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Mobile only users powered by Fixed-Mobile substitution*

Julienne Liang[†], Marc Petulowa

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Abstract

In a context of partial fixed-mobile substitution, we analyze fixed-mobile bundling and mobile-to-fixed offloading in a duopoly model in which consumers buy one or two products. A joint purchase discount mitigates fixed-mobile substitutability and consequently reduces "mobile-only" and "fixed-only" consumers. Practises like introducing a small discount, applied on a bundle of multiple service or mobile-to-fixed offloading by both operators are analysed. We find that such practises do not have negative impacts on the profits of whole market and lead to both consumers' surplus and welfare gains. The investment incentives in fixed network are positive and can be boosted by FM bundling without considering regulatory intervention and before taking into account of fixed costs. The investment incentives in mobile network are more likely a situation of prisoners' dilemma where operators should invest as long as there are "mobile-only"-consumers.

1 Introduction

Deregulation of the telecommunication sector in the late 90's enabled the entry new market actors and introduced competition. Former natural and state-owned monopolies have to face competitors that are investing in new technologies. At the beginning, mobile networks have been preferred by new market players¹.

In the same time, the fixed telephony network was also subject to major technological progress, reaching from local loop unbundling over deployment of networks with a larger bandwidth ((A)DSL) to the deployment of a fibre based network (FTTx²). In most countries, the latter is still in the phase of deployment and the former is most widespread³.

Whereas consumers do not notice a drastic change in telephony services⁴, the Internet service have been strongly impacted by these technological evolutions. Downloading and uploading have reached new speed records as new technologies have been deployed. As illustration operators announce a maximum speed for ADSL of up to 20 Mbits/s for downloads and up to 800 kbits/s for uploads. Fibre network allows a maximum speed of 100 Mbits/s for downloading and up to 10Mbits/s for uploading. The consumer will thus notice these differences, because requests on the Internet will be executed faster (e.g.: cloud services which upload speed as high as download speed).

Mobile networks also allow the consumer to connect to the Internet and they have also been subject to major progress in terms of Internet speed. Several norms exists in mobile Internet access: GSM (2 G), GPRS (2.5 G), EDGE (2.75 G), UMTS (3 G) and HSPA (3.5 G). The latest and largest diffused technology is the 3 G. Again, consumers might not remark a change in voice matters, while the responsiveness of the Internet is sharply increased.

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¹A reason for this might be the fact that developing a proper mobile infrastructure avoids the payment of more or less important access charges to the incumbent's fixed-line network. Furthermore, competing in services based on an all new technology seems to be simpler than competing in services that are also offered by a firm that already has solid experience in a given technology. In this latter case, an entrant would have many problems to offer attractive, differentiated services.

²The reader can find a list of the different acronyms used in this paper in appendix 1.

³On a European level, the digital agenda, adopted in 2010, is part of the European economic strategy 2020 which replaces the former Lisbon strategy. It seeks to establish a highspeed broadband network by 2020.

⁴Some customer even are not aware of the fact that their phone calls using fixed lines now use the Internet protocol (VoIP).

fixed line possibly renders the subscription of both accesses more sustainable.

Since distinct consumer patterns exist ("mobile-only", "fixed-only" and subscription of both) the accesses are neither completely substitutable, nor totally complementary. Indeed, would both be perfectly substitutable, paying for both accesses would be senseless. On the other side, perfect complementarity would eliminate all single service customers (without taking any budget constraints into consideration).

As mentioned above, fixed and mobile access technologies coexist and may compete against each other. The aim of this paper is thus (i) to formalize and to understand what variables impact the share of the single service customers⁶ and (ii) to analyze the profitability of several practices such as product bundling, off-loading and investing in one or the other network.

In particular, we will try to answer the following questions relying on roughly three topics:

- the evolution and impacts on the supply-side, as well as on the demand-side: which consumption pattern will emerge? Will there be more "mobile-only", "fixed-only" or adoption of both? How will technological progress influence the substitutability on the supply-side?
- the operators' possible pricing strategies: does the fixed-mobile bundling consent to reduce the substitution, i.e. is bundling a good strategy to reduce the market share of "mobile-only"-consumer? What is the profitability of introducing a discount on fixed-mobile bundling for a multi-service operator?
- the operators' investment incentives: What is the impact of FMS on profits and the investment incentives? Does bundling encourage investments in the one or the other network?
- Mobile-to-fixed off-loading is used to off-load data from mobile network onto fixed network. What is the impact on profits of operators?
- How do bundling and mobile-to-fixed off-loading, as well as investments impact the social welfare?

Finally, as we will see in the review of existing results, there exists a lack in the literature concerning the practice of bundling in a duopoly setting. As far as we are aware, all the duopoly papers assume (i) fully served markets and/or (ii) that consumers always buy two products (whether as a bundle or not). To our knowledge, there exist no paper dealing with these issues at the same time. Moreover, this is a bit distant from the reality in the telecommunications sector where markets do not obey to these assumptions. Indeed, the existence of single service customers indicates that both hypotheses do not always hold.

Our aim is thus to fill this gap and attempt to provide understanding of the market functioning when these assumptions are loosened.

The rest of this paper is organized as follows: Section 2 will provide a brief review of the literature relative to (i) FMS and (ii) profitability of bundling. Section 3 will present the model specification. In section 4, we focus closer on "mobile-only"-market in order to identify the variables of interest in FMS matters. Section 5 discusses the incentives to adopt the different strategies mentioned above (bundling, off-load, investment in fixed or mobile networks) and their subsequent profitability. Consumer surplus is analyzed in section 6. Concluding remarks precede the appendix.

2 Review of the existing literature

This paper aims to provide insights into the recent developments in the telecommunications market, especially in what concerns Internet access. It invokes concepts like FMS, bundling and off-loading. In this section, a brief review of existing results in the different domains is made.

Since off-loading is rather a marketing practice, it has not been examined in depth by economic papers. Therefore, this subject will not be treated here in the review.

In FMS matters, most retrieved papers are empirical and treat the problematic in the context of voice services or access and reveal emerging policy implications. However, Liang (2010) elaborated a theoretical paper treating FMS. In her paper, two single-service operators face competition based on mobility and

⁶We refer to single service consumers by denominated "mobile-only"-consumers or "fixed-only"-consumers.

data volume between fixed and mobile broadband access. This paper allowed the investigation of some interdependencies between fixed and mobile broadband markets.

More empirical, Rodini, Ward and Woroch (2003) estimate the substitutability of fixed and mobile services access and find that second fixed line can be substituted by mobile access. They used an US household survey in order to estimate demand functions for mobile access, as well as for fixed line access for 2000 and 2001. Then they derived own-price and cross-price elasticities allowing them to state that demand elasticities of second fixed line subscription is elastic, suggesting that an price increase results in a demand decrease. Also, a positive cross-price elasticity between fixed line and mobile has been found, with higher values for second line access. Hence, in these early years of mobile telephony, market power held by fixed telephony operators has already been constraint to "an economically significant" (Rodini, Ward and Woroch, 2003, p. 475) degree by mobile operators. The authors concluded further that, since the development of mobile telephony had yet to come, regulators should take account of these results.

An interesting opposition between two articles involves Schejter *et al.* (2009) and Briglauer, Schwarz and Zulehner (2009). Both articles deal with the relation FMS and market regulation.

The former is an cluster analysis (based on years 2004 to 2006) conducted in the US which allowed to separate their data set into two subsamples of fixed line users and into 5 subsamples of wireless customers. This segmentation has been realized using six variables describing consumption behavior. Their main results are that (i) "mobile and wireline consumers are indeed distinct groups" (Schejter *et al.*, 2009, p.10), (ii) taking consumer satisfaction into account may help to understand FMS, (iii) US consumers "do not perceive necessarily [mobile phones] as substitutes to wireline services, but rather as separate, perhaps complementary products [...] (*op. cit.* Schejter *et al.*, p.12)". Thus, separate regulatory schemes should be adopted for both products.

Briglauer, Schwarz and Zulehner (2009) estimate one demand function for national calls and another for access of residential customers using a time series (2002-2007) analysis for the Austrian market. In order to address the problem of substitution between fixed and mobile, they derive short/long run own/cross price elasticities for both services. In terms of long run estimations, the authors find significant positive own price elasticities and negative cross price elasticities for calls, suggesting that fixed mobile phone calls are substitute for fixed calls. In contrast, short run cross price elasticities turn out non-significant, which can be due to consumer lock-in linked to minimum contract duration.

The next step of their study was to undertake an hypothetical monopolist test ⁷ in order to check whether fixed and mobile calls, respectively access, belong to the same market. The conclusion was that "mobile *access* is not part of the same market as fixed access for residential customers" (Briglauer, Schwarz and Zulehner, 2009, p.21)⁸, while "fixed and mobile national *calls* of private users are part of the same market" (*op. cit.* Briglauer, Schwarz and Zulehner, p.23). The resulting implications for Austrian policy makers are in sharp contrast to the conclusions drawn by Schejter *et al.* for the US market. Briglauer, Schwarz and Zulehner indicate that:

Given the existence of four mobile operators, this would also indicate that a retail regulation for fixed network national calls of private users is no longer needed. [...] (*ibid*)

Finally, the authors point out that:

The results should therefore [*because of the techniques used to*] be interpreted with caution interpolate the mobile price data from quarterly to monthly data for a large part of the sample and in the context of other evidence available. For Austria, such additional evidence point in the same direction as our estimates, however. (*op. cit.* Briglauer, Schwarz and Zulehner, p.24)

Using a data set consisting of all EU members, Barth and Heimeshoff (2011) empirically analyze the substitution between mobile and fixed line subscription of private users on a time period going from 2003 to 2009. Their result show, as in most studies, a one way substitution from fixed to mobile. This asymmetric substitution entails that mobile can perfectly substitute to fixed line access, while the substitutability in the other direction is usually not confirmed.

⁷The aim of a hypothetical monopolist test is determine whether two products belong to the same market. This is done by comparing the own-price elasticities to a given threshold.

⁸Emphasis added.

As Briglauer, Schwarz and Zulehner, the authors consider possible implications for regulation of the telecommunications sector. In particular, they point out that, due to substitutability,

"[...] it would be difficult to appropriate rents as a consequence of significant market power in fixed or mobile markets only" (Barth, Heimeshoff, 2011, p.12).

Hence, tests for significant market power should be revisited.

Several regulatory organs in the world begin to consider FMS in market definition matters. Already in 2005, the Austrian regulatory authority for broadcasting and telecommunication, RTR, defined DSL and CATV broadband access as being substitutes and thus belong to the same market. In 2009, RTR enlarged the market towards mobile access, arguing that CATV and mobile broadband access are close enough substitutes⁹.

The BEREC report on the impact of FMS on market definition (Berec, 2011) is interesting in what it gathers how national regulators deal with FMS.

