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Rational Expectations Models with Anticipated Shocks and Optimal Policy

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Abstract

This paper investigates optimal policy in the presence of anticipated (or news) shocks. We determine the optimal unrestricted and restricted policy response in a general rational expectations model and show that, if shocks are news shocks, the optimal unrestricted control rule under commitment contains a forward-looking element. As an example, we lay out a micro-founded hybrid New Keynesian model and show i) that anticipated cost-push shocks entail higher welfare losses than unanticipated shocks of equal size and ii) that the inclusion of forward-looking elements enhances distinctly the performance of simple optimized interest rate rules.

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1 Introduction

Recently, a number of macroeconometric studies emphasized the role of anticipated shocks as sources of macroeconomic fluctuations. Beaudry and Portier (2006) find that more than one-half of business cycle fluctuations are caused by news concerning future technological opportunities. Davis (2007) and Fujiwara, Hirose, and Shintani (2008) analyze the importance of anticipated shocks in medium-scale New Keynesian DSGE models and find that these disturbances are important components of aggregate fluctuations. Schmitt-Grohé and Uribe (2008) conduct a Bayesian estimation of a real-business cycle model and find that anticipated shocks are the most important source of aggregate fluctuations. In particular, they show that anticipated shocks explain two thirds of the volatility in consumption, output, investment, and employment.

In light of these findings, we investigate optimal policy in a rational expectations models with news shocks. In particular, we asked whether the anticipation of future shocks has a stabilizing and therefore welfare-enhancing effect when compared to unanticipated shocks. Furthermore, we investigate whether news shocks change the structure of optimal policy rules. In order to conduct an analysis of the (welfare) effects of anticipated shocks, this paper presents a general solution method for linear dynamic rational expectations models with anticipated shocks and optimal policy. Our method extends the work of Söderlind (1999), who uses the generalized Schur decomposition method, advocated by Klein (2000), to solve linear rational expectations models with optimal policy. However, Söderlind (1999) only considers stochastic models with white noise shocks which are, by definition, unpredictable.

As an economic example, we lay out a calibrated New Keynesian model for a closed and cashless economy with internal habit formation in consumption preferences, a variant of Calvo price staggering with partial indexation to past inflation and a time-varying wage mark-up which represents a typical cost-push shock. We compare the effects of mark-up shocks under optimal monetary policy for different lengths of the anticipation period. We find i) that anticipated cost shocks entail higher welfare losses than unanticipated cost disturbances of equal magnitude and ii) that forward-looking elements enhances the

performance of simple optimized interest rate rules when agents learn about future disturbances in advance. We provide a rationale for this result by demonstrating that, if shocks are news shocks, the optimal unrestricted control rule under commitment contains as a basic principle a forward-looking element.

The paper is organized as follows. Section 2 discusses optimal policies in RE models with *anticipated* temporary shocks. We first determine the optimal unrestricted policy under precommitment and calculate the minimum value of the intertemporal loss function. We then consider (optimal) simple rules and demonstrate how the Schur decomposition can be used to solve the model under these conditions. Section 3 presents the hybrid New Keynesian model and discusses the effects of anticipated and unanticipated cost-push shocks. Finally, Section 4 provides concluding remarks.

2 The Model

In this paper we discuss the following linear rational expectational model

$$A \begin{pmatrix} w_{t+1} \\ \mathbf{E}_t v_{t+1} \end{pmatrix} = B \begin{pmatrix} w_t \\ v_t \end{pmatrix} + C u_t + D \nu_{t+1}, \quad (1)$$

where w_t is an $n_1 \times 1$ vector of predetermined variables, assuming w_0 given, v_t an $n_2 \times 1$ vector of non-predetermined variables, u_t an $m \times 1$ vector of policy instruments, and ν_{t+1} an $r \times 1$ vector of exogenous shocks. The matrices A and B are $n \times n$ (where $n = n_1 + n_2$), while the matrices C and D are $n \times m$ and $n \times r$ respectively. We allow matrix A to be singular which is the case if static (intratemporal) equations are included within the dynamic relationships. The vector w , composed of backward-looking variables, can include exogenous variables, following autoregressive processes. $\mathbf{E}_t v_{t+1}$ denotes model consistent (rational) expectations of v_{t+1} formed at time t . We assume that the shocks are anticipated

by the public in advance and take the following form

$$\nu_t = \begin{cases} \bar{\nu} & \text{for } t = \tau > 0 \\ 0 & \text{for } t \neq \tau, \end{cases} \quad (2)$$

where $\bar{\nu} = (\bar{\nu}_1, \dots, \bar{\nu}_r)'$ is a constant non-zero $r \times 1$ vector. It is assumed that at time $t = 0$ the public anticipates a shock of the form outlined in (2) to take place at some future date $\tau > 0$. Note that τ also defines the lengths of the anticipation period. Since shocks are anticipated by the public we have $E_t \nu_{t+1} = \nu_{t+1}$. For notational convenience, we define the $n \times 1$ vector $k_t = (w'_t, v'_t)'$. Assume that the policy maker's welfare loss at time t is given by

$$J_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \lambda^i \{k'_{t+i} \tilde{W} k_{t+i} + 2k'_{t+i} P u_{t+i} + u'_{t+i} R u_{t+i}\}, \quad (3)$$

where \tilde{W} and R are symmetric and non-negative definite, P is $n \times m$, and $0 < \lambda \leq 1$.

