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**The Equivalence of Strict  
Liability and Negligence Rule:  
A « Trompe l'œil »  
Perspective**

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#### A « Trompe l'œil » Perspective

By Gérard Mondello, University of Nice Sophia Antipolis, CREDECO, GREDEG, UMR 7321, CNRS

#### Summary

This paper analyzes the difficulties of comparing the respective effectiveness of two among the most important liability regimes in tort law: rule of negligence and strict liability. Starting from the standard Shavellian unilateral accident scheme, I show that matching up liability regime on their capacity to provide the highest level of safety is ineffective. This demonstration lies on two components. The first one gathers some results drawn from literature that introduces uncertainty. The second one takes into consideration the beliefs of agents and their aversion to ambiguity. The model applies uncertainty to the level of maximum damage. This demonstration reinforces the previous result. Hence, both regimes apply on specific tort question and comparing their individual efficiency needs to call for other components as the transaction costs associated to the burden of evidence, the fairness between victims and injurers, etc.

**Keywords:** Strict Liability, Negligence Rule, Ambiguity Theory, Uncertainty, Accident Model

**JEL Classification** K0, K32, Q01, Q58

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# The Equivalence of Strict Liability and Negligence Rule: A « Trompe l'œil » Perspective

Gérard Mondello<sup>1</sup>

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## Summary

*This paper analyzes the difficulties of comparing the respective effectiveness of two among the most important liability regimes in tort law: rule of negligence and strict liability. Starting from the standard Shavellian unilateral accident scheme, I show that matching up liability regime on their capacity to provide the highest level of safety is ineffective. This demonstration lies on two components. The first one gathers some results drawn from literature that introduces uncertainty. The second one takes into consideration the beliefs of agents and their aversion to ambiguity. The model applies uncertainty to the level of maximum damage. This demonstration reinforces the previous result. Hence, both regime applies on specific tort question and comparing their individual efficiency needs to call for other components as the transaction costs associated to the burden of evidence, the fairness between victims and injurers, etc.*

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## 0. Introduction

Since Ronald Coase's pioneering paper "The social Cost" (Coase (1960)), scholars have particularly studied and formalized the economic incidences of tort law<sup>2</sup> and particularly the legal liability question which is now an important regulatory policy instrument. Legal liability threatens potential injurers with having to pay for the harms they cause, even if insurance can also provide compensation more cheaply. Hence, for economists, the primary social function of liability system is to provide incentives to induce potential injurers to increase their level of prevention. More particularly, one among the major research trends centers on the comparison of the different liability regimes to achieve this task. Steven Shavell (Shavell (1985), Shavell (1987)) fathered the most popular touchstone accident model used by modern literature. Hence, to minimize the social costs of a major harm, a rational regulator can enforce either a liability regime based on fault (as a rule of negligence), or on a no-fault regime (strict liability), this, according the relative performance of each in providing safety.

Broadly speaking, under negligence, injurers can escape liability if they have taken due care while strict liability regimes induce the injurers' responsibility whatever their safety effort and independently of any fault. In practice, strict liability is much less used than negligence (Cantu (2001)) and is applied for Environment protection, ultra-hazardous activities and products defaults. Negligence is invoked for the whole remaining fields. Despite this division of role, determining the most appropriate liability scheme generates keen debates among economists. The choice criterion is efficiency in the providing of the highest level of care. In the eighties, authors<sup>3</sup> showed that both regimes perform equivalently and minimize primary costs, i.e., the sum of the cost of care and of expected accident losses. These results are reached under specific assumptions as certainty about the level of maximum damage, no consideration for the activity level, etc. However, if stating equivalence needs strong assumptions, relaxing them opens the Pandora's box of ambivalent results about the kind of regime to enforce. Actual debates bear upon whether uncertainty should favor either strict liability or negligence. For instance, Newman and Wright (1990) demonstrate that in the presence or absence of moral hazard within the firm, strict liability induces the principal to

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<sup>2</sup> According Kaplow and Shavell (1999), Bentham in the XIXth Century conceived the modern analysis of law and Coase in 1960 extended this analysis to probabilistic externalities. Beyond Coase's (1960) article, the sixties and seventies expanded this field with Becker's (1968) article on crime and law enforcement. The first systematic work on accident law has been provided by Calabresi (1970) while Posner (1972) studied economic analysis of law.

