

Paloyo, Alfredo R.

**Working Paper**

## When Did We Begin to Spell "Heteros\*edasticity" Correctly?

Ruhr Economic Papers, No. 300

**Provided in Cooperation with:**

RWI – Leibniz-Institut für Wirtschaftsforschung, Essen

*Suggested Citation:* Paloyo, Alfredo R. (2011) : When Did We Begin to Spell "Heteros\*edasticity"  
Correctly?, Ruhr Economic Papers, No. 300, ISBN 978-3-86788-345-0, Rheinisch-Westfälisches  
Institut für Wirtschaftsforschung (RWI), Essen

This Version is available at:

<https://hdl.handle.net/10419/61670>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



# RUHR

ECONOMIC PAPERS

Alfredo R. Paloyo

## When Did We Begin to Spell “Heteros\*edasticity” Correctly?

# Imprint

## Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics  
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences  
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics  
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)  
Hohenzollernstr. 1-3, 45128 Essen, Germany

## Editors

Prof. Dr. Thomas K. Bauer  
RUB, Department of Economics, Empirical Economics  
Phone: +49 (0) 234/3 22 83 41, e-mail: [thomas.bauer@rub.de](mailto:thomas.bauer@rub.de)

Prof. Dr. Wolfgang Leininger  
Technische Universität Dortmund, Department of Economic and Social Sciences  
Economics – Microeconomics  
Phone: +49 (0) 231/7 55-3297, email: [W.Leininger@wiso.uni-dortmund.de](mailto:W.Leininger@wiso.uni-dortmund.de)

Prof. Dr. Volker Clausen  
University of Duisburg-Essen, Department of Economics  
International Economics  
Phone: +49 (0) 201/1 83-3655, e-mail: [vclausen@vwl.uni-due.de](mailto:vclausen@vwl.uni-due.de)

Prof. Dr. Christoph M. Schmidt  
RWI, Phone: +49 (0) 201/81 49-227, e-mail: [christoph.schmidt@rwi-essen.de](mailto:christoph.schmidt@rwi-essen.de)

## Editorial Office

Joachim Schmidt  
RWI, Phone: +49 (0) 201/81 49-292, e-mail: [joachim.schmidt@rwi-essen.de](mailto:joachim.schmidt@rwi-essen.de)

## Ruhr Economic Papers #300

Responsible Editor: Christoph M. Schmidt

All rights reserved. Bochum, Dortmund, Duisburg, Essen, Germany, 2011

ISSN 1864-4872 (online) – ISBN 978-3-86788-345-0

The working papers published in the Series constitute work in progress circulated to stimulate discussion and critical comments. Views expressed represent exclusively the authors' own opinions and do not necessarily reflect those of the editors.

---

**Ruhr Economic Papers #300**

Alfredo R. Paloyo

**When Did We Begin to Spell  
“Heteros\*edasticity” Correctly?**

## Bibliografische Informationen der Deutschen Nationalbibliothek

---

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über:  
*<http://dnb.d-nb.de>* abrufbar.

ISSN 1864-4872 (online)  
ISBN 978-3-86788-345-0

---

Alfredo R. Paloyo<sup>1</sup>

## When Did We Begin to Spell “Heteros\*edasticity” Correctly?

### Abstract

*Using digitized texts scanned by Google and subjected to optical character recognition, I show that heteroskedasticity overtook heteroscedasticity as the preferred spelling in 2001 and has continued to dominate, except for 2005, up to 2008. The latest trends indicate that writers are moving toward the k variant. However, for words such as homoskedasticity, heteroskedastic, and homoskedastic, the corresponding spellings using c are still overwhelmingly dominant, albeit slowly shifting.*

*JEL Classification: A20, B19, B29*

*Keywords: Heteroskedasticity; Culturomics; Google Books; econometric orthography; philology*

*December 2011*

---

<sup>1</sup> RWI. – I thank Vaia Karapanou for comments with respect to the Greek words in this manuscript, and Dominik Cremer-Schulte and Florent Fremigacci for help with the French ones. Thanks are also due to Ronald Bachmann, Christoph M. Schmidt, and Colin Vance for constructive suggestions, as well as Trevor Breusch, Jerry Hausman, Kenneth D. West, and Halbert White for responding to queries about their influences concerning their orthographic preferences. I am extremely grateful to J. Huston McCulloch and Lilian Coronel, who both provided an extended commentary on an earlier draft which substantially improved the paper. – All correspondence to Alfredo R. Paloyo, RWI, Hohenzollernstr. 1-3, 45128 Essen, Germany. E-mail: [alfredo.paloyo@rwi-essen.de](mailto:alfredo.paloyo@rwi-essen.de).

# 1 Introduction

In a brief article in *Econometrica* [1985b], J. Huston McCulloch advanced that “[t]he most pressing issue in econometric orthography today is whether heteros\*edasticity should be spelled with a k or with a c.” At the time his note went into press, the majority of published manuscripts spelled it as *heteroscedasticity*. Arguing that since it was coined directly from Greek source words into English without French or Latin distillation, he declared confidently that “[h]eteroskedasticity is therefore the proper English spelling.”

Within an econometric context, heteroskedasticity is the phenomenon where the random disturbance term exhibits a nonconstant conditional variance. Consider the linear regression model in matrix notation:  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ . Let the observation index  $i$  run from  $1, \dots, N$ . Heteroskedasticity is present if the conditional variance of the error term,  $\boldsymbol{\epsilon}$ , is  $\text{Var}[\epsilon_i|\mathbf{X}] = \sigma_i^2$ , i.e., if the spread or dispersion of  $\boldsymbol{\epsilon}$  is a function of specific values of  $\mathbf{X}$ . A stock example is the variance of food expenditure conditional on income, where higher income corresponds to a higher variance in food outlay. This has implications for the consistent estimation of the standard errors of the estimated coefficients,  $\widehat{\text{s.e.}}(\widehat{\boldsymbol{\beta}})$ . The details are beyond the scope of this article but a textbook treatment of the topic is readily available for both novices [Gujarati and Porter 2008] and experts [Wooldridge 2010].

Using digitized texts scanned by Google and subjected to optical character recognition (OCR), I show that *heteroskedasticity* did in fact eventually overtake *heteroscedasticity* as the preferred spelling but it only did so consistently 15 years after the publication of McCulloch [1985b]. However, for the noun *homoskedasticity* and the adjectives *heteroskedastic* and *homoskedastic*, the corresponding spellings using *c* still dominate although the recent trend is to move toward the *k* variants. For instance, *homoskedasticity* recently surpassed *homoscedasticity* in published books.