2.1 Optimal Policy with Precommitment

We are now going to develop the policy maker's optimal policy rule at time $t = 0$. It is assumed that the policy maker is able to commit to such a rule. From the Lagrangian

$$\mathcal{L}_0 = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \lambda^t \{k'_t \tilde{W} k_t + 2k'_t P u_t + u'_t R u_t + 2\rho'_{t+1} [Bk_t + C u_t + D \nu_{t+1} - A k_{t+1}]\} \quad (4)$$

with the $n \times 1$ multiplier ρ_{t+1} , we get the first-order conditions with respect to ρ_{t+1} , k_t , and u_t :

$$\begin{pmatrix} A & 0_{n \times m} & 0_{n \times n} \\ 0_{n \times n} & 0_{n \times m} & \lambda B' \\ 0_{m \times n} & 0_{m \times m} & -C' \end{pmatrix} \begin{pmatrix} k_{t+1} \\ u_{t+1} \\ \rho_{t+1} \end{pmatrix} = \begin{pmatrix} B & C & 0_{n \times n} \\ -\lambda \tilde{W} & -\lambda P & A' \\ P' & R & 0_{m \times n} \end{pmatrix} \begin{pmatrix} k_t \\ u_t \\ \rho_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} \nu_{t+1}. \quad (5)$$

To solve the system of equations in (5), expand the state and costate vector k_t and ρ_t as $(w'_t, v'_t)'$ and $(\rho'_{wt}, \rho'_{vt})'$ respectively and rearrange the rows of the $(2n + m) \times 1$ vector

$(k'_t, u'_t, \rho'_t)'$ by placing the predetermined vector ρ_{vt} after w_t . Since v_t is forward-looking with an arbitrarily chosen initial value v_0 , the corresponding Lagrange multiplier ρ_{vt} is predetermined with an initial value $\rho_{v0} = 0$. Rearrange the columns of the $(2n + m) \times (2n + m)$ matrices in (5) according to the re-ordering of $(k'_t, u'_t, \rho_t)'$ and write the result as

$$F \begin{pmatrix} \tilde{w}_{t+1} \\ \tilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} \nu_{t+1}, \quad (6)$$

where $\tilde{w}_t = (w'_t, \rho'_{vt})'$ and $\tilde{v}_t = (v'_t, u'_t, \rho'_{wt})'$. The $n \times 1$ vector \tilde{w}_t contains the 'backward-looking' variables of (5) while the $(n + m) \times 1$ vector \tilde{v}_t contains the 'forward-looking' variables.

Equation (5) implies that the $(2n+m) \times (2n+m)$ matrix F is singular. To solve equation (6) we apply the generalized Schur decomposition method (Söderlind, 1999; Klein, 2000). The decomposition of the square matrices F and G is given by $F = \bar{Q}' S \bar{Z}'$, $G = \bar{Q}' T \bar{Z}'$ or equivalently $Q F Z = S$, $Q G Z = T$, where Q, Z, S , and T are square matrices of complex numbers, S and T are upper triangular and Q and Z are unitary, i.e. $Q \cdot \bar{Q}' = \bar{Q}' \cdot Q = I_{(2n+m) \times (2n+m)} = Z \cdot \bar{Z}' = \bar{Z}' \cdot Z$, where the non-singular matrix \bar{Q}' is the transpose of \bar{Q} , which denotes the complex conjugate of Q . \bar{Z}' is the transpose of the complex conjugate of Z . The matrices S and T can be arranged in such a way that the block with the stable generalized eigenvalues (the i th diagonal element of T divided by the i th diagonal element of S) comes first. Premultiply both sides of equation (6) with Q and define auxiliary variables \tilde{z}_t and \tilde{x}_t so that

$$\begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} = \bar{Z}' \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix}. \quad (7)$$

Partitioning the triangular matrices S and T in order to conform with \tilde{z} and \tilde{x} . Then set

$$Q \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}, \quad (8)$$

where Q_1 is $n \times r$ and Q_2 is $(n+m) \times r$. We then obtain the equivalent system

$$\begin{pmatrix} S_{11} & S_{12} \\ 0_{(n+m) \times n} & S_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0_{(n+m) \times n} & T_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \nu_{t+1}, \quad (9)$$

where the $n \times n$ matrix S_{11} and the $(n+m) \times (n+m)$ matrix T_{22} are invertible while S_{22} is singular. The square matrix T_{11} may also be singular. The lower block of equation (9) contains the unstable generalized eigenvalues and must be solved forward. Since

$$\tilde{x}_{t+s} = M_2 \tilde{x}_{t+s+1} - T_{22}^{-1} Q_2 \nu_{t+s+1} \quad (s = 0, 1, 2, \dots), \quad (10)$$

where $M_2 = T_{22}^{-1} S_{22}$, the unique stable solution for \tilde{x}_t is given by

$$\tilde{x}_t = - \sum_{s=0}^{\infty} M_2^s T_{22}^{-1} Q_2 \nu_{t+s+1} \quad (11)$$

The upper block of (9) contains the stable generalized eigenvalues and can be solved backward. Since

$$\tilde{z}_{t+1} = M_1 \tilde{z}_t + S_{11}^{-1} (T_{12} \tilde{x}_t - S_{12} \tilde{x}_{t+1}) + S_{11}^{-1} Q_1 \nu_{t+1}, \quad (12)$$

where $M_1 = S_{11}^{-1} T_{11}$ (which in general is not invertible), the general solution is given by

$$\tilde{z}_t = M_1^t K + \sum_{s=0}^{t-1} M_1^{t-s-1} S_{11}^{-1} (T_{12} \tilde{x}_s - S_{12} \tilde{x}_{s+1} + Q_1 \nu_{s+1}) \quad (13)$$