<sup>3</sup> Shavell (1980b), Shavell (1987b), Landes and Posner (1987), Shavell (1980), Landes and Posner (1981), Pozzo (1996).

offer a contract which gives rise to a socially optimal level of care. This level varies according to the presence or the absence of moral hazard. Conversely, Demougin and Fluet (1999) show that when the agent earns a positive rent, strict liability will generate an under provision of care. Then negligence rule is more efficient than strict liability. Asymmetric information is *typically* vicarious liability which is the liability of one party, generally the ‘principal’ for wrongdoing of another party, the ‘agent’ (here the operator)<sup>4</sup>.

Most of these contributions tend to consider as relevant the fact of comparing liability schemes on the grounds of their efficacy. However, a close examination of the legal foundation of each regime shows that switching from one to the other one can hardly be done on this basis. Hence, this paper shows that many other factors should be considered as for instance fairness, transaction costs in seeking evidence, etc.

Indeed, uncertainty means not only that agents ignore the value of fundamental variables but, also, that they form beliefs about them and determine their choice on them. Consequently, they express either preference or aversion for ambiguity, optimism or pessimism. In a seminal work, J.C. Teitelbaum (2007) introduced ambiguity theory in the basic accident model. This author felt unsatisfied with the expected utility theory used in the basic unilateral accident model. Basically, this model lets aside Knightian uncertainty that deals with unknown or ambiguous probability distributions. Consequently, Teitelbaum introduced attitudes towards uncertainty as optimism (ambiguity lovers) or pessimism (ambiguity aversion). He shows that sensitive operators to ambiguity will either over-invest in prevention (pessimistic) or under-invest in it (optimistic). In both cases, the required socially optimal level of prevention is never met.

Our goals and treatment of ambiguity will be quite different. In our approach, agents are uncertain on the scale of the maximum damage induced by the major harm. This point is quite realistic and follows Cooter (1984) or Beard (1990) that conceived that major harm may be considered as a random variable. Indeed, the consequences of an accident are generally unknown. For instance, the explosion of a chemical plant will have different impact on population according to the moment it happens in the day, whether it induces some disastrous blaze or not, whether the weather is rainy, etc, this independently of the maximum level of care effort. From this basis, our main goal consists in characterizing the feature of each liability regime through uncertainty.

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<sup>4</sup> Concerning this field, we can consider the works of Sykes (1984), Sykes (1988) and Sykes (1998). We refer equally to Kornhauser (1982) or still Polinsky and Shavell (1993), Schmitz (2000), Dari-Mattiacci, and Parisi (2003), Dari-Mattiacci (2006). We can mention too Segerson and Tietenberg, (1992), Menell (1991). For a global analysis see also Larsson (1999).

Ambiguity theory helps in putting into evidence more deeply the fundamental differences between the liability regimes that prevent considering that switching from one to another one is only a matter of efficiency. In few words, we show that choosing among liability scheme cannot be done on the usual criteria of efficiency, but rather on some other richer grounds. This opens the door to elements such as fairness between victims and polluters, the minimization of transaction costs, etc.

In the first part of this paper we recall the main results reached by literature about the equivalence question of both regimes under certainty and uncertainty. We study the assumptions under which are reached the main results. A second part analyzes the comparison of strict liability and negligence rule under ambiguity and we put into evidence the theoretical impossibility to achieve such a task. A third part concludes.

## **1. The equivalence between liability regimes is not robust under uncertainty**

Concerning the standard accident model applied to environmental matters, liability regimes are equivalent only under assumptions (Shavell (1985), (1987) or Landes and Posner (1987)). Essentially, agents (polluters and victims) are gifted with Von Neumann Morgenstern utility functions and are supposed neutral to risk. In addition, the probability distribution of accidents is common knowledge. Only unilateral preventive actions of the polluter are considered because victims cannot do it. It is few mentioned that comparing regime is made considering efficiency only and not fairness as Shavell (1982, p.121) highlights it when dealing with the victims: “(.) *under the negligence rule injurers do not bear risk – if they are not negligent, they will not have to pay damages when involved in accidents – and victims do bear risk*”.

### **1.1 The equivalence between strict liability and negligence rule**

Here, we do consider the case of certainty about the scale of damage and we present the standard model (Cooter (1984), Shavell (1980) and (1987)) as a benchmark.