The literature of bundling is very large. The practice of bundling can be defined as the combined sale of two products at price less than the sum of the two individual prices. Bundling can be defined in two ways: pure bundling (i.e. offering two products only as a bundle) and mixed bundling (i.e. selling two products separately as well as a bundle at discounted price). This practice has been largely examined in the context of a multiproduct monopolist as well as of duopoly market structure.

At the outset, note that two major doctrines emerged. The first one sees bundling as a means to price discriminate, i.e. propose a price structure that induces consumers to choose the most appropriate good or basket of goods and thus reveal private information about their preferences. This allows firms to extract more rents from consumers whose surplus is therefore reduced. The second doctrine believes that bundling enables firms to leverage market power from one market to another market.

Schmalensee (1984) and McAfee, McMillan and Winston (1989) offered a generalization of a commonly cited paper by Adams and Yellen (1976). The latter analyzed the discriminatory effect of bundling through a set of examples. Adams and Yellen showed that, in a setting of multiproduct monopolist and atomistic distribution of consumers' reservation prices for both products, bundling has several effects: allocative and distributive inefficiency and profitability of introducing bundles, which is due to increased segmentation of buyers. Schmalensee (1984) made a first generalization of Adams and Yellen's paper by using a joint normal distribution. Then, McAfee, McMillan and Winston (1989) demonstrate in an even more generalized setting (general distribution of reservation values) that bundling remains the optimal strategy. However, under certain conditions on the correlation between reservation values, independent pricing arises as Nash equilibrium in a duopoly setting. These papers also note that pure bundling, is never an optimal strategy, because a firm can do at least as good with mixed bundling.

In a more competitive setting, Matutes and Régibeau (1992) consider a duopoly where the two firms produce both two components that the consumer can assemble in order to build a system. Then they verify whether bundling is profitable if complementary goods are sold together and check the incentives of the duopolists to introduce compatibility between rival components. Their model highlights a positive "variety increasing effect" when components are compatible. The result of this effect is then to pass from pure intra-house systems to inter-house systems and thus doubling the number of possible combinations. It is also shown that with compatible components, cutting the price of one product does not necessarily increase the demand and thus not profits, because the compatibility effect is internalized by the firms which then compete more fiercely in prices. The extent of market coverage amplifies this negative internalization effect on prices.

Reisinger (2004) explains, using a double Salopian circle, each circle representing one product, why bundling reduces profits in a duopoly. He places emphasis on the fact that, when reservation values are negatively correlated, bundling reduces consumer heterogeneity, which induces higher competition in the market.

This reduction is due to the fact that firms position their bundles close to each other and therefore compete fiercer. He states that:

The initial idea of the bundle, namely to price discriminate in a more skilful manner, is dominated by the business-stealing effect. [...] In this [...] case firms are in a prisoner's dilemma situation. It would be better for both of them not to bundle (Reisinger, 2004, p. 4).

⁹For both of these reports, the RTR's conclusions on substitutability concern private users and differ when business customers are considered.

Armstrong (2010) gave some interesting insights into the unilateral incentive to introduce a discount when products are substitutes. His analysis shows that even duopolists, each one producing one good, have a unilateral incentive to offer a price reduction on its product whenever its customer has already bought the rival's partial substitute. A sufficient condition for this to occur is that "[...] demand for the bundle is more elastic than demand for [the firm's own] stand-alone product[...]" (Armstrong, 2010, p.15). This "inter-firm discount" may have two faces: (i) only one firm has an incentive to offer a discount and (ii) both firm are animated to do so. In the latter case, however, since products are supplied separately and the discount chosen in a non-cooperatively fashion, "double marginalization will result and the overall discount will be too small" (op. cit. Armstrong, p. 16). In an extension, the author shows that the form of substitution, e.g. substitution induce by time constraints, separate sellers may prefer not to offer discounts and even increase the price of its stand-alone product when the consumer has bought the rival product. Should a consumer want to buy the second product even so, the firm will consider her/him as belonging to a distinct consumer group on which it will exert his market power.

A final feature of this paper is that the discount acts to reduce substitutability between products and may even, under certain circumstances, turn products into complements. This may happen because the discount, acting like the implicit price¹⁰ of the least desired good, reduces the "disutility of joint consumption" (op. cit. Armstrong, p. 3).

After this review, we will now describe our model specifications.

3 Model setup

We assume two multi-service operators, indexed by $k = A, B$. Both are able to commercialize fixed telecommunications services, mobile services, as well as a bundle of both. The marginal costs for fixed services, c_f , and for mobile services, c_m , incurred by the operators are normalized to 0.

Consumers are supposed to buy at least one service. This is not such a strong hypothesis as empirical evidence shows that the proportion of the population not having subscribed to neither service is very small (e.g. about one percent in the US (NCHS), and in France (Credoc, 2011)). The subscription of a second service depends on the additional valuation the consumer concedes to it. Suppose for example the consumer firstly subscribes to mobile services and then thinks about buying fixed services. Her additional utility will then stem from the additional volume¹¹. On the other hand, if mobile services are bought as supplementary service, the additional utility is linked to the additional mobility. We think that the subscription of a second service amounts to anticipate the total utility gained from having both services. We will take account of the issue by the introduction of the concept of expected utility, denoted EU . A simple interpretation of EU is to behold it as a learning-by-doing effect: only after having used a given stand-alone service, the consumer will recognize his real needs for either data volume or mobility¹². Introducing the concept of expected utility constitutes a novelty in the literature on bundling and allows us to draw different conclusions as the existing ones. Our paper shows that bundling in a duopoly market actually can be profitable or at least does not have a negative impact on industry's profits.

Before presenting the consumers' preferences and associated utility functions, we announce the timing of the game in order to allow a better understanding of the choices our consumers are supposed to make. The timing of our game is as follows:

Stage 0: Operators fix the prices for their mobile and fixed services ($p_{m_A}, p_{f_A}, p_{m_B}, p_{f_B}$)

Stage 1: Consumers buy the first service (either fixed or mobile).

Stage 2: Consumers choose whether to purchase additional services within the same house and benefit from the discount, to contract with the other operator, or not to purchase at all at this stage.

¹⁰ An implicit price corresponds to the price of the product causing a disutility, on which the totality of the discount is passed.

¹¹ However, the fixed services may also offer increased mobility when the WiFi Community is developed.

¹² Assuming that consumers anticipate future purchase of imperfect substitutable services is not unrealistic, since, following a marketing survey, the majority of respondents sees the possibility of subscribing to new services.

The preferences of the consumers are heterogeneous and we opt for a representation à la Hotelling. Each consumer has to make one choice per service. However, choices are made in different stages (specifically stage 1 and 2) and each stage is represented by one Hotelling line. Figure 2 below illustrates this modelling.

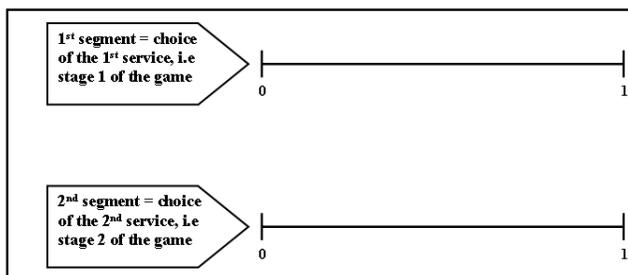


Figure 2: Sequentiality of consumers' choices.

All consumers are supposed to actually buy when they have to do their first choice. The assumption that consumers subscribe to at least one service is thus satisfied. For their second choice, the consumers have 3 possibilities: buying the second service from the same operator as for their first service, buying the second service from the other operator or do not buy at all. This latter case finds its rationale in the empirical presence of single service consumers, i.e. "mobile-only" or "fixed-only"¹³. If the consumer chooses the same operator for his second service, and thus making a bundle out of her services, she will be granted a discount. Otherwise, should the consumer buy from the other operator, full prices of each service have to be paid.

Whether fixed services are chosen in the first place or mobile services depends on the consumers' characteristics or more specifically on their preferences. An exogenous parameter $\sigma \in [0; 1]$ captures these characteristics. Two groups can then be distinguished. The first group (group σ) subscribes to mobile services in the first place. Its interpretation is quite intuitive: this may be the group containing young people who have subscribed to a mobile offer, but do not (yet) benefit from any fixed services¹⁴. After having left the parents' house, they might need a higher data volume and therefore be willing to subscribe to an additional fixed services offer. The second group, group $(1 - \sigma)$, could represent the population that is more bound to their domicile (e.g. older people) and therefore use fixed services for sure. Their second option is thus to also adopt mobile services or not to buy any further services.

On each segment, the positions of our two operators are exogenously given: operator A is located at 0, and operator B at 1. Following the standard Hotelling setting, there are transportation costs, denoted t . Figure 3 represents the general situation in the telecommunications sector. It depicts the consumers whose first choice (the upper segment) is either mobile services (group σ) or fixed services (group $(1 - \sigma)$) as well as the consumers' second choice (lower segment). On the lower segment, we also show how "mobile-only" or "fixed-only" consumers appear.

We are now able to describe the total utility in a general way. For σ -consumers, we have

$$U = U_m + EU_{mf} \quad (1)$$

whereas for the $(1 - \sigma)$ -consumers, total utility is

$$U = U_f + EU_{fm} \quad (2)$$

In these expressions, U_m , resp. U_f are the certain part of the total utility for the corresponding group of consumers¹⁵ and stem from the choice of the first service. $EU_{ij}, \forall i, j = f, m$ and $i \neq j$ is the expected utility coming with the subscription of the second service for the concerned consumer group.

U_m and U_f can further be defined as follows: \forall operator $k = A, B$

$$U_{m_k} = g_{m_k} + v_{m_k} - p_{m_k} - t|y_k - y| \quad (3)$$

$$U_{f_k} = g_{f_k} + v_{f_k} - p_{f_k} - t|y_k - y| \quad (4)$$

¹³These consumers are satisfied enough with the service chosen in the first place and do not contract any further service.

¹⁴We refer here to fixed services they have subscribed on their own and not those where the parents are the subscribers.

¹⁵This also reflects the assumption that the consumer surely buys at least one service.