The constant K can be determined using the initial value of the predetermined vector \tilde{w} . By premultiplying equation (7) with Z and by partitioning the matrix Z to conform

with the dimension of \tilde{z} and \tilde{x} , we obtain

$$\begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} \quad (14)$$

and therefore $\tilde{w}_0 = Z_{11}\tilde{z}_0 + Z_{12}\tilde{x}_0$ with $\tilde{w}_0 = (w'_0, 0'_{n_2 \times 1})'$, $\tilde{z}_0 = K$, and $\tilde{x}_0 = -M_2^{\tau-1}T_{22}^{-1}Q_2\bar{v}$, where it is assumed that $\tau > 0$.¹ The constant K is then given by $K = Z_{11}^{-1}\tilde{w}_0 - Z_{11}^{-1}Z_{12}\tilde{x}_0$, provided the inverse Z_{11}^{-1} exists. A necessary condition is that the dynamic system (6) has the saddle path property, i.e., that the number of backward-looking variables ($n_1 + n_2 = n$) coincides with the number of stable generalized eigenvalues (Söderlind, 1999; Klein, 2000).

If Z_{11} is invertible, equation (14) implies

$$\tilde{v}_t = N\tilde{w}_t + \hat{Z}\tilde{x}_t, \quad (15)$$

where $N = Z_{21}Z_{11}^{-1}$ and $\hat{Z} = Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}$. Write equation (15) as

$$\begin{pmatrix} v_t \\ u_t \\ \rho_{wt} \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \\ N_{31} & N_{32} \end{pmatrix} \begin{pmatrix} w_t \\ \rho_{vt} \end{pmatrix} + \begin{pmatrix} \hat{Z}_1 \\ \hat{Z}_2 \\ \hat{Z}_3 \end{pmatrix} \tilde{x}_t \quad (16)$$

and assume the $n_2 \times n_2$ matrix N_{12} is invertible. The optimal control rule under commitment or implicit instrument rule can then be written as

$$u_t = N_{21}w_t + N_{22}\rho_{vt} + \hat{Z}_2\tilde{x}_t, \quad (17)$$

To obtain a convergent adjustment process for $t \geq \tau$, \tilde{x}_t must be set equal to zero. Hence, for $t \geq \tau$, the vector of policy instruments, u_t , depends only on backward-looking state and costate variables.² For $t < \tau$, however, u_t depends on the auxiliary (jump) state

¹In the special case $\tau = 0$ (unanticipated shocks) we have $\tilde{x}_0 = 0$ and $\tilde{z}_t = (S_{11}^{-1}T_{11})^t K + (S_{11}^{-1}T_{11})^t S_{11}^{-1}Q_1\bar{v}$ implying $\tilde{z}_0 = K + S_{11}^{-1}Q_1\bar{v}$ and $K = Z_{11}^{-1}\tilde{w}_0 - S_{11}^{-1}Q_1\bar{v}$ with $w_0 \neq 0$. By contrast, the initial value w_0 can be normalized to zero if $\tau > 0$.

²For the case of unanticipated shocks ($\tau = 0$) the history dependence of the optimal control rule was already shown by Currie and Levine (1993).

variable \tilde{x}_t of the generalized Schur-transformed system (9).³

It is well-known, however, that such an optimal unrestricted control rule can not be implemented as an explicit instrument rule for two reasons. First, it leads to an indeterminacy problem. Second, the rule is rather complicated since it depends on all endogenous model variables as well as on the exogenous shock processes. However, from the structure of the optimal control rule we can infer that an optimal simple monetary policy rule should also contain a forward-looking element. We demonstrate that this conjecture is indeed true by evaluating optimal simple rules for a hybrid New Keynesian model.

To determine the minimum value of the loss function J_t at time $t = 0$, we express J_t as function of \tilde{w} and \tilde{v} . The loss function (3) can be written as

$$J_t = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i (k'_{t+i}, u'_{t+i}) H \begin{pmatrix} k_{t+i} \\ u_{t+i} \end{pmatrix} = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i (w'_{t+i}, v'_{t+i}, u'_{t+i}) H \begin{pmatrix} w_{t+i} \\ v_{t+i} \\ u_{t+i} \end{pmatrix}, \quad (18)$$

where the $(n + m) \times (n + m)$ matrix H is given by

$$H = \begin{pmatrix} \tilde{W} & P \\ P' & R \end{pmatrix} \quad (19)$$

with $H = H'$.

2.2 (Optimal) Simple Rules

The policy maker could alternatively commit to a suboptimal simple rule of the form

$$u_t = \Lambda k_t + \Psi E_t k_{t+1}, \quad (20)$$

³If the $n_2 \times n_2$ matrix N_{12} is invertible it is possible to substitute the predetermined costate variables $p_{v,t}$ by the original state variables v_t and w_t . The implicit instrument rule can then alternatively be expressed as $\dot{i}_t = \Gamma_v v_t + \Gamma_w w_t + \Gamma_{\tilde{x}} \tilde{x}_t$, where $\Gamma_v = N_{22} N_{12}^{-1}$, $\Gamma_w = N_{21} - N_{22} N_{12}^{-1} N_{11}$, and $\Gamma_{\tilde{x}} = -\hat{Z}_2 + N_{22} N_{12}^{-1} \hat{Z}_1$. Since $\tilde{x}_t = 0$ for $t \geq \tau$, the optimal control rule is only a linear function of the original state variables v_t and w_t . For $t < \tau$, the auxiliary jump variable \tilde{x}_t can not be eliminated from the optimal implicit instrument rule.