Notations :

- $x$ , the level of effort of prevention
- $\{d, D\}$ , the amount of maximum damage, where  $d < D$ , which means that for a given effort level, the maximum damage is poorly known by Society.
- $\pi(x)$ , probability of a major accident which depends on the achieved level of prevention, where  $\pi'(x) < 0$ ,  $\pi''(x) \geq 0$ .

- In the following we denote by “NR” the negligence rule regime, no index means “strict liability”. The index “P” denotes the injurer or polluter, “A” the victims.

We present the standard unilateral accident model which bears no uncertainty on the scale of damage ( $d = D$ ). The injurer’s looks at minimizing the expected prevention costs:

$$(1) \quad \min_x EC_P(x) = x + D\pi(x) \text{ s.c. } x \geq 0$$

While the victim’s expect costs of the harm under the strong assumption that damages are fully compensated:

$$(2) \quad EC_A(x) = (D - D)\pi(x) = 0$$

The social cost from the regulator view point  $ECS$  is then (1)+(2):

$$(3) \quad \min_x ECS(x) = x + D\pi(x) + 0 = x + D\pi(x) \text{ s.c. } x \geq 0$$

We have to note the perfect correspondence between the objective of the regulator and the one of the injurer. Indeed, the first order conditions give:

$$ECS' = EC_P' = 0 \text{ for } x^* > 0 \text{ such that } \pi'(x^*) = -\frac{1}{D}$$

$x^*$  is the socially optimal level of safety effort. This result is well known and constitutes the yardstick for this analysis.

We consider now the case of negligence rule. Following Richard Posner (2007, p.167-71), the understanding of negligence corresponds to the Hand Formula which is a “reasonable precaution” or precaution which is cost justified. Under the certain case about damages ( $D = d$ ), the expected prevention costs depend on the effective compliance with the optimal prevention cost. Consequently, they correspond to this classical presentation:

$$(4) \quad EC^{NR}_P = \begin{cases} x^\mu & \text{if } x^\mu \geq x^* \\ x^S + \pi(x^S)D & \text{if } x^S < x^* \end{cases}$$

The operator will comply by engaging a level of prevention equal or higher to  $x^\mu = x^*$  (i.e; the socially optimal safety effort). As a consequence, for victims, the expected costs of a risky activity write as:

$$(5) \quad EC^{NR}_A = \begin{cases} \pi(x^\mu)D & \text{if } x^\mu \geq x^* \\ 0 & \text{if } x^S < x^* \end{cases} \text{ whenever } (D = d)$$

Indeed, when the harm occurs, the victim bears its full consequences when the operator complies with the optimum level of safety. Consequently, for the regulator, the social cost writes as

$$(6) \quad ECS^{NR} = \begin{cases} x^\mu + \pi(x^\mu)D & \text{if } x^\mu \geq x^* \\ x^S + \pi(x^S)D & \text{if } x^S < x^* \end{cases}$$

A rational injurer supplies the socially optimal level of care and spends  $x^\mu = x^*$ . As a consequence, the expected social cost will settled at:  $ECS^* = x^* + \pi(x^*)D$ .

Hence, the equivalence of both schemes holds because the social costs under either plan are identical and because agents are led to choose the same socially optimal level of effort, that is to say  $x^* > 0$ . The only difference is that, under negligence, liability waiver occurs only if the polluter achieves this level of effort.

Undermining the assumption of certainty about the maximum damage leads to a sub-optimal situation because the polluters are not encouraged to supply the optimal level of effort (Cooter (1984)).

## 1.2 Uncertainty and the lack of equivalence between liability regimes

Now, let us consider the general case according which victim and polluter are uncertain about the effective level of the due compensation that may be either  $D$  (i.e. the effective level of damage), or  $d$ , ( $d < D$ ) where  $d$  is the level assessed by the court. This discrepancy is due for instance to the scientific uncertainty about the extent of damages. Hence, the underestimation of the harm is made with a probability equal to  $p$ ,  $1 > p > 0$ , and the effective assessment appears with a probability of  $1 - p$ . We analyze successively the strict liability regime and the negligence rule.

