete mismatch between the model that the monetary authority uses for the conduct of its monetary policy and the true data generating process of the economy. The monetary authority is completely ignorant in terms of realizing that there is a mismatch between the assumed transmission mechanism and the actual one. As this ignorance is time-invariant, any issues related to learning are ruled out. Hence, even as time $t$ approaches infinity and the monetary authority obtains all available information, it will never be able to learn about the true data generating process and it will therefore never update its model used for policy determination accordingly.

We take an intermediate point of view where some degree of learning is assumed to have already taken place in the past such that the monetary authority is only faced with uncertainty on the side of the parameters of the true data generating process due to, for instance, insufficient estimation techniques, rather than the entire transmission mechanism itself.[1] Hence, we consider certainty with respect to the structure of the economy but uncertainty of the monetary authority about model parameters which clearly influences the effectiveness of its monetary policy. The question then arises whether the robustifying nature of Taylor rule cross-checking under complete model uncertainty in the spirit of Røisland and Sveen (2011) carries over to the case of parameter uncertainty. In particular, we are interested in the issue

[1]For other approaches on parameter uncertainty, see for instance Giannoni (2007), Söderström (2002), or Tillmann (2011).

how much attention the monetary authority should pay to the information resulting from this kind of cross-checking.

We find that whether or not Taylor rule cross-checking is beneficial for the monetary authority in terms of a reduction of loss incurred from inflation and output deviating from the respective steady-state values crucially hinges on the functional form and also the degree of the parameter misspecification. In particular, much attention of the monetary authority should be paid to choosing the appropriate relative weight $\lambda_\Delta$ which it attaches to the information resulting from Taylor rule cross-checking. Increasing this weight involves a trade-off for the monetary authority in selected specifications. On the one hand, an increase in the weight is associated with an increase in the average gain for the monetary authority in terms of a reduction of loss. On the other hand, increasing the weight increases the standard deviation of the resulting relative losses. Hence, the choice of $\lambda_\Delta$ will also heavily depend on the risk aversion and also on the information set of the monetary authority, in other words its knowledge about the functional form of the parameter misspecification.

In cases where the monetary authority is not able to credibly commit to an announced policy, it may in principle be beneficial only to selectively adjust its policy using the information resulting from Taylor rule cross-checking. The monetary authority should do so when it has reasons to believe that the true data generating process differs from the assumed one. In this framework, the detection of potential regime switches is of particular importance.

# 2 The theoretical framework

## 2.1 The conduct of monetary policy

In most cases, a Taylor-type rule is specified in order to close a DSGE model where the nominal interest rate is a function of inflation and some measure of economic activity. However, in our case, this step is obsolete because our aim is to replace an otherwise exogenously specified policy rule by a policy rule that is obtained from the optimization problem of the monetary authority. We assume that the monetary policy objective can be summarized by a simple quadratic loss function. That is to say that the monetary authority wants to minimize the weighted sum of the variances of certain target variables. This approach is standard and for example presented in Clarida et al. (1999). However, we note that this loss function is not derived from welfare-theoretical considerations. On the contrary, it is an ad-hoc objective function trying to describe preferences of the monetary authority. One could also think of this loss function as a way to model flexible inflation targeting as introduced by Svensson (1999) where the monetary authority seeks to stabilize inflation, output, and potentially other target variables simultaneously. We define the "traditional" per period loss function as

$$L_t \equiv \lambda_\pi \hat{\pi}_t^2 + \lambda_y \hat{y}_t^2, \tag{1}$$

where the parameters $\lambda_\pi$ and $\lambda_y$ capture the importance of stabilizing inflation $\hat{\pi}_t$ and output $\hat{y}_t$, respectively. We set $\lambda_\pi = 1$ so that $\lambda_y$ captures the relative importance of stabilizing output to stabilizing inflation. Variables

with circumflex denote log-deviations from the steady-state.

Motivated by the statements about the usefulness of the Taylor rule quoted in the introduction, one might think that equation (1) does not capture the actual objective of the monetary authority. More precisely, the "traditional" per period loss function does not incorporate deviations of the interest rate implied by optimal monetary policy from the interest rate implied by a Taylor-type rule. Hence, in the standard approach, such interest rate deviations are considered irrelevant for the conduct of monetary policy. However, previous research suggests that the monetary authority might be better off following a Taylor-type rule. In the case of uncertainty, the monetary authority may want to insure itself against model misspecification, meaning that it seeks to robustify its policy. Røisland and Sveen (2011) show that the Taylor rule can indeed robustify monetary policy in case of model uncertainty, in other words in the case of a complete mismatch between the model that the monetary authority uses in order to determine its monetary policy and the true data generating process of the economy.

We assume that there are two types of models. The first model is referred to as the *reference model* of the monetary authority which reflects its belief of how the economy is structured and what the model parameters are. In principal, the reference model may or may not entirely reflect the true data generating process of the economy. This gives rise to the second type of model, which we call the *true model*. This model describes the true structure of the economy and may differ from the reference model. In fact, we assume that both the reference model and the true model are structurally identical and therefore reflect the same monetary policy transmission mechanism.

However, there is a misspecification on the side of the model parameters as the monetary authority is not able to perfectly estimate the model parameters. This approach is realistic in the sense that we do not believe the monetary authority (at least in the long run) to get it wrong in terms of the reference model which is the basic implication of Røisland and Sveen (2011). Hence, we let the monetary authority at least optimize using the correct structural model.

The monetary authority uses the reference model to determine its optimal policy in the sense that this policy is the result of an optimization problem and knows about its biased view of the world. It is therefore crucial to note that the resulting policy is "optimal" just in the reference model. In case of parameter misspecification the policy may very well turn out to be suboptimal and it is difficult to judge ex ante, what the quantitative consequences of a mismatch between the monetary authority's reference model and the true data generating process in terms of loss will be. Hence, it might be beneficial to find some way to insure against those misspecification as the exact functional form of the misspecification is assumed to be unknown.