where the constant matrices Λ and Ψ are $m \times n$. Assuming rational expectations and exogenous shocks of the form (2) which are anticipated in $t = 0$, we obtain the dynamic system

$$\begin{pmatrix} A & 0_{n \times m} \\ \Psi & 0_{m \times m} \end{pmatrix} \begin{pmatrix} k_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} B & C \\ -\Lambda & I_{m \times m} \end{pmatrix} \begin{pmatrix} k_t \\ u_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix} \nu_{t+1}. \quad (21)$$

The generalized Schur decomposition yields the system of equations

$$F \begin{pmatrix} \tilde{w}_{t+1} \\ \tilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix} \nu_{t+1}, \quad (22)$$

where $\tilde{w} = w$ is an $n_1 \times 1$ vector, $\tilde{v} = (v', u)'$ is an $(n_2 + m) \times 1$ vector and where the square matrices F and G are $(n + m) \times (n + m)$ with the decomposition $QFZ = S$ and $QGZ = T$, where Q , Z , S , and T are $(n + m) \times (n + m)$ matrices. Since

$$\begin{pmatrix} \tilde{w} \\ \tilde{v} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \tilde{z} \\ \tilde{x} \end{pmatrix}, \quad (23)$$

the matrices Z_{11} , Z_{12} , Z_{21} , and Z_{22} are now $n_1 \times n_1$, $n_1 \times (n_2 + m)$, $(n_2 + m) \times n_1$, and $(n_2 + m) \times (n_2 + m)$ respectively. The auxiliary variables \tilde{z} and \tilde{x} satisfy the following system of equations

$$\begin{pmatrix} S_{11} & S_{12} \\ 0_{(n_2+m) \times n_1} & S_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0_{(n_2+m) \times n_1} & T_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \nu_{t+1}, \quad (24)$$

where S_{11} and T_{11} are $n_1 \times n_1$ matrices, S_{22} and T_{22} are $(n_2 + m) \times (n_2 + m)$ and S_{12} and T_{12} are $n_1 \times (n_2 + m)$. The matrices Q_1 and Q_2 are $n_1 \times r$ and $(n_2 + m) \times r$ respectively with

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = Q \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix}. \quad (25)$$

The solution of (24) is given by (11) and (13). For $t \geq \tau$, we obtain $\tilde{v}_t = N\tilde{w}_t = Nw_t$, where $N = Z_{21}Z_{11}^{-1}$ is now an $(n_2 + m) \times n_1$ matrix.

The loss function (18) simplifies to

$$J_t = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i(w'_{t+i}, \tilde{v}'_{t+i}) H \begin{pmatrix} w_{t+i} \\ \tilde{v}_{t+i} \end{pmatrix}. \quad (26)$$

The minimization of J_0 with respect to the coefficients of the matrices Λ and Ψ yields an optimal simple rule of the form (20).

3 Example: A Hybrid New Keynesian Model

The model is a standard New Keynesian model for a closed and cashless economy with monopolistic competition in goods and labor markets and with the additional features of internal habit formation in consumption preferences, a variant of the Calvo (1983) mechanism with partial indexation of non-optimized prices to past inflation, and a time-varying wage mark-up which represents a cost-push shock.⁴

After log-linearization, the model consists of hybrid IS and Phillips curves. The hybrid IS curve is given by

$$\hat{Y}_t = \kappa_1 \hat{Y}_{t-1} + \kappa_2 E_t \hat{Y}_{t+1} - \kappa_3 E_t \hat{Y}_{t+2} - \kappa_4 (\hat{R}_t - E_t \hat{\pi}_{t+1}), \quad (27)$$

where \hat{Y}_t is the percentage deviation of output from its steady state, \hat{R}_t is the percentage deviation of the nominal interest rate, and $\hat{\pi}_t$ is the percentage deviation of inflation.

The parameters are as follows: $\kappa_1 = \frac{h}{1+h+\beta h^2}$, $\kappa_2 = \frac{1+\beta h+\beta h^2}{1+h+\beta h^2}$, $\kappa_3 = \frac{\beta h}{1+h+\beta h^2}$, and $\kappa_4 = \frac{(1-h)(1-\beta h)}{\sigma(1+h+\beta h^2)}$. β is the discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $h \geq 0$ measures the degree of internal habit formation in consumption preferences.

Note that the term $-\kappa_3 E_t \hat{Y}_{t+2}$ results from the assumption of internal habit formation. In the alternative case of external habit formation (or "keeping up with the Joneses") the IS curve will be a linear function only of \hat{Y}_{t-1} , $E_t \hat{Y}_{t+1}$, and of the

⁴Similar models are used by Smets and Wouters (2003), Giannoni and Woodford (2004), or Casares (2006).

real interest rate, $\hat{R}_t - E_t \hat{\pi}_{t+1}$. Note that for $h = 0$, we obtain the purely forward-looking New Keynesian IS curve.

Under the assumption of Calvo price setting with partial indexation of non-optimized prices to past inflation, the log-linearized price setting equation can be written as

$$\hat{\pi}_t = \omega_1 E_t \hat{\pi}_{t+1} + \omega_2 \hat{\pi}_{t-1} + \Theta \hat{m}c_t, \quad (28)$$

where $\hat{m}c_t$ denotes linearized real marginal costs, $\omega_1 = \frac{\beta}{1+\beta\gamma}$, $\omega_2 = \frac{\gamma}{1+\beta\gamma}$, and $\Theta = \frac{(1-\beta\theta)(1-\theta)}{\theta(1+\beta\gamma)}$. γ is the degree of price indexation and θ is the usual Calvo parameter. Note that for $\gamma = 0$, equation (28) collapses into a purely forward-looking price setting equation.

