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## Home Voters, House Prices, and the Political Economy of Zoning

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# Home Voters, House Prices and the Political Geography of Zoning

– June 25th, 2012 –

Kristof Dascher

**Abstract:** A number of countries have seen large booms in house prices over recent years. Some authors argue that these booms have also been driven by the desire of homevoters – voters who are homeowners – to restrict supply, or zone. Only, empirical studies inform us that homevoters’ observed role in these booms is ambiguous at best. This paper explores three straightforward reasons for why homevoters do not appear to visibly drive house prices in cross-sectional studies. The paper’s model argues that (i) rent spillovers from one city to the next, (ii) zoning spillovers from one homevoter-ruled city to all other homevoter-ruled cities and (iii) compensation payments from homevoter-ruled cities to their tenant-ruled neighbors (in exchange for extra zoning) all contribute to obscuring an existing homevoter-rent-nexus.

**JEL - Classifications:** R31, R52, D72

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# 1 Introduction

A number of countries have seen large house price increases over the years preceding the recent real estate bust. As has been suggested by Glaeser et al. (2003), Fischel (2005) and Ortalo-Magné/Pratt (2008), these booms may have also been driven by the desire of homevoters – voters who are homeowners – to restrict supply, or “zone”. Only, the empirical evidence in support of homevoters’ role for the price of housing is ambiguous at best. This paper gives three straightforward reasons for why homevoters do not appear to drive house prices in cross-sectional studies. Its model argues that (i) rent spillovers from one city to the next, (ii) zoning spillovers from one homevoter-ruled city to all other homevoter-ruled cities and even (iii) compensation payments from homevoter-ruled cities to their tenant-ruled neighbors (in exchange for extra zoning) all contribute to obscuring an existing homevoter-rent-nexus.

In the empirical literature on homevoters, zoning and rent, demonstrating a link either (i) between zoning and rent or (ii) between homevoters and zoning has not been a straightforward exercise, let alone identifying (iii) a causal relationship running all the way from homevoters to the price of housing. Earlier studies have frequently failed to identify a significantly positive effect of intensifying land use regulation on the price of housing (surveyed in Quigley/Rosenthal (2005)). It is only in more recent work that tighter regulation consistently appears to drive house prices (e.g. Glaeser/Gyourko/Saks (2003), Ihlanfeldt (2005), Zabel/Dalton (2011), Magliocca et al. (2012)). However, the available evidence on the relationship between zoning and homeownership (as a proxy of homevotership) still remains ambiguous. Hilber/Robert-Nicoud (2009) find weak support of a positive relationship, but Glaeser/Ward (2009) actually identify a weakly negative relationship.

When appreciating these results the case for homevoters’ meddling with housing markets seems weak at best. Yet this paper argues that homevoters may exert substantial influence on rent nonetheless. Its model brings together three explanations – one well-known, two apparently overlooked – of why a link between homevoters and house prices need not be easily observed in empirical work, even as theory suggests that it exist. All three explanations depart from assuming that cities are open. If cities are open then rent increases in any given city drive away the more mobile of that city’s tenants. Homevoters’ strength in one city may tempt that city to zone, but the rent increases thus triggered will spill into every other city, too. Rent changes dissipate, and this dissipation not just makes it difficult to trace rising rent back to its geographical origin.

Also, rent dissipation is one well known explanation of why cross-sectional differences in rent belittle the true underlying impact of zoning on rent, and hence of why a city’s rent may not be driven visibly by that city’s homeownership (e.g. White (1975), Hamilton (1978), Glaeser/Ward (2009)). Here, and in addition to this, the paper’s model offers two extra explanations that, if intimately related, do seem novel and are distinct. The first of these is that homeowners’ strength in one city may imply supply regulations showing up in – *another*. If homeowner-dominated cities alternate with tenant-dominated cities, then homeowners strong in one city should find it advantageous to bribe tenants strong

in its neighbor, to have extra restrictions imposed there that they themselves either have put in place already or choose not to put in place at all. As emphasized by the literature on fiscal externalities and federalism (e.g. Oates (2011)), such decentralized trades are helped along by geographical proximity. For instance, homeowners in an agglomeration's suburbs plus tenants in that agglomeration's center might coalesce over extra regulation in the tenant-dominated center, complementing suburbs' own zoning code.

The second extra explanation is related to a non-linearity inherent to the relationship between homeowners and rent. When the initial homeowner share in a city's constituency is small then a small further rise in homeowners obviously fails to establish political control, and hence does nothing to raise rent. If the homeowner share is close to the 50 percent threshold the same small increase in homeowner numbers will secure political control and may jumpstart zoning and rent. And for an even higher homeowner share any further increase in homeowners will drive rents – *down*. If homeowners are particularly numerous then tenants must be particularly few, and the homeowner majority's incentive to exploit this minority of tenants must be weaker. Non-cooperative zoning equilibrium exhibits a non-linear relationship between homeownership and rent. In this context, a broad increase in homeowner numbers may produce successive housing boom and bust, all without any concomitant change in “fundamentals”.

From a positive perspective, the paper's model is an attempt to explain why homeowners, zoning and land rent empirically do not go together in obvious ways. From a normative perspective, and as more than just a side aspect of the analysis, the paper's model also explores the welfare effects of zoning, effects that are non-too-obvious either. In the setup suggested just above the preceding paragraph, for instance, a tenant-ruled center is bribed into zoning by the landlord-ruled suburbs. This makes better off center tenants (who receive transfers that more than make up for the inconvenience of rising rent), makes better off suburb landlords (whose benefits from rising real estate incomes exceed those transfers) joint with landlords elsewhere, yet makes worse off suburb tenants (who face a rising cost of living). In short, zoning affects the different groups of society in different ways, and these differences need not emerge along the fault lines of the absentee landlord single open city model.

If one way to introduce this paper's two contributions is to point to their links to the literatures on zoning or on fiscal externalities and fiscal federalism, yet another way could be to emphasize their role in the debate on the social merits of homeownership. While homeownership does generate a number of positive externalities (e.g. Glaeser/diPasquale (1999)), at least one critical piece of the literature also suggests that pervasive homeownership leaves households more exposed to adverse shocks on the labor market (Oswald (1996)). – Now, a number of building blocks of the earlier theoretical literature enter this paper's modeling setup as well. From Brueckner (1998) this paper's model receives the idea that housing supply restrictions in one city interact with restrictions imposed in another. From Ortalo-Magné/Pratt (2007) and Hilber/Nicoud (2009) this paper inherits the explicit emphasis on the political struggle between the various stakeholders in the urban economy. And as Brueckner (1998) this paper also assumes congestion externalities away (and thus zoning's potential role in correcting these).

At the same time, in contrast to Brueckner (1998) and Hilber/Nicoud (2009), landlords are not absentee (i.e. non-resident) but are a subset of the local electorate. In contrast to Hilber/Nicoud (2009), restrictions to the local housing supply are not assumed ad hoc but are explicitly modeled. In contrast to Ortalo-Magné/Pratt (2007), tenants are not assumed immobile but may migrate between cities. Finally, in contrast to Calabrese/Epple/Romano (2007) as an important contribution to the political economy of zoning, households are not heterogeneous in terms of their incomes, to the effect that zoning here is never meant to deter poorer households. – At the more technical level, a distinct feature of the model presented here is its extensive use of circulant matrices, matrices that are not commonly encountered in the economics literature. Not only are circulant matrices a natural reflection of this model’s circular setup. Circulants also facilitate the rigorous exposition of equilibrium’s existence and properties. – Section 3 presents the model. Extensions and short applications follow in section 4. Section 5 concludes.

## 2 Model

Subsection 2.1 sets out an urban system composed of many cities, each of which is open to individuals from neighboring cities. Cities are arranged on a circle, giving rise to an external spatial structure reminiscent of the Hotelling/Salop model. At the same time each city’s internal spatial structure conforms to the standard v. Thünen layout. Cities are monocentric, and land rents only settle at households’ maximum bids. Into this economic geography subsection 2.2 inserts a political antagonism between landlords and tenants that goes beyond the standard economic conflict over rent typical of voluntary transactions in the housing market. In contrast to parts of the urban economics literature landlords are not absentee but do reside within the city. Landlords and tenants wrestle for control over the decision on whether to limit land supply, or “zone”.

Cities with a tenant majority never ponder introducing such zoning but cities with a majority of landlords do. In Nash equilibrium, all cities with a landlord majority zone. Subsection 2.3 shows how equilibrium rents respond to variations in the number and spatial allocation of landlords. As one special case, rents may first increase with, to later fall in, landlords’ prevalence, thereby creating a “rent bubble”. Subsection 2.4 explores the dissipation of the rent effect of one landlord-ruled city’s zoning throughout the urban system. Rather than zone to increase her own land revenues, a tenant-ruled city may zone to receive payment for increasing her neighbors’ revenues. Matters are complicated further if local governments and federal government are both ruled by landlords, as in subsection 2.5.

### 2.1 Economic Geography

A total of  $n$  cities are laid out on a circle, where  $n$  is even throughout. Generally it is true that cities have at least two neighbors. Our circular setup retains this property’s essence, assigning two neighbors to each city. Cities are indexed  $i = 0, \dots, n - 1$ , in clockwise fashion. If  $n$  is 8 then city 0 has cities 7 and 1 as immediate neighbors. More generally, if  $s$  is a positive integer we address city  $i$ ’s two neighbors reached after moving  $s$  cities either

in clockwise fashion (“down the circle”, as a shorthand) or in counterclockwise fashion (“up the circle”). Then on the one hand  $i + s$  identifies city  $i$ ’s neighbor reached after moving  $s$  cities down provided we properly redefine  $i + s$  as the remainder when  $i + s$  is divided by  $n$ , or  $(i + s) | n$ . On the other hand,  $i - s$  properly identifies city  $i$ ’s neighbor reached after moving  $s$  cities up as long as we reinterpret  $i - s$  as the – positive – remainder when  $i - s + n$  is divided by  $n$ , or  $(i - s) | n$  for short. As an example of computing this positive remainder, moving 7 cities up from city 3 takes us to city  $4 = ((3 - 7) + 8) | 8$ .