In what follows, we do not argue that the monetary authority should completely and mechanically follow the Taylor rule in setting the interest rate. Still, it might be favorable to perform cross-checking in the spirit of Røisland and Sveen (2011) and let the information resulting from Taylor rule cross-checking influence the conduct of monetary policy. In other words, the monetary authority should be able to adjust its monetary policy according to the signals that it receives from performing this kind of cross-checking. Therefore, we redefine the monetary authority's objective by an augmented

loss function $\tilde{L}_t$. Consider that the monetary authority also reacts to deviations of the policy instrument from the Taylor rule-implied interest rate. We define this spread as

$$\Delta_t \equiv \hat{i}_t - \hat{i}_t^T, \tag{2}$$

where $\hat{i}_t^T$ denotes an interest rate implied by the Taylor rule. The specific form that we use is standard and reads

$$\hat{i}_t^T \equiv \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t, \tag{3}$$

where we use the original parameter values from Taylor (1993), in other words $\phi_\pi = 1.5$ and $\phi_y = 0.5$. Then, we augment the standard loss function (1) by a cross-checking term representing the squared interest rate spread and a corresponding weighting parameter $\lambda_\Delta$. Hence,

$$\tilde{L}_t \equiv \lambda_\pi \hat{\pi}_t^2 + \lambda_y \hat{y}_t^2 + \lambda_\Delta \Delta_t^2. \tag{4}$$

Equation (4) belongs to the class of "modified" loss functions, with the most well-known examples presented in Rogoff (1985) and Walsh (1995). In order to empirically motivate this approach, consider figure 1 where we plot the squared value of $\Delta_t$ against the loss resulting from the period loss function (1). We compute those series from actual quarterly U.S. data. The figure suggests that there is a relationship between the monetary authority's loss and deviations of the federal funds rate from the Taylor rule-implied

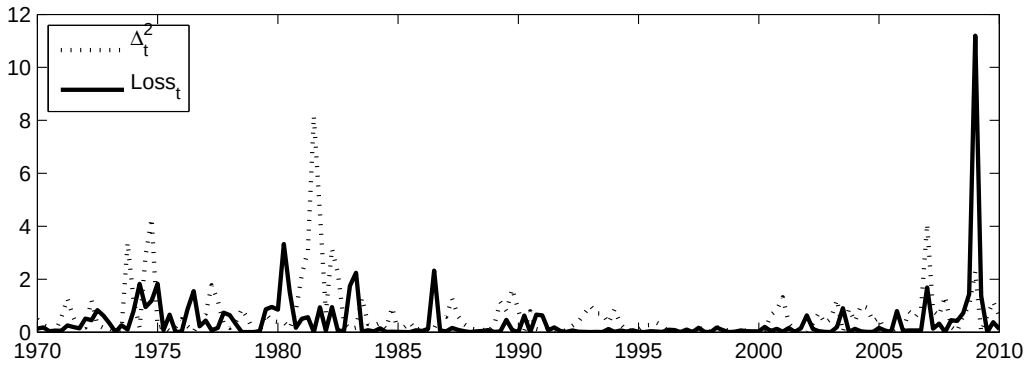


Figure 1: Losses and squared deviations between the federal funds rate and the Taylor rule-implied interest rate for the U.S. .

interest rate. Both series are positively correlated. Therefore, it seems to be the case that the economy experiences higher losses when deviations of the policy instrument from the Taylor rule-implied interest rate are large and vice versa.

Since the monetary authority faces a dynamic problem, it minimizes a discounted "lifetime" loss function

$$\mathscr{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} (1-\beta)\beta^t \tilde{L}_t, \tag{5}$$

where $0 < \beta < 1$ is the discount factor subject to the equations characterizing the reference model. The standard approach of flexible inflation targeting is nested by setting $\lambda_\Delta = 0$ in (4). The assumption of a lifetime loss function becomes mainly important when we consider optimal monetary policy under commitment. Since under commitment the monetary authority is able to credibly convince the public that it will stick to a particular policy, it is able to influence the agents' expectations. This enables the monetary authority, compared to a discretionary policy maker, to obtain lower future losses at

the cost of higher losses today. We employ a numerical approach to calculate the optimal monetary policy under commitment. In particular, we follow Svensson (2010), who also shows how to solve a linear quadratic regulator (LQR) problem with rational expectations.[2]

Let the linear dynamic model equations be

$$\begin{bmatrix} \hat{X}_{t+1} \\ H\hat{x}_{t+1|t} \end{bmatrix} = A \begin{bmatrix} \hat{X}_t \\ \hat{x}_t \end{bmatrix} + B\hat{i}_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}, \tag{6}$$

where $\hat{X}_t$ is an $(n_X \times 1)$-vector of predetermined variables, $\hat{x}_t$ is an $(n_x \times 1)$-vector of non-predetermined variables, $\varepsilon_{t+1}$ is an $(n_\varepsilon \times 1)$-vector of i.i.d. shocks with mean zero, and $\hat{x}_{t+1|t}$ is the expectation of $\hat{x}_{t+1}$ conditional on information available at time $t$.

Consider the case of commitment. Minimizing the loss function (5) subject to the linear dynamic model equations (6) with respect to $\hat{X}_t$, $\hat{x}_t$, and $\hat{i}_t$ yields $n_X + n_x + 1$ first-order conditions represented by

$$\bar{A}' \begin{bmatrix} \xi_{t+1|t} \\ \Xi_t \end{bmatrix} = W \begin{bmatrix} \hat{X}_t \\ \hat{x}_t \\ \hat{i}_t \end{bmatrix} + \frac{1}{\beta} \bar{H}' \begin{bmatrix} \xi_t \\ \Xi_{t-1} \end{bmatrix}, \tag{7}$$

where $\xi_t$ and $\Xi_{t-1}$ are $(n_X \times 1)$- and $(n_x \times 1)$-vectors of non-predetermined and predetermined Lagrange multipliers, respectively. $\bar{A}$ and $\bar{H}$ are matrices conformable with the vectors of stacked Lagrange multipliers and $W$ is the diagonal weighting matrix of the LQR problem. It will in general be the case

[2] Dennis (2007), Dennis (2004), or Söderlind (1999) suggest alternative solution methods to the LQR problem which are equivalent to the method in Svensson (2010).

that the optimal monetary policy under commitment resulting from setting up the Lagrangian function and taking the first-order conditions necessarily relates the interest rate $\hat{i}_t$ to the predetermined variables of the model. Hence, optimal monetary policy under commitment relates the interest rate to $\tilde{X}_t \equiv \begin{bmatrix} \hat{X}_t' & \Xi_{t-1}' \end{bmatrix}'$. The unique equilibrium explicit instrument rule is then given by

$$\hat{i}_t = F_i \tilde{X}_t, \tag{8}$$

where $F_i$ is an $(1 \times n_{\tilde{X}})$-vector and $n_{\tilde{X}} \equiv n_X + n_x$. Different instrument rules, however, have different determinacy properties (Svensson, 2010). As such, it may be necessary to generate an implicit instrument rule which relates the policy instrument also to the non-predetermined variables of the model in order to ensure determinacy. Define an arbitrary $(1 \times n_{\tilde{x}})$-vector $K$ where $n_{\tilde{x}} \equiv n_X + n_x$ and also for convenience assume that $K(1, n_x + 1) = 0$. Then, a non-unique equilibrium implicit instrument rule is given by

$$\hat{i}_t = F_i \tilde{X}_t = F_i \tilde{X}_t + K(\tilde{x}_t - F\tilde{X}_t) = \tilde{F}\tilde{X}_t + K\tilde{x}_t,$$

where $\tilde{x}_t \equiv \begin{bmatrix} \hat{x}_t' & \hat{i}_t' & \xi_t' \end{bmatrix}'$.