## 2 The etymology of heteroskedasticity

Heteroskedasticity has its roots in two Greek words, *éteros*, meaning “other” or “different” and *skedánnymi*, meaning “to scatter” [McCulloch 1985b].<sup>1</sup> The word *skedánnymi* actually comes from Ancient Greek; its Modern Greek rendition is *skorpízo*. There is no contention regarding the first part of the word but other sources attribute different root words for *-skedasticity*, such as *skedastós* (“capable of being scattered”<sup>2</sup>) and *skédasis* (“scattering” or “dispersion”<sup>3</sup>). In its entry on *homoscedasticity*, the Merriam–Webster dictionary refers to *skedastikos* (“able to disperse”), which comes from *skedannynai* (“to disperse”).<sup>4</sup> Clearly, however, these alternative root words are merely derivatives of the same idea. The word in question is therefore a neoclassical compound for it is a combination of Ancient Greek words (what the Oxford English Dictionary calls “combining forms”) and is imbued with a specific technical meaning.

McCulloch mentioned in a footnote that he found the earliest use of either *heteroskedasticity* or *heteroscedasticity* in a 1923 statistics text by Truman L. Kelley. He does not quote the original text at the request of the editor but he presumably referred to the following instance on Sec. 48 (“Properties of correlation surfaces”) of the book *Statistical Method*, p. 172: “If the standard deviation of the successive  $x$  arrays are equal, the distribution is homoscedastic in the  $x$  variable...”. Aldrich (see Footnote 2) and David [1998]<sup>5</sup> write, however, that the term was introduced earlier by Karl Pearson in 1905 in the article “On the General Theory of Skew Correlation and Non-linear Regression”.<sup>6</sup>

---

<sup>1</sup>McCulloch spells it as *skedánnymi* but I render the Greek letter  $\nu$  as  $\gamma$  to emphasize its earlier correspondence with the Latin  $\gamma$ . This is also the transliteration recommended by ISO 843:1997. In Greek, heteroskedasticity is *heteroskedastikótita*.

<sup>2</sup>This is from the contribution of John Aldrich to the website *Earliest Known Uses of Some of the Words of Mathematics* maintained by Miller [2011]: <http://jeff560.tripod.com/h.html>.

<sup>3</sup>This is from the Wikipedia entry on *heteroscedasticity*, which incidentally uses the  $c$  variant (replacing the first  $c$  with a  $k$  in the following will redirect the browser to the former): <http://en.wikipedia.org/wiki/Heteroscedasticity>.

<sup>4</sup>See <http://www.merriam-webster.com/dictionary/homoscedasticity>. Schwartzman [1994] traces the roots of the Greek words even further to the Indo-European *sek*, *sked*, and *skel*, which ultimately gave rise to the English words such as *schizophrenic*, *shatter*, *scoliosis*, and *isosceles*.

<sup>5</sup>An update is available at <http://www.stat.iastate.edu/preprint/articles/2011-10.pdf>.

<sup>6</sup>The Merriam–Webster dictionary also lists 1905 as the year of first known use of *homoscedasticity* but does not mention its source. Oddly, Van Looc [2007] writes—in a footnote explaining the etymology of heteroskedasticity—something extremely similar to what Aldrich contributes to *Earliest Known Uses*



A search through [Google Books](#) (see [Sec. 3](#)) returned results which indicated that the first instance appeared in an earlier manuscript by [Pearson](#), “On the Theory of Contingency and its Relation to Association and Normal Correlation”, which was published in 1904. This is an obvious technical mistake since searching through that particular paper does not reveal any variant of the words *heteroskedasticity* or *heteroscedasticity*. Thus, I confirm that the [Pearson \[1905\]](#) publication contains the first incarnation of the term. Specifically, [Pearson](#) introduces the term for the first time as follows:

[T]he variability of an array, *i.e.*, the standard deviation of an array... may or may not be the same for all arrays. If it is the same, or all arrays are *equally scattered* about their means, I shall speak of the system as a *homoscedastic* system, otherwise it is a *heteroscedastic* system. [[Pearson 1905](#)]

Since the origin of *heteroscedasticity* is clearly Greek, it is useful to briefly enumerate the three channels through which the Greek language has contributed to the modern English lexicon. First, a Greek word could be a direct donor, such as *isosceles* from the Greek *isoskeles* (“with equal sides”; from *isos*, “equal”, and *skelos*, “leg”). Second, it may come to English through an intermediate language, such as Latin<sup>7</sup> or French<sup>8</sup>. [Aldrich \[2011\]](#) cites the example of *geometry*, which was Greek *geometria* (from *gē*, “earth”, and *metrēō*, “to measure”), then Latin *geometria*, and then French *géométrie*, after which it was adopted into English. This percolation through the sieves of various languages naturally has implications for both the word’s pronunciation and its orthographic manifestation. Third, a lexical gap in English may be filled by coining a modern word using Greek roots: the neoclassical compound.<sup>9</sup> It is this third way that *heteroscedasticity* came to English.

---

*of Some of the Words of Mathematics* [[Miller 2011](#)] but instead of referencing [Aldrich](#), he cites [McCulloch \[1985b\]](#), who did not mention [Pearson](#) at all and who referred to *skedánnymi* instead of *skedastós*.

<sup>7</sup>Latin was primarily introduced by the Christian missionaries beginning in the 5th to 6th centuries. It was also, at that time, the *lingua franca* of Europe.

<sup>8</sup>French was brought in via the Norman conquest of England when William, Duke of Normandy, defeated King Harold II of England at the Battle of Hastings in 1066.

<sup>9</sup>For the special case of neoclassical compounds in English morphology, see [Bauer \[2005\]](#).

Aldrich [2011] notes that, in the context of coining new technical words in mathematics and statistics, Pearson’s “specialty was the Greek-based neologism” and gives the examples of *histrogram*<sup>10</sup> and *heteroscedasticity*. Pearson introduced many other such terms in two lecture series in 1892 while he was Professor of Geometry at the prestigious Gresham College: “The syllabi show an abundance of fancy terminology: stigmograms, euthygrams, epipedograms, histograms, chartograms, hormograms, topograms, stereograms, radiograms, and isodemotic lines,” writes Stigler [1986]. A search through Miller’s [2011] compendium of *Earliest Known Uses of Some of the Words of Mathematics* reveals the substantial contribution of Pearson to modern statistical terminology.