### *Strict liability*

Under this regime, the injurer's program is:

$$(7) \quad \min_x EC_P(x) = (x + (pd + (1 - p)D)\pi(x) + x(1 - \pi(x))) = x + (D + p(d - D))\pi(x) \text{ s.t. } x \geq 0. \text{ And, for the victims:}$$

$$(8) \quad EC_A = (p(D - d) + (1 - p)(D - D)\pi(x) + 0(1 - \pi(x))) = p(D - d)\pi(x),$$

where  $d < D$ . Then, the social planner's program expresses as:

$$(9) \quad \min_x ECS(x) = x + (D + p(d - D))\pi(x) + p(D - d)\pi(x) = x + D\pi(x) \text{ s.t. } x \geq 0.$$

In spite of the assumption of two levels of damage, the social cost integrates only the higher value of the harm. This different program will give different solutions. Proposition 1 shows that if  $x^*$  is the optimal social level of care, and if  $x^0$  is the optimum level of safety for the injurer, then  $x^* > x^0$ .

**Proposition 1:** *Under uncertainty about the level of maximum damages and under a strict liability regime, the level of prevention effort supplied by the potential injurer is socially sub-optimal.*

**Proof:** Indeed,  $ECS' = 0 \Rightarrow \exists x^*: \pi'(x^*) = -\frac{1}{D}$  and  $EC_P' = 0 \Rightarrow \exists x^*: \pi'(x^0) = -\frac{1}{(D+p(d-D))}$  with obviously  $(D + p(d - D)) < D$  because  $1 \geq p > 0$  and  $d < D$ ,

then  $\pi'(x^*) = -\frac{1}{D} > \pi'(x^0) = -\frac{1}{(D+p(d-D))}$  because by assumption  $\pi''(x) > 0$ , and, as a consequence  $x^* > x^0$  as shown in figure 1.

[insert figure 1]

Uncertainty about the value of damage creates an agency relationship between the operator and the regulator: the firm's interest is to not comply with the socially optimal level of prevention<sup>5</sup>. Indeed, under a strict liability rule, it is only necessary to bring it to that level that minimizes its prevention cost level. Indeed, the regulator would like that society performs the highest level of prevention at the minimum social cost level which is not the case for  $x = x^0$ . Consequently, there is a clear discrepancy between the objectives of the firms and the ones of the regulator. To implement the level  $x^*$  the regulator has to design a specific mechanism. This is the road followed by Newman and Wright (1992). By another argument we find again the result of Cooter (1984). Does this result consecrate the superiority of the negligence rule in this context? This is the point to study now.

#### ***Uncertainty and negligence rule***

Conversely to strict liability, under negligence, injurers are held liable in tort if they did not take reasonable precautions only. Hence, the judge must not only seek the causal link between the harm and the polluting activity, but also assess the adequacy of prevention compared to the damage scale. Then, for some type of repeated accidents (road traffic, work accident) the process may be detrimental for victims. As Calabresi (1970) showed it, negligence involves high transaction costs, especially considering automobile accident. Resorting to strict liability is preferable because it limits the time devoted to prove the fault existence. The prevalence of one regime compared to another one depends on specific circumstances (sector of activity, frequency of accidents, their scale, etc.). The economic analysis of law considers that strict liability and negligence rule are substitutes. Hence, negligence rule could appear as a natural shelter if strict liability cannot be enforced.

Based on existing literature, the rule of negligence cannot lead enforcing an optimal level of prevention. Consequently, the expected accident cost corresponds to the following lines:

$$(10) \quad EC^{NR}_P = \begin{cases} x^\mu & \text{if } x^\mu \geq x^* \\ x^S + \pi(x^S)(D + p(d - D)) & \text{if } x^S < x^* \end{cases}$$

The consequences for victims are underestimated because the social cost is not affected by negligence rule. The operator complies by engaging a level of prevention equal to

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<sup>5</sup> See for instance the analysis of Newman and Wright (1990) that induced a trend of researches in this area.

$x^\mu = x^*$ , Cooter (1984). As a consequence, for victims, the expected costs of a risky activity write as:

$$(11) \quad EC^{NR}_A = \begin{cases} \pi(x^\mu)D & \text{if } x^\mu \geq x^* \\ \pi(x^S)(p(D-d)) & \text{if } x^S < x^* \end{cases}$$

Hence, the victim bears the full consequences of the harm occurrence when the court recognizes that the operator complied with the optimum level of safety ( $D$ ). Furthermore, when the injurer does not fulfill the prevention level and if the court undervalues damages, then, the victims run the risk of not being entirely reimbursed for the incurred damage. Consequently, a rational operator has interest to comply with the socially optimal level of care and will spend  $x^\mu = x^*$ . Consequently, under a rule of negligence, even when the level of damage is uncertain, the optimum level of preventions settles at its highest level.