When one wants to implement a policy which is optimal in a certain model with one set of parameters into a different potentially misspecified model, the first-order condition of the Lagrangian with respect to the interest rate $\hat{i}_t$ (in other words the last row of (7)) is replaced by the policy resulting from the optimization problem of the monetary authority.

In the case of discretionary policy, the Lagrange multipliers will not play a role for optimal monetary policy. The interest rate will only be a linear function of the predetermined variables $\hat{X}_t$ and implementing a policy only involves replacing an otherwise exogenously specified policy rule. A non-unique equilibrium implicit instrument rule can be constructed in a similar way as before (Svensson, 2010).

## 2.2 The model economy

In order to determine the optimal monetary policy on the basis of the reference model and to simulate data using the true model, we use a standard DSGE model incorporating sticky wages and prices. The linearized model we employ is closely related to the one developed by Smets and Wouters (2003). Hence, we use a model that is on the one hand accepted in the profession, and on the other hand captures the most relevant frictions necessary to fit actual data. Our calibration can be found in tables 1 and 2 and mostly follows the results in Smets and Wouters (2003) for their estimated Euro area model.

In what follows, we will just give a brief and mostly non-technical overview of the model features.[3] We focus on the general structure of the model commenting on the frictions implemented. The linearized model equations are summarized in the Appendix.

The economy is inhabited by a continuum of households who maximize their expected lifetime utility. Those households decide upon their intertemporal allocation of consumption and are subject to external habit formation

[3]For readers interested in the details and the derivation of the model, we recommend to consult Smets and Wouters (2003).

| Parameter | Description | Value | |
|---|---|---|---|
| $\beta$ | discount factor | 0.999 | |
| $\tau$ | depreciation rate of capital | 0.025 | $*$ |
| $\alpha$ | capital output ratio | 0.3 | $*$ |
| $1/\psi$ | elasticity of cap. util. cost | 0.169 | $*$ |
| $\gamma_p$ | degree of partial indexation prices | 0.469 | $*$ |
| $\gamma_w$ | degree of partial indexation wages | 0.763 | $*$ |
| $\xi_p$ | Calvo price stickiness | 0.908 | $*$ |
| $\xi_w$ | Calvo wage stickiness | 0.737 | $*$ |
| $\lambda_w$ | markup in wage setting | 0.5 | $*$ |
| $\sigma_l$ | inverse elasticity of labor supply | 2.400 | $*$ |
| $\sigma_c$ | coeff. of relative risk aversion | 1.353 | $*$ |
| $h$ | habit portion of past consumption | 0.573 | $*$ |
| $\phi$ | 1+ share of fixed cost in prod. | 1.408 | $*$ |
| $1/\varphi$ | investment adj. cost | 6.771 | $*$ |
| $inv_y$ | investment share of GDP | $8.8\tau$ | $(*)$ |
| $c_y$ | consumption share of GDP | $1 - 0.18 - inv_y$ | $(*)$ |
| $\bar{r}_k$ | steady-state return on capital | $1/\beta - 1 + \tau$ | $(*)$ |

Table 1: Calibrated model parameters. Parameters marked with "$*$" are directly considered for misspecification whereas those marked with "$(*)$" will be affected indirectly.

| Parameter | Description | Value |
|---|---|---|
| $\rho_{\epsilon_l}$ | AR for labor supply shock | 0.889 |
| $\rho_{\epsilon_a}$ | AR for productivity shock | 0.823 |
| $\rho_{\epsilon_b}$ | AR for preference shock | 0.855 |
| $\rho_{\epsilon_g}$ | AR for government expenditure shock | 0.949 |
| $\rho_{\epsilon_{inv}}$ | AR for investment shock | 0.927 |
| $\rho_w$ | AR for wage markup shock (i.i.d.) | 0 |
| $\rho_q$ | AR for return on equity shock (i.i.d.) | 0 |
| $\rho_{\lambda_p}$ | AR for price markup shock (i.i.d.) | 0 |
| $\sigma_{\epsilon_l}$ | stand. dev. of labor supply shock | 3.520 |
| $\sigma_{\epsilon_a}$ | stand. dev. of productivity shock | 0.598 |
| $\sigma_{\epsilon_b}$ | stand. dev. of preference shock | 0.336 |
| $\sigma_g$ | stand. dev. of government expenditures shock | 0.325 |
| $\sigma_{\epsilon_{inv}}$ | stand. dev. of investment shock | 0.085 |
| $\sigma_{\lambda_p}$ | stand. dev. of price markup shock | 0.160 |
| $\sigma_{\lambda_w}$ | stand. dev. of wage markup shock | 0.289 |
| $\sigma_{\epsilon_q}$ | stand. dev. of equity premium shock | 0.604 |

Table 2: Calibrated shock processes

meaning that today's utility depends not only on today's consumption but also on last period's aggregate consumption. Technically, consumption habits work as if one assumed consumption adjustment cost, thus they induce consumers to adjust consumption levels more gradually. According to Abel (1990), this effect is sometimes referred to as "catching up with the Joneses", capturing the idea that households compare their consumption level to the one of neighboring households'. Furthermore, they intratemproally face a labor/leisure decision. A shock to the discount factor as well as a shock to preferences are added to the households' optimization problem. Households face a budget constraint which allows them to shift funds intertemporally via riskless bonds and have labor income, income from investment into state-contingent securities, and income from capital investments. Note

that a variable capital utilization rate is assumed which in turn affects households' return on capital and improves upon the persistence of the variables in sticky prices general equilibrium models (Dotsey and King, 2006). Therefore, it might be preferable to first increase the utilization rate before extending the existing capital stock.

Wages are set in a staggered way following Erceg et al. (2000). With a fixed and exogenous probability $1 - \xi_w$ wages can be reoptimized whereas with the converse probability, wages cannot be adjusted. As a result, wages are set in a forward looking manner such that future expectations of wages also become relevant for current wages. It is assumed that those wages which cannot be reoptimized are subject to partial indexation which makes current wages also depend on past wages.

On the one hand, households decide about their investment into the capital stock. This investment will be available for production with a one-period lag. On the other hand, households influence the capital utilization rate which determines how intensively the existing capital stock is used. This is of particular importance as households face capital adjustment costs that induce a wedge between the marginal product of capital and its rental rate, introducing a variable price for capital.