Direct Greek contributions to modern scientific terms in English are relatively recent phenomena. Before the Renaissance (which began around the 14th century), Greek-sourced words came into English primarily after having already undergone a Latin or French transliteration. At the dawn of the Industrial Revolution around the 18th century, terms from earlier Greek works were being directly imported into English along with Latin terms with Greek roots to represent new knowledge. Modern coinages (e.g., *heteroscedasticity*), particularly classical compounds (which were popular in scientific and technical applications), reached their peak in the 19th century [Aldrich 2011].

Since Pearson invented the term in 1905, McCulloch is therefore correct in noting that *heteroscedasticity* is a modern coinage and that “[t]he letter in question is...the transliteration of the Greek kappa ( $\kappa$ ).” How, then, is  $\kappa$  transliterated into English? While certain Greek letters, such as  $\gamma$ ,  $\delta$ ,  $\nu$ , and  $\chi$  have varying transliterations,  $\kappa$  is nowadays invariably represented with the letter *k* in English. The Beta Code<sup>11</sup>, the BGN/PCGN romanization<sup>12</sup>, the ISO 843:1997 system<sup>13</sup>, the UN romanization system<sup>14</sup>,

---

<sup>10</sup>The term has Greek root words, specifically *istos* (“mast”, as on a ship) and *gramma* (“something written”). See Ioannidis [2004].

<sup>11</sup>Developed by David W. Packard in the 1970s, the Beta Code is a system of representing ancient Greek using ASCII (American Standard Code for Information Interchange) characters.

<sup>12</sup>The US Board on Geographic Names (BGN) and the Permanent Committee on Geographic Names (PCGN) for British Official Use has its own transliteration convention for geographic names.

<sup>13</sup>This system applies to Greek script irrespective of the period in which it was used.

<sup>14</sup>This is based on the ELOT 743 conversion system of the Greek Standardization Organization, which also forms the basis for ISO 843:1997.

and the US Library of Congress transliteration chart (for both Ancient/Medieval and Modern Greek) all recommend transliterating  $\kappa$  as *k* (or *K*, as the Beta Code recommends capital letters). However, these attempts at standardization are all recent initiatives and were codified after Pearson had coined the term.

Nonetheless, Greek words with  $\kappa$  directly imported into English typically would have had the  $\kappa$  replaced with a *k* even before formal attempts at standardization (a notable exception is the aforementioned *isosceles*). McCulloch provides the examples *skeptic* and *skeleton* but these words were not lifted directly from Greek. *Skeptic*<sup>15</sup> entered via French (*sceptique* from the Latin *scepticus*) while *skeleton* appeared in Late Latin (c.300–c.700) as *sceletus*. A more appropriate example would have been *kinesis* or *kinetic* from the Greek *kinesis*.

As one can already see, when Greek words entered Latin or French, the  $\kappa$  was transliterated as *c*. It is through this channel that the original  $\kappa$  in Greek may appear in English as *c* subject to palatalization in Late Latin.<sup>16</sup> McCulloch provides *scepter* (Old French (c.900–c.1400) *sceptre*, Latin *sceptrum*, Greek *skeptron*), *scene* (Middle French (c.1400–c.1600) *scène*, Latin *scaena*, Greek *skené*), and *cyclic* (French *cyclique*, Latin *cyclicus*, Greek *kyklikos*) as relevant instances.<sup>17</sup>

The lack of consensus over the spelling of heteroskedasticity has carried over to its pronunciation. Within the scientific community, the questionable *k* or *c* is typically enunciated as a voiceless velar plosive [k], irrespective of how the word is actually spelled. McCulloch reiterates this in *Econometrica*. Merriam–Webster, however, indicates that it is the voiceless alveolar sibilant (grooved fricative, [s]), consistent with its suggested spelling of *homoscedasticity*. This is commonly known as the soft *c*, which is the usual

---

<sup>15</sup>This is the preferred spelling in American English; the variant *sceptic* is predominantly found in British English. Indeed, since the word entered through French, it used to be spelled with a *c* on both sides of the Atlantic. However, largely successful US spelling reforms initiated by Noah Webster (of Merriam–Webster fame) emphasized phonetic faithfulness (hence, *color* (AE) for *colour* (BE), among many others). Oddly enough, despite the soft *c* [s] pronunciation in French, the persistent pronunciation in English is [k].

<sup>16</sup>In phonetic terminology, there are two different phonemes here: [k] and [c], where the former is a voiceless velar plosive and the latter is a voiceless palatal plosive.

<sup>17</sup>See the Online Etymology Dictionary maintained by Douglas Harper for details: <http://www.etymonline.com/>.

phoneme when *c* appears before *e*, *i*, and *y* (*sceptic* in British English is a notable exception). Schwartzman [1994] is more permissive: “The word *heteroscedastic* may be pronounced as if the first *c* were a *k* or as if the first *c* were omitted.” The question of pronunciation even found its way to *The Math Forum @ Drexel*, a leading online resource for teaching mathematics and statistics, when Jeff Miller of *Earliest Known Uses* [Miller 2011] raised the matter, citing the soft *c* recommendation of the instructor’s guide to the book *Statistics in Action*.<sup>18</sup>

It seems that the confusion in spelling and its inevitable consequences for pronunciation over the last century could have been avoided had Pearson simply conformed with orthographic convention. Was he inspired by how uniquely *isosceles* entered English? Perhaps, but regardless of his motivation, the fact is that he did not follow standard practice, so we are now left with the etymologically sound *heteroskedasticity* and its progenitor, *heteroscedasticity*, which has a pedigree all the way to the coiner (whose authority and influence over mathematical and statistical neologisms is hard to overstate).