Similarly, if we move from  $i$  to  $j$  in clockwise fashion then the distance between these two cities is  $j - i$ , where  $j - i$  implicitly is properly redefined as

$$j - i = (j - i) | n \tag{1}$$

If city  $i$  is 6 and city  $j$  is 3, then the clockwise distance between cities 6 and 3 is  $5 = (3 - 6) + 8 | 8$ . Alternatively, if we move from  $i$  to  $j$  in counterclockwise fashion then the counterclockwise distance is  $i - j$ , again if properly redefined. In the example this is  $3 = ((6 - 3)) | 8$ . – All city indices arising from sums or differences of indices are thought to have undergone prior proper adjustment.

Into any given city a total of  $L_i$  landlords and  $T_i$  tenants are born, making  $P_i = L_i + T_i$  its population total. Landlords are immobile across cities, while tenants are assumed mobile. However, and as explained in detail below, half of the tenants will only ever migrate to the upper neighbor, while the other half will only ever consider the lower one. And while all tenants are mobile in principle, they do differ with respect to their individual migration costs. We assume that individual migration cost  $m$  is distributed uniformly on  $[0, \bar{m}_i/2]$ , with  $\bar{m}_i > 0$ . Tenants do not own any land. Instead landlords own, and between them equally share the proceeds of, all urban land. Moreover, the landlord class also own all undeveloped land in the city’s vicinity. Cities are spaced sufficiently far apart from each other to make this assumption meaningful.

In that landlords own the land that their tenants occupy, they truly are landlords. Yet in that landlords also own the land they themselves occupy, they should rightfully also be considered homevoters. This dual landlord-homevoter role is not an unpleasant artefact of the model’s ambition to simplify but must be seen as an implication of housing ownership’s not being divided up equally in society. If some households are tenants only, then some other household(s) must be both homeowner and landlord. Only in that we suggest that tenants’ plots are owned by homeowners resident in the same, rather than some other, city do we impose an assumption that might be subject to controversy later. For now, in view of landlords’ dual role the paper refers to landlords as homevoters whenever that latter concept is more appropriate.

The city’s CBD, or central business district, is where everybody must go for work. From the CBD the city extends into one direction, at unit width. There are no structures, to the effect that housing is synonymous for land. Thus if the marginal plot is  $r$  units away from the CBD then  $r$  units of housing could be occupied altogether. All workers need to commute to the CBD from where they reside. Commuting cost per unit of distance travelled daily is  $2t$ , and the daily urban wage earned at the CBD is  $w$  and constant across cities. Rent paid (or earned) at distance  $r$  from the CBD is  $q_i(r)$  for the single unit of

housing that households occupy. Rent at the CBD is  $q_i(0)$ , which is shortened to  $q_i$ . In the city's internal spatial equilibrium, all tenants must be equally well off, so that the urban-wide structure of rents is given by  $q_i(r) = q_i - 2tr$ .

The model assigns landlords an important role in local elections and for this it must treat these landlords as residents. But dispensing with the absentee landlord framework raises the question of where exactly landlords reside. In this paper landlords always populate the city's inner ring, while tenants are relegated to the outer ring. (We might assume that landlords were born into the city first, populating the plots closest to the CBD.) Of course, a landlord may be made exchange his inner ring plot with any tenant's outer ring plot, in order to collect greater income from rent. But in our simple framework the attendant gain (represented by the tenant's greater rent bid) is just offset by the loss wrought by added travel (imposed on the landlord-household now having to commute longer). We conclude that landlords have no incentive to forego their plots in the inner ring.<sup>1</sup>

As agricultural bid rent is set to zero both, the urban boundary and the city's total supply of housing, equal  $q_i/2t$ . The  $L_i$  units closest to the CBD are inhabited by the city's landlords-homevoters. Local politics might decree to not let the remainder be rented out in its entirety, to protect part of the remaining urban area from settlement. Such zoning specifies the size of the area  $z_i$  to be withheld from settlement. It seems natural to assume that the zoned area comprises all the land between the urban fringe,  $q_i/2t$ , on the one hand and  $q_i/2t - z_i$  on the other.<sup>2</sup> With this, net housing supply to the tenant class becomes  $q_i/(2t) - L_i - z_i$ . Tenant utility is  $w - q_i$ , while landlord utility is  $w$  plus the representative landlord's share in overall urban rent.

We turn to tenants' location decisions. We assume that one half of tenants are oriented towards city  $i - 1$ : Should these tenants ever emigrate, they would only do so to city  $i - 1$ . The other half of tenants, correspondingly, are oriented towards to city  $i + 1$ . Now first suppose that  $q_{i-1} < q_i$ . A tenant native to  $i$ , oriented towards  $i - 1$ , endowed with migration cost  $m$  and swapping  $i - 1$  for  $i$  now receives  $w - q_{i-1} - m$  instead of  $w - q_i$ . Thus a tenant native to  $i$ , oriented towards to  $i - 1$  and endowed with migration cost equal to just  $q_i - q_{i-1}$  is indifferent between leaving and staying while every tenant native to  $i$ , oriented towards  $i - 1$  and endowed with migration cost short of this rent differential leaves for  $i - 1$ . Total emigration out of  $i$  and into  $i - 1$ , denoted  $M_{i,i-1}$ , becomes

$$M_{i,i-1} = (q_i - q_{i-1})T_i/\bar{m}_i. \quad (2)$$

Alternatively, suppose that  $q_i < q_{i-1}$ . Then total immigration from  $i - 1$  into  $i$  is  $(q_{i-1} - q_i)T_{i-1}/\bar{m}_{i-1}$ . A reasonable assumption here is that migration cost's variance is increasing in the tenant class' size. While some cities may have larger tenant classes than others, these cities also exhibit greater  $\bar{m}_i$ . Specifically, we assume that the ratio  $T_i/\bar{m}_i$  is

<sup>1</sup>This assumed pattern certainly does not suit every city, and thus might deter some readers. It should be emphasized, therefore, that all subsequent results could also be obtained for the more general case where landlords/homevoters and tenants mingle (at the cost of a more cumbersome exposition.)

<sup>2</sup>That homevoter-induced zoning is dynamically assigned with respect to the urban fringe, must also make sense from the perspective of homevoters' revenue maximization objective, introduced shortly. Why locate the zoned area anywhere else but on the city's least valuable land? – In view of the plethora of ways to zone (Quigley/Rosenthal (2005)), this paper's approach to zoning only captures zoning's supply-constraining aspect.

constant for all  $i = 0, \dots, n - 1$ . But then  $M_{i,i-1}$  not just represents the migration flow from city  $i$  to  $i - 1$ , if positive. Also, it represents the migration flow from city  $i - 1$  to  $i$ , if negative.

Total demand for  $i$ 's housing comes from native landlords, native tenants still present, and immigrant tenants, and can be written as natives  $P_i$  plus overall net immigration ( $M_{i+1,i} - M_{i,i-1}$ ). Equating this sum with net housing supply  $q_i/2t - z_i$  and making use of the migration flow definition in (2) twice yields the market clearing condition in  $i$

$$q_i/(2t) - (T_i/\bar{m}_i) (q_{i-1} + q_{i+1} - 2q_i) = P_i + z_i. \quad (3)$$

Letting  $i$  run from 0 through  $n - 1$  traces out all  $n$  conditions for local housing market clearing. These conditions can only be solved simultaneously. Since the  $n$  left hand sides in (3) are linear in all of the urban system's rents  $q' = (q_0, \dots, q_{n-1})$  we define the coefficient matrix  $A$  with entries  $a_{ij}$  such that

$$a_{ij} = \begin{cases} a = 2T_i/\bar{m}_i + 1/(2t) & \text{if } i = j \\ -b = -T_i/\bar{m}_i & \text{if } i = j + 1 \text{ or if } i = j - 1 \\ 0 & \text{else.} \end{cases} \quad (4)$$

with  $i, j = 0, \dots, n - 1$ . (For an illustration of this matrix see (17) in the Appendix.)

$A$  is an  $n \times n$  matrix with a strong pattern. Non-zero entries are only found on the main diagonal, on the two diagonals below or above the main diagonal, and as elements  $a_{n-1,0}$  and  $a_{0,n-1}$ . All other elements are zero. As the Appendix shows,  $A$  is symmetric, diagonally dominant, and circulant (Lemma 1), where  $A$  being circulant means that elements along any given diagonal are constant. These three properties confer a great deal of structure on the inverse of  $A$ , and the various other matrices built around that inverse later. For example,  $A$ 's dominant diagonal implies that all of its inverse's entries are strictly positive; and  $A$ 's being circulant implies, among other things, that the sum of the elements of any of its inverse's rows (or columns) is the same.