The production sector consists of final and intermediate goods producers. Final goods producers construct consumption goods using intermediate goods and sell them to households. Furthermore, they are subject to cost-push shocks. The intermediate goods sector uses utilized capital and labor for production. In order to motivate price setting on the side of the firms, they act under monopolistic competition. Hence, firms have some degree of market

power. Prices are set according to Calvo (1983), in other words, firms are able to reoptimize prices with a fixed and exogenous probability $1 - \xi_p$ whereas the non-optimized prices are partially indexed to last period's inflation. This induces price setting to be forward and backward looking at the same time which results in a hybrid version of the New Keynesian Phillips curve.

As indicated before, we do not adopt the monetary policy rule used in Smets and Wouters (2003) since it is our goal to implement what would have been the optimal policy in one model into a different potentially misspecified model.

# 3 Simulation

## 3.1 Simulation setup

As pointed out earlier, we assume that some degree of learning has already taken place in the past such that the monetary authority is only faced with uncertainty on the side of the parameters of the true data generating process. Consequently, the monetary authority is completely aware of the true structure of the economy but does not know all relevant parameters entirely.

At a first stage, we assume that the parameters of the reference model show a misspecification of a systematic form. That is to say that the monetary authority for some reason systematically over- or underestimates some parameters of the true model. Here, we assume that the estimation bias is of magnitude 2.5% or 5.0%. The parameters that we consider for misspecification are marked with "$*$" in table 1. The reason for this choice is twofold.

First, the standard deviations of the shocks that are incorporated in the reference model do not influence optimal monetary policy. This is the so-called certainty equivalence property (Svensson, 2010). Second, we exclude those parameters that might exceed critical thresholds when they are increased or decreased in an ad-hoc way. This is for instance of particular importance for the parameters indicating the autocorrelation coefficients of the shock processes and also the discount factor $\beta$. Parameters marked with "$(*)$" are affected indirectly. Subsequently, we think of the misspecification as being of a random nature. In this case, parameters of the true model are randomly either overestimated, underestimated, or pinned down correctly.

Optimal monetary policy under commitment is obtained using the reference model of the monetary authority. For all subsequent analyses, we fix the weighting parameters $\lambda_\pi$ and $\lambda_y$ to 1 and 2/3, respectively. As we are interested in the relative importance of the squared interest rate spread, we choose $\lambda_\Delta \in [0; 0.25]$. For the model simulations, the true model is used which is closed using the policy obtained from the optimization problem of the monetary authority using the reference model. Since squared deviations of the interest rate from the Taylor rule-implied interest rate are irrelevant from a welfare-theoretical perspective, there is no reason why one should evaluate the monetary authority's loss using the per period loss function given by equation (4) with $\lambda_\Delta \neq 0$. A reasonable alternative is to compute the loss with respect to the traditional per period loss function (1) even though the optimal policy is determined using (4). Therefore, it is important to note that for model evaluation and loss determination $\lambda_\Delta$ is set equal to zero in all cases. This is in line with Røisland and Sveen (2011) and ensures

comparability of the simulation results.

We perform simulations of the true model for $\lambda_\Delta \in [0; 0.25]$. For each value of $\lambda_\Delta$, we perform a set of $N = 100$ simulations, each containing $T = 5,000$ simulated quarters. By doing so, we ensure that for each set of simulated time series, simulated quarters that are more than $T$ periods ahead are negligible for loss evaluation.

## 3.2 Simulation results – commitment

Figures 2 up to 6 show the simulation results where figures 2 and 3 refer to the case where the true model parameters are systematically overestimated compared to the reference model that is used to determine monetary policy, whereas figures 4 and 5 represent the case of a systematic underestimation. Furthermore, we consider the case where the parameters are randomly either overestimated, underestimated, or pinned down correctly by the monetary authority in figure 6. Within each figure, we plot in the upper panel the average relative loss between using the traditional loss function (1) for the determination of monetary policy and using loss function (4) that is adjusted for a Taylor rule cross-checking term. When the respective values exceeds 100%, adding the cross-checking term is on average beneficial in terms of the monetary authority's objective. Since we simulate the true model for each value of $\lambda_\Delta$ $N = 100$ times, we end up with a whole distribution of relative losses such that we are able to compute the standard deviations of the relative losses. The respective plot can be found in the lower left panel. Furthermore, conditional on assuming Gaussian relative losses, we plot the

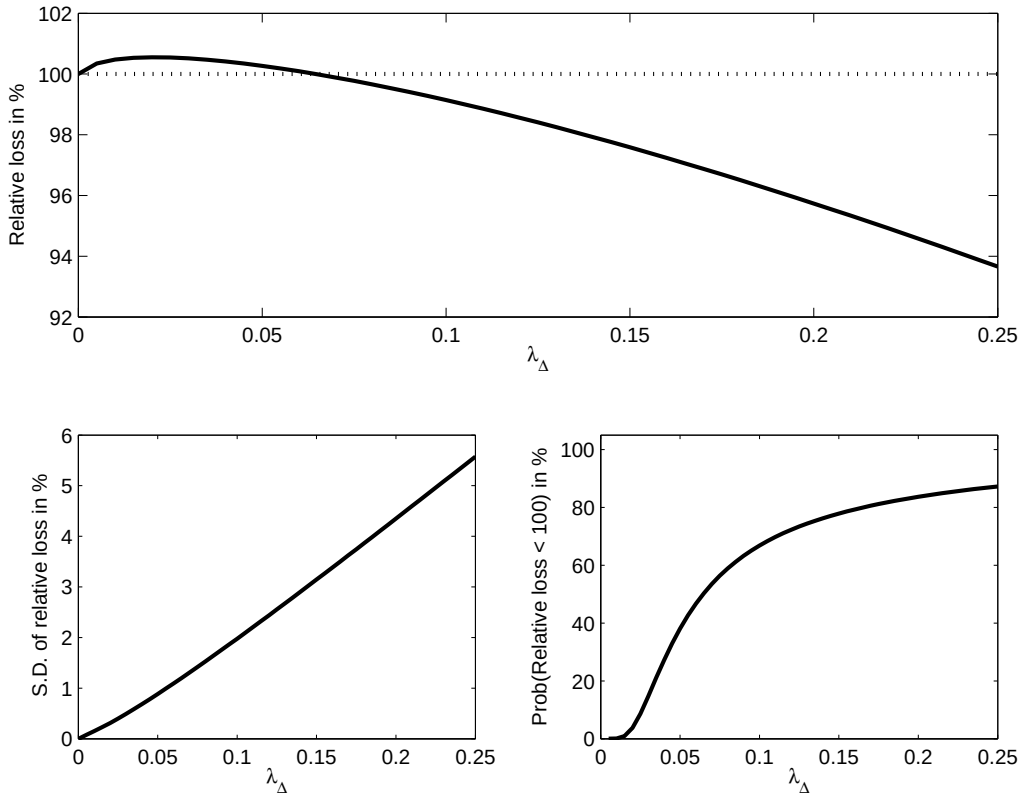


Figure 2: True parameters are 5% lower compared to the reference model.

probability that Taylor rule cross-checking is *not* beneficial for the monetary authority, in other words the probability that the relative loss will be smaller than 100%.