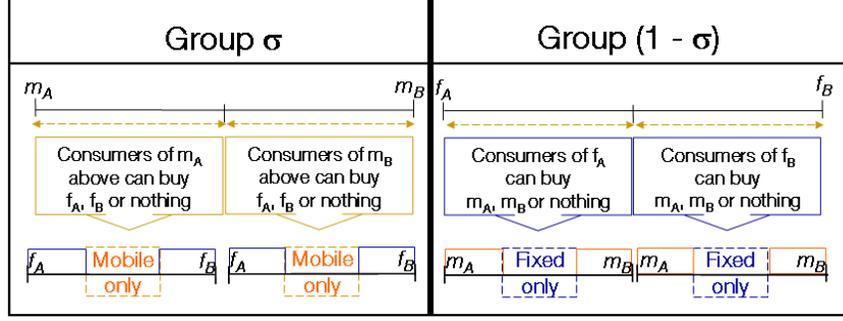


Figure 3: General situation in the telecommunications sector.

expressions where $y \in [0, 1]$ denotes the consumer's position and y_k represents operator k 's position on the Hotelling line. For operator A , y_k takes the value of 0 and for operator B , y_k is equal to 1. v_{i_k} is the valuation accorded to the data volume coming with services $i = f, m$ and g_{i_k} the valuation accorded to the mobility feature of service i . p_{m_k} , resp. p_{f_k} , is the price of mobile, resp. fixed services, of operator k .

When it comes to the second choice, the consumer's decision is based on her expected utility EU_{ij} of having an additional service. There are two possibilities of writing EU_{ij} , based on the group considered. Indeed, if the fixed services are bought as additional service, then additional utility stems for additional data volume. Otherwise, additional utility comes from the supplement in mobility.

Furthermore, in the last stage of the game, the consumer has again the choice between operator A and operator B . But, she also has the choice to benefit from a discount if, and only if, she is buying the second service from the same operator that already provides the first service. This discount, denoted $\delta_k, \forall k = A, B$, will be applied to the price of the second service. Moreover, on this second Hotelling line, the operators are again exogenously located as before. Therefore, if a member of group σ chooses operator k for both of his services, we can write EU_{ij} as follows¹⁶

$$\begin{aligned}
 EU_{m_k f_k} &= \int [U_{m_k f_k} - U_{m_k}] dx \\
 &= \int [g_{f_k} + (v_{f_k} - v_{m_k}) - (p_{f_k} - \delta_k) - t|x_k - x|] dx
 \end{aligned} \tag{5}$$

Otherwise,

$$\begin{aligned}
 EU_{m_k f_{-k}} &= \int [U_{m_k f_{-k}} - U_{m_k}] dx \\
 &= \int [g_{f_{-k}} + (v_{f_{-k}} - v_{m_k}) - p_{f_{-k}} - t|x_k - x|] dx
 \end{aligned} \tag{6}$$

In the equations above, $x \in [0, 1]$ is the consumer's position and x_k is operator k 's position on this second Hotelling line. Note that in equation 6, the consumer prefers cross-buying and therefore no discount is applied. The mobility feature of the fixed service, g_f , can be left aside, since its valuation is virtually equal to 0¹⁷.

For the $(1 - \sigma)$ -group, similar expressions are used:

$$\begin{aligned}
 EU_{f_k m_k} &= \int [U_{f_k m_k} - U_{f_k}] dx \\
 &= \int [v_{m_k} + (g_{m_k} - g_{f_k}) - (p_{m_k} - \delta_k) - t|x_k - x|] dx
 \end{aligned} \tag{7}$$

¹⁶The reader should pay attention to the ordering of the subscripts. The first one indicates the service that has been chosen in the first place: e.g.: mf means that mobile services have been the consumer's first choice and the fixed ones are bought later on.

¹⁷However, when WiFi Community is available, g_f will be strictly positive. See section XXXX where g_f is positive in order to treat the profitability of off-loading.

and

$$\begin{aligned}
EU_{f_k m_{-k}} &= \int [U_{f_k m_{-k}} - U_{f_k}] dx \\
&= \int [v_{m_{-k}} + (g_{m_{-k}} - g_{f_k}) - p_{m_{-k}} - t|x_k - x|] dx
\end{aligned} \tag{8}$$

Similar as for EU_{mf} , the additional data volume (gained by subscribing to an mobile offer in the last stage) in EU_{fm} is negligible. Although the reason here is not that v_m approaches 0, but is rather small compared to the data data volume provided by the fixed service and it seems hardly plausible that this additional data volume constitutes the decisive factor encouraging to subscribe to a mobile services offer.

Finally, when the consumer has to make her choice about choosing a second service or not, the final trade-off is made vis-à-vis all the possible combinations of consuming the services. This is, we have 6 consumption possibilities for each group¹⁸ and the total trade-off is made by summing both integrals of the group σ or group $(1 - \sigma)$:

$$EU_{mf} = EU_{m_k f_k} + EU_{m_k f_{-k}} \tag{9}$$

$$EU_{fm} = EU_{f_k m_k} + EU_{f_k m_{-k}} \tag{10}$$

Until now, the model illustrates consumers buying both products. In the introduction of this paper however, it has been mentioned that a part of the population uses only one service. The part of "mobile-only"-consumers is increasing, while the number of "fixed-only"-consumers becomes less.

Our modelling is able to consider this characteristic. Indeed, the possibility that a consumer does not value the additional service high enough allows for having an unoccupied sub-segment on the second Hotelling line. The occupation of this second line depends thus on the magnitude of the transportation costs t in relation to the valuation of the second service. In a general manner, excessive transportation costs can be linked to transaction costs like filtering all the existing mobile offers, etc. In the case of σ -consumers, the blank space on the second line corresponds to "mobile-only"-consumers. Such a consumption pattern may appear because t is too high compared to $(v_f - v_m)$, so that the consumers' willingness to pay for additional data volume, although positive, is insufficient to cover the disutility. Another possible reason could be that $(v_f - v_m)$ actually is 0 or even negative. Several reasons may explain why this latter case could occur:

- technological progress in the mobile infrastructure that has sharply increased the data capacity and the consumers do not wish or even need additional data volume stemming from a fixed offer,
- unprofitable local loop unbundling (if operators do not offer fixed services in some geographic regions and thus v_f can be 0)
- poor quality of fixed technology (mobile services might be as good, or even better than fixed services.)

Similar explanations for "fixed-only"-consumers can be found: the anticipated additional mobility might be evaluated too low so that transportation costs are not covered or the technological progress in the fixed network annihilates the mobility feature of mobile services (e.g.: WiFi-Community).

Solving the game in its general structure hides several interesting insights. Therefore, in the following sections we make some additional assumptions, so that our model allows us to answer the two questions mentioned above. We begin with the analysis of the variables that impact the market share of single service consumers and focus on "mobile-only"-consumers.

4 "Mobile-only"-market analysis

For the sake of simplicity, we make additional assumptions. First, as noted in the model setup, "mobile-only"-consumers belong to the group σ . Hence, understanding the variables acting on the part of "mobile-only"-consumers is easier when σ is assumed to be equal to 1. Only the left hand-side of figure 3 is then relevant. Second, we suppose the mobile market to be fully covered.

Our analysis begins with a first case where only one operator offers fixed services. This monopoly-case in the market for fixed services shows that FMS is a means to reduce the operator's market power in the

¹⁸E.g. for group σ : two possibilities of single service consumption (m_A and m_B) and four multi-service consumptions ($m_A f_A$, $m_A f_B$, $m_B f_A$ and $m_B f_B$).

fixed market. FMS will be represented by the increase of v_m relative to v_f . In a second case, both operators offer the two services and it can be demonstrated that the FMS has a negative impact on profits of both operators, whereas bundling mitigates the FMS and reduces the share of "mobile-only"-consumers.

4.1 Impact of FMS on operator's profits

Monopoly in the market of fixed services The analysis of the "mobile-only"-market is only possible if not all the consumers subscribe to fixed services. Therefore, the market for fixed services is not fully covered. Firm A is supposed to be the only one that offers fixed services as represented in figure 4.

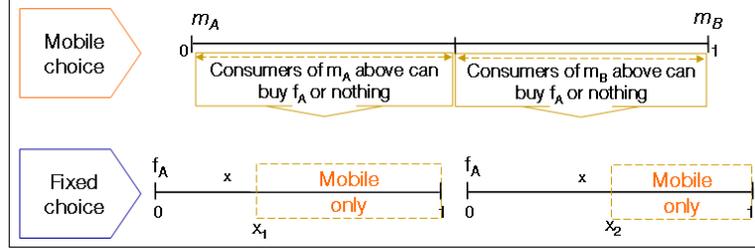


Figure 4: Market structure if operator A has a monopoly in the fixed market and faces competition from operator B in the mobile services market.

At stage 2, consumers choose to subscribe or not to an additional service. In this case, the additional service corresponds to the fixed services offered by operator A . Standard Hotelling calculations lead to oppose multi-service users to "mobile-only"-consumers, which amounts to check whether additional utility stemming from additional volume is valued high enough. Hence, if additional utility is nil, the corresponding consumer will remain single-service user and more precisely "mobile-only"-consumer.

The point of nullity of additional utility is the point of indifference of the marginal consumer. There are, however, two points of indifference to identify since consumers can have mobile subscription from either operator A or B ¹⁹. In the case treated here, these points are easily identified by the following expressions:

$$\begin{aligned} x_1 &\Leftrightarrow U_{m_A f_A} - U_{m_A} = 0 \\ &\Leftrightarrow (v_{f_A} - v_{m_A}) - (p_{f_A} - \delta_A) - tx_1 = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} x_2 &\Leftrightarrow U_{m_B f_A} - U_{m_B} = 0 \\ &\Leftrightarrow (v_{f_A} - v_{m_B}) - p_{f_A} - tx_2 = 0 \end{aligned} \quad (12)$$

Consumers between 0 and x_1 buy fixed services from operator A and enjoy a discount δ_A (given that they have subscribed a mobile offer from operator A), whereas consumers between 0 and x_2 buy from operator B and do not get a discount on their additional subscription. Finally, "mobile-only"-consumers lie between x_k and 1, $\forall k = A, B$.

As we have mentioned above, consumers' choice in the second stage of the game rely upon their expected utility of having a second service. Using equations (11) and (12), expected utility can thus be written as:

$$\begin{aligned} EU_{m_A f_A} &= \int_0^{x_1} (U_{m_A f_A} - U_{m_A}) dx_1 \\ &= \int_0^{x_1} [(v_{f_A} - v_{m_A}) - (p_{f_A} - \delta_A) - tx_1] dx_1 \end{aligned} \quad (13)$$

$$\begin{aligned} EU_{m_B f_A} &= \int_0^{x_2} (U_{m_B f_A} - U_{m_B}) dx_2 \\ &= \int_0^{x_2} [(v_{f_A} - v_{m_B}) - p_{f_A} - tx_2] dx_2 \end{aligned} \quad (14)$$

¹⁹Recall that buying both services from one and the same operator allows for a discount on their subscription of fixed services.

The expected utilities EU_{mf} above²⁰ are formed in stage 1 of the game. In that stage, which we analyze now, consumers choose their mobile service provider.