### 3 Word history and prevalence using Google Books

The data used in this paper is made publicly available by Google through its [Google Books](#) project and the multi-institutional team behind [Culturomics](#). Originally known as Google Print that started in 2004, the service allows users to search the full text of—as of October 2010—“15 million books from more than 100 countries in over 400 languages” [Crawford 2010]. This represents about 12 percent of all published books. Searching through the contents of each book is made possible by subjecting the digitized copies to optical character recognition (OCR). [Google Books](#) holds perhaps the largest corpus of collected human knowledge and it is still rapidly growing as even more published materials

---

<sup>18</sup>See <http://mathforum.org/kb/message.jspa?messageID=5906833> and the discussion therein. Like John Aldrich, as narrated by Miller, I have never heard the soft *c* pronunciation in English. [McCulloch](#) rightly points out that it is *hétéroscédasticité* in French and is consequently pronounced as [s] in that language. A French colleague informs me that when French economists speak in English, they are much more likely to use [s] in this case.

are scanned.<sup>19</sup>

Starting with the collection of [Google Books](#), [Michel et al. \[2011\]](#)—the “Culturomics team”—selected about five million books based on the quality of the resulting OCR output and the metadata. The assembled database contains over 500 billion words, of which 361 billion are English. For a particular word to appear in the dataset, it must have appeared in at least 40 books. Although the dataset begins in the 1500s, I restricted the subsequent analysis to published works beginning in 1905, the year when Pearson coined the term *heteroscedasticity*. The series ends in 2008. I used the English corpus<sup>20</sup> for the graphs below.

Figure 1 shows the prevalence of the words *heteroskedasticity* and *heteroscedasticity* over time, with the horizontal axis representing years and the vertical axis representing the share of these two words in the body of published (English) words for a particular year. The lines have been smoothed using a three-period moving average (MA(3)). For much of the 20th century, the *heteroscedasticity* variant coined by Pearson outnumbered the *k* variant.

It was not until 1974 when the *k* variant overtook the *c* spelling, as seen in Figure 2, where I restrict the time dimension to 1968–2008 and remove the smoothing to show the raw counts. This *k*-dominance, however, lasted only one year. Subsequently, *heteroskedasticity* briefly appeared more times than *heteroscedasticity* for the years 1986, 1992, and 1993. Except for 2005, all years after 2001 show a consistent preference for *heteroskedasticity*.

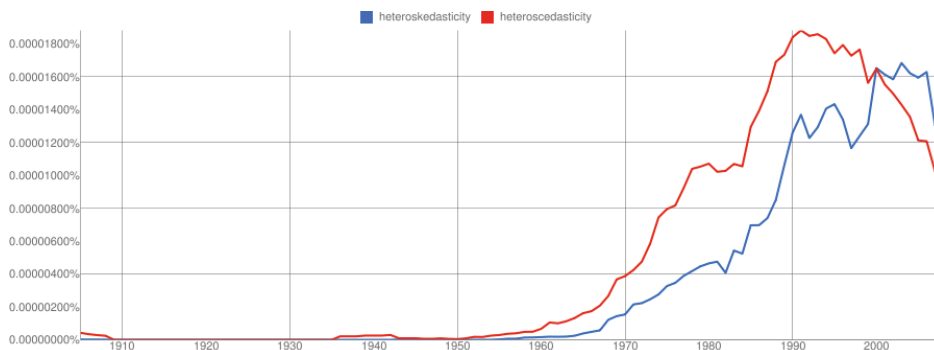
Interestingly, for the phenomenon of homoskedasticity, the *c* spelling dominates for all but one year (see Figure 3). Although not evident from the figure because of the MA(3) smoothing, *homoskedasticity* only appeared more than *homoscedasticity* for the final year

---

<sup>19</sup>Sections of the following materials were retrieved using Google Books: *Statistical Method* [Kelley 1923], *The History of Statistics* [Stigler 1986], *The Words of Mathematics* [Schwartzman 1994], “On the General Theory of Skew Correlation and Non-linear Regression” [Pearson 1905], *Problems in Education* [Western Reserve University 1927], “The borderline between derivation and compounding” [Bauer 2005], and *History of Friedrich II of Prussia* [Carlyle 1858].

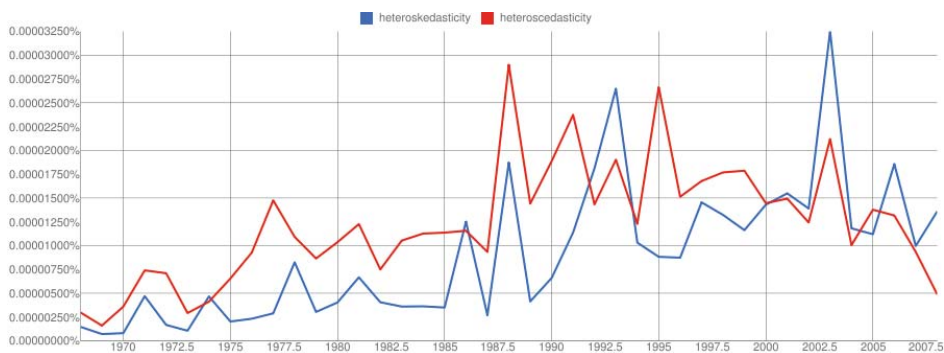
<sup>20</sup>The other corpora are American English, British English, Chinese (simplified), English Fiction, English One Million, French, German, Hebrew, Spanish, and Russian.

FIGURE 1  
 HETEROSKEDASTICITY AND HETEROSCEDASTICITY, 1905–2008, MA(3)



Note: The graph was generated on 24 November 2011.  
 Source: Google Books Ngram Viewer, <http://books.google.com/ngrams>.

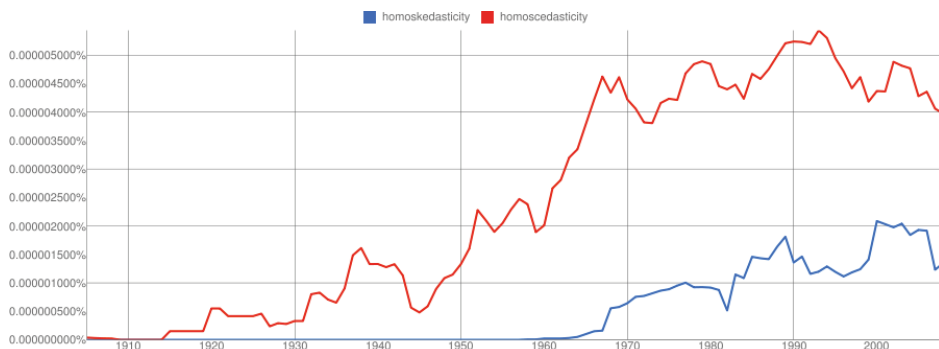
FIGURE 2  
 HETEROSKEDASTICITY AND HETEROSCEDASTICITY, 1968–2008, MA(0)



Note: The graph was generated on 25 November 2011.  
 Source: Google Books Ngram Viewer, <http://books.google.com/ngrams>.

of the dataset (2008). It seems, therefore, that *heteroskedasticity* is leading the way and that *homoskedasticity* is catching up with changes in spelling preference. Perhaps this is because *heteroskedasticity* tends to be more often used than *homoskedasticity*, anyway, so any changes in spelling will be reflected in the former ahead of the latter.