By assuming monopolistic competition in labor markets and a constant returns to scale technology with labor as the only input factor in production, we obtain the following relation between real marginal costs, $\hat{m}c_t$, the real wage, \hat{w}_t , the marginal rate of substitution between consumption and leisure, $\hat{m}\hat{r}s_t$, and the output gap \hat{Y}_t .⁵

$$\hat{m}c_t = \hat{w}_t = \hat{\lambda}_{w,t} + \hat{m}\hat{r}s_t = \hat{\lambda}_{w,t} + (\eta + \delta_1)\hat{Y}_t - \delta_2\hat{Y}_{t-1} - \beta\delta_2 E \hat{Y}_{t+1}, \quad (29)$$

where $\eta > 0$ is the inverse of the Frisch elasticity of labor supply, $\delta_1 = \frac{\sigma(1+\beta h^2)}{(1-h)(1-\beta h)}$, and $\delta_2 = \frac{h\sigma}{(1-h)(1-\beta h)}$. The log-linearized wage mark-up $\hat{\lambda}_{w,t}$ is described by the AR(1) process

$$\hat{\lambda}_{w,t} = \xi_w \hat{\lambda}_{w,t-1} + e_t. \quad (30)$$

The dependence of real marginal costs on past and future output is driven by the assumption of internal habit formation in consumption. Note that for $h = 0$, we obtain $\hat{m}c_t = \hat{\lambda}_{w,t} + (\eta + \sigma)\hat{Y}_t$.

By inserting equation (29) into equation (28), we obtain a hybrid New Keynesian

⁵Note that in our model the level of output in the absence of nominal rigidities (the natural level) Y_t^n is constant. Thus, the linearized output \hat{Y}_t coincides with the linearized output gap $\hat{Y}_t^g = \hat{Y}_t - \hat{Y}_t^n$, where $\hat{Y}_t^n = 0$.

Phillips curve of the following form

$$\hat{\pi}_t = \omega_1 \text{E}_t \hat{\pi}_{t+1} + \omega_2 \hat{\pi}_{t-1} + \omega_3 \hat{Y}_t - \omega_4 \hat{Y}_{t-1} - \beta \omega_4 \text{E}_t \hat{Y}_{t+1} + \Theta \hat{\lambda}_{w,t}, \quad (31)$$

where $\omega_3 = \Theta(\eta + \delta_1)$ and $\omega_4 = \Theta\delta_2$. Note that for $\gamma = h = 0$, this equation collapses into the purely forward-looking New Keynesian Phillips curve.

Following Woodford (2003, Ch. 6) and Giannoni and Woodford (2004), a second-order approximation to the households' utility yields a loss function of the form

$$J_0 = \text{E}_0 \sum_{t=0}^{\infty} \lambda^t \left((\hat{\pi}_t - \gamma \hat{\pi}_{t-1})^2 + \alpha_Y (\hat{Y}_t - \delta \hat{Y}_{t-1})^2 \right), \quad (32)$$

where $\alpha_Y = \frac{\Theta h \sigma \lambda_p}{(1 + \lambda_p) \delta (1 - \beta h) (1 - h)}$, $(1 + \lambda_p)$ is the mark-up in the goods market, and δ is the smaller root of the quadratic equation

$$\frac{h\sigma}{(1 - \beta h)(1 - h)} (1 + \beta \delta^2) = \left(\eta + \frac{\sigma}{(1 - \beta h)(1 - h)} (1 + \beta h^2) \right) \delta. \quad (33)$$

We follow Giannoni and Woodford (2004) and Casares (2006) by assuming that the monetary authority is concerned about the volatility of the nominal interest rate. Therefore, we augment the welfare-theoretic loss function by the additional term $\alpha_R \hat{R}_t^2$, where α_R measures the weight on interest rate stabilization.

The monetary authority then seeks to minimize the loss function

$$J_0 = \text{E}_0 \sum_{t=0}^{\infty} \lambda^t \left((\hat{\pi}_t - \gamma \hat{\pi}_{t-1})^2 + \alpha_Y (\hat{Y}_t - \delta \hat{Y}_{t-1})^2 + \alpha_R \hat{R}_t^2 \right), \quad (34)$$

subjected to the equations (30), (31), and (27). Note that in our model, the discount factor of the policy-maker, λ , is equal to the household's discount factor β .

In order to solve the model by using the methods outlined in Section 2, we define the policy objective parameters $\hat{Y}_t^o = \hat{Y}_t - \delta \hat{Y}_{t-1}$ and $\hat{\pi}_t^o = \hat{\pi}_t - \gamma \hat{\pi}_{t-1}$. Furthermore, we define the auxiliary variables $\tilde{\pi}_t = \hat{\pi}_{t-1}$, $\tilde{Y}_t = \hat{Y}_{t-1}$, and $s_t = \text{E}_t \hat{Y}_{t+1}$. If we add the definition of the real interest rate $\hat{r}_t = \hat{R}_t - \text{E}_t \hat{\pi}_{t+1}$, we finally obtain a 3×1 vector w_t of predetermined

variables given by $w_t = (\hat{\lambda}_{w,t}, \tilde{\pi}_t, \tilde{Y}_t)'$, a 6×1 vector v_t of non-predetermined variables given by $v_t = (\hat{\pi}_t, \hat{Y}_t, s_t, \hat{r}_t, \hat{\pi}_t^o, \hat{Y}_t^o)'$, the vector of policy instruments u_t which is simply the scalar $u_t = \hat{R}_t$, and the 1×1 shock vector $\nu_t = e_t$.