Both operators compete in a fully covered market for mobile services. They are again located at the two extremes of the Hotelling line. Thus, the consumer's choice is built on comparing total utility levels gained from either operator. Said otherwise, the position of the indifferent marginal consumer, \tilde{y} , is derived by

$$\begin{aligned}\tilde{y} &\Leftrightarrow U_{m_A} + EU_{m_A f_A} = U_{m_B} + EU_{m_B f_A} \\ &\Leftrightarrow g_{m_A} + v_{m_A} - p_{m_A} - ty + EU_{m_A f_A} = g_{m_B} + v_{m_B} - p_{m_B} - t(1 - y) + EU_{m_B f_A}\end{aligned}\quad (15)$$

Thus, the demand for mobile services addressed to operator A stems from every consumers located between 0 and \tilde{y} . Alternatively, consumers between \tilde{y} and 1 subscribe to operator B .

Solutions to stage 1 and 2 allow to completely characterize the different consumer types that emerge in our setting. The table below summarizes the different types.

	$[0; \tilde{y}]$	$[\tilde{y}; 1]$
$[0; x_1]$	Bundled service user	/
$[0; x_2]$	/	Cross-purchaser
$[x_k; 1], \forall k = A, B$	"Mobile-only"-consumers	

The different consumer types in the case of monopoly in the fixed service market.

Once we have identified the different consumer patterns, we can turn to stage 0 of the game, where operators fix their prices for all their services. In the present case, profits are given by the following functions:

$$\pi_A = (p_{m_A} - c_m)\tilde{y} + (p_{f_A} - c_f - \delta_A)\tilde{y}x_1 + (p_{f_A} - c_f)[(1 - \tilde{y})x_2] \quad (16)$$

$$\pi_B = (p_{m_B} - c_m)(1 - \tilde{y}) \quad (17)$$

The first term in equation (16) is revenues from "mobile-only"-consumers having subscribed to operator A , the second corresponds to revenues from bundle-consumers and finally, the last term represents income from cross-purchasers.

In order to solve stage 0, we suppose the data volume of mobile service to be equal, i.e. $v_{m_A} = v_{m_B} = v_m$ ²¹. Recalling also that c_f and c_m are set equal to 0, then equilibrium prices are

$$p_{f_A}^* = \frac{(v_f - v_m)}{2} + \frac{\delta_A}{4} \quad (18)$$

$$p_{m_A}^* = t + \frac{(v_f - v_m)}{6t}\delta_A \quad (19)$$

$$p_{m_B}^* = t - \frac{(v_f - v_m)}{6t}\delta_A \quad (20)$$

We are in presence of inter-platform competition, because the price for fixed services exhibits a competitive pressure stemming from mobile services. Indeed, $p_{f_A}^*$ is negatively impacted by v_m . This is the (*inter-platform*) *competition effect* stemming from FMS. Substitution of fixed by mobile services has thus a negative impact on the business of fixed services. Moreover, the negative impact means that the more the consumer values the volume coming with a mobile offer, the more the monopoly in the fixed market has to set an attractive price for his fixed services. Another direct interpretation is that the more the consumer values the volume in her mobile offer, the less she is tempted to subscribe to an additional service which would only offer more volume.

In the mobile market, prices exhibit a rather strong price competition. Operator B has to price his mobile services more aggressive if his rival is offering a discount δ_A so as not to loose too many market shares in

²⁰Note that these expressions are equivalent to equations (5) and (6) given in the model setup, but (*i*) where the mobility feature of fixed services, g_f , is left aside (see associated discussion on why g_f can be left aside) and (*ii*) where the second integral is left aside. The reason for this is that in the present case here, operator B does not offer fixed services and thus no trade-off towards f_B has to be considered and $\int [U_{m_A f_B} - U_{m_A}] dx$ does not exist.

²¹This assumption is quite intuitive and realistic. Indeed, due to the race for competition the telecommunications operators are engaged in, it seems implausible that a competitor is able to remain reasonably competitive using an *older* technology then its rival.

the fully covered mobile market. This interaction is the *business-stealing effect*. This price competition is furthermore transferred on the market shares of "mobile-only"-consumers and fixed services. Total demand for mobile stand-alone services are given by

$$D_{MO} = 1 - \frac{v_{f_A} - v_m}{t} - \frac{\delta_A}{4t} \quad (21)$$

A necessary condition for the "mobile-only"-market to exist is

$$D_{MO} > 0 \Leftrightarrow t > \frac{1}{4} [2(v_{f_A} - v_m) + \delta_A] \quad (22)$$

This condition states that whenever the disutility of having both services is too high compared to the valuation of additional volume, the concerned consumer do never subscribed to an additional services.

The demand for fixed services is

$$D_F = \frac{v_{f_A} - v_m}{t} + \frac{\delta_A}{4t} \quad (23)$$

A first response element to the question of what variables influence the market share of "mobile-only"-consumers can thus be given. The discount δ_A induces a decrease of this type of consumption, as we can see with D_{MO} (more specifically, $\frac{\partial D_{MO}}{\partial \delta_A} < 0$). Therefore, the business-stealing effect plays in favor of the consumption of multiple services (this is, $\frac{\partial D_F}{\partial \delta_A} > 0$), even though mobile and fixed services are (imperfect) substitutes. Most notably, consumers who value the additional data volume, but who incur, say too high a transaction cost, are tempted to actually buy a second service, because of the discount. An possible interpretation of this is that the discount acts like a compensation of such excessive transportation costs.

A second influencing variable is the difference in valuation for volume ($v_{f_A} - v_m$), i.e. the inter-platform competition effect. The demand for mobile as stand-alone service is positively impacted by the competition effect. Indeed, the more both valuations of volumes are equivalent, the more D_{MO} is stimulated.

Not surprisingly, profits manifest the same impacts of both of the effects described above. Profits at equilibrium are given by

$$\pi_A^* = \frac{t}{2} + \frac{(v_{f_A} - v_m)^2}{4t} + \frac{v_{f_A} - v_m}{6t} \delta_A \quad (24)$$

$$\pi_B^* = \frac{t}{2} - \frac{v_{f_A} - v_m}{6t} \delta_A \quad (25)$$

Proposition 1 *If the market for mobile services is fully covered and consumers' valuation for fixed services too low, the competition effect will negatively impact the multi-service operator.*

Proof. Although the demand for mobile as stand-alone service is stimulated by FMS, the competition effect exerted by FMS on prices for mobile services induces a decrease in profits of a multi-service operator. Indeed, checking the derivative of π_A^* with respect to v_m shows that its sign is negative: $\frac{\partial \pi_A^*}{\partial v_m} = -\frac{v_{f_A} - v_m}{2t} - \frac{\delta_A}{6t} < 0$. Therefore, a multi-service operator who is in a monopoly position in the market for fixed services is constrained by competition from mobile services²². ■

Corollary 2 *Substitution of fixed by mobile services is a means to reduce the market power of the multi-service operator who has market power in the fixed market.*

It is trivial that the business-stealing effect affects negatively the single-service operator (operator B): the higher δ_A is, the more important are caused losses.

The business-stealing effect stems from two sources. Firstly, the asymmetry between operators accords a competitive advantage to the multi-service operator who can soften competition by leveraging its market power from the fixed market to the mobile market. Secondly, consumers anticipate their utility of having two services at the time they choose the provider of their mobile service. They integrate the discount in their proper calculations and might therefore choose the multi-service operator in the first place.

²²One should note that since the multi-service operator is facing competition by a rival firm in the mobile service market as well as by its own subsidiary that provides mobile services, cannibalisation can be another concern of the multi-service operator. However, we do not adress this issue here which is beyond the scope of this paper.

Proposition 3 *The domination of either the business-stealing effect or the competition effect depends on whether the concerned operator is favored by the asymmetry or not.*

Proof. Because of the asymmetry, we have to check for each operator independently which effect dominates. Let us first check for the single-service operator. The difference between derivative with respect to δ_A and v_m is always negative:

$$\frac{\partial \pi_B^*}{\partial \delta_A} - \frac{\partial \pi_B^*}{\partial v_m} = -\frac{v_{f_A} - v_m + \delta_A}{6t} < 0 \quad (26)$$

Unless operator B occupies only geographic regions where local loop unbundling is barely offered, the competition effect does not allow to compensate losses inflicted by the business-stealing effect.

In what concerns the multi-service operator, determining the domination of either effect reveals that the business-stealing effect always dominates the competition effect:

$$\frac{\partial \pi_A^*}{\partial \delta_A} - \frac{\partial \pi_A^*}{\partial v_m} = \frac{(3+t)(v_{f_A} - v_m) + t\delta_A}{6t^2} > 0 \quad (27)$$

The opposite signs prove the proposition. The most favored operator, here operator A , benefits more from the business-stealing effect than the competition effect harms him. ■

Nevertheless, relaxing the assumption of asymmetry between operators annihilates the business-stealing effect.

Multi-service duopoly The assumption of asymmetry is relaxed and both operators now offer fixed and mobile telecommunications services. Also, both operators propose a discount δ to consumers who buy both services from the same operator.

In stage 2, calculations slightly differ from those in the previous case, because two more combinations of multiple service consumption are now possible: a bundle of fixed and mobile services from operator B (denoted $m_B f_B$) and cross-purchasing where mobile consumers of operator A choose operator B as provider of their fixed services (denoted $m_A f_B$). Therefore, we distinguish 4 points of indifference in the last stage of the game:

$$x_1 \Leftrightarrow U_{m_A f_A} - U_{m_A} = 0 \quad (28)$$

$$x_2 \Leftrightarrow U_{m_B f_A} - U_{m_B} = 0 \quad (29)$$

$$x_3 \Leftrightarrow U_{m_A f_B} - U_{m_A} = 0 \quad (30)$$

$$x_4 \Leftrightarrow U_{m_B f_B} - U_{m_B} = 0 \quad (31)$$

Given their choice of mobile service provider in stage 2, we identify consumers who want to buy fixed services from

* operator A by $x \in [0; x_1]$ or $x \in [0; x_2]$,

* operator B by $x \in [x_3; 1]$ or $x \in [x_4; 1]$,

* neither operator and stay "mobile-only"-consumers by $x \in [x_1; x_3]$ or $x \in [x_2; x_4]$.

We solve for these points and integrate them to form expected utilities which are written as follows:

$$\begin{aligned} EU &= EU_{m_A f_A} + EU_{m_A f_B} \\ &= \int_0^{x_1} [U_{m_A f_A} - U_{m_A}] dx + \int_{x_2}^1 [U_{m_A f_B} - U_{m_A}] dx \end{aligned} \quad (32)$$

$$\begin{aligned} EU &= EU_{m_B f_A} + EU_{m_B f_B} \\ &= \int_0^{x_3} [U_{m_B f_A} - U_{m_B}] dx + \int_{x_4}^1 [U_{m_B f_B} - U_{m_B}] dx \end{aligned} \quad (33)$$

Stage 1 is solved using the same approach as in the previous case and details are therefore omitted. The emerging consumption patterns are detailed in the table below.