If we let  $e' = (P_0 + z_0, \dots, P_{n-1} + z_{n-1})$  be the vector of right-hand sides in (3) we can rewrite this system more compactly as  $Aq = e$ . Due to our assuming the ratio  $T_i/\bar{m}_i$  to be constant across all  $i$  (even as  $T_i$  and  $\bar{m}_i$  are free to vary), none of the elements of  $A$  will be determined within the model. Moreover,  $A$ 's inverse  $A^{-1}$ , henceforth denoted  $C$ , exists so that the solution to (3) simply is  $Ce$ . Rent in any city  $i$  is a linear function not just of the parameters governing city  $i$  but of the parameters governing all cities. Specifically, rent in city  $i$  equals

$$q_i = \sum_{j=0}^{n-1} c_{ij} (P_j + z_j) \quad (5)$$

where the  $c_{i0}, \dots, c_{i,n-1}$  are the  $n$  successive entries in  $C$ 's row  $i$ . In the case where the urban population  $P_j$  is the same in every city and equal to  $P$  and where there is no zoning anywhere, the rent vector emerging for housing market clearance equals  $P \sum_{j=0}^{n-1} c_{ij}$ . In the Appendix it is shown that for every row of  $C$  the sum of entries, as the second term in this latter product, equals  $2t$  (Lemma 2, Property (iv)). So each city's rent for such no-zoning housing market clearing is  $2tP$ .



Rent's response in city  $i$  to extra zoning in city  $j$  is just the entry found in row  $i$  and column  $j$  of  $C$ . But then additional zoning in city  $j$  drives up rents throughout the entire urban system because, in view of (5),  $\Delta q_i/\Delta z_j > 0$  for all  $i, j = 0, \dots, n-1$ . Not just does extra zoning in city  $j$  raise that city's rent. Also, extra zoning in city  $j$  lifts rent everywhere else, despite migration linking a city with its immediate neighbors only rather than linking it with every other city. Much as in the filtering literature (e.g. Sweeney (1974)), rent changes originating in one housing segment (quality or spatial) diffuse into other segments, too. Of course, this has long been recognized in the "monopoly zoning"-literature discussing the extent to which communities are substitutes to each other (e.g. White (1975), Hamilton (1978), Pollakowski/Wachter (1990)).

Other properties follow from the setup introduced so far. (For instance, the spillover from a zoning city  $k$  felt  $s$  cities down from  $k$  is the same as the spillover felt  $s$  cities up.) To focus on the paper's theme, we merely note that the fact that rent spills over into neighboring cities raises two fundamental questions dear to the theory of positive externalities. On the one hand, if inter city cooperation is little (i.e. if transaction costs are large) then spillovers should take away from a given city's incentive to zone. On the other hand, if inter city cooperation is intense (i.e. transaction costs are small) then the disincentive tied to these spillovers may be mitigated by inter city negotiations on Pareto-improving transfers. Both these themes are pursued below, with Proposition 3 exploring the first theme and Proposition 4 addressing the second. – First, however, Proposition 1 analyzes the extent to which rent changes dissipate in the urban system, and how this dissipation depends on inter city mobility. (Proofs of all propositions are in the Appendix.)

**Proposition 1: (Distance Decay and Inter City Mobility)**

*Responses of rents to extra zoning are as follows:*

$$\begin{aligned}
 (i) \quad & \frac{\Delta q_i}{\Delta z_k} > \frac{\Delta q_j}{\Delta z_k} && \text{if } \min\{k-i, i-k\} < \min\{k-j, j-k\} \\
 (ii) \quad & \frac{\Delta q_i}{\Delta z_k} \rightarrow \frac{2t}{n} && \text{if } b \rightarrow \infty, \text{ and for all } i.
 \end{aligned}$$

(i) (*Distance Decay*) In response to extra zoning in city  $k$ , rent in any city  $i$  close to  $k$  changes by more than rent in any other city  $j \neq i$  further away from  $k$ .

(ii) (*Inter City Mobility*) As intercity mobility becomes ever greater ( $b = T_i/\bar{m}_i$  tends to infinity as all the  $\bar{m}_i$  fall suitably), any city's rent changes induced by city  $k$ 's extra zoning converge to  $2t/n$ .

If city  $k$  zones and  $i$  is closer to  $k$  than  $j$  is the rent change induced in  $i$  will exceed that triggered off in  $j$  (Property (i)). More generally, rent increases gradually die out as one moves away from the source of extra zoning. The degree to which this is true is not fixed. We may roughly distinguish between two cases. If mobility is limited such that the  $\bar{m}_i$  are large and  $b$  consequently is small then rent increases fade away quickly and near their source, with the impact of extra zoning in  $k$  largely being absorbed by its closer neighbors. In contrast, if mobility is nearly unlimited so that  $b$  is very large, rent increases travel throughout the urban system and are even evenly distributed across cities (Property (ii)). We return to this latter property when discussing whether inter city trades in spillovers can really arise, showing that such trades do occur if household mobility is sufficiently strong.

**An Example:** Let  $n = 8$ ,  $a = 10$ ,  $b = 4.5$  and hence  $2t = 10$ . Then matrix  $C$ 's fourth column,  $C_3$ , looks as follows:

$$C'_3 = (0.08, 0.11, 0.16, 0.24, 0.16, 0.11, 0.08, 0.07) \quad (6)$$

Following this column, one extra unit of zoning in city 3 will drive up all cities' rents, given that all of the column's entries are strictly positive. Rent goes up strongest at the source, by 0.24 Euro. City 2's and city 4's rent increases are next largest, at 0.16 Euro each. Rent increases in other cities yet further away are smaller still, equal to 11 cents only. Finally, the rent increase is smallest in the city opposite city 3, generating a mere 7 cents extra.

## 2.2 Political Geography

As noted above, if the native population is the same in every city and there is no zoning then competitive rents are  $2tP$ . In such an equilibrium there is no inter city migration. Depending on the initial respective sizes of landlord class and tenant class,  $L_i$  and  $T_i$ , either landlords or tenants govern  $i$ . If  $T_i > L_i$  then tenants rule, and to city  $i$  we will refer to as a tenant city. A tenant city will not want to restrict urban land, will set  $z_i$  to zero, and will have competitive rent obtain. Conversely, if  $T_i < L_i$  then homevoter-landlords rule, and city  $i$  becomes what we will call a homevoter city. There landlords toy with the idea of zoning. Because restraining access to developable land raises rent on those remaining units ultimately rented out, such "monopoly zoning" may raise landlords' aggregate rental income as long as remaining tenants are sufficiently numerous.

There is no restriction with regards to whether or not any particular city is landlord ruled. Nor do we restrict the overall number of homevoter cities. But we do assume that homevoter cities are spaced equally far apart. That is, the distance when going from one homevoter city to the next is always the same. For example, if  $n = 8$  we either may have eight homevoter cities, four homevoter cities spaced two cities apart, or two homevoter cities spaced four cities apart. We address these equidistant homevoter cities by defining an ordered set  $I$  that collects homevoter cities' indices, in the ordering implied by starting with the smallest homevoter city index and then sliding down the urban circle. For example, if cities 5, 3, 7 and 1 are homevoter cities then  $I = \{1, 3, 5, 7\}$ .

When focusing on some homevoter city  $i$  in  $I$ , let  $\delta_i$  be the cost of zoning one unit of land there (and monitoring it so that it stays vacant). The landlord class's revenues are

$$\left( q_i/(2t) - L_i - z_i \right) \cdot \left( 2tz_i + (q_i - 2tL_i) \right) / 2 - \delta_i z_i \quad (7)$$

The first term in brackets gives housing actually rented out. The second term in brackets represents average rent earned on this latter stock when making use of the following facts: (i) the lowest rent earned on any plot is  $2tz_i$  (at the border between tenant occupied housing and zoned, peripheral land), (ii) the highest rent earned on any plot is  $q_i - 2tL_i$  (at the boundary between landlord occupied housing and tenant occupied housing), and (iii) the urban rent gradient  $q_i(r) = q_i - 2tr$  is linear so that average rent is just the average of the two rent extremes.

The landlord class in a given homevoter city maximize total rent (7) with respect to  $z_i$ , taking zoning efforts in all other homevoter cities as given. Prior to finding revenue's

first derivative, simplifying it and setting it equal to zero we briefly recall that by (5) rent's derivative  $dq_i/dz_i$  or  $\Delta q_i/\Delta z_i$  is a constant, and can be written as  $c_{ii}$ . Moreover, given that  $C$  is a circulant  $c_{ii}$  does not depend on  $i$ , and can even more briefly written as  $c_0$ . Employing this notation we find homevoter city  $i$ 's first order condition to be

$$q_i c_0/(2t) - 2tz_i = \delta_i + c_0 L_i, \quad (8)$$

from which we see that zoning efforts are strategic complements (as in Brueckner (1995)). Extra zoning in some homevoter city other than  $i$  results in greater rents anywhere, and hence also in  $i$ . But then the marginal benefit to extra zoning in  $i$  rises, too. Further, note that the second derivative of aggregate rental income with respect to zoning  $z_i$  equals  $c_0^2/(2t) - 2t$ , which is easily seen to be strictly negative.<sup>3</sup> With the s.o.c. strictly negative, the f.o.c. does characterize optimum zoning on the part of  $i$ 's government.

We next replace  $q_i$  featuring in (8) by the expression obtained for housing market clearing, spelt out in (5). Noting that  $z_j = 0$  whenever  $j \notin I$  and rearranging the resulting condition translates into

$$c_0/(2t) \left[ \sum_{j \in I} c_{ij} z_j \right] - 2tz_i = \delta_i + c_0 \left[ L_i - (1/2t) \cdot \sum_{j=0}^{n-1} c_{ij} P_j \right] \quad (9)$$

for homevoter city  $i \in I$ .