FIGURE 3  
HOMOSKEDASTICITY AND HOMOSCEDASTICITY, 1905–2008, MA(3)



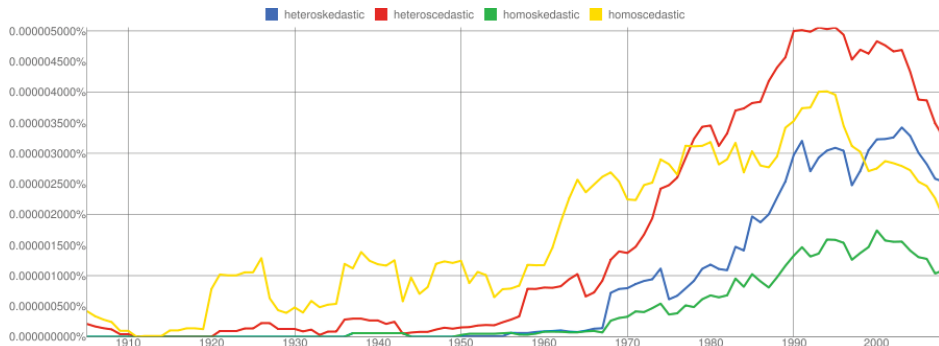
Note: The graph was generated on 24 November 2011.

Source: Google Books Ngram Viewer, <http://books.google.com/ngrams>.

In Figure 4, I plot the corresponding adjectives over time with MA(3) smoothing. Here, it is quite clear that the *c* variant dominates for the whole period. It is rather odd that, while the noun *heteroskedasticity* has overtaken its *c* variant, the adjective seems to have lagged so far. However, the overall trend seems to indicate that the two variants of the adjective will converge some time soon.

Is there a difference in the prevalence of each variant of heteroskedasticity between American and British usage? After all, Pearson is British and the influential economists who used *k* are American (see Sec. 4). To see this, I graph *heteroskedasticity* and *heteroscedasticity* separately for the American and British English corpora of Google Books for the period 1985–2008 in Figure 5. These corpora are restricted by the place of publication of the book in the dataset. Here, one can see that British publications are only recently moving toward *heteroskedasticity* and recent years are showing a rapid decline in the usage of *heteroscedasticity*. In comparison, American orthography has been more accepting of the newer *k* variant and is shifting away from Pearson’s original spelling.

FIGURE 4  
 HETEROSKEDASTIC, HETEROSCEDASTIC, HOMOSKEDASTIC, AND HOMOSCEDASTIC, 1905–2008,  
 MA(3)



Note: The graph was generated on 24 November 2011.

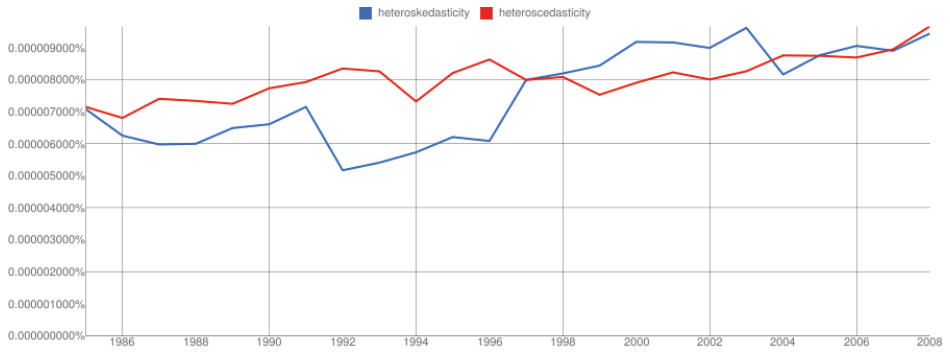
Source: Google Books Ngram Viewer, <http://books.google.com/ngrams>.

The corpus of English texts made available by the Culturomics team is limited because it is only a sampling of the books that the Google Books project have subjected to OCR. Other materials were excluded, such as journals, periodicals, magazines, and the like. Moreover, material found on the internet is not included. Thus, I tabulate the number of search hits in Google and Google Scholar, which indexes only scholarly literature, of the terms of interest to complement the numbers retrieved from Google Books (see Table 1). Similar to the figures above, search data from Google indicate that it is only with the noun *heteroskedasticity* that the  $k$  variant has become more common. For the other instances, the  $c$  variant is still heavily dominant (in particular, for the noun *homoscedasticity*).

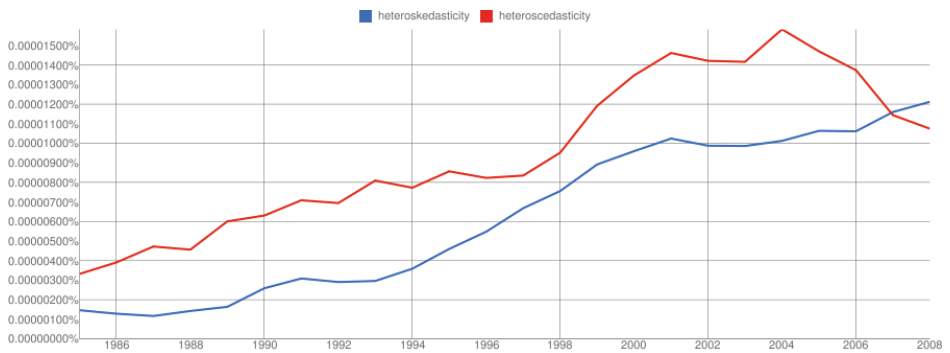
I also queried JSTOR for the relevant terms to examine their prevalence in the academic journals in JSTOR’s archive. Column (1) of Table 2 is the result of queries in the JSTOR database of Economics, Mathematics, Political Science, Sociology, and Statistics journals. Column (2) is restricted only to Economics and Statistics journals. In this case, it can be seen that the academic literature is still largely faithful to the original spelling of Pearson. However, when I restrict the sample to the years 2000 and after, the dominance of *heteroskedasticity* once again becomes apparent (see Column (3), which is the post-1999 restricted version of (1)).



FIGURE 5  
 HETEROSKEDASTICITY AND HETEROSCEDASTICITY, 1985–2008, MA(3), AMERICAN AND BRITISH  
 ENGLISH CORPORA



(a) American English



(b) British English

Note: The graphs were generated on 13 December 2011.