Consider first the case where the parameters of the true model are systematically 5% lower compared to the parameter values of the reference model in figure 2. We find that the monetary authority is on average able to improve the resulting loss in the region where $\lambda_\Delta = [0; 0.06]$. Hence, there is only a small window for choosing $\lambda_\Delta$ in a beneficial way. Therefore, the monetary authority has to be cautious as attaching too much weight on the information resulting from Taylor rule cross-checking is not harmless. Choosing higher

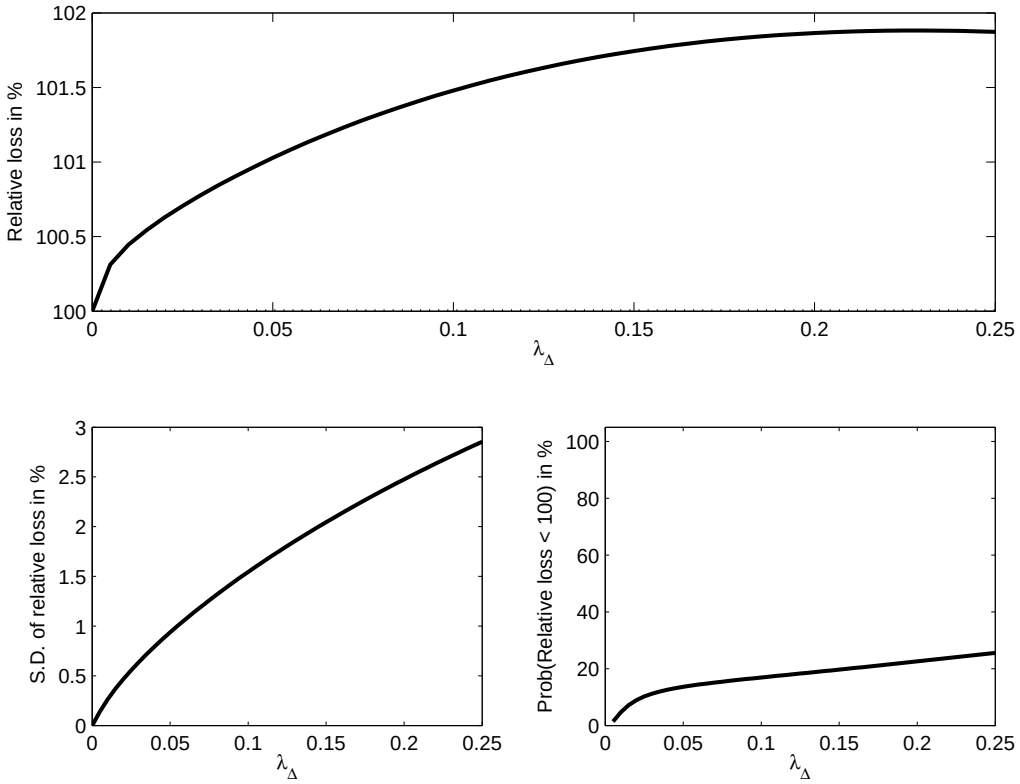


Figure 3: True parameters are 2.5% lower compared to the reference model.

values of $\lambda_\Delta$ increases the volatility of the relative losses. Hence, it becomes riskier for the monetary authority to increase the relative weight.

Figure 3 shows a more optimistic picture. When the true parameters are overestimated and in fact 2.5% lower compared to the parameters of the reference model, the monetary authority is on average able to generate improvements in terms of the relative loss over the whole range $\lambda_\Delta \in [0; 0.25]$ with a maximum average gain at $\lambda_\Delta = 0.23$. Hence, here the monetary authority would be better off adjusting its policy instrument into the direction of the Taylor rule-implied interest rate. The standard deviation of the relative losses is lower compared to the previous case and also increases steadily

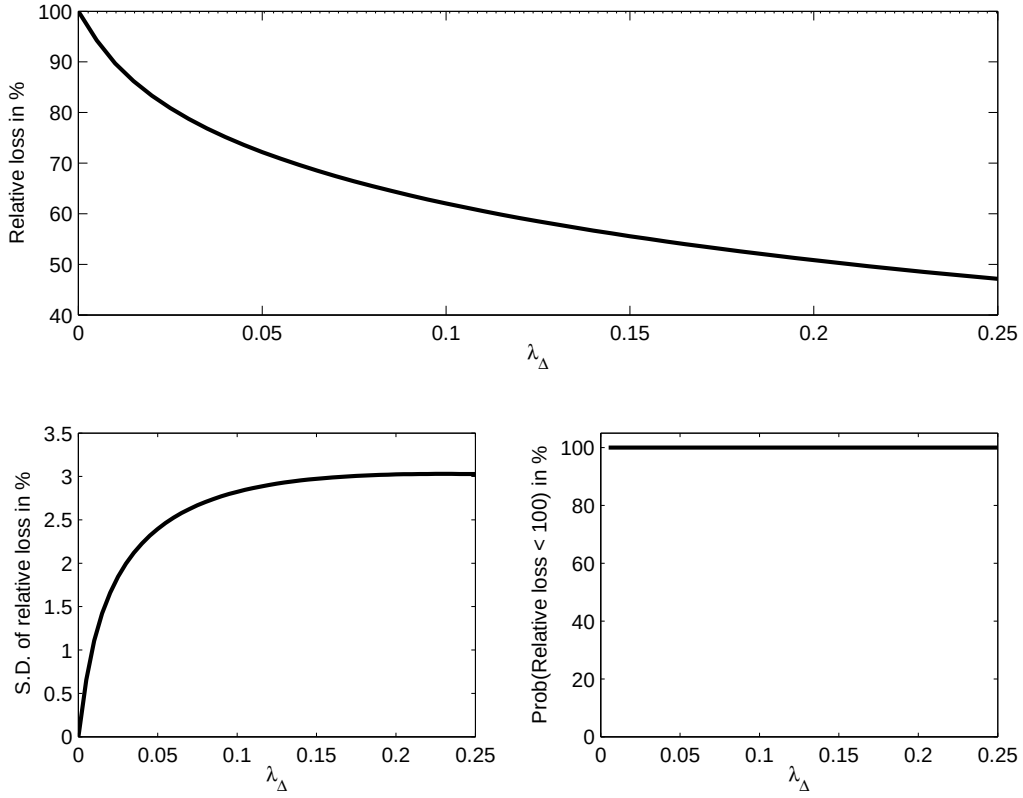


Figure 4: True parameters are 5% higher compared to the reference model.

with $\lambda_\Delta$. This impression is also recovered from the probability of ending up worse compared to the baseline case. The probability is rather low and in fact only increases gradually.