The resulting system has  $\bar{n} = |I|$  equations, where  $\bar{n} \leq n$ . To rewrite these as a matrix equation we first define the coefficient matrix  $D$  with entry  $d_{ij}$ ,

$$d_{ij} = \begin{cases} c_{ij} \cdot c_0/(2t) - 2t & \text{if } i = j \\ c_{ij} \cdot c_0/(2t) & \text{else,} \end{cases} \quad (10)$$

where  $i$  and  $j$  are restricted to indices contained in  $I$ , making us retain the initial city labels in what follows. Moreover, let  $g$  be the  $\bar{n} \times 1$  vector of right hand sides in (9). And denote by  $z$  the  $\bar{n} \times 1$  vector of zoning efforts across homevoter cities. Then (9) can more compactly be rewritten as  $Dz = g$ . Let  $F$  denote  $D$ 's inverse (provided it exists). Then this system's solution  $Fg$ , or  $\tilde{z}$  more briefly, is our zoning equilibrium.

Equilibrium CBD rents  $\tilde{q}$  are found by substituting  $\tilde{z}$  back into housing market clearing conditions (5). Specifically, let  $P' = (P_0, \dots, P_{n-1})$  be the city population vector,  $L' = (L_0, \dots, L_{n-1})$  the allocation of homevoters and  $Q$  be the  $\bar{n} \times n$  matrix one obtains from the  $n \times n$  identity matrix  $E_n$  after eliminating all rows of  $E_n$  with indices not in  $I$ .<sup>4</sup> As  $q = Ce$  (representing (5)),  $e = P + Q'\tilde{z}$  by definition,  $\tilde{z} = Fg$  (representing (12)),

$$\tilde{q} = C(P + Q'Fg), \quad (11)$$

with  $g$  as defined above or  $c_0 L - (c_0/2t)QCP$  in the special case of no zoning cost. Proposition 2 has further details.

<sup>3</sup>As mentioned, the sum of all elements of any row of  $C$  is  $2t$  (Lemma 2, Property (iv)). But then  $c_0$ , as just one entry in that row, is smaller than  $2t$ .

<sup>4</sup>Depending on the context,  $P$  thus has two interpretations, being each city's constant local population or representing the vector of all cities' (not necessarily identical) populations.

**Proposition 2: (Zoning: Existence, Uniqueness, Symmetry, Rents and Welfare)**

Let  $\delta_i + c_0 \left[ L_i - (1/2t) \sum_{j=0}^{n-1} c_{ij} P_j \right]$  be the same negative constant for all  $i \in I$ .

(i) (Zoning Existence and Uniqueness) A zoning equilibrium exists in which homevoter cities' zoning efforts are strictly positive. This equilibrium is unique.

(ii) (Zoning Symmetry) Homevoter cities' uniform zoning efforts are explicitly equal to

$$\tilde{z}_i = \frac{c_0 \left[ (1/2t) \cdot \sum_{j=0}^{n-1} c_{ij} P_j - L_i \right] - \delta_i}{2t - c_0 / (2t) \cdot \sum_{j \in I} c_{ij}}. \quad (12)$$

(iii) (Zoning and Homevoter City Rents) Rents in homevoter cities are identical.

(iv) (Zoning and Tenant City Rents) A tenant city's rent falls short of homevoter cities' rent. Further, a tenant city's rent is decreasing in distance to the nearest homevoter city.

(v) (Zoning and Welfare) If homevoter cities and tenant cities coexist (mutually exclude each other), a symmetric zoning equilibrium is Pareto-inefficient (Pareto-efficient).

Proposition 2 states that there exists a unique Nash equilibrium in zoning efforts (Property (i)).<sup>5</sup> Existence of a non-cooperative equilibrium suggests that homevoter cities may indeed artificially restrict the supply of housing to sustain income from local housing, even as their “monopoly zoning power” is constrained by households' potential flight to neighboring cities nearby. Following (5), rents obtained in Nash-equilibrium clearly exceed those obtained under perfect competition. Under the assumption given in the proposition, zoning efforts are the same across homevoter cities (Property (ii)). (This occurs, e.g., in the special case of  $P_j = P$  for all  $j = 0, \dots, n - 1$  and  $L_i = L$ ,  $\delta_i = \delta$  for all  $i \in I$ , so that the numerator in (12) reduces to the simple constant  $c_0 T_i - \delta_i$ .)

From (12), a given homevoter city  $i$ 's zoning effort is increasing in the number of its tenants. After all,  $\Delta \tilde{z}_i / \Delta P_i > 0$ . The same is true, if to a lesser degree, for zoning's response to an increase in rivalling cities' tenants:  $\Delta \tilde{z}_i / \Delta P_j > 0$ . (Subsection 3.5 thus inquires into whether immigration into the urban system may even be endogenous.) Next note that obviously any increase in  $\delta_i$  pulls down  $\tilde{z}_i$ . This is a reassuring feature of the model but if  $\delta_i$  is too large then  $Fg$  becomes negative.  $Fg$  represents a zoning equilibrium only if it is strictly positive, else  $\tilde{z}_i$  is set to zero. Can we be sure that cities that start out as homevoter cities continue to be homevoter cities in zoning equilibrium? Since in equilibrium homevoter cities are more expensive, very mobile tenants will have left where immobile landlords will have stayed on. This asymmetry reinforces the preexisting homevoter majorities in homevoter cities.

Not only do all homevoter cities charge identical CBD rent (Property (iii)). Also, homevoter cities charge higher CBD rent than tenant cities do. Moreover, the smaller a tenant city's distance to its next nearest homevoter neighbor, the greater that tenant city's rent (Property (iv)). This variation in rent translates into inefficiency. Assume that all cities initially have the same landlord population and let there be both, homevoter cities and tenant cities ( $0 < \bar{n} < n$ ). Then homevoter cities house a smaller overall population even as their developable area is larger. This reflects the loss of habitat imposed by local

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<sup>5</sup>To prove this we do not need to impose the assumption that the numerator of the fraction in (12) is constant across cities; the numerator merely needs to be strictly positive.

zoning. Tenant cities house tenants further out than homevoter cities do, to the effect that tenant cities' marginal residents (those living right on the fringe) could be made better off (without making anyone else worse off) once reallocated to that part of any homevoter city's zoned area that is closest (and also closer) to a CBD (Property (v)).<sup>6</sup>

Alternatively, let there be either no tenant cities ( $\bar{n} = n$ ) or tenant cities only ( $\bar{n} = 0$ ). Then all cities' marginal residents are equally far away from the CBD; the equilibrium is efficient because no excess commuting is observed. In our model at least, having homevoters rule every city is just as efficient as having them rule no city; and either is always better than having homevoters rule a mere subset of the urban system's city total. The following example illustrates a number of Proposition 2's properties. In particular, this example demonstrates how the cross-sectional rent change observed in the data once the zoning equilibrium has emerged may very much (roughly ten times here) understate the rent change that the more adequate city-wise before-after comparison would put to zoning.

**The Example Continued:** Suppose that there are  $\bar{n} = 4$  homevoter cities, cities 1, 3, 5 and 7. Suppose further that  $\delta_i = 0$ ,  $P = 7$  and that  $L_i = 4$  for all  $i \in I$ . Then equilibrium zoning amounts to 0.83 units of housing in each of the four homevoter cities. Corresponding equilibrium rents are

$$q' = (7.39, 7.43, 7.39, 7.43, 7.39, 7.43, 7.39, 7.43) \quad (13)$$

Due to the dissipation of the impact of the four homevoter cities' zoning on rents, homeowner cities are not much more expensive than tenant cities. At the same time, had zoning not taken place then rents would be 7 Euro throughout, hence much smaller.

Different types of mobility affect zoning and rent in different ways. While changes in parameter  $b$  capture changes in maximum migration cost  $\bar{m}_i$  and hence the distribution of inter city mobility, changes in parameter  $t$  reflect changes in commuting cost and hence intra city mobility. Suppose all cities are tenant cities ( $\bar{n} = 0$ ) such that equilibrium rents in (5) are  $2tP$  only. An increase in  $b$  will change the coefficients of  $C$  but will leave intact their row sums, of  $2t$  always (Lemma 2, Property (iv)). So rents do not look different. In contrast, a decrease in  $t$  obviously depresses those equilibrium rents of  $2tP$ . To summarize, for  $\bar{n} = 0$  rents do not change as tenants become more mobile across, yet fall as these tenants become more mobile within, cities. – Alternatively, suppose all cities are homevoter cities ( $\bar{n} = n$ ). Then adjustments are more involved, as Proposition 3 reports.

**Proposition 3: (Zoning Equilibrium and Inter- and Intracity Mobility)**

*Suppose all cities are homevoter cities ( $\bar{n} = n$ ) and that zoning costs  $\delta_i$  are zero.*

*(i) (Inter City Mobility, Zoning and Rent) A general increase in inter city mobility, as captured by an increase in  $b$ , causes zoning efforts  $\tilde{z}_i$  and rents  $\tilde{q}_i$  to retreat.*

*(ii) (Intracity Mobility and Zoning) A general increase in intra city mobility, in contrast, and as captured by a decrease in  $t$ , causes zoning efforts  $\tilde{z}_i$  to intensify.*

On the one hand, as tenants roam the urban system more freely competition among homevoter cities grows in intensity, making them relent in their efforts to zone. Rent schedules shift inwards, causing rents to drop uniformly (Property (i)). Given that extra

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<sup>6</sup>Admittedly, the zoning equilibrium's Pareto-inefficiency is partly due to the fact that no externalities in need of correction distort the no-zoning equilibrium.

inter city mobility means greater rental spillovers (see Lemma 2, Property (ix)) Proposition 3 confirms the intuitive notion that a city's externality-generating activity decreases in the extent of the spillover. On the other hand, as tenant travel to the CBD gets ever easier local governments' incentive to zone only grows, as a fall in commuting cost extends cities' developable areas (Property (ii)). We can ascertain the zoning response here but establishing the rent response is not straightforward. The rent-attenuating effect of lower commuting costs may or may not be offset by the rent-strengthening effect of the extra zoning induced.