Source: Google Books Ngram Viewer, <http://books.google.com/ngrams>.

TABLE 1  
GOOGLE AND GOOGLE SCHOLAR SEARCH HITS

Word	Google	Google Scholar
Heteroskedasticity	866,000	72,900
Heteroscedasticity	460,000	65,300
Homoskedasticity	48,000	8,840
Homoscedasticity	212,000	30,800
Hetereoskedastic	152,000	22,100
Heteroscedastic	209,000	30,600
Homoskedastic	59,200	7,720
Homoscedastic	80,100	11,600

Note: Data were retrieved on 24 November 2011.

TABLE 2  
JSTOR SEARCH HITS

Word	(1)	(2)	(3)
Heteroskedasticity	4,729	4,659	1,821
Heteroscedasticity	5,244	5,176	1,349
Homoskedasticity	498	492	160
Homoscedasticity	1,298	1,266	205
Hetereoskedastic	1,583	1,566	538
Heteroscedastic	2,764	2,739	774
Homoskedastic	583	576	203
Homoscedastic	1,438	1,426	322

Note: Data were retrieved on 24 November 2011.

Incidentally, a comparison between Columns (1) and (2) of Table 2 illustrates that the concept of heteroskedasticity is almost exclusively the concern only of economists and statisticians. It has yet to substantially cross-pollinate the allied social sciences with a significant quantitative subfield, such as Political Science and Sociology.

Overall, the term *heteroskedasticity* has been the lone shooting star among the  $k$  siblings. While *heteroskedasticity* has overtaken *heteroscedasticity* and has been steadily outnumbering its  $c$  rival, the other  $k$ 's have consistently failed to surpass their  $c$  cousins. The trends from the figures above, however, seem to indicate that the profession is marching toward the  $k$  variant, albeit perhaps tentatively. One reason for this transition is presumably the phonetic faithfulness of the  $k$  spelling and, to a rather minor extent, the standardized transliteration of the Greek letter  $\kappa$  into  $k$ , which the  $k$  crusaders such as McCulloch like to point out. Of course, incredibly influential papers such as White [1980] massively contribute to this trend.

## 4 Let the econometricians speak!

Although one cannot conclude with absolute certainty, the spike of *heteroskedasticity* in 1986 (Figure 2) may be the result of McCulloch's 1985b publication. In private correspondence, Prof. McCulloch mentions that the original motivation for the *Econometrica* article is the insistence of an editor of the *Journal of Banking and Finance* to spell heteroskedasticity with a  $c$  in McCulloch [1985a], "Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates", where ACH means adaptive conditional heteroskedasticity. McCulloch wanted to show that  $k$  is "not only acceptable, but preferable." The journal then went along with  $k$  and he sent his note to *Econometrica*.

However, five years before the publication of McCulloch, Halbert White [1980] already published his influential paper, "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", in *Econometrica*. According to Kim, Morse and Zingales [2006], this is the most cited paper in economics since 1970. By

their count based on a selection of top general-interest and field journals, [White \[1980\]](#) has been cited 4,318 times, with the next-most-popular paper cited a little less than 300 times White’s citation count. [Google Scholar](#)<sup>21</sup> reports 15,344 citations. Computing “White standard errors” (or “robust standard errors”) is now standard in most applications.

In [Kim, Morse and Zingales \[2006\]](#)’s paper, “What Has Mattered to Economics Since 1970”, *heteroskedasticity* appears five times in the titles of their list of influential papers. Those before 1990 include the aforementioned [White \[1980\]](#) paper and also [Newey and West \[1987\]](#) and [Bollerslev \[1986\]](#), cited 8,603 and 10,996 times, respectively, in [Google Scholar](#). *Heteroscedasticity* appears in two papers, namely in [Engle \[1982\]](#) and [Breusch and Pagan \[1979\]](#) for a combined [Google Scholar](#) citation count of 13,590, which is 23,353 counts less than just the pre-1990 influential *heteroskedasticity* papers.

In an email, Prof. White suggests that he had perhaps three sources for his preference for the  $k$  variant: first, his PhD supervisor at MIT, Jerry Hausman; second, the *Econometric Theory* textbook of Arthur S. [Goldberger \[1964\]](#); third, the textbook of Jan [Kmenta \[1971\]](#), *Elements of Econometrics*, which he “very much liked”. He believes that he most probably picked it up from [Goldberger \[1964\]](#) since he was “strongly influenced” by that book. Among other factors, Prof. White’s choice of using  $k$  and the subsequent popularity of his 1980 publication most certainly pulled the profession away from the  $c$  variants.

Prof. Hausman writes that he did, in fact, use  $k$  in his lectures at MIT. He does not recall how exactly he came to prefer that spelling but notes that he might have followed “the Greek approach.” He could have also influenced other people at MIT, such as Whitney Newey and Kenneth D. West (now at the University of Wisconsin), both of whom were his students in econometrics and who jointly developed a technique to estimate heteroskedasticity- and autocorrelation-consistent (HAC) standard errors.<sup>22</sup> Although

---

<sup>21</sup>The citation numbers from [Google Scholar](#) reported here and the next paragraph were retrieved on 01 December 2011.

<sup>22</sup>Prof. West also acknowledges Daniel McFadden (now at the University of California at Berkeley) and Franklin M. Fisher as his former econometrics professors. Both McFadden and Fisher seem to have

Prof. West notes that they were familiar with [White \[1980\]](#) and the [White \[1984\]](#) textbook, *Asymptotic Theory for Econometricians*, there was, to the best of his recollection, no discussion between him and Prof. Newey on how to spell heteroskedasticity: spelling it with a *k* just seemed “the natural one.”

Of course, what was natural in the '80s may not have been so in the '70s. Prof. Trevor Breusch recalls that the *c* variant was dominant at the time he and Prof. Adrian Pagan wrote [Breusch and Pagan \[1979\]](#). This is apparent in the references cited in their paper. The majority of the textbooks he owned also used the *c* spelling and thus their decision preferring *c* was not entirely a deliberate one. As an indication of the enduring influence of the [McCulloch \[1985b\]](#) publication, Prof. Breusch writes that he was convinced by the arguments laid out therein and has since switched to the *k* variant.