We complete the description of the model by presenting the calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$, implying a quarterly steady-state real interest rate of approximately one percent. The intertemporal elasticity of substitution is assumed to be $\sigma = 2$. We follow Casares (2006) and set the habit formation parameter to $h = 0.85$ implying that the weight on lagged output in the IS equation is $1/3$. The calibrated $\eta = 3$ implies a labor supply elasticity with respect to the real wage of $1/3$. λ_p is set to $8/7$ which implies a steady-state mark-up in the goods market of approximately 14 percent. We assume the linearized wage mark-up $\hat{\lambda}_{w,t}$ to be persistent and choose ξ_w equal to 0.8. In our baseline scenario, the Calvo parameter θ is set to 0.75 implying an average duration of price contracts of one year. In order to check whether our welfare results will also hold in the case of flexible price adjustment, we set $\theta = 0.05$. The price indexation parameter γ is set to 0.45 which is roughly equal to the value reported by Smets and Wouters (2003). This implies that the weight on lagged inflation in the Phillips curve equation is 0.31.

The parameter values chosen for our model imply a weight on output in the policy-makers' objective function of approximately $\alpha_Y = 0.69$. Following Casares (2006), we set $\alpha_R = 0.0088$ implying a small preference for interest rate smoothing.

For the analysis concerning anticipated and unanticipated shocks, we assume that the economy is in a deterministic steady-state until period $t = 0$. In the case of an unanticipated shock, the mark-up $\hat{\lambda}_{w,t}$ jumps by one percent in period $t = 0$ and begins to fall thereafter. In the case of an anticipated shock, the agents anticipate in period $t = 0$ that a one percent increase in the mark-up will take place at some future date $\tau > 0$. They also know that the mark-up will subsequently decline according to the autoregressive process (30), where now $e_t = 1$ for $t = \tau$ and $e_t = 0$ for $t \neq \tau$. Note that τ also defines the length of the anticipation period or the time interval between $t = 0$ and $t = \tau$. In order to obtain impulse response functions and welfare results, we simulate the dynamic

adjustment paths and the welfare loss function by using the methods outlined in Section 2.

3.1 Unrestricted Optimal Monetary Policy

Figure 1 depicts the impulse response functions of inflation, output, nominal, and real interest rates under unrestricted optimal monetary policy. The solid lines with circles represent the responses to an unforeseen cost-push shock that emerged in period $t = 0$. The solid lines with squares, triangles, and stars represent responses to a cost-push shock whose realization in period $\tau = 1$, $\tau = 2$, or $\tau = 3$ is anticipated in period $t = 0$.

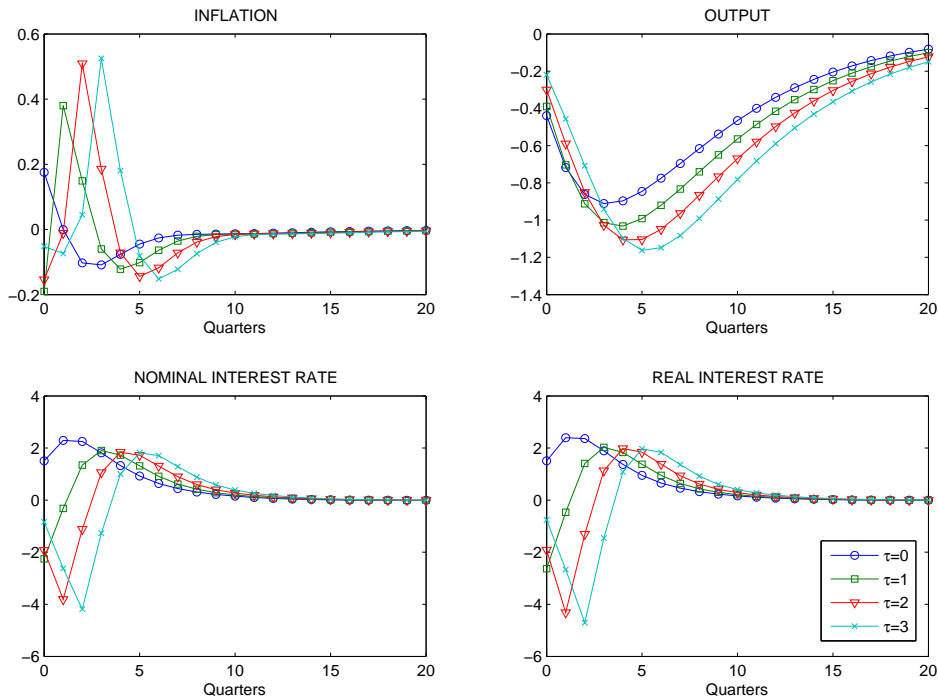


Figure 1: Impulse response functions under unrestricted optimal monetary policy.

Notes: Solid lines with circles denote responses to an unanticipated cost-push shock, solid lines with squares, triangles, and stars denote responses to an anticipated cost-push shock taking place in period $\tau = 1$, $\tau = 2$, and $\tau = 3$.

An *unanticipated* rise in the wage mark-up puts upward pressure on the prices of intermediate goods and hence on inflation. Despite the instantaneous jump in inflation, the real interest rate rises due to the sharp increase in the nominal interest rate. The increase in the real interest rate induces households to postpone consumption implying

an abrupt drop in output. Subsequently, the nominal interest rate continues to rise. This leads – in conjunction with the decline in inflation – to hump-shaped response functions of the real interest rate and output.

In the case of *anticipated* shocks, the optimal policy calls for a decline in nominal and real interest rates in response to the anticipation of a future rise in marginal costs. At the latest with the occurrence of the anticipated shock in period τ , the nominal and real interest rates start to rise and display a hump-shaped development. Inflation declines in response to the anticipation of the future rise in marginal costs. After this initial decline, inflation starts to rise and peaks in the period when the anticipated shock materializes. Output displays a hump-shaped downturn, starting at the point of anticipation, $t = 0$. The drop in output is thereby amplified by the lengths of the anticipation period τ .