### 2.3 Zoning Trading

If the number of homevoter cities equals  $n/2$  homevoter cities alternate with tenant cities. The pair of homevoter cities now encircling each tenant city could be viewed as an agglomeration's suburbs or periphery, with the tenant city that periphery's center. (It is true that such agglomerations overlap yet this does not alter the analysis that follows.) The positive fiscal externality associated with rent spilling over into neighboring jurisdictions' revenues, the geographical fact that all three cities are close to each other and the assumption that it is a jurisdiction's well-defined right to zone jointly suggest that homevoter suburbs offer a transfer to the tenant center.

**The Example Continued:** In terms of our earlier example of four homevoter cities altogether, if the center zones one extra unit of land then homevoters earn an extra 16 cents on every plot rented out while tenants resident with the tenant city zoning lose 24 cents (see (6)). If the center's tenant majority is tiny, if the peripheries' homevoter majorities are tiny also and if native populations all equal  $P$ , then the tenant city's aggregate loss in tenant welfare is  $0.5\dot{P} \cdot 0.24$  while the periphery's aggregate homevoter welfare gain is  $2 \cdot 0.5P \cdot 0.16$  – which is larger. Homevoter suburbs could offer a transfer equal to some sum within  $[0.12P, 0.16P]$  in exchange for one unit of zoning in the center.

The example points to the environment that is fertile in encouraging this type of trade, an environment featuring not just similar homevoter numbers but also suburb governments' disregard for their respective tenant minorities.<sup>7</sup> More generally, consider any given tenant city  $i$  (the center) and the two homevoter cities  $i - 1$  and  $i + 1$  adjacent to it (the periphery). Suppose that the center zones  $z_i$  units of housing there. Departing from zoning equilibrium  $\tilde{z}$  makes suburb  $i - 1$  gain  $t(q_{i-1}/(2t) - L_{i-1})^2 - t(\tilde{q}_{i-1}/(2t) - L_{i-1})^2$ , where  $q_{i-1}$  ( $\tilde{q}_{i-1}$ ) is the rent obtained after (before) the center has zoned.

A similar gain applies to the other suburb,  $i + 1$ . And given that in zoning equilibrium no center tenant remaining will want to flee to the periphery (which is more expensive according to Property (iv) of Proposition 2), all of those  $\tilde{q}_i/(2t) - L_i$  initial center tenants lose  $q_i - \tilde{q}_i$  each. Summing the two welfare terms corresponding to the two suburbs (one of them spelt out above), subtracting center tenants' loss, adding zoning cost, taking the first derivative of the resulting sum with respect to  $z_i$  and evaluating that derivative at the

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<sup>7</sup>The principle outlined in the example may be generalized. If homevoter neighbors 3, 5 or even  $s$  ( $s$  odd) cities away from  $i$  are part of the coalition also then the compensation paid out to the center may be even bigger. Our objective here is to show that trading in zoning occurs even if the number of bribers is small, provided that mobility is sufficiently high and transaction costs are sufficiently low.

zoning equilibrium gives the net benefit from marginal “Coasian zoning” in the center,

$$\left(\tilde{q}_{i+1}/(2t) - L_{i+1}\right) c_1 + \left(\tilde{q}_{i-1}/(2t) - L_{i-1}\right) c_1 - \left(\tilde{q}_i/(2t) - L_i\right) c_0 - \delta_i \quad (14)$$

where  $c_1$  is shorthand for  $dq_{i-1}/dz_i = dq_{i+1}/dz_i$ . The first two terms indicate the two suburbs’ rent gains, approximately equal to these cities’ outer rings multiplied by the rise in rent. The third term captures the center’s welfare loss, equal to that city’s initial number of tenants times the rent rise these tenants have gone through. If rent spillovers from center zoning  $c_1$  are sufficiently large then suburbs indeed have an incentive to pay the center for starting to zone.

As Proposition 1 has established (Property (ii)) the spillover  $c_1$  can always be made arbitrarily close to the local rent change  $c_0$  by having inter city mobility be sufficiently high. Proposition 4 thus explains how inter city mobility may contribute to making tenant city zoning more likely.

**Proposition 4: (Zoning Trading)**

*Suppose  $\bar{n} = n/2$  and  $L_i = L_{i-1} = L_{i+1} = P/2$ . A tenant city  $i$  will zone in exchange for a transfer from the two homevoter cities  $i - 1$  and  $i + 1$  surrounding it if mobility is sufficiently large and zoning costs sufficiently small. This trilateral trade makes majority voters in each of the three cities involved better off, at the expense of all those tenants in the urban system that are not resident in  $i$ .*

Once trading in zoning occurs, the cross-sectional distinction between homevoter cities’ zoning and tenant cities’ zoning becomes blurred. (Admittedly, Proposition 4 does not suggest that tenant cities zone more than homevoter cities do.) At the same time, if local government needs to observe minority (tenant) welfare also suburbs have a diminished incentive to bribe the center. This distinction could also provide the basis for additional statements on the relationship between a country’s political institutions and local governments’ trading in zoning, not pursued here. Finally, note that this subsection’s fiscal externality not necessarily provides a rationale for merging zoning boards (or even jurisdictions) if suburbs cannot credibly commit to those transfers to center tenants (for case studies on credibility issues see Fischel (1980) and Fischel (1995)).

**2.4 Housing Bubble**

Both the spatial structure and pervasiveness of homevoter dominance play into the comparative statics of zoning and rents. First we analyze the number of homevoter cities  $\bar{n}$  getting larger, then we permit homevoter cities’ homevoters  $L_i$  to be more numerous.

**Proposition 5: (Zoning Equilibrium and Homevoter Strength)**

- (i) (Homevoter Cities’ Number, Zoning and Rents) Let  $I_1$  and  $I_2$  be two sets of equidistant homevoter cities, with  $I_1 \subset I_2$ . Then  $\tilde{z}_i < \tilde{z}_j$  and  $\tilde{q}_i < \tilde{q}_j$  for all  $i \in I_1$  and  $j \in I_2$ .*
- (ii) (Homevoter Cities’ Homevoter Numbers, Zoning and Rents) Consider some homevoter city set  $I$ . If for all  $i \in I$  the number of homevoters  $L_i$  is greater while  $P_i$  is the same (so that tenants  $T_i$  are less), then both  $\tilde{z}_i$  and  $\tilde{q}_i$  are smaller.*

On the one hand, an urban system with a large number of cities with homevoter majority will settle at more intense zoning than an urban system starting out with a smaller such number (Property (i)). This essentially reflects the strategic complementarity between zoning efforts mentioned before. On the other hand, for an urban system in which homevoter cities harbor a smaller number of tenants, zoning and rents are lower (Property (ii)). From this perspective, growth in the number of homevoter cities is very different from growth in homevoter cities' homevoters. Now, both types of comparative statics are essentially cross-sectional, showing how the Nash-equilibrium changes from one urban system (i.e. country) to another. The sequel to our example illustrates Property (i):

**The Example Continued:** If there are merely  $\bar{n} = 2$  homevoter cities (cities 2 and 6, say), then zoned areas in these two cities equal  $0.78 < 0.83$  units of housing, and the resulting equilibrium rents are

$$q' = (7.17, 7.18, 7.24, 7.18, 7.17, 7.18, 7.24, 7.18) \quad (15)$$

This illustrates how with less homevoter cities rents are lower than in the previous example (see (13)) throughout – though still in excess of the 7 Euro that would be observed in the absence of any zoning. Moreover, we see how rent changes in tenant cities decay as we move away from one homevoter city – from 7.24 via 7.18 to 7.17 – yet rise again – from 7.17 via 7.18 back to 7.24 – as we approach the next. Finally, observe how the variation in rent rises as the number of homevoter cities falls.

Proposition 5 may also be instructive from a longitudinal perspective. Suppose changes in homevoter numbers are exogenously driven and consider the following explicit “timing” here. First let us suppress homevoter influence altogether (because it may need time to build up) and simply observe housing market clearing rents. These rents are the same in every city and equal  $2tP$ . Then let homevoters usurp power in those cities in which they command a majority of votes and have zoning equilibrium obtain. Rents then exceed  $2tP$  since zoning is strictly positive in all homevoter cities. Finally and third, convert all of every city’s tenants into homevoters. The equilibrium observed for a homevoter-only society is one where zoning has reverted to zero and rents are  $2tP$  again.

The third step sidesteps the question of how and whether tenants become homevoters yet this sequence of snapshots may nonetheless give a fair account of events if the tenure decision is shaped by forces outside the model, e.g. subsidies to homeownership, expansionary monetary policy and low interest etc. (rather than by the level of rent, determined within the model.)<sup>8</sup> In this special case, a uniform increase in homevoter numbers will first push system rent beyond the no-zoning system rent  $2tP$ , to eventually pull system rent back down to the homevoter-only system rent  $2tP$ . Without overstressing the model’s interpretation, the overall impression is one of increasing homevoter numbers first raising rents beyond what is warranted by “fundamentals”  $t$  and  $P$  (a “boom”) ultimately to be succeeded by a realignment of rents to these fundamentals (a “bust”).

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<sup>8</sup>Two methodological difficulties need to be conceded here. First, here we compare two equilibria with each other while previously we merely compared an initial allocation with that in equilibrium. Following equilibrium over time appropriately requires making assumptions on repeat migration. A full analysis would benefit from a model of repeat migration joint with endogenous tenure. And second, increasing  $L_i$  while reducing  $T_i$  (such that  $P_i$  can remain the same) also implies concomitant reductions in  $\bar{m}_i$  (so that  $b$  can stay constant, as assumed.) I.e. we implicitly assume that those tenants-turned-homevoters are those with largest migration cost. (This is not entirely unplausible.)