Evidently, while the authority of Karl Pearson over Greek-based mathematical and statistical neologisms is redoubtable, his coinages are not immune to orthographic mutations, especially when such mutations are driven by a well argued position published in a reputable scholarly journal. Indeed, *Caesar non supra grammaticos*—Caesar is not above the grammarians.<sup>23</sup>

## 5 Conclusion

Writing for the Financial Times in 1998, economist John [Kay](#) says, “Every serious subject has its jargon. Economists need to know about heteroscedasticity. I take this example because it is virtually impossible to pronounce, and impossible to use the word in front of a class without everyone bursting out into laughter. Indeed, most spell-check programmes

---

oscillated between the *c* [[McFadden 1987](#); [Fisher et al. 1966](#)] and *k* [[McFadden 1974](#); [Fisher, McGowan and Evans 1980](#)] variants, though. But, in fact, so did Hausman in 1987 when he and Paul A. Ruud spelled it with a *c* in the same issue of the *Journal of Econometrics* where [McFadden \[1987\]](#) appeared. So, perhaps that was the call of the editor, Richard Blundell, who predominantly writes with a *c*.

<sup>23</sup>In opening the Council of Constance (1414–1418), Sigismund, Holy Roman Emperor, had used the word *schisma* as feminine instead of neuter: “*Date operam, ut illa nefanda schisma eradicetur*,” referring to Hussite Bohemia during the Western Schism (otherwise known as the Three-Popes Controversy). A cardinal reminded him, “*Domine, ‘schisma’ est generis neutrius* [Your Majesty, *schisma* is neuter].” To which Sigismund replied, “*Ego sum Rex Romanus, et super grammaticam* [I am King of the Romans, and above grammar]!” [[Carlyle 1858](#)]

reject it, and offer improbable or embarrassing alternatives. Yet heteroscedasticity is an important concept.” With this, I agree, which is why I think McCulloch’s early attempt at settling an important issue of orthography in the profession is commendable and why I believe documenting its philological development remains relevant, especially as an example of how knowledge is diffused within the scientific community and eventually to popular literature.<sup>24</sup>

Based on the millions of books digitized by the Google Books project, the answer to the question posed in the title, “When did we begin to spell *heteros\*edasticity* correctly?”, is 1927, when Western Reserve University<sup>25</sup> published *Problems in Education*, where the authors noted—quite serendipitously in our context—that teaching spelling “is an enigma.” The specific instance appeared in the following question: “Correlation, concentration, apperception, interest are not strange terms, but what about standard deviations, I.Q.’s, accomplishment quotients, tetrachoric  $r$ , multiple correlation or heteroskedasticity?” Much earlier, David [1998] had already noted that 1927 was the debut year of *heteroskedasticity* but in a different work, that of Frank M. Weida’s, who wrote in the *Annals of Mathematics*, p. 303:

If the limited mean error of  $y$  remains constant for all possible values of  $x$ , the connection of  $y$  with  $x$  is said to be *homoskedastic*; and if the limited mean error of  $y$  does not remain constant for all possible values of  $x$ , the connection of  $y$  with  $x$  is said to be *heteroskedastic*. [Weida 1927]

For the most part of recent history, however, the  $k$  variant of the word heteroskedasticity was never mentioned more than its  $c$  counterpart. It was only in 2001 when it has consistently dominated (with the exception of 2005). If the trends since the turn of the century persist, then we can expect *heteroskedasticity* to remain the dominant spelling in published works in succeeding years. Whether this will influence the spelling of the

---

<sup>24</sup>By any reckoning, it is hard to overlook the institutional contribution of MIT in advancing standard statistical inference in the presence of nonspherical errors, considering the potent “one-two combo” of White [1980] and Newey and West [1987].

<sup>25</sup>The university is now known as Case Western Reserve University after it merged with the Case Institute of Technology in 1967.

related words *homoskedasticity*, *heteroskedastic*, and *homoskedastic*, all of which are still outnumbered by their *c* variants, remains to be seen. There are, however, indications that it has already had an effect. Notably, *homoskedasticity* overtook *homoscedasticity* for the first time in 2008.

This being an article about language, I end by acknowledging my sloppy use of language. Admittedly through my own fault, the reader might surmise that implicit in this discussion is the notion that *heteroskedasticity* is indeed the “correct” spelling. However, orthographic issues are hardly ever black and white (e.g., “a history” or “an history”?). As E.B. White is wont to remind us, “The language is perpetually in flux: it is a living stream, shifting, changing, receiving new strength from a thousand tributaries, losing old forms in the backwaters of time.” [Strunk Jr. and White 1959]

Dictionaries list *heteroscedasticity* as an acceptable variant and it is certainly still used in modern textbooks both at the undergraduate (e.g., Gujarati and Porter [2008]) and graduate (e.g., Greene [2011]) levels. The textbook of Greene, in particular, remains the standard textbook for a first-year graduate econometrics sequence. It will undoubtedly have an enduring influence on how future economists will spell heteroskedasticity. Moreover, that Wikipedia prefers *c* is significant in an age when information is increasingly retrieved from the internet, especially for the nonpractitioner.<sup>26</sup>

The aim of this manuscript is to be descriptive as opposed to prescriptive.<sup>27</sup> Despite appearances, it is not my objective to elevate one orthographic manifestation over the other. More modestly, I only describe trends in spelling variations of an important concept in statistics and econometrics. The present *communis opinio doctorum* is that both spellings are correct. No etymological undertaking can overrule that. The real

---

<sup>26</sup>A journalist will more likely head over to Wikipedia to learn about heteroskedasticity rather than pick up White [1980]. Indeed, since Merriam–Webster and Wikipedia both seem to prefer *heteroscedasticity* while the academe is moving toward *heteroskedasticity*, it is conceivable that the orthography in academic journals will be different from popular literature in the future.

<sup>27</sup>McCulloch, Guy Judge (University of Portsmouth), and David E.A. Giles (University of Victoria) are the latter: using *k* is the “proper English spelling”, according to McCulloch, “you spell it with a *k* and not a *c*!” is the call of Judge [2007], and “Yes, this word should indeed be spelled with a ‘*k*’, and not another ‘*c*,’” says Giles [2011]. Both Judge and Giles cite McCulloch [1985*b*]. Prof. McCulloch says, however, that as an editor or reviewer, he would never insist on an author spelling these words with a *k*.

answer to the question in the title, therefore, is that we have spelled (spelt!) it correctly all the time.