A visual inspection of the impulse response functions suggests that the anticipation of future shocks leads to an increase in the volatility of inflation, output as well as nominal and real interest rates which is in turn increasing with the lead time τ . This observation is in line with the results of Fève, Matheron, and Sahuc (2009). They consider a stylized one-dimensional purely forward-looking rational expectations model with an anticipated shock and demonstrate that the volatility of the endogenous variable is an increasing function of the length of the time period between the anticipation and the realization of the shock, τ .

The destabilizing effect of anticipations does not imply necessarily that the welfare loss which is a positive function of economic volatility is also increasing with the lead time τ . The reason is the opposite effect of discounting the future impacts of the shock from period τ to period $t = 0$.⁶

The welfare effects of anticipations are shown in Figure 2 which displays the welfare loss, J_0 , as a function of the time span between the anticipation and the occurrence of the cost-push shock (a) in our baseline model; (b) in the case of no price indexation, $\gamma = 0$; (c) in the case of no habit formation in consumption, $h = 0$; (d) in the case of a

⁶This is true although the impact or anticipation effect which measures the initial jump of a variable taking place at the time of anticipation is inversely related to the time span between the anticipation and the realization of the cost-push shock.

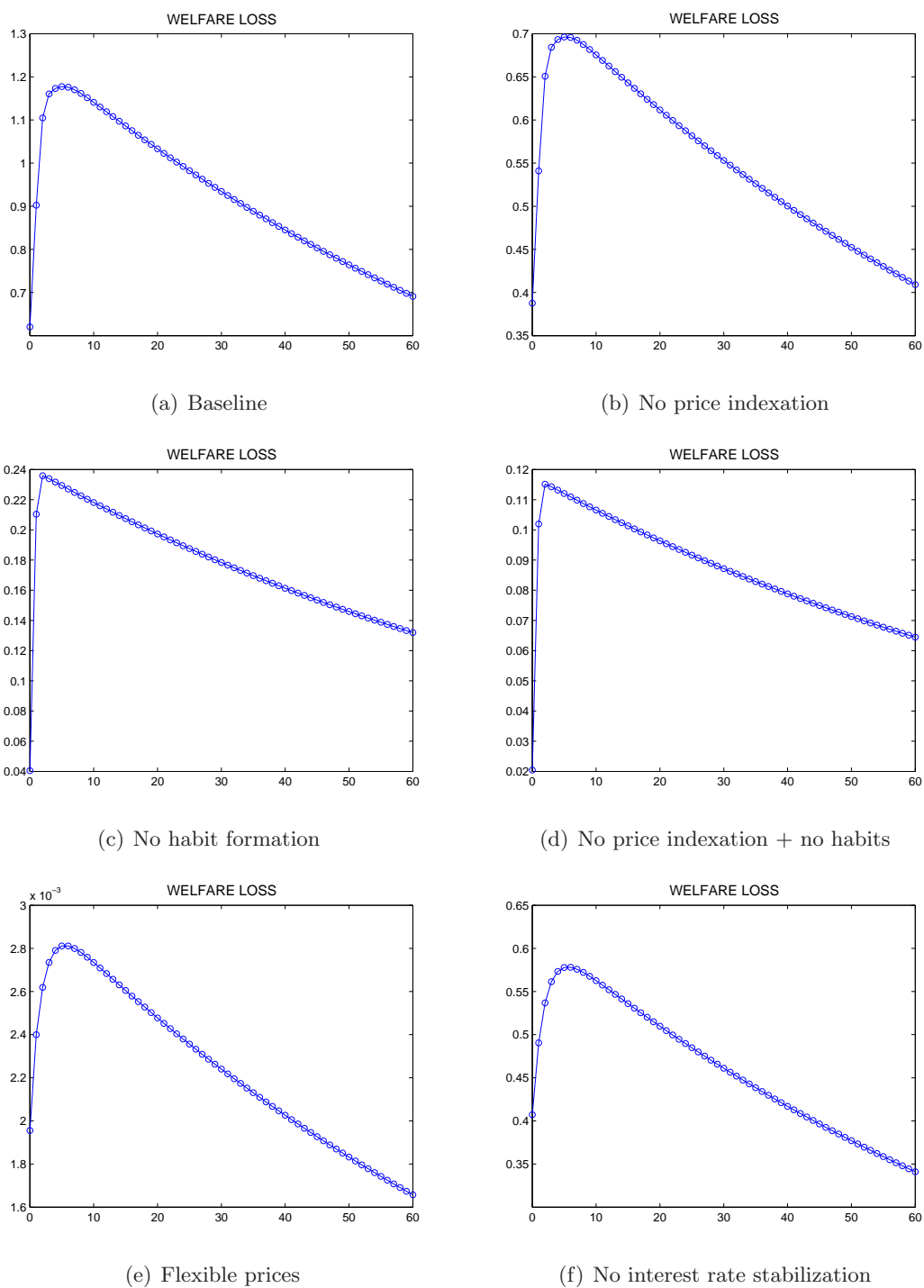


Figure 2: Welfare loss for different lengths of the anticipation period under unrestricted optimal monetary policy

purely forward-looking New Keynesian model, $\gamma = h = 0$; (d) in the case of highly flexible prices, $\theta = 0.05$; (e) in the case where the policy maker does not care about interest rate

stabilization, $\alpha_R = 0$.

The welfare function exhibits – independent of the model specifications – a hump-shaped pattern implying that for a realistic time span between the anticipation and the realization of cost-push shocks, anticipated disturbances entail *higher* welfare losses than unanticipated disturbances of equal size. The rationale is that the discounting effect is dominated by the volatility effect. A welfare gain from anticipating can only be achieved for very large (and unrealistic) values of τ .