## 2.5 Zoning and Immigration

Previous analysis shows that an increase in population will drive up zoning and rents as long as the overall number of cities  $n$  is given (rather than endogenous). If homevoters rule at the federal level, too, what can be said about their incentive to relax barriers to immigration from other countries as to let  $P$  rise? Let  $\iota = (1, \dots, 1)$  be a list of ones of length  $n$ , and suppose  $L'\iota > T'\iota$ . Then the national homevoter majority could ponder increasing the tenant total almost up to  $L'\iota$ , by allowing extra tenants in. Intuitively, this has two effects on rents. First, rents rise because extra population directly boosts the demand for housing (see the effect of  $P_i$  on rents in (5)). And second, rents rise because extra population indirectly incites homevoter cities to zone even more (see the effect of  $P_i$  on zoning via (12)), adding to the pressure on rents.

For simplicity, let zoning costs equal zero and recall that then  $g = c_0L - (c_0/2t)QCP$ . Substituting  $g$  into (11) and observing that  $C$  is symmetric gives

$$\tilde{q} = c_0 \cdot C'Q'FL + \left( C + (c_0/2t) \cdot C'Q'(-F)QC \right) P \quad (16)$$

The term within brackets is the “rent multiplier” of immigration. Proposition 6 explains how this multiplier depends on the number of homevoter cities. Federal government’s incentive to zone interacts with homevoter cities’ strength within the urban system.

### **Proposition 6: (Federal-Local-Government Interaction in Immigration/Zoning)**

*The derivative of CBD rent  $\tilde{q}$  with respect to population  $P$ ,  $\partial\tilde{q}/\partial P$ , is strictly increasing in the number of homevoter cities,  $\bar{n}$ .*

## 3 Discussion

A number of the model’s assumptions could be relaxed without changing the essence of the propositions. For instance, exogenous wages could easily vary across cities, and if local public goods entered utility in additive fashion then local public goods could be included, too. (Including wages and local public goods would only add to the right hand sides of (3), leaving intact the comparative statics discussed until now.)

Other changes, it is true, may alter the model’s results. These are (i) endogenizing wages (introducing businesses as yet another interest group with a stake in the urban zoning decision, e.g. Cheshire/Hilber (2008)), (ii) permitting city creation (tying tenant utility more closely to some reference utility obtainable in nearby cities newly developed), (iii) endogenizing housing tenure (making supply of and demand for a share in local land depend on the vector of local rents), (iv) and giving landlords the right to exit the city, too (creating even stronger incentives to zone, if at the cost of less clout).

The three case studies below illustrate the potential contribution of a homevoter focus to providing a fresh perspective on each of the policy examples given (but naturally are not meant to substitute for rigorous discussion). Each of these examples explores one exogenous change analyzed in the model. The case study on the US studies an exogenous increase in the number of homevoters and homevoter cities; the case study on Germany

investigates an exogenous drop in inter city migration cost; and the case study on Ireland addresses the impact of a homevoter majority at the federal level.

### **3.1 US 1990-2008 (Homevoter Proliferation)**

As mentioned, numerous authors have argued that housing supply regulations have multiplied in many US states over the last two decades, tending to make housing more expensive there. As Proposition 5 argues, pointing to the increase in homevoters may not just be helpful in explaining the initial increase in house prices but may also be helpful in explaining their subsequent decrease. As Proposition 5 shows, as homeownership rates tend to 1 local governments become less inclined to zone. (A similar story might be told for Spain and Ireland, two other countries with high initial homeownership, even higher later homeownership and a boom bust pattern in house prices.)

### **3.2 Germany 1990-2012 (Migration Facilitation)**

Let Germany be divided up roughly into the following four equal-sized regions: West (Northrhine-Westphalia, Rhineland-Palatinate, Hestia) and East (Saxony, Saxony-Anhalt, Thuringia, Mecklenburg-Vorpommern, Berlin) should justly count as tenant-ruled regions, while somewhat less urbanized South (Bavaria, Baden-Wuerttemberg, Saarland) and North (Lower Saxony, Schleswig-Holstein, Hamburg, Bremen) could be considered homeowner-ruled. Proposition 4 might be taken to suggest that once the wall had fallen (a sudden decrease in inter city mobility cost), South and North bribed East to zone, to the benefit of homevoters everywhere and at the expense of tenants in West, South and North. From this perspective those well-known tremendous fiscal West-to-East transfers could also reflect Eastern tenants' compensation for risen rent.

### **3.3 Ireland 1990-2008 (Immigration Capitalization)**

Ireland's housing markets have witnessed impressive rises in rents. These increases have typically been associated with strong economic growth at the time of the "Celtic Tiger" (Dascher (2000)). At the same time, Ireland has been one of the few EU countries that very early threw open the gates to immigration. In line with Proposition 6, the decision to boost population by more than half a million tenants may not merely coincide with, or simply respond to the needs of, economic growth, but may instead reflect homevoters' influence at all tiers of government. To allow immigration was attractive to the homevoter-ruled federal government because immigration fueled rents and house prices not just through the extra housing demand implied but – also – via the extra zoning induced in those many homevoter ruled local governments (McDonald (2000)).

## **4 Conclusions**

The literature emphasizing the importance of zoning for the price of housing is growing, but the evidence on homevoters' part in this remains hazy. The paper's model is an

attempt to bridge this gap, arguing that even as homevoters do drive zoning and rent their responsibility may be clouded in various ways. The origins of rising rent are difficult to identify; rent dissipation may induce tenant-ruled cities to zone at the behalf of homevoter-ruled neighbors; and rent dissipation places homevoter-ruled cities into a mutual strategic interaction in which one homevoter city's zoning stimulates another's zoning, even as it may not always be preferable to push for added zoning oneself.

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## 6 Appendix

### Lemma 1: (Properties of the Coefficient Matrix, $A$ )

The coefficient matrix  $A$  can be written

$$\begin{pmatrix} a & -b & 0 & 0 & \dots & 0 & 0 & -b \\ -b & a & -b & 0 & \dots & 0 & 0 & 0 \\ 0 & -b & a & -b & \dots & 0 & 0 & 0 \\ & & & & \vdots & & & \\ 0 & 0 & 0 & -b & \dots & a & -b & 0 \\ 0 & 0 & 0 & 0 & \dots & -b & a & -b \\ -b & 0 & 0 & 0 & \dots & 0 & -b & a \end{pmatrix} \quad (17)$$

once we set  $b = T_i/\overline{m}_i$  and  $a = 2T_i/\overline{m}_i + 1/(2t) = 2b + 1/(2t)$ . Note that  $a - 2b$ , or  $1/(2t)$ , is the sum of all elements of any given row or column. Now,  $A$  has the following three properties (stated without proof):

- (i)  $A$  is symmetric.
- (ii)  $A$  is diagonally dominant.
- (iii)  $A$  is circulant. That is, element  $a_{ij}$  equals  $a_{j-i}$  or, as stated in the text and emphasized here one more time, may simply be written as  $a_{(j-i)|n}$  (Graybill (1983), section 8.10).

### Lemma 2: (Properties of the Inverse of the Coefficient Matrix, $C$ )

Consider the inverse of  $A$ , with  $A^{-1}$  denoted  $C$  as in the text.

- (i)  $C$  exists.
- (ii)  $C$  is symmetric.
- (iii)  $C$  is circulant. That is,  $c_{ij} = c_{j-i}$ .
- (iv) The sum of all elements of any column or row of  $C$  is the reciprocal of the sum of all elements of any row or column of  $A$ , and hence  $2t$ .
- (v) For any given row of  $C$ , elements satisfy  $c_k = c_{n-k}$  for  $k = 1, \dots, n-1$ .
- (vi) The ratio  $c_{k-1}/c_{k-2}$  can recursively be determined via

$$c_{k-1}/c_{k-2} = \alpha/(1 - \alpha \cdot (c_k/c_{k-1})), \quad (18)$$

where  $k = 2, \dots, n/2$  and  $\alpha = b/a$ .

- (vii) Elements can be ranked as follows:

$$c_0 > c_1 > \dots > c_{n/2}. \quad (19)$$

- (viii)  $C$  has strictly positive elements only.

- (ix) Element  $c_0$  is decreasing in  $b$ .