The case where the true parameter values are in fact 5% larger compared to the reference model is depicted in figure 4. Putting even a very low weight on the Taylor rule cross-checking term worsens the situation of the monetary authority in terms of the relative loss. The average relative loss immediately falls below 100%, reflecting that the loss when sticking to the non-adjusted policy is smaller. A weight of $\lambda_\Delta = 0.25$, for instance, results in a loss which is on average twice as large compared to the baseline case. The standard

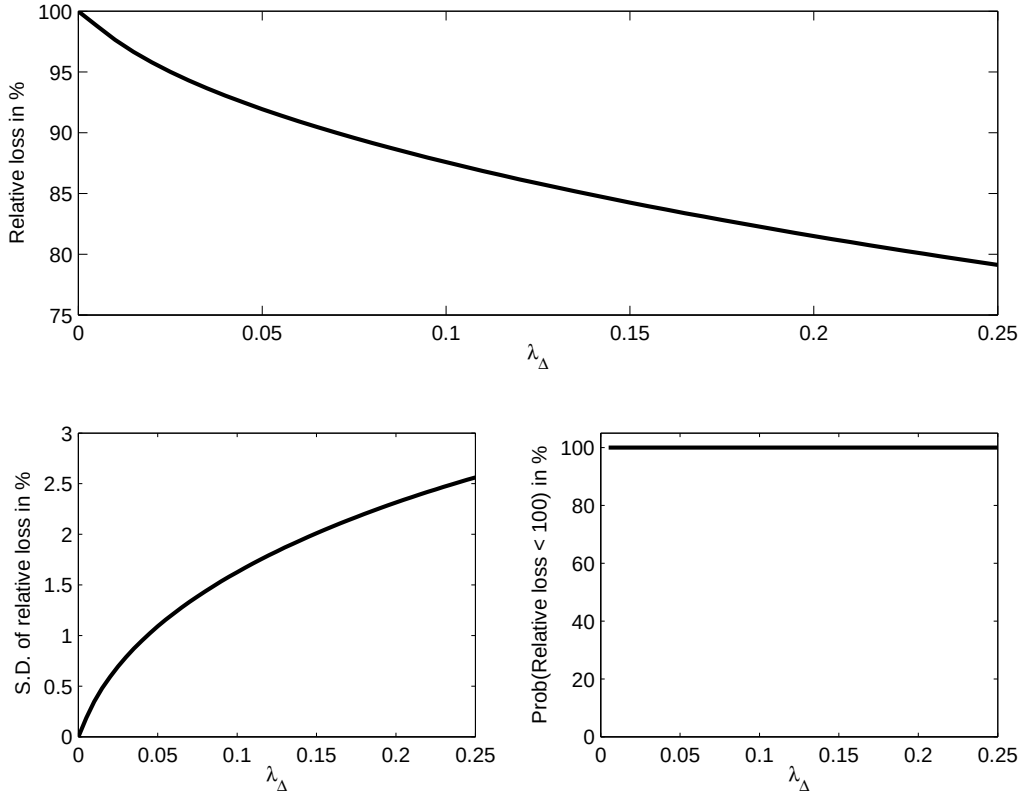


Figure 5: True parameters are 2.5% higher compared to the reference model.

deviation increases and approaches a value of 3%. The lower right panel emphasizes that putting a positive weight on $\lambda_\Delta$ will almost always increase the loss. Hence, for this parameter specification, the monetary authority should not use the Taylor rule when deciding about its monetary policy as this has on average strong negative effects on the associated losses.

Next, we analyze the case where the true parameter values are 2.5% larger compared to the reference model. In figure 5, the results are qualitatively similar compared to the previous case, in other words adjusting monetary policy for Taylor rule cross-checking deteriorates the monetary authority's loss. However, the effect is weaker. The standard deviation approaches a

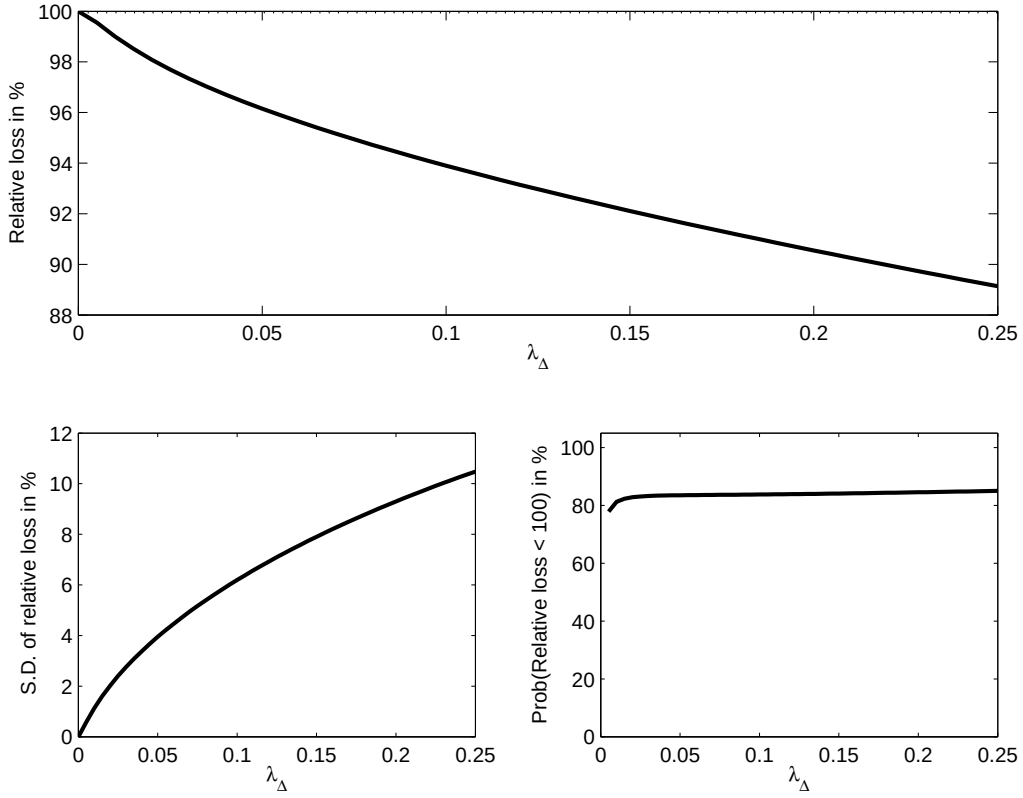


Figure 6: True parameters are either 2.5% lower, 2.5% higher, or pinned down correctly compared to the reference model.

similar value of about 2.5%. The probability of ending up worse compared to the baseline case is around 100% and can hardly be distinguished from the certainty case.