3.2 Optimal Simple Monetary Policy Rules

In the following, we will check correctness of the conjecture that simple rules which include forward-looking elements perform better when the economy is hit by news shock. In order to do so, we consider a set of possible simple interest rate rules and minimize the policy maker’s loss function with respect to the coefficients of the respective rule. The rules considered are variants of the interest rate rule proposed by Taylor (1993) which describes the nominal interest rate as a linear function, f , of current inflation, π_t , and the current output gap, \hat{Y}_t . We employ the following forward-looking variants of the Taylor rule: i) $\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{\pi}_{t+1})$, ii) $\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{Y}_{t+1})$, iii) $\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$, iv) $\hat{R}_t = f(\hat{Y}_t, E_t \hat{\pi}_{t+1})$, v) $\hat{R}_t = f(\hat{\pi}_t, E_t \hat{Y}_{t+1})$, and vi) $\hat{R}_t = f(E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$. We consider three different values for the length of the anticipation period, τ : $\tau = 0$, $\tau = 3$, and $\tau = 8$. $\tau = 0$ implies an unanticipated shock, $\tau = 3$ and $\tau = 8$ imply that agents learn about the exogenous disturbance three and eight quarters in advance, respectively.⁷

The results of our numerical simulations are shown in Table 1. It is shown that the inclusion of forward-looking elements is significantly welfare-improving if the occurrence of the shock is anticipated in advance. Take, for instance, the rule $\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$, where the interest rate reacts not solely to current inflation and output gap, but also to the expected future values $\hat{\pi}_{t+1}$ and \hat{Y}_{t+1} . This rule performs significantly better than the original Taylor rule for both $\tau = 3$ and $\tau = 8$. Note, however,

⁷Note that a rule which is found to be optimal in the case of an unanticipated shock will not be optimal in the case of, for instance, $\tau = 3$. Therefore, we reoptimize the coefficients of a given rule when τ changes. This approach is necessary for a reasonable comparison of different optimal simple rules given a specific timing of the exogenous disturbance.

Table 1: Performance of optimal simple rules

Rule	$\tau = 0$		$\tau = 3$		$\tau = 8$	
	Loss	Relative loss	Loss	Relative loss	Loss	Relative loss
Unrestricted optimal policy	0.62	100.00	1.16	100.00	1.16	100.00
$\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t)$	0.67	108.71	1.74	150.33	1.75	150.21
$\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{\pi}_{t+1})$	0.67	107.26	1.51	130.09	1.58	136.24
$\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{Y}_{t+1})$	0.65	104.86	1.54	132.41	1.62	139.62
$\hat{R}_t = f(\hat{\pi}_t, \hat{Y}_t, E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$	0.65	105.31	1.40	120.77	1.41	121.21
$\hat{R}_t = f(\hat{Y}_t, E_t \hat{\pi}_{t+1})$	0.69	110.96	4.11	353.91	5.07	436.59
$\hat{R}_t = f(\hat{\pi}_t, E_t \hat{Y}_{t+1})$	0.68	109.23	1.74	149.50	1.74	149.53
$\hat{R}_t = f(E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$	0.69	111.53	4.04	348.11	4.98	428.45

Note: The relative loss is the percentage of the absolute loss from the simple rule relative to the absolute loss from the optimal unrestricted monetary policy.

that purely forward-looking rules that react not at all on current economic conditions such as $\hat{R}_t = f(E_t \hat{\pi}_{t+1}, E_t \hat{Y}_{t+1})$ perform remarkably worse than an optimized standard Taylor rule. But this is completely in line with the conclusion drawn from the inspection of the optimal control rule (17) which contains current state variables *and* forward-looking elements if the policy maker is faced with anticipated shocks. It is worth noticing that rules which do not include the actual level of inflation perform distinctly worse.

4 Conclusion

In this paper, we presented a method to solve linear dynamic rational expectations models with anticipated shocks and optimal policy by using the generalized Schur decomposition method. Furthermore, we determine the optimal unrestricted and restricted policy responses to anticipated shocks. Our approach also allows for the evaluation of the widely discussed case of unpredictable shocks and can therefore be seen as a generalization of the methods summarized by Söderlind (1999).

We demonstrated our method by means of a calibrated New Keynesian model with internal habit formation in consumption preferences, a variant of Calvo price staggering with partial indexation to past inflation, a time-varying wage mark-up which represents a typical cost-push shock, and a utility-based loss function. We simulated the model economy's responses to unanticipated and anticipated cost-push shocks under the unrestricted optimal monetary policy. We showed that anticipated shocks amplify both, the

stagflationary effects of cost-push shocks and the overall welfare loss.

This paper offered a novel insight about the optimal conduct of monetary policy by demonstrating that anticipated shocks provide a rationale for the inclusion of forward-looking elements in optimal monetary policy rules. We demonstrated that the optimal implicit instrument rule derived from the solution of an optimal control problem comprises a forward-looking element when disturbances are anticipated in advance. In the standard case of unanticipated shocks, this rule is only a linear function of backward-looking state variables. We infer from this general result that optimal simple (monetary) policy rules should also contain a forward-looking element. We show that this conjecture is indeed true by evaluating a set of optimal simple rules within the hybrid New Keynesian model. For anticipated shocks, we find that partly forward-looking simple rules are welfare-enhancing when compared to a standard optimized Taylor rule. However, consistent with our theoretical result, the inclusion of forward-looking elements does not significantly enhance the performance of optimal simple rules if shocks occur unexpectedly.

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