	$(0; \tilde{y})$	$(\tilde{y}; 1)$
$[0; x_1]$	Bundle from operator A	/
$[0; x_2]$	/	Cross-purchaser
$[x_3; 1]$	Cross-purchaser	/
$[x_4; 1]$	/	Bundle from operator B
$[x_1; x_3]$ and $[x_2; x_4]$	"Mobile-only"-consumer	

The different consumer types in the duopoly case.

In stage 0, the operators face the following profit-maximisation problems:

$$\max_{p_{f_A}, p_{m_A}} \pi_A = (p_{m_A} - c_m) \tilde{y} + (p_{f_A} - c_f - \delta_A) \tilde{y} x_1 + (p_{f_A} - c_f) (1 - \tilde{y}) x_2 \quad (34)$$

$$\begin{aligned} \max_{p_{f_B}, p_{m_B}} \pi_B &= (p_{m_B} - c_m) (1 - \tilde{y}) + (p_{f_B} - c_f - \delta_B) (1 - \tilde{y}) (1 - x_4) \\ &+ (p_{f_B} - c_f) (1 - \tilde{y}) (1 - x_3) \end{aligned} \quad (35)$$

In both expressions above, the first term is the revenue from "mobile-only"-consumers, the second term corresponds to revenues stemming from bundled sales and the last term is the revenue from cross-purchasers.

Since the operators are now symmetric in all respects²³, equilibrium prices and profits in this duopoly setting are given by

$$p_f^{**} = \frac{(v_f - v_m)}{2} + \frac{\delta}{4} \quad (36)$$

$$p_m^{**} = t \quad (37)$$

$$\pi^{**} = \frac{t}{2} + \frac{(v_f - v_m)^2}{4t} \quad (38)$$

Moreover, demands²⁴ for mobile as stand-alone services, as well as for multiple services are

$$D_{MO}^{**} = 1 - \frac{(v_f - v_m)}{t} - \frac{\delta}{2t} \quad (39)$$

$$D_F^{**} = \frac{(v_f - v_m)}{t} + \frac{\delta}{2t} \quad (40)$$

A first thing to notice is that profits are the same for each operator and independent of the discount. Thus, the business-stealing effect is softened because both operators have the same means to leverage their market power onto the mobile market. The operators use the discount in order to bait consumers when they choose their provider of mobile services (i.e. in stage 1) hoping that they buy a second service from them. The competition for these consumer is fierce, as we can see when prices for mobile services are compared. Indeed, compared to the situation of asymmetric operators, prices for mobile offer sharply decrease (for the "former" multi-service operator from the asymmetric case).

To readdress the questions about the effects of FMS, the results of this second case confirm our proposition 1. The results of the asymmetric case can thus be generalized.

Conclusion 4 *FMS reduces prices for fixed services, demand for fixed services and total profits, whereas it stimulates the demand for mobile services as a stand-alone product.*

The "mobile-only"-market analysis has shown that the number of "mobile-only"-consumers increases as the consumers' valuation for volume coming with mobile services v_m increases, while bundling induces an opposite effect. Furthermore, our analysis has also shown that competition between a multi-service operator and a single-service operator is beneficial for the former. This is so because of the business-stealing effect. The multi-service operator is then able to leverage market power from its monopolized service to the competitive market.

²³We assume that v_f , resp. v_m (and the mobility feature g_m) are equally evaluated by the consumers. We omit therefore the subscript k in the expressions of the equilibria in order to simplify the reading.

²⁴The existence condition for in the multi-service duopoly case is $t > v_f - v_m + \frac{\delta}{2}$. More general demands (i.e. with non-equal valuations v_f , v_m , g_f and g_m) and existence conditions are given in section 6, treating consumer surplus.

The next step of our analyzed hitherto was to rerun the exercise but with two multi-service operators. It has been shown that the inter-platform competition effect is still at work, but the business-stealing effect lost its strength. This raises (partly) the second question addressed by this paper. Is bundling in an environment, where two operators offer imperfect substitutes, a profitable strategy? This analysis is broadened on practises like investing in mobile or fixed network and off-loading.

5 Possible reactions of multi-service operators

In the section above, we have seen that FMS has an negative impact on multi-service operators' profits. As a consequence, the operators can choose different strategies that mitigate this impact. A first strategy has already been mentioned: bundling two imperfect substitutes.

In this section, we will look at the incentive multi-service duopolists have to offer a price reduction on bundled sales. Further focus will lie on the profitability of this strategy. However, bundling is not the only possible reaction. Indeed, operators can propose to partly off-load data traffic from the mobile to the fixed network. Moreover, investing in either the fixed or the mobile network can be possible strategies to play in order to influence the valuations of either mobility (by introducing traffic off-load) or of data volume (by investing) and thus to mitigate the impact of FMS.

In order to address these issues, we will first derive profits at equilibrium when operators do not adopt any of the discussed strategies²⁵. Using maximisation programs given by (34) and (35), *basic* profits at equilibrium²⁶ are given by

$$\pi_A = \pi_B = \pi_k = \frac{t}{2} + \frac{(v_{f-k} - v_m)^2}{4t} + \frac{(v_{f-k} - v_m)(v_{f_k} - v_{f-k})}{2t} \quad (41)$$

Each strategy will either increase or decrease final profit outcomes. In the rest of this section, these final profits will be denoted by Π_k .

5.1 Incentives and profitability of bundling

The case of a multi-service duopoly showed that both operators enjoy the same level of profits and that profits are as high as in the situation where neither operator offers a discount for bundled sales²⁷. The question is thus to know if an operator does have an incentive to bundle.

In order to answer this question, four situations will be distinguished: neither operator offers a discount ($\delta_k = \delta_{-k} = 0$), both offer the same discount ($\delta_k = \delta_{-k} = \delta > 0$) and only one operator bundles ($\delta_k > 0, \delta_{-k} = 0$). Profits are summarised in the "pay-off table" below:

	$\delta_{-k} = 0$	$\delta_{-k} > 0$
$\delta_k = 0$	$\Pi_k = \pi_k;$ $\Pi_{-k} = \pi_{-k}$	$\Pi_k = \pi_k - \frac{(v_{f-k} + v_m)}{6t} \delta;$ $\Pi_{-k} = \pi_{-k} + \frac{(v_{f-k} + v_m)}{6t} \delta$
$\delta_k > 0$	$\Pi_k = \pi_k + \frac{(v_{f_k} + v_m)}{6t} \delta;$ $\Pi_{-k} = \pi_{-k} - \frac{(v_{f_k} + v_m)}{6t} \delta$	$\Pi_k = \pi_k + \frac{(v_{f_k} - v_{f-k})}{6t} \delta;$ $\Pi_{-k} = \pi_{-k} + \frac{(v_{f-k} - v_{f_k})}{6t} \delta$

The table shows that each operator individually has an incentive to introduce a discount: $\forall k = A, B$

$$\Pi_k|_{\delta_k > 0, \delta_{-k} = 0} > \Pi_k|_{\delta_k = \delta_{-k} = 0} \quad (42)$$

²⁵The assumption of equal valuation for volume of fixed services is relaxed (i.e. $v_{f_A} \neq v_{f_B}$), whereas the assumption that $v_{m_A} = v_{m_B} = v_m$ is maintained.

²⁶The expressions of *basic* profits at equilibrium will reappear several times and according to the strategy under consideration, some more specific terms have to be added. Moreover, this expression is equivalent to equation (38) where v_{f_A} and v_{f_B} have been set equal.

²⁷To see this latter result, it suffices to set δ equal to 0 and $v_{f_A} = v_{f_B} = v_f$ in equation (24) giving the profits of a multi-service operator that is in a monopoly position for one of the services.

As a result, both operators offer a discount δ . This equilibrium even holds regardless the valuation for data volume of fixed services.

Proposition 5 *Even if $v_{f_k} = v_{f_{-k}}$, the only credible outcome is $(\delta_k > 0; \delta_{-k} > 0)$, i.e. both operators offer a discount.*

Proof. As stated above, if data volumes of either operators are equally valued, the situation where both operators offer price reductions for the subscription of a second service (here fixed services) procures the same level of profits as if no operator would offer a discount. This latter outcome however is not credible, since each operator can increase its profits by introducing a bundle discount. Hence, if one operator offers δ , the other has to respond by the same means, in order not to lose profits. Playing the strategy of no discount ($\delta = 0$) while the competitor does the opposite is thus an incredible strategy. The outcome $(\delta_k > 0; \delta_{-k} > 0)$ is therefore the only credible equilibrium. ■

The equilibrium outcome also exhibits a competition between both operators, where the operator enjoying a higher valuation for his fixed network actually gains in profits, whereas the other one loses profits. But, it can be demonstrated that even in the case of diverging valuations, both operators will offer a discount.

Proposition 6 *Both operators offer a discount even if the consumers' valuation for data volume of one operator is stronger than for the other operator.*

Proof. We show that $(\delta_k > 0; \delta_{-k} = 0)$ never arises in equilibrium. We proceed by contradiction. For this purpose, suppose $v_{f_k} > v_{f_{-k}}$. In this case, operator $-k$ would have to make a trade-off between offering a discount or not, given that operator k offers one. In the first case, operator $-k$'s profits are equal to $\pi_{-k} - \frac{(v_{f_k} + v_m)}{6t}\delta$ and $\pi_{-k} + \frac{(v_{f_{-k}} - v_{f_k})}{6t}\delta$ otherwise. For the outcome $(\delta_k > 0; \delta_{-k} = 0)$ to emerge, $\Pi_{-k}|_{\delta_k > 0, \delta_{-k} = 0}$ has to be greater than $\Pi_{-k}|_{\delta_k = \delta_{-k} = \delta > 0}$. The trade-off amounts then to

$$\begin{aligned} \pi_{-k} - \frac{(v_{f_k} + v_m)}{6t}\delta &> \pi_{-k} + \frac{(v_{f_{-k}} - v_{f_k})}{6t}\delta \\ v_m + v_{f_{-k}} &< 0 \end{aligned}$$

Since v_m and v_f are non-negative, the above inequality is an impossibility and therefore $(\delta_k > 0; \delta_{-k} = 0)$ can never arise in equilibrium. ■

Each operator individually has thus the incentive to introduce a discount and the operator with the higher valuation gains in every situation, whether his rival offers a discount or not. Therefore, we conclude that bundling is a profitable strategy. Furthermore, as it has been shown in the previous section, bundling has also an (inter-platform) competition weakening effect.

The next possible strategy is to off-load data traffic from the mobile to the fixed network. This issue is analyzed in the subsequent point.