After having discussed a systematic over- and underestimation of the model parameters we now turn to the case where the misspecification is of a random nature. More precisely, we assume that a given parameter is overestimated with a probability of 40%, underestimated with a probability of 40%, and pinned down correctly with the converse probability.

In this setup we have to undertake an intermediate step in order to ensure determinacy of the model in all simulations. Since we do not a priori know

whether a certain combination of parameters leads to a determinate solution of the model, the parameter combinations have to be checked for determinacy first. If a combination yields an indeterminate solution, the random draw is not accepted and another combination is generated. We then end up with a set of 100 misspecified parameter combinations at our disposal. The subsequent analysis is the same as before such that the the corresponding graphs can be found in figure 6.

The results are qualitatively similar to the ones obtained in figures 4 and 5. Hence, adding the cross-checking term will on average generate a higher loss for the monetary authority. The impact, however, seems to be smaller in the sense that the relative loss does not decrease as much. The probability of ending worse compared to the baseline case is slightly above 80%.

Summing up, we find that whether or not cross-checking is beneficial for the monetary authority crucially hinges on the type and also the degree of the parameter misspecification. This makes it necessary for the monetary authority to determine which parameters are subject to uncertainty, in other words which parameters are likely to be estimated with error. Hence, whether or not cross-checking is beneficial also depends on the information set of the monetary authority. This point is of particular importance as our results suggest that for some parameter specifications, Taylor rule cross-checking may even have severe effects on the relative loss and therefore on the monetary policy objective.

## 3.3 Simulation results – discretion

If the monetary authority is able to commit to a certain policy, it may or may not be beneficial to let the information resulting from Taylor rule cross-checking influence the conduct of monetary policy depending on the type of parameter misspecification. In what follows, we drop this assumption and assume instead that no technology is available such that the monetary authority is not able to credibly commit to an announced policy. Whether or not commitment or discretion is a more adequate depiction of reality is not obvious (Friedman and Woodford, 2010). Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2007), for instance, argue that an intermediate case may perhaps be more realistic. Besides the fact that discretionary policy may or may not describe actual monetary policy, analyzing this case enables us to consider non-constant model parameters.

Recall that a drawback of Røisland and Sveen (2011) results from the fact that the monetary authority is ignorant in terms of realizing that there is a mismatch between the assumed transmission mechanism and the actual one. The same argument, even though in an alleviated form, also applies to our analysis so far. It is reasonable to believe that the monetary authority realizes in the long run that there is a missmatch on the side of the parameters of the true data generating process compared to its reference model.

As in the previous section, we assume that there are two types of models. The first model is the reference model of the monetary authority which reflects its belief of how the economy is structured. The second model differs from the reference model in the sense that it may contain a different set

of model parameters. The true underlying economy and therefore the true data generating process are now assumed to be instationary in the sense that the true data generating process potentially switches from one period to the next. To make this operational, we assume that the set of model parameters switches according to a Markov process. We argue that this feature makes the setup of the previous analysis more realistic as it is hard to believe that the monetary authority does not learn the true parameters over time if the true model parameters persistently differ from the reference model parameters. In this new environment, even if the monetary authority was somehow able to ex-post assess whether or not its reference model was identical to the true underlying economy in the previous period, this does not provide any valuable information for the conduct of its future monetary policy as the transmission mechanism may change in the subsequent period according to the assumed Markov process.

Furthermore, reconsidering the quotes mentioned in the introduction, it seems more plausible that the monetary authority selectively adjusts its policy using the information resulting from Taylor rule cross-checking. Hence, when no commitment is possible and the monetary authority is able to reoptimize in each period, it may be optimal to adjust its monetary policy only if the monetary authority is for some reason convinced that the reference model differs from the true model economy.

As a starting point, we assume that the monetary authority perfectly detects regime switches. That is to say that the monetary authority knows for sure whether or not its reference model is currently identical to the true economy. It is important, however, that the parameters of the true data gen-

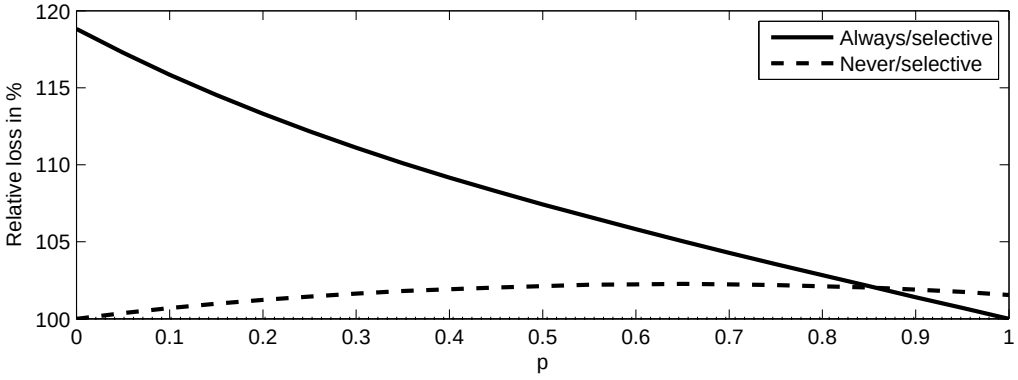


Figure 7: Relative losses for different model probabilities $p$.

erating process are still assumed to be unknown and that precise estimation is not feasible.

Since our previous results suggest that for the commitment case, adjusting optimal policy according to a Taylor rule is not always advantageous, we would expect a similar result to materialize under discretionary policy. For this reason, we only consider a case where Taylor rule cross-checking actually helped to improve losses. We therefore look at the case where the true parameters are 2.5% lower compared to the reference model and set $\lambda_\Delta = 0.23$. Subsequently, we let the probability $p$ of the Markov process that the true model economy is not identical to the monetary authority's reference model increase from 0 up to 1. The loss evaluation is carried out in the same form as before. We present the simulation results with respect to the relative losses in figure 7. The solid line represents the loss when the monetary authority always uses the augmented loss function in order to determine its monetary policy divided by the loss when it only selectively uses the augmented loss function. We call the latter case "selective monetary policy adjustment". The dashed line represents the relative loss when the monetary authority never