## References

- Aldrich, John. 2011. "Mathematical Words: Origins and Sources." <http://www.economics.soton.ac.uk/staff/aldrich/Mathematical%20Words.htm>.
- Bauer, Laurie. 2005. The borderline between derivation and compounding. In *Morphology and its Demarcations: Selected Papers from the 11th Morphology Meeting, Vienna, February 2004*, ed. Wolfgang U. Dressler. John Benjamins Publishing Company.
- Bollerslev, Tim. 1986. "Generalized Autoregressive Conditional Heteroskedasticity." *Journal of Econometrics* 31(3):307–327.
- Breusch, Trevor and Adrian Pagan. 1979. "A Simple Test for Heteroscedasticity and Random Coefficient Variation." *Econometrica* 47(5):1287–1294.
- Carlyle, Thomas. 1858. *History of Friedrich II of Prussia, Called Frederick the Great. Book II: Of Brandenburg and the Hohenzollerns (928–1417)*. Project Gutenberg 2008. <http://www.gutenberg.org/dirs/2/1/0/2102/2102.txt>.
- Crawford, James. 2010. "On the future of books." Inside Google Books: <http://booksearch.blogspot.com/2010/10/on-future-of-books.html>.
- David, Herbert A. 1998. "First (?) Occurrence of Common Terms in Probability and Statistics—A Second List, with Corrections." *The American Statistician* 52(1):36–40.
- Engle, Robert F. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50(4):987–1007.
- Fisher, Franklin M., John J. McGowan and David S. Evans. 1980. "The Audience–Revenue Relationship for Local Television Stations." *The Bell Journal of Economics* 11(2):694–708.
- Fisher, Franklin M., Victor E. Ferrall, Jr., David Belsley and Bridger M. Mitchell. 1966. "Community Antenna Television Systems and Local Television Station Audience." *The Quarterly Journal of Economics* 80(2):227–251.
- Giles, David E.A. 2011. "The Second-Longest Word in the Econometrics Dictionary." *Econometrics Beat: David Giles' Blog*: <http://davegiles.blogspot.com/2011/03/second-longest-word-in-econometrics.html>.
- Goldberger, Arthur S. 1964. *Econometric Theory*. Wiley.
- Greene, William H. 2011. *Econometric Analysis*. 7th ed. Pearson Education Limited.
- Gujarati, Damodar N. and Dawn C. Porter. 2008. *Basic Econometrics*. 5th ed. McGraw-Hill.
- Hausman, Jerry A. and Paul A. Ruud. 1987. "Specifying and Testing Econometric Models for Rank-Ordered Data." *Journal of Econometrics* 34(1–2):83–104.

- Ioannidis, Yannis. 2004. The History of Histograms (Abridged). In *Proceedings of the 30th International Conference on Very Large Datasets*, ed. Mario A. Nascimento, M. Tamer Özsu, Donald Kossman, Renée J. Miller, José A. Blakeley and Berni Schiefer. Morgan Kaufmann.
- Judge, Guy. 2007. “How do you spell ‘heteroskedasticity’?” Guy’s Econometrics Blog: <http://econometricsstuff.blogspot.com/2007/03/how-do-you-spell-heteroskedasticity.html>.
- Kay, John. 1998. “In defence of endogenous growth theory.” <http://www.johnkay.com/1998/03/02/in-defence-of-endogenous-growth-theory>.
- Kelley, Truman L. 1923. *Statistical Method*. The Macmillan Company.
- Kim, E. Han, Adair Morse and Luigi Zingales. 2006. “What Has Mattered to Economics Since 1970.” *Journal of Economic Perspectives* 20(4):189–202.
- Kmenta, Jan. 1971. *Elements of Econometrics*. Macmillan.
- McCulloch, J. Huston. 1985a. “Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates.” *Journal of Banking and Finance* 9(1):137–156.
- McCulloch, J. Huston. 1985b. “Miscellanea: On Heteros\*edasticity.” *Econometrica* 53(2):483.
- McFadden, Daniel. 1974. Conditional Logit Analysis of Qualitative Choice Behavior. In *Frontiers in Econometrics*, ed. Paul Zarembka. Academic Press.
- McFadden, Daniel. 1987. “Regression-Based Specification Tests for the Multinomial Logit Model.” *Journal of Econometrics* 34(1–2):63–82.
- Michel, Jean-Baptiste, Yuan Kui Shen, Aviva Presser Aiden, Adrian Veres, Matthew K. Gray, The Goolge Books Team, Joseph P. Pickett, Dale Hoiberg, Dan Clancy, Peter Norvig, Jon Orwant, Steven Pinker, Martin A. Novak and Erez Lieberman Aiden. 2011. “Quantitative Analysis of Culture Using Millions of Digitized Books.” *Science* 331(6014):176–182.
- Miller, Jeff. 2011. “Earliest Known Uses of Some of the Words of Mathematics.” <http://jeff560.tripod.com/mathword.html>.
- Newey, Whitney K. and Kenneth D. West. 1987. “A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix.” *Econometrica* 55(3):703–708.
- Pearson, Karl. 1904. Mathematical Contributions to the Theory of Evolution. XIII. On the Theory of Contingency and its Relation to Association and Normal Correlation. In *Draper’s Company Research Memoirs: Biometric Series (Vol. I)*. Dulau and Co.
- Pearson, Karl. 1905. Mathematical Contributions to the Theory of Evolution. XIV. On the General Theory of Skew Correlation and Non-linear Regression. In *Draper’s Company Research Memoirs: Biometric Series (Vol. II)*. Dulau and Co.

- Schwartzman, Steven. 1994. *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English*. Mathematical Association of America.
- Stigler, Stephen M. 1986. *The History of Statistics: The Measurement of Uncertainty Before 1900*. Harvard University Press.
- Strunk Jr., William and E.B. White. 1959. *The Elements of Style*. Macmillan.
- Van Loco, Joris. 2007. Method Validation for Food Analysis: Concepts and Use of Statistical Techniques. In *The Determinants of Chemical Elements in Food: Applications for Atomic and Mass Spectrometry*, ed. Sergio Caroli. John Wiley & Sons.
- Weida, Frank M. 1927. "On Various Conceptions of Correlation." *Annals of Mathematics (Second Series)* 29(1/4):276–312.
- Western Reserve University. 1927. *Problems in Education*. Western Reserve University Press.
- White, Halbert. 1980. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity." *Econometrica* 48(4):817–838.
- White, Halbert. 1984. *Asymptotic Theory for Econometricians*. 5th ed. Academic Press.
- Wooldridge, Jeffrey M. 2010. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. MIT Press.