5.2 Incentive to introduce off-load

Due to an increasing need of data volume and simultaneously, due to the scarcity of radio frequency resources, operators can propose their consumers to off-load traffic from the mobile network to the fixed network. The principle of off-loading lies in discharging data traffic from the mobile network to the fixed network. A consumer who has access to a mobile network usually enjoys limited data capacity. In order to increase her data capacity with her handset, she can connect her mobile device to a fixed modem and use Internet services with her handset, but using a fixed network. Off-loading is thus a means to confer a strictly positive valuation for mobility to the fixed network²⁸.

In order to capture the impact of off-loading alone, we make the following assumptions. First, valuations for mobile services (mobility and data volume) as well as for mobility of fixed network are the same for both operators ($g_{m_k} = g_{m_{-k}} = g_m$, $v_{m_k} = v_{m_{-k}} = v_m$ and $g_{f_k} = g_{f_{-k}} = g_f$). Second, we abstract from any discount ($\delta_k = \delta_{-k} = 0$). Finally, it is assumed that off-loading reduces the perceived marginal costs of mobile data traffic²⁹, which can be translated by $\hat{c}_{m_k} = (1 - \frac{g_{f_k}}{g_{m_k}})c_m$. This definition tells us that for a given

²⁸The analysis below also applies to the introduction of a WiFi-Community, where operators "open" the modems of fixed services so that mobile user can connect with their mobile devices. Recall that the difference, between off-load and WiFi-community is that with the latter, the consumer does not dispose any mobile service.

²⁹We relax the assumption that c_m is equal to 0.

value accorded to mobility of a mobile offer, the higher the valuation for mobility of a fixed offer is, the less high is the marginal cost of mobile data traffic.

Under the given assumptions and the four possible scenarii (neither operator introduces off-loading, both or only one operator introduces it), profit outcomes are as follows

	$\hat{c}_{m_k} = c_m$	$\hat{c}_{m_k} = (1 - \frac{g_f}{g_m})c_m$
$\hat{c}_{m_k} = c_m$	$\Pi_k = \pi_k; \Pi_{-k} = \pi_{-k}$	$\Pi_k = \pi_k - c_m \frac{g_f}{3g_m}; \Pi_{-k} = \pi_{-k} + c_m \frac{g_f}{3g_m}$
$\hat{c}_{m_k} = (1 - \frac{g_f}{g_m})c_m$	$\Pi_k = \pi_k + c_m \frac{g_f}{3g_m}; \Pi_{-k} = \pi_{-k} - c_m \frac{g_f}{3g_m}$	$\Pi_k = \pi_k; \Pi_{-k} = \pi_{-k}$

In line with the incentives to bundle, each multi-service operator has an individual incentive to propose off-load. But, if both operators do so, profits fall back to their initial levels. Furthermore, would marginal costs of mobile data traffic be equal to 0, the two multi-service operators would be perfectly indifferent between introducing off-loading and not to introduce. Profits would never be impacted.

Again, the outcome where both operators off-load data traffic is the only credible outcome since, playing the strategy of no off-load ($c'_{m_k} = c_m$) as a response to an operator that does off-load ($c'_{m_{-k}} = (1 - \frac{g_f}{g_m})c_m$) induces a decrease in profits. Such a decrease would be even stronger the more the rival off-loads, i.e. the higher $\frac{g_f}{g_m}$ is.

In FMS matters however, no impact (other than the *standard* impact on basic profits) on profits nor market share of "mobile-only"-consumers appears.

5.3 Incentive to invest in fixed and mobile networks

The increasing need of data volume can also be satisfied by an investment in either the fixed or mobile network in order to increase the capacity to transfer data. As mentioned in the introduction, mobile networks will evolve towards LTE technology, which increases substitutability between fixed and mobile services. On the other hand, investing in the fixed network by deploying a FTTx network has an opposite effect.

In this section, we check whether multi-service operators have an incentive to invest in either network, given the impact of FMS on its profits

Investment in FTTx Without taking any regulatory or financial constraints into account, we assume that an operator k can invest in the fixed network and increase thus the valuation consumers accord to the capacity of data traffic of fixed service by $\Delta v_{f_k} > 0$. The table below reports the different profit outcomes for the situations where (i) no operator invests ($\Delta v_{f_k} = \Delta v_{f_{-k}} = 0$), (ii) only one operator invests ($\Delta v_{f_k} > 0$ and $\Delta v_{f_{-k}} = 0$), (iii) both invest ($\Delta v_{f_k} = \Delta v_{f_{-k}} > 0$). It shows that in equilibrium both operators invest³⁰:

	$\Delta v_{f_{-k}} = 0$	$\Delta v_{f_{-k}} > 0$
$\Delta v_{f_k} = 0$	$\Pi_k = \pi_k; \Pi_{-k} = \pi_{-k}$	$\Pi_k = \pi_k; \Pi_{-k} = \pi_{-k} + \frac{(v_{f_{-k}} - v_m)}{2t} \Delta v_{f_{-k}}$
$\Delta v_{f_k} > 0$	$\Pi_k = \pi_k + \frac{(v_{f_k} - v_m)}{2t} \Delta v_{f_k}; \Pi_{-k} = \pi_{-k}$	$\Pi_k = \pi_k + \frac{(v_{f_k} - v_m)}{2t} \Delta v_{f_k}; \Pi_{-k} = \pi_{-k} + \frac{(v_{f_{-k}} - v_m)}{2t} \Delta v_{f_{-k}}$

In contrast to the two cases above, the action of operator k has no impact on operator $-k$. However, each operator still faces the inter-platform competition effect i.e. $\frac{(v_{f_k} - v_m)}{2t} \Delta v_{f_k}$. Hence, not only is there an incentive to invest in an infrastructure for which the market is yet not fully served (indeed, profits are higher compared to a situation of no investment), but the competition effect is weakened, too. The difference of derivatives of profits with respect to v_m , given that Δv_{f_k} is either positive or 0, shows up negative:

$$\left. \frac{\partial \Pi_k}{\partial v_m} \right|_{\Delta v_{f_k} > 0} - \left. \frac{\partial \Pi_k}{\partial v_m} \right|_{\Delta v_{f_k} = 0} = -\frac{\Delta v_{f_k}}{2t} \quad (43)$$

³⁰Following assumptions were made: $v_{m_k} = v_{m_{-k}} = v_m$, $g_{m_k} = g_{m_{-k}} = g_m$ and finally $g_{f_k} = g_{f_{-k}} = 0$. Also, we abstract from any discount ($\delta_k = \delta_{-k} = 0$). For a discussion on incentive to invest when there is a positive δ , see at the end of this section, above conclusion 7 on page 19.

This expression is always negative since t and Δv_{f_k} are non-negative. It can thus be concluded that investing in an infrastructure whose market is not fully covered relaxes inter-platform competition and is a profitable strategy for a multi-service operator for mitigating the impacts of FMS on its profits.

However, our multi-service operators also have the opportunity to invest in mobile networks in order to increase the consumer's valuation v_m . This case will be treated next.

Investment in mobile networks Similar to the case above, we want to analyse whether an multi-service operator has an incentive to invest in his mobile network infrastructure when FMS is a concern of both. It is supposed that each operator can invest an amount so as to positively influence the consumers' valuation for data volume stemming from a mobile offer by Δv_{m_k} . The analysed situations are (i) $\Delta v_{m_k} = \Delta v_{m_{-k}} = 0$, (ii) $\Delta v_{m_k} > 0$ and $\Delta v_{m_{-k}} = 0$, (iii) $\Delta v_{m_k} = \Delta v_{m_{-k}} > 0$. Under the assumptions that valuations for data volume and mobility are equal between the two operators, profits yield the following expressions³¹:

	$\Delta v_{m_{-k}} = 0$	$\Delta v_{m_{-k}} > 0$
$\Delta v_{m_k} = 0$	$\Pi_k = \pi_k$ $\Pi_{-k} = \pi_{-k}$	$\Pi_k = \pi_k - \frac{2t - (v_f - v_m)}{6t} \Delta v_{m_{-k}}$ $\Pi_{-k} = \pi_{-k} + \frac{t - 2(v_f - v_m)}{3t} \Delta v_{m_{-k}}$
$\Delta v_{m_k} > 0$	$\Pi_k = \pi_k + \frac{t - 2(v_f - v_m)}{3t} \Delta v_{m_k}$ $\Pi_{-k} = \pi_{-k} - \frac{2t - (v_f - v_m)}{6t} \Delta v_{m_k}$	$\Pi_k = \pi_k - \frac{(v_f - v_m)}{2t} \Delta v_{m_k}$ $\Pi_{-k} = \pi_{-k} - \frac{(v_f - v_m)}{2t} \Delta v_{m_{-k}}$

Individually, operator k would gain in profits if he was the sole firm to invest in his mobile infrastructure. Therefore, the equilibrium predicts that both operators invest. But they face a prisoner's dilemma, because in this equilibrium outcome, profits decrease compared to the situation where no one invest. The outcome ($\Delta v_{m_k} = \Delta v_{m_{-k}} = 0$) is thus the pareto-optimal equilibrium. However, since each operator plays non-cooperatively, they invest individually in their mobile networks. And since the mobile market is fully covered, competition on this market is even more fierce which induces the decrease in profits.

Though the emergence of the prisoner's dilemma is subject to a condition. This condition is most visible (and its interpretation more clear cut) if the invested amounts are supposed equal, i.e. $\Delta v_{m_k} = \Delta v_{m_{-k}}$. The prisoner's dilemma appears when the following trade-off is satisfied:

$$\begin{aligned}
\Pi_k = \Big|_{\Delta v_{m_k}=0, \Delta v_{m_{-k}}>0} &< \Pi_k = \Big|_{\Delta v_{m_k}>0, \Delta v_{m_{-k}}>0} \\
\pi_{-k} - \frac{2t - (v_f - v_m)}{6t} \Delta v_{m_k} &< \pi_{-k} - \frac{(v_f - v_m)}{2t} \Delta v_{m_{-k}}
\end{aligned} \tag{44}$$

The inequality is satisfied when $t > 2(v_f - v_m)$. This means that whenever the consumers do not value high enough the additional data volume stemming from fixed services, the part of "mobile-only"-consumers is too important. Because of the obligation to invest, combined with the already fierce competition in the mobile market, the operators may experience difficulties to recover their investments. This conclusion refers to an *intra-platform* competition effect, which, however, is somewhat hidden by the assumption of equal valuation of data volume of mobile services³².

In what concerns FMS, the impact on profits of an increase in v_m is undoubtedly increased by

$$\left. \frac{\partial \Pi_k}{\partial v_m} \right|_{\Delta v_{m_k}>0, \Delta v_{m_{-k}}>0} = \frac{\Delta v_{m_k}}{2t} \tag{45}$$

, compared to the situation where no one invests. Thus, investing in mobile infrastructure reinforces the inter-platform competition effect.