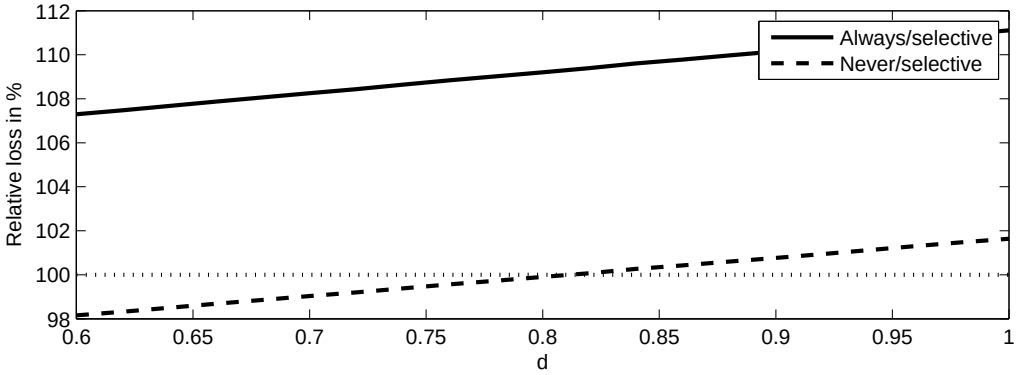


Figure 8: Relative losses for different detection probabilities $d$.

adjusts the objective function for a cross-checking term divided by the selective monetary policy adjustment loss. Values greater than 100% indicate that selective adjustment is beneficial compared to the respective alternative. Figure 7 shows that if $p$ is small, the monetary authority gains a lot in terms of loss when it adjusts its policy selectively instead of always. We find the maximum gain at $p = 0$ where the monetary authority is able to improve the loss by about 18%. When $p = 1$, in other words when the true model is not equal to the reference model, there is no gain as both policies are identical by construction. For $0 < p < 1$, the relative loss decreases almost linearly to 100%. The results are similar but less pronounced when comparing the never with the selective adjustment policy. We find that over the whole range of $p$, the relative losses are always greater than 100% except for the case where $p = 0$. In the latter case, the two policies are again identical by construction. However, the maximum gain is 2% and rather small. Under this parameter constellation, the monetary authority does not do harm when using selective monetary policy adjustment.

Finally, we relax the in some way unrealistic assumption that the mon-

etary authority is able to perfectly detect regime switches. In this respect, we assume that there is an exogenous probability $d$ of detecting the current parameter state. In figure 8 we present the simulation results for $d \in [0.6; 1]$. Again, we compare selective monetary policy adjustment to an optimal policy where the monetary authority always adjusts its policy according to a Taylor rule and an optimal policy where it never does so. Overall, we find that the selective adjustment policy turns out to be beneficial on average as long as the detection probability is greater or equal to 0.82. Hence, the monetary authority does not necessarily need to detect regime switches with certainty.

# 4 Conclusion

This paper builds upon Røisland and Sveen (2011) and sheds light on the question whether the robustifying nature of Taylor rule cross-checking in their spirit carries over to the case of parameter uncertainty. We consider certainty with respect to the structure of the economy but uncertainty of the monetary authority about model parameters. In particular, we also examine how much attention the monetary authority should pay to choosing the relative weight $\lambda_\Delta$ for the conduct of its monetary policy.

Considering optimal commitment policy, our results suggest that whether or not cross-checking is beneficial for the monetary authority in terms of a reduction of loss incurred from inflation and output deviating from the respective steady-state values crucially hinges on the functional form and also the degree of the parameter misspecification. Hence, it is necessary for

the monetary authority to determine which parameters are mostly subject to uncertainty, in other words are likely to be estimated with error. This renders the information set of the monetary authority relevant. This point is pivotal as we find that for some parameter specifications, Taylor rule cross-checking may have severe effects on the monetary policy objective.

Therefore, much attention of the monetary authority should be paid to choosing the appropriate relative weight $\lambda_\Delta$ which it attaches to the information resulting from Taylor rule cross-checking. Increasing the weight involves a trade-off in some specifications as an increase in the weight is associated with an increase in both the average gain for the monetary authority and the standard deviation of the relative losses. Hence, the choice of the relative weight will also depend on the risk aversion of the monetary authority.

When no technology is available such that the monetary authority is not able to credibly commit to an announced policy, we consider the case of discretionary policy. This enables us to analyze the case where the monetary authority adjusts its policy only in times when there are reasons to believe that the true data generating process differs from the assumed one. For our particular parameter constellation, we find that selective monetary policy adjustment can be beneficial in terms of relative losses even though the current parameter state can only be detected imperfectly. It is up to future research to propose reliable mechanisms that enable the monetary authority to detect regime switches with sufficient accuracy.

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# Appendix

This section summarizes the linearized model equations that are used in order to determine optimal policy and to simulate the true model. The model is closely related to the one introduced by Smets and Wouters (2003) but it lacks a monetary policy reaction function which is replaced by a policy resulting from the optimization problem of the monetary authority. Variables with circumflex denote log-deviations from the steady-state.

Capital accumulation in the economy evolves according to

$$\hat{k}_t = (1-\tau)\hat{k}_{t-1} + \tau\widehat{inv}_{t-1}.$$

Labor demand is summarized by

$$\hat{l}_t = -\hat{w}_t + (1-\psi)\hat{r}_t^k + \hat{k}_{t-1}.$$

The goods market clearing condition is

$$\hat{y}_t = (1-\tau k_y - g_y)\hat{c}_t + \tau k_y\widehat{inv}_t + \epsilon_t^g.$$

Production in the economy is described by

$$\hat{y}_t = \phi\epsilon_t^a + \phi\alpha\hat{k}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{l}_t.$$

Intertemporal allocation of consumption features habit formation and is determined by

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\mathbb{E}_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma_c}\left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) + \frac{1-h}{(1+h)\,\sigma_c}\hat{\epsilon}_t^b.$$

Investment is related to Tobin's $q$ and follows

$$\widehat{inv}_t = \frac{1}{1+\beta}\widehat{inv}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\widehat{inv}_{t+1} + \frac{\varphi}{1+\beta}\hat{q}_t + \hat{\epsilon}_t^{inv}.$$

Tobin's $q$ evolves according to

$$\hat{q}_t = \left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) + \frac{1-\tau}{1-\tau+\bar{r}^k}\mathbb{E}_t\hat{q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}\mathbb{E}_t\hat{r}_{t+1}^k + \eta_t^q.$$

The hybrid New Keynesian Phillips curve reads

$$\begin{aligned}\hat{\pi}_t =& \frac{\beta}{1+\beta\gamma_p}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} \\ &+ \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\epsilon}_t^a\right] + \eta_t^p.\end{aligned}$$

Wages are described by

$$\begin{aligned}\hat{w}_t =& \frac{\beta}{1+\beta}\mathbb{E}_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t \\ &+ \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \\ &\times\left[\hat{w}_t - \sigma_l\hat{l}_t - \frac{\sigma_c}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) + \hat{\epsilon}_t^l\right] + \eta_t^w.\end{aligned}$$