### Proof of Lemma 2

- (i) Since  $A$  is diagonally dominant it is non-singular and hence  $C$  exists (Graybill (1983, Theorem 8.11.2)).  $\square$
- (ii) Since  $A$  is symmetric, so is its inverse.  $\square$
- (iii) Since  $A$  is circulant,  $C$  is circulant also (Graybill (1983, Theorem 8.10.4)).  $\square$

(iv) Let  $\iota$  be a column vector of ones, of dimension  $n$ . We form the matrix product  $\iota' A C_0$ , where  $C_0$  is column 0 of  $C$ . This matrix product is a number (i.e.  $1 \times 1$ -matrix), and this number must equal 1, by  $C$  being  $A$ 's inverse. Moreover, this number also equals

$$\sum_{i=0}^{n-1} \sum_{k=0}^{n-1} a_{ik} c_{k0} = \sum_{k=0}^{n-1} c_{k0} \sum_{i=0}^{n-1} a_{ik} = \frac{1}{2t} \sum_{k=0}^{n-1} c_{k0}$$

where the second equality follows from the fact that each column of  $A$  sums to  $1/2t$  (see Lemma 1). But then  $\sum_{k=0}^{n-1} c_{k0} = 2t$ . The proof for any other column of  $C$  is similar.  $\square$

(v) Because  $C$  is circulant (Property (iii)), row 0 is  $(c_0, c_1, \dots, c_{n-1})$  while column 0 is  $(c_0, c_{n-1}, \dots, c_1)'$ . But then, because  $C$  is symmetric (Property (ii)),  $c_1 = c_{n-1}$ ,  $c_2 = c_{n-2}$ , etc. To summarize,

$$c_k = c_{n-k} \quad \text{for} \quad k = 1, \dots, n-1. \quad \square \quad (20)$$

(vi) Consider matrix  $A$  as in (17). Consider  $n/2$  as the index of the city "facing" city 0, in the circular context. Information on this city's housing market is contained in row  $n/2$  of  $A$ . Multiply this row by the first column of  $C$ , i.e. by  $(c_0, c_{n-1}, \dots, c_1)'$ . This gives  $-b(c_{n/2-1} + c_{n/2+1}) + ac_{n/2} = 0$ . Property (v) reveals that  $c_{n/2+1} = c_{n/2-1}$ . Rearranging and replacing  $b/a$  by  $\alpha$  yields

$$c_{n/2}/c_{n/2-1} = 2\alpha. \quad (21)$$

Next multiply row  $n/2 - 1$  by column 0 of  $C$ . This gives  $-b(c_{n/2-2} + c_{n/2}) + ac_{n/2-1} = 0$  or, after making use of (21) and rearranging,

$$c_{n/2-1}/c_{n/2-2} = \alpha/(1 - 2\alpha^2). \quad (22)$$

More generally, suppose that ratio  $c_k/c_{k-1}$  is known and that  $c_{k-1}/c_{k-2}$  is sought for. Multiplying row  $k - 1$  by the first column of  $C$  gives  $-b(c_{k-2} + c_k) + ac_{k-1} = 0$ . Dividing through by  $bc_{k-1}$  and rearranging gives (18).  $\square$

(vii) Since  $a - 2b > 0$  we have  $\alpha < 1/2$ . Then by (21),  $c_{n/2-1} > c_{n/2}$ . Next, given  $\alpha < 1/2$  we have  $1 - 2\alpha^2 > 1/2$ . But then  $\alpha < 1 - 2\alpha^2$  and hence by (22),  $c_{n/2-2} > c_{n/2-1}$ .

More generally, if  $c_k/c_{k-1}$  is known to be strictly smaller than 1, then  $c_{k-1}/c_{k-2}$  can swiftly be shown to be strictly smaller than one, too, consulting (18).  $\square$

(viii) Since  $A$  is diagonally dominant, all of  $C$ 's elements are non-negative (Graybill (1983), Theorem 11.4.2 (Property (7)) joint with Theorem 11.43. (Property (3))). It remains to confirm that all entries in  $C$  are strictly positive.

By this Lemma's Property (iii) it is sufficient to prove that all elements of the first column of  $C$  are strictly positive. Let us focus on column 0 thus. Multiplying row 0 of  $A$  by column 0 of its inverse  $C$ , and making use of this Lemma's Property (v), gives  $ac_0 - 2bc_1 = 1$ .

Given that  $c_1 \geq 0$  it is not possible that  $c_0 = 0$ . Hence  $c_0 > 0$ . Multiplying the second row of  $A$  by the first column of its inverse  $C$  and again making use of Property (v) gives  $ac_1 - b(c_0 + c_2) = 0$ . Given  $c_0 > 0$  and  $c_2 \geq 0$  it is not possible that  $c_1 = 0$ . Hence  $c_1 > 0$ . Etc. Continuing in this fashion reveals that all entries of  $C$ 's first column are strictly positive.  $\square$

(ix) Note first that distance decay – each element  $c_i$  in the list  $\{c_0, \dots, c_{n/2}\}$  is smaller than its predecessor  $c_{i-1}$  (Property (vi)) – becomes less and less pronounced as  $b$  rises. After all,  $c_{n/2}/c_{n/2-1}$  as in (21) is increasing in  $b$ , as then, successively, are all preceding coefficient ratios via (18).

Suppose that  $c_0$  is not decreasing in  $b$ . Then increasing  $b$  must at least once fail to depress  $c_0$ . But then given that distance decay becomes less pronounced with  $b$  rising, the sum of elements in the set  $\{c_0, \dots, c_{n/2}\}$ , and hence also the sum of all entries of any of  $C$ 's columns, grows. This is in contradiction to that sum being equal to  $2t$  throughout (Property (iv)). Thus  $c_0$  must be decreasing in  $b$ .  $\square$

**Proof of Proposition 1: (Distance Decay and Inter City Mobility)**

(i) (*Distance Decay*) The ratios mentioned in the proposition,  $\Delta q_i/\Delta z_k$  and  $\Delta q_j/\Delta z_k$ , equal  $c_{ik} = c_{k-i}$  and  $c_{jk} = c_{k-j}$ , respectively. If either  $k - i$ ,  $k - j$ , or even both, exceed  $n/2$  we map those indices in excess of  $n/2$  into indices smaller than  $n/2$  by invoking Lemma 2's Property (v). So without loss of generality let us assume that  $k - i$  and  $k - j$  are  $n/2$  or smaller. But then the assumption  $k - i < k - j$  joint with the ranking (19) implies  $c_{ik} = c_{k-i} < c_{k-j} = c_{jk}$ .  $\square$

(ii) (*Inter City Mobility*) From (21) we see that the ratio  $c_{n/2}/c_{n/2-1}$ , or  $2\alpha = 2b/a = 2b/(2b + 1/(2t))$ , not only grows but also converges to 1 as  $b \rightarrow \infty$ . But then, following (18) through recursively, all subsequent ratios  $c_{n/2}/c_{n/2-1}$ ,  $c_{n/2-1}/c_{n/2-2}$ , etc. grow, and converge to 1, too, as  $b$  tends to infinity. Put differently, all of  $C$ 's elements become arbitrarily close to one another.

By Lemma 2's Property (ix),  $c_0$  is decreasing in  $b$ . Moreover,  $c_0$  is bounded from below, by zero. By the convergence of bounded monotone sequences,  $c_0$  must converge to some limit as  $b$  tends to infinity. Yet if  $c_0$  is convergent then  $c_1$  must be convergent, too, and must converge to the same limit. After all,  $c_1 = c_0 \cdot (c_1/c_0)$  is the product of two convergent sequences. By the rules on convergent sequences,  $c_1$  converges to the products of the limits of  $c_0$  and  $c_1/c_0$ , and hence to the limit of  $c_0$ .

Continuing on in this fashion shows that  $c_0, \dots, c_{n/2}$  not only converge but even share the same limit. Let  $\bar{c}$  denote this common limit. Since  $\lim(c_0 + 2c_1 + \dots + 2c_{n/2-1} + c_{n/2}) = n\bar{c} = 2t$ ,  $\bar{c}$  must equal  $2t/n$ .  $\square$

**Lemma 3: (Circulant Symmetric Submatrix)**

We construct a submatrix of  $C$ , called  $\bar{C}$ , that is circulant and symmetric, too. Let homevoter cities be evenly spaced such that the common distance between any two neighboring homevoter cities,  $s = n/\bar{n}$ , is an integer. Let  $k$  be the city index of the first homevoter city encountered when starting at city 0 and moving down clockwise. Then cities  $k, k + s, k + 2s, \dots, k + n - s$  all are homevoter cities. Their indices are collected in  $I$ , the ordered set of  $\bar{n}$  homevoter cities' indices.

Now let us reduce  $C$ , by eliminating all rows and columns that do not have an index in  $I$ . This Lemma claims that the resulting  $\bar{n} \times \bar{n}$ -submatrix  $\bar{C} = QCQ'$  is circulant and symmetric.

**Proof of Lemma 3:**

We label rows and columns in  $\bar{C}$  by indices  $r = 0, \dots, \bar{n} - 1$  and  $p = 0, \dots, \bar{n} - 1$ , respectively. The element in row  $r$  and column  $p$  of  $\bar{C}$  is  $c_{k+rs, k+ps}$  or, given that  $C$  is circulant,  $c_{(p-r)s}$ . We show that elements on the same diagonal are the same, too. Now, the element  $\sigma$  rows further down (or up, if  $\sigma < 0$ ) and  $\sigma$  columns further to the right (or to the left, if  $\sigma < 0$ ) is  $c_{k+(r+\sigma)s, k+(p+\sigma)s}$  or, given that  $C$  is a circulant,  $c_{(p-r)s}$ .  $\bar{C}$  is symmetric because  $(QCQ')' = QCQ'$  in view of Lemma 2, Property (ii).  $\square$

**Proof of Proposition 2: (Zoning Efforts in Zoning Equilibrium)**

(i) (*Zoning Existence*) We first show that  $D$  is diagonally dominant. Given that  $c_{ii} < 2t$ , main diagonal elements must be strictly negative; while given that  $c_{ij} > 0$  (Lemma 2, Property (viii)) off-diagonal elements are strictly positive. Now,  $\sum_{i=0}^{n-1} c_{ij}$  just equals  $2t$  (Lemma 2, Property (iv)). Omitting one (or more) of these elements (corresponding to tenant cities) while making remaining elements smaller (by multiplying them with  $c_0/2t$ , a number smaller than one) implies

$$c_0/(2t) \sum_{i \in I, i \neq j} c_{ij} < 2t - c_{ii}c_0/(2t) = |c_{ii}c_0/(2t) - 2t|. \tag{23}$$

For any given column, the sum of off-diagonal elements is dominated by the absolute value of the main diagonal element.