A final feature we want to offer is how profits react relative to the latter three strategies when both operators offer a positive discount. We find that the incentive to invest in the fixed network is positively impacted. Using profit expression of operator k 's³³ and computing the derivative with respect to v_{f_k} shows

³¹ Assumptions: $v_{f_A} = v_{f_B} = v_f$, $v_{m_A} = v_{m_B} = v_m$, $g_{f_A} = g_{f_B} = 0$, $g_{m_A} = g_{m_B} = 0$ and $\delta = 0$.

³² In appendix 3, we analyze the profit function when this assumption is relaxed.

³³ See table relative to bundling incentives on page 15.

up positive: $\frac{\partial \Pi_k |_{\delta_k > 0, \delta_{-k} = 0}}{\partial v_{f_k}} = \frac{\partial \Pi_k |_{\delta_k > 0, \delta_{-k} > 0}}{\partial v_{f_k}} = \frac{v_{f_{-k}} - v_m}{2t} + \frac{\delta}{6t}$. This result is due to the assumption that the market for fixed services is not fully covered. Indeed, the introduction of δ does not impact the incentive to invest in mobile networks, nor does it impact the incentive to off-load.

Before turning to the analysis of consumer surplus, we briefly conclude this section.

Conclusion 7 *In a duopolistic environment, multi-service operators individually have the incentive to introduce a bundle discount, as well as to offer the possibility to off-load data traffic. Furthermore, investing in either network is individually a profitable strategy and allows to mitigate the inter-platform competition effect of FMS. At the end however, the profitability of bundling depends on which operator enjoys the higher valuation of his fixed service's data volume capacity, whereas in equilibrium both operators are indifferent between introducing off-load or not (in the sense that profits are at the same level). The investment in the fixed network is always profitable (and even more when there is a positive discount), but investment in the mobile infrastructure exhibits a situation of the prisoner's dilemma. Here, the pareto-suboptimal outcome is chosen in equilibrium.*

6 Analysis of consumer surplus

The previous section depicted the impacts on profits of multi-service operators if strategies like bundling, off-loading or investing are employed. In the following, the consumer surplus is analyzed and checked for eventual beneficial or detrimental impacts of the discussed strategies.

The analysis begins by deriving the referee case, i.e. the equilibrium prices and the consumer surplus in a situation where neither strategy is adopted. This situation can be interpreted as an initial case, where it is assumed that, in terms of data volume, fixed services are higher evaluated than mobile services. Hence, in the expressions below, we have that $v_f - v_m > 0$. Then, taking the different strategies into account, the subsequent prices and consumer surplus will be compared to the referee case³⁴.

6.1 The initial situation

In this case, neither multi-service operator does adopt one of the discussed strategies. If the assumption $v_{m_k} = v_{m_{-k}} = v_m$ holds, prices are given by, $\forall k = A, B$

$$p_{f_k}^* = \frac{v_{f_k} - v_m}{2} \quad (46)$$

$$p_{m_k}^* = t \quad (47)$$

These prices allow for the following demands

$$D_{MO} = 1 - \frac{v_{f_k} + v_{f_{-k}} - 2v_m}{2t} \quad (48)$$

$$D_F = \frac{v_{f_k} + v_{f_{-k}} - 2v_m}{2t} \quad (49)$$

Notice that the condition for D_{MO} to exist is given by

$$D_{MO} > 0 \Leftrightarrow t > \frac{1}{2} (v_{f_k} + v_{f_{-k}}) - v_m \quad (50)$$

Consumer surplus is derived using equation (1) on page 7 and plugging equilibrium prices into

$$CS^* = U_m(p_{f_k}^*, p_{m_k}^*) + EU_{mf}(p_{f_k}^*, p_{m_k}^*) \quad (51)$$

³⁴In order to warrant comparability, the same assumptions made for the derivation of the equilibrium with a given strategy will be applied to the referee case.

In this case, CS is then equal to

$$CS^* = (g_m + v_m) + \frac{v_{f_k}^2 + v_{f_{-k}}^2 + 2v_m^2}{4t} - \frac{5t}{4} - \frac{2v_m(v_{f_k} + v_{f_{-k}})}{4t} \quad (52)$$

For CS^* to be positive, it is needed that

$$CS^* > 0 \quad (53)$$

$$\Leftrightarrow t < \frac{1}{5} \left[2(g_m + v_m) + \sqrt{4g_m^2 + 5(v_{f_k}^2 + v_{f_{-k}}^2) + 8g_mv_m - 10(v_{f_k} + v_{f_{-k}})v_m + 14v_m^2} \right]$$

This is a less strong condition than for having $D_{MO} > 0$. Thus, whenever $t > \frac{1}{2}(v_{f_k} + v_{f_{-k}}) - v_m$, the consumer surplus is positive.

6.2 Consumer surplus and the different strategies

Bundling has a positive impact on consumer surplus Bundling has shown up profitable for the operator that enjoys the higher valuation for the volume of its fixed service. This fact stems from the higher prices charged for stand-alone sales. Indeed, the differences in prices and demands, compared to the referee case, are as follows³⁵:

$$\Delta p_{f_k}^B = \frac{\delta}{4} \quad (54)$$

$$\Delta p_{m_k}^B = \frac{(v_{f_k} - v_{f_{-k}})\delta}{6t} \quad (55)$$

$$\Delta D_{MO} = -\frac{\delta}{2t} \quad (56)$$

$$\Delta D_F = \frac{\delta}{2t} \quad (57)$$

The discount plays the central role in the profitability of the strategy of bundling. This strategy is also beneficial for consumers. In effect, consumer surplus increases by

$$\Delta CS^B = \frac{(v_{f_k} + v_{f_{-k}} - 2v_m)\delta}{4t} \quad (58)$$

Off-loading enhances consumer surplus

The analysis of incentives showed that in equilibrium off-loading data traffic from the mobile onto the fixed network does not negatively impact multi-service operators. The only impact on prices concerns the price of the mobile offer, which decreases by $\Delta p_{m_k} = -c_m \frac{g_f}{g_m}$. Furthermore, demands do not vary, too. As a result of non-varying demands and a decreasing price for mobile services, the gain in consumer surplus induced by off-loading is equal to³⁶:

$$\Delta CS^O = c_m \frac{g_f}{g_m} \quad (59)$$

It can be seen that the increase of consumer surplus correspond exactly to the price decrease. Hence, no competitive effect (either inter or intra-platform) affects this one-to-one translation.

The beneficial impact of investments

³⁵ Assumptions: $v_{m_k} = v_{m_{-k}} = v_m$, $v_{f_k} \neq v_{f_{-k}}$, $\delta_k = \delta_{-k} = \delta = 0$ and all mobility valuations are set equal to 0.

³⁶ Assumptions: $v_{m_k} = v_{m_{-k}} = v_m$, $v_{f_k} \neq v_{f_{-k}}$, $\delta_k = \delta_{-k} = 0$, $g_{m_k} = g_{m_{-k}} = g_m$, $g_{f_k} = g_{f_{-k}} = g_f$. Exceptionnally, $c_m > 0$.

Investment in fixed networks We have seen that operators gain in profits by positively influencing the consumers' valuation for volume stemming from fixed networks. This increase in valuation is reflected by the price variation for fixed services which increases by half the amount of the additional valuation, i.e. $\Delta p_{f_k} = \frac{\Delta v_{f_k}}{2}$. As could be expected, an increase in valuation of volume of fixed services has a positive effect on the demand for fixed services ($\Delta D_F = \frac{\Delta v_{f_k} + \Delta v_{f_{-k}}}{2t}$), whereas mobile services are negatively impacted ($\Delta D_{MO} = -\frac{\Delta v_{f_k} + \Delta v_{f_{-k}}}{2t}$). Since a part of the "mobile-only"-consumers switch to multi-service users, the increase in valuation of data volume of fixed services is to interpret as a compensation for excessive transportation costs. Furthermore, consumer surplus increases by³⁷

$$\Delta CS^{IF} = \frac{1}{2t} [\Delta v_{f_k}(v_{f_k} - v_m) + \Delta v_{f_{-k}}(v_{f_{-k}} - v_m)] + \frac{1}{4t} (\Delta v_{f_k}^2 + \Delta v_{f_{-k}}^2) \quad (60)$$

Since we start from an initial situation with $v_{f_k} > v_m$, the expression above is strictly positive.

Investing in mobile networks Similar to the results above, investing in a mobile network will impact the valuation of volume stemming from a mobile offer. Although there is inter-platform competition and fierce competition on the mobile segment, the prices of a mobile offer remain unaffected. The prices of fixed services on the other hand decrease by $\Delta p_{f_k} = -\frac{\Delta v_m}{2}$. In demand matters, an equal shift from multi-service users to single-service consumers is to notice³⁸:

$$\Delta D_{MO} = \frac{\Delta v_m}{t} \quad (61)$$

$$\Delta D_F = -\frac{\Delta v_m}{t} \quad (62)$$

The variation of consumer surplus is given by

$$\Delta CS^{IM} = \frac{\Delta v_m}{2t} [2(t - v_f - v_m) + \Delta v_m] \quad (63)$$

In this expression, we find the existence condition for a positive "mobile-only"-demand, $t > v_f - v_m$. It can be seen that, even if there would be no "mobile-only"-consumers, i.e. $t < (v_f - v_m)$, investment in mobile network infrastructure would always increase consumer surplus, since $\Delta v_m > 0$ (except for very low transportation costs).

Conclusion 8 *It has been shown that each strategy is beneficial to the consumers. Social welfare considerations, defined as the sum of profits and consumer surplus, are straightforward. Bundling and investing in fixed network infrastructure clearly augment operators' profits and consumer surplus. Thus, social welfare is increased. Off-loading does not negatively impact profits, but increases CS, increasing in turn social welfare. Investment in mobile networks, however, induce a decrease in profits and the impact on consumer surplus is likely (but not necessarily) to be positive. The impact on social welfare depends not only on transportation cost t , but also on Δv_m .*

7 Concluding remarks

This paper draws upon a duopoly model where two multi-service operators offer mobile services, fixed services and a bundle of both. This model allowed us to answer two questions. First, we were interested in the variables that affect the market share for only one of the two offered services³⁹. Second, the incentives and profitability of strategies like bundling, off-loading and investment were analyzed. Our focus lied on

³⁷ Assumptions: $v_{m_k} = v_{m_{-k}} = v_m$, $g_{m_k} = g_{m_{-k}} = 0$, $g_{f_k} = g_{f_{-k}} = 0$ and $\delta = \delta_{-k} = 0$.

³⁸ Assumptions: valuations for volume and mobility are all equal between both operators and there is no discount.

³⁹ Such a setting fills thus a gap in the bundling literature, where consumers are always supposed to buy the two products. We assume that consumers can either buy one stand-alone product or buy the two services.

"mobile-only"-consumers and we used a Hotelling scenario where consumers subscribe to a second (imperfectly substitutable) service only if their *expected utility of having both services* is positive.