### 1.3 Rule of negligence: Some specific results

Judges have to be very cautious about the determination of the prevention level. For instance, courts might wish restoring fairness and balancing the weight of the damage against the marginal increase in safety. Thus, they might not follow the regulator's will. Besides, Craswell and Calfee (1986) were the first to underline of the possibility of mistakes from the Courts side. Indeed, Courts might err in determining what the first best level of care should be. However, this is not from this barrier side that we will analyze the point. Indeed, these points are well known and will not be developed here. We will study rather this increase of information that might get the judge when investigating the causes of an accident.

#### *Negligence and the determination of the effective level of prevention*

Hence, even if the injurer has reached this level of safety deemed as the first best by the regulator, the judge's investigation can find evidence showing that this level is insufficient. For instance, the court could prove that the accident has been caused by human negligence, this, even if the operator took the first best care level. For example, this may be the case for human shortcomings (a sudden sick or drunk supervisor of a system at the origin of a disaster). Hence, under negligence, we can never take for granted the polluter's exemption even if the required amount of prevention is similar to the strict liability scheme.

Consequently, even if an operator complies with the optimal level of prevention, the probability of escaping liability is no longer equal to 1. The operator can consider that the court will be favorable to him with a probability equal to  $\lambda$ , and will produce a negative judgment with a probability  $1 - \lambda$ . Consequently, he can expect the following issue:

$$(12) \quad EC^{NR}_P = \begin{cases} \lambda x^\mu + (1 - \lambda)(x^\mu + D\pi(x^\mu)) = x^\mu + ((1 - \lambda)D\pi(x^\mu)) & \text{if } x^\mu \geq x^* \\ x^S + \pi(x^S)D & \text{if } x^S < x^* \end{cases}$$

This writing shows that complying with the first order level of prevention is no longer an insurance against the involvement of the operator's liability. As a consequence, the operator may be tempted to undersize the level of prevention. Indeed, we can check that he is not induced to supply the efficient level of care. To see that, it is sufficient to compute the first order conditions of  $x^\mu + ((1 - \lambda)D\pi(x^\mu))$ :

$$(13) \quad EC^{NR}'_P = 0 \Rightarrow \pi'(x^{\mu*}) = -\frac{1}{(1-\lambda)D}$$

Obviously, by the same argument than for proposition 1, we find that  $x^{\mu*} < x^*$  (i.e. the operator is not induced to achieve the same level of prevention effort). However, we have a paradoxical result, because, if  $x^{\mu*} < x^*$ , then the cost of prevention will not be  $x^{\mu*} + ((1 - \lambda)D\pi(x^{\mu*}))$  but, rather,  $x^S + \pi(x^S)D$ . His interest could then to conform to the optimum level. However, this is the case only if  $x^{\mu*} + ((1 - \lambda)D\pi(x^{\mu*})) > x^{S*} + \pi(x^{S*})D$  where  $x^{S*}$  is that effort of safety which maximizes  $x^S + \pi(x^S)D$ . Hence, we can consider that the operator will comply if the following relationship is verified:

$$\frac{x^{\mu*} - x^{S*}}{\pi(x^{S*}) - ((1 - \lambda)D\pi(x^{\mu*}))} > D$$

As  $\lambda$  increases, i.e. as the probability to involve a compliant operator becomes higher, the conditions that verify the above relationship become weaker.

### ***Negligence and the suing costs***

Let us assume now that plaintiffs undergo sue costs and taxes if they lose. We expect that the defendant is solvent and that she can reimburse the amount  $D$  when the plaintiffs win. Does this state of matter have consequences on the social cost function? Let us assume that the plaintiff sue the tortfeasor every time that he can check any damage. He incurs the total cost  $D + K$  if they lose and gain  $D$  in the opposite case. Then the probability of loosing is  $\lambda$  (i.e. the probability of winning for a compliant operator) and  $(1 - \lambda)$  in case of winning these are simplified assumption compared to Shavell and Polinsky (1989). Then, for the victims, the expected cost are the following:

$$(14