But a diagonally dominant matrix is non-singular (Graybill (1983), Theorem 8.11.2) and hence invertible. Moreover,  $F = D^{-1}$  has but negative elements. Combining this with the fact that vector  $g$  has a constant expression as typical element, for  $z = Fg$  to be positive it is sufficient that

this constant expression is negative. – Uniqueness follows from  $D$  being non-singular.  $\square$

(ii) (*Zoning Symmetry*) Matrix  $D$  equals  $(c_0/2t) \cdot \bar{C} - 2t \cdot E$ , with  $E$  an  $\bar{n} \times \bar{n}$  identity matrix.  $D$  is circulant, because  $\bar{C}$  and  $E$  are (Lemma 3) and because a weighted sum of circulant matrices is also circulant (Graybill (1983), Theorem 8.10.1). Moreover, all elements in row  $i$  of matrix  $D$  sum to  $c_0/(2t) \cdot \sum_{j \in I} c_{ij} - 2t$ .

Since  $D$  is circulant, this latter sum is the same in every row. Moreover, since the inverse of a circulant also is a circulant (Graybill (1983), Theorem 8.10.4)  $F$  is a circulant, too. The sum of elements of any row or column of  $F = D^{-1}$  is the reciprocal of the sum of elements of any row or column of  $D$  (Lemma 2, Property (iv)). Hence the elements of any given row or column of  $F$  sum to  $(c_0/(2t) \cdot \sum_{j \in I} c_{ij} - 2t)^{-1}$ . Multiplying  $F$  with  $g$  gives  $\tilde{z}$ , the typical entry of which is shown in (12).  $\square$

(iii) (*Zoning and Homevoter City Rents*) According to (11),  $\tilde{q} = C(P + Q'Fg)$ . If all entries in  $g$  are the same, and because  $F$  is circulant (Property (ii)), all entries in  $Fg$  are the same, too. Premultiplying  $Fg$  by  $Q'$  for each homevoter city adds the same number to each city's base rent  $2tP$ .  $\square$

(iv) (*Zoning and Tenant City Rents*) The proof is in two steps. First we establish that rents in tenant cities of the same distance to the next nearest homevoter city are the same. Then we show that tenant cities closer to the next-nearest homevoter city are dearer.

We first claim  $q_k = q_{k+s} > q_{k+1} = q_{k+s-1}$  with  $k, k+s \in I$ . I.e. consider the tenant cities bounded by two successive landlord cities,  $k$  and  $k+s$ . Suppose one moves one city away from  $k$  (away from  $k+s$ ), in the direction of  $k+s$  (in the direction of  $k$ ).

To find the overall effects of the various zoning efforts onto these two tenant cities note that we may ignore the effects of zoning on the part of all homevoter cities other than  $k$  and  $k+s$  onto the two tenant cities  $k+1$  and  $k+s-1$  because these latter effects will be the same anyway, given overall symmetry.

Moreover, note that not just does the effect of zoning in  $k$  on  $k+1$  equal the effect of zoning in  $k+s$  on  $k+s-1$ . (After all, in both cases distance is the same, equal to one city.) Also, the effect of zoning in  $k$  on  $k+s-1$  equals the effect of zoning in  $k+s$  on  $k+1$ . (Again, in both cases the distance is the same, now equal to  $s-1$  cities.) Formally,

$$c_{k+1,k} = c_{n-1} = c_1 = c_{k+s-1,k+s} \quad (24)$$

$$c_{k+s-1,k} = c_{n-s+1} = c_{s-1} = c_{k+1,k+s} \quad (25)$$

where the equality in the middle of (24) or (25) follows from Lemma 2's Property (v) and where all other equalities merely apply the definition of a matrix being circulant. We conclude that  $\tilde{q}_{k+1} = \tilde{q}_{k+s-1}$ . Repeating this argument reveals that for any  $l \in \{1, \dots, s/2 - 1\}$  we must have  $q_{k+l} = q_{k+s-l}$

Next consider the tenant city located at the center of the segment  $\{k, \dots, k+s\}$ , city  $k+s/2$ . Also, assume that adjacent cities  $k+s/2-1$  and  $k+s/2+1$  are tenant cities, too, lest the proof is complete. These latter two tenant cities must exhibit the same rent, as explained. But then

$$q_{k+s/2} < q_{k+s/2+1} = q_{k+s/2-1} \quad (26)$$

must be true. Suppose not. Then while the introduction of zoning in homevoter cities would make rents, and also housing supply, go up, housing demand would not go up because neighboring cities' rents would be cheaper, and hence would pull mobile residents away. The housing market in  $k+s/2$  would no longer balance. Hence (26) must be true. Continuing this argument proves the ranking indicated in the proposition.  $\square$

(v) *Zoning and Welfare*: Let us focus on (any) one homevoter city and (any) one tenant city. In the model's zoning equilibrium, a tenant city is more expensive than any homevoter city (Property (iv)). Hence the tenant city's population exceeds the population of any homevoter city. This in turn gives  $\tilde{q}_j/(2t) > \tilde{q}_i/(2t) - \tilde{z}_i$  for  $j \notin I$  and  $i \in I$ .



Thus there are locations  $r'$  in the homevoter city's zoned area that are closer to the CBD than is the tenant city's marginal plot at the tenant city's urban boundary  $\tilde{q}_j/(2t)$ . Reassigning a perfectly mobile tenant (who surely exists given that until now there has only been immigration into the tenant city) currently at the tenant city's boundary to the vacant plot  $r'$  units away from the homevoter city's CBD will release part of that tenant's commuting cost while making noone worse off.  $\square$

**Proof of Proposition 3: (Zoning Equilibrium and Mobility)**

(i) (*Intercity Mobility*) Uniform equilibrium zoning efforts (12) reduce to  $\tilde{z}_i = T_i/(2t/c_0 - 1)$ . Combine this with the fact that an increase in  $b$  drives  $c_0$  down (Lemma 2, Property (ix)). Then  $\tilde{z}_i$  surely falls as  $b$  rises. Via (5), so do equilibrium rents.  $\square$

(ii) (*Intracity Mobility*) Uniform equilibrium zoning efforts are as in the previous property's proof. The rise in  $t$  causes  $a$  to fall and hence  $\alpha$  to rise. According to (21),  $c_{n/2}/c_{n/2-1}$  rises, and hence via (18) so does  $c_{n/2-1}/c_{n/2-2}$ . Continuing on in this fashion shows that all successive ratios rise. Moreover, each element in  $\{c_1, \dots, c_{n/2}\}$  grows faster than  $c_0$  does. But then  $2t/c_0$  must rise, too. We conclude that equilibrium zoning efforts fall.  $\square$

**Proof of Proposition 5: (Zoning Equilibrium and Homevoter Strength)**

(i) (*Homevoter Cities' Number, Zoning and Rents*) Index set  $I_2$  includes even more indices than set  $I_1$  and, hence, the sum in the denominator on the right hand side of (12), or  $c_0/(2t) \cdot \sum_{j \in I} c_{ij}$ , now includes even more elements. Moreover, as homevoter cities necessarily now are positioned closer to each other each element that was included in that sum beforehand now gets replaced by an element that is larger.

The proof of rents rising in every city follows the same idea as the proof of zoning rising in every city, and thus is only sketched. If there are extra homevoter cities that zone, incumbent homevoter cities not only receive an extra housing price stimulus from these extra zoners. Also, given greater closeness to each other they also receive even greater stimuli from incumbent zoners.  $\square$

(ii) (*Homevoter Cities' Homevoter Numbers, Zoning and Rents*) According to (12),  $\tilde{z}_i$  is strictly decreasing in  $L_i$ . Moreover, given the fact that  $g$  is increasing in  $L_i$  while  $F$  has negative elements only, equilibrium rents in (11) must be strictly decreasing in  $L_i$ , also.  $\square$

**Proof of Proposition 4: (Tenant-City-Zoning in Homevoter-Tenant-Coalitions)**

Exploiting the fact that  $\tilde{q}_{i-1} = \tilde{q}_{i+1}$ , making use of  $L_{i-1} = L_i$ , dividing (14) through by  $c_0$  and rearranging for  $c_1/c_0$  yields a strictly positive residual iff

$$\frac{c_1}{c_0} > \frac{\tilde{q}_i/(2t) - L_i}{2(\tilde{q}_{i-1}/(2t) - L_i)} \tag{27}$$

On the left hand side, as  $b \rightarrow \infty$  the ratio approaches 1 (Proposition 1, Proof of Property (ii)). On the right hand side, as  $b \rightarrow \infty$  both  $\tilde{q}_i$  and  $\tilde{q}_{i-1}$  tend to  $2t(P + (\bar{n}/n)\tilde{z}_i)$ , in view of (5 and Proposition 1's Property (ii)). Thus the numerator tends to  $(P + (\bar{n}/n)\tilde{z}_i) - L_i$ , while the denominator tends to  $2(P + (\bar{n}/n)\tilde{z}_i) - L_i$ . So on the right hand side the ratio approaches one half.  $\square$

**Proof of Proposition 6: (Federal/Local Government Interaction)**

From (16), the derivative  $\partial\tilde{q}/\partial P$  equals  $C + (c_0/2t) \cdot C'Q'(-F)QC$ . For the matrix product contained in the second term,  $C'Q'(-F)QC$ , the entry in row  $i$  and column  $l$  is

$$\sum_{j \in I} \sum_{k \in I} c_{ij}(-f_{jk})c_{kl},$$

for  $i, l = 0, \dots, n - 1$ . As the number of homevoter cities  $\bar{n}$  expands, said double sum becomes longer and hence, given that all of the double sum's terms involved are positive only, larger.  $\square$