We find that when bundling the two services together is possible, the market share of "mobile-only"-consumers reduces, because the discount acts like either a partial compensation for excessive transportation costs or reduces the substitutability by reducing the valuation for data volume of mobile services. Alternatively, the more the consumer values the data volume of a mobile offer relative to a data volume of a fixed offer, the more the "mobile-only"-market share increases. The effect stems from Fixed-to-Mobile substitution and is called the inter-platform competition effect. This is, the dominant competitive effect stems from competition between fixed and mobile network, rather than between the two operators.

In what concerns the incentive and profitability of strategies like bundling, off-loading and investment, we find that operators have the incentive to bundle and to invest in fixed networks. Profits strictly increase if these strategies are adopted. The profitability of bundling in duopolistic environment has (to our knowledge) always been reported as unprofitable. In this paper, however, bundling could be profitable. This contrasting result is due to the introduction of expected utility of the subscription of a second service (which, if bought from the same operator as the first service, allows for a price reduction). When consumers anticipate their utility of having a second service and possibly a discount, they are more inclined to actually subscribe to both, even if services are imperfect substitutes.

Off-loading does not increase, nor decrease profits in equilibrium. However, operators are individually incentivized to propose off-load. This leads to a situation where both operators introduce off-loading, even though equilibrium profits do not differ from the situation where they do not.

Only the investment in mobile networks shows up unprofitable. Indeed, the outcome is such that both multi-service operators invest in mobile infrastructures, but enjoy lower profits than if they did not invest. Thus, operators clearly face a prisoner's dilemma situation.

Finally, we checked for impacts on consumer surplus and find that each strategy is desirable from the consumers' point of view. This leads to the conclusion that, except for investments in mobile networks, for which the conclusion is more ambiguous, social welfare always increases.

The modelling in this paper did not take regulatory issues into consideration, nor eventual fixed cost to incur while investing. It could thus be interesting to analyse a situation where an incumbent operator offers or is obligated to offer wholesale access to its fixed network to a retail competitor. This access pricing problem, combined with the different competition effects identified in this papers, can be subject to further research.

8 Appendix

Appendix 1: List of acronyms used in the paper

ADSL	Asymmetric Digital Subscriber Line
BEREC	Body of European Regulators of Electronic Communications
CATV	Cable Television
DSL	Digital Subscriber Line
EDGE	Enhanced Data-rates for Global Evolution
FMS	Fixed-Mobile Substitution
FTTx	Fibre to the x ⁴⁰
Gbits/s	Gigabits per second
GPRS	General Packet Radio Service
GSM	Global System for Mobile Communications
HSPA	High Speed Packet Access ⁴¹
kbits/s	kilobits per second
LTE	Long Term Evolution
Mbits/s	Megabits per second
RTR	Rundfunk & Telekom Regulierungs-GmbH
UMTS	Universal Mobile Telecommunications System
VoIP	Voice over Internet Protocol

Appendix 2: Connection Speed of mobile technologies

Technology	Theoretical Download speed
GSM	9 kbits/s
GPRS	33 kbits/s
EDGE	50-60 kbits/s
UMTS	128-350 kbits/s
HSDPA	up to 14 Mbits/s ⁴²
HSPA+	42 Mbits/s ⁴³
LTE	up to 1 Gbits/s ⁴⁴

Appendix 3: The intra-platform competition effect in the mobile market

In section 5.3 on page 5.3, we analyzed the incentive to invest in mobile networks. The aim of such an investment is to increase the valuation accorded to data volume stemming from a mobile offer. One of the assumption was that $v_{m_k} = v_{m_{-k}} = v_m$, which implicitly hid the intra-platform competition effect in the mobile market. This effect induces both operators to invest even though, in the final outcome, both incur losses after having invested.

Here, we relax the mentioned assumption in order to explicit this effect. The intra-platform competition effect in the mobile market will be denoted by $\mu = v_{m_k} - v_{m_{-k}}$. Also, the inter-platform competition effect is still present and it is denoted by $\eta = 2v_f - v_{m_k} - v_{m_{-k}}$. The equilibrium profits are given then by

$$\begin{aligned} \Pi_k = & \frac{t}{2} + \frac{1}{3}\mu + \frac{1}{18t^3}\mu^2\eta(\mu - 4\Delta v_m) + \frac{1}{9t^2}\mu^2(\eta + 2\Delta v_m) \\ & + \frac{1}{36t} \left[\begin{aligned} & 9v_f^2 + 14v_{m_k}^2 - 6v_f(4v_{m_k} - v_{m_{-k}} + 3\Delta v_m) \\ & - 4v_{m_k}(v_{m_{-k}} - 6\Delta v_m) - v(m_{-k}v_{m_{-k}} + 6\Delta v_m) \end{aligned} \right] \end{aligned} \quad (64)$$

⁴⁰x stands for the fibre network's endpoint. This can be either the enduser's home (FTTH), the last amplifier (FTTLa), the building (FTTB) or, if the enduser is a firm, to a cabinet (FTTC).

⁴¹There exist several norms of HSPA, like HSDPA (HS downlink PA) or HSPA+ (a version offer higher connection speed).

⁴²This is a theoretical speed, because it has to be shared between all the users who are connected in the same time.

⁴³*idem*.

⁴⁴This is valid for the most ideal conditions.

Operator k 's profits are negatively impacted by intra-platform competition (this is, $\frac{\partial \Pi_k}{\partial \mu} < 0$) whenever $v_{m_{-k}}$ exceeds v_{m_k} by at least $\varepsilon = \frac{3t^3}{\eta(2t+\eta)+4\Delta v_m(t-\eta)}$ ⁴⁵. This term is positively impacted by Δv_m , inducing an stronger intra-platform competition effect. This is, the more ε approaches 0, the more intra-platform competition is fierce. Since, investing in mobile networks does not vary the price of mobile services p_{m_k} (possibly due to the full coverage of the mobile market), the multi-service operator have to act on prices of their fixed services in order to persuade consumer to buy the two services from the same operator. This explains why equilibrium profits decrease when operators invest in mobile infrastructure. Finally, for the impact of Δv_m on ε to be positive, we must have that $t > \eta$, i.e. the condition for a positive demand for mobile as stand-alone service:

$$\frac{\partial \varepsilon}{\partial \Delta v_m} = \frac{12t^3(t-\eta)}{(\eta(\eta-4\Delta v_m)+2t(\eta+\Delta v_m))^2} \quad (65)$$

Hence, the intra-platform competition effect is conditioned by the inter-platform competition effect.

⁴⁵This has to be understood as follows: $\frac{\partial \Pi_k}{\partial \mu} < 0 \Leftrightarrow \mu < -\frac{3t^3}{\eta(2t+\eta)+4\Delta v_m(t-\eta)}$.

References

- [1] Armstrong M., 2010: Bundling Revisited: Substitute Products and Inter-Firm Discounts, Economics Series Working Papers 574, University of Oxford, Department of Economics.
- [2] Adams W.J., Yellen J.L. Aug., 1976: Bundling and the Burden of Monopoly, *The Quarterly Journal of Economics*, **90** (3), pp. 475-498.
- [3] Albon R., 2006: Fixed-to-Mobile Substitution, Complementarity and Convergence, *Agenda*, **13** (4), pp.309-322.
- [4] Barth A., Heimeshoff U., 2011: Does the growth of mobile markets cause the demise of fixed networks? – Evidence from the European Union.
- [5] Berec, 2011: Report on Impact of Fixed-Mobile Substitution on Market definition, December 2011, downloadable at www.erg.eu.int/doc/berec/bor/bor11_54_FMS.pdf.
- [6] Briglauer W., Schwarz A., Zulehner C., 2009: Is Fixed-Mobile Substitution strong enough to de-regulate Fixed Voice Telephony? Evidence from the Austrian Markets, Working Papers / Research Institute for Regulatory Economics, 2009,2. Forschungsinstitut für Regulierungsökonomie, WU Vienna University of Economics and Business, Vienna, (2009).
- [7] Carbajo J., De Meza D., Seidmann D. J., 1990: A Strategic Motivation For Commodity Bundling, *The Journal of Industrial Economics*, **38** (3), pp. 283-298.
- [8] Chen Y., 1997: Equilibrium Product Bundling, *Journal of Business*, **70** (1), pp. 85-103.
- [9] Gans J. S., King S. P., 2006: Paying for Loyalty: Product Bundling in Oligopoly, *The Journal of Industrial Economics*, **54** (1), pp. 43-62.
- [10] Liang J., 2010: Competition between fixed and mobile broadband access based on mobility and data volume, ITS Sept. 2010 downloadable on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1684019. to be published in IJMNE
- [11] Matutes C., Régibeau P. 1992: Compatibility and Bundling of Complementary Goods in a Duopoly, *The Journal of Industrial Economics*, **40** (1), pp. 37-54.
- [12] McAfee R. P., McMillan J., Whinston M., 1989: Multiproduct Monopoly, Commodity Bundling and Correlation of Values, *The Quarterly Journal of Economics*, **104** (2), pp. 371-384.
- [13] Nalebuff B., 1998: Competing against Bundles, Working Paper No. 7, Yale School of Management.
- [14] Nalebuff B., 2004: Bundling as an Entry Barrier, *The Quarterly Journal of Economics*, **119** (1), pp. 159-188.
- [15] Niedermayer A: Does a Platform Monopolist Want Competition? Dec. 2006 downloadable on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=893617.
- [16] Reisinger M., 2006: The Effects of Product Bundling in Duopoly, mimeo Munich.
- [17] Rodini M., Ward M. R., Woroch G. A., 2003: Going Mobile: Substitutability between Fixed and Mobile Access, *Telecommunications Policy*, **27**, pp. 457-476.
- [18] RTR, 2005: Abgrenzung des Marktes für Breitbandigen Zugang auf Vorleistungsebene, March 2005, downloadable at www.rtr.at/uploads/media/3072_Marktabgrenzung_Breitband_nat_Konsultation.pdf
- [19] RTR, 2009: Abgrenzung des Marktes für Breitbandigen Zugang auf Vorleistungsebene, March 2005, downloadable at www.rtr.at/de/tk/TKMV2008/Begleittext_TKMV_2008.pdf.
- [20] Schejter A.M. *et al.*, 2009: Policy implications of market segmentation as a determinant of fixed-mobile service substitution: What it means for carriers and policy makers, *Telemat Informat.* (2009), doi:10.1016/j.tele.2009/05/002.

- [21] Thanassoulis J., 2007: Competitive Mixed Bundling and Consumer Surplus, *Journal of Economics & Management Strategy*, **16** (2), pp. 437-467.
- [22] Vogelsang I., 2010: The relationship between mobile and fixed-line communications: A survey, *Information Economics and Policy*, 22, pp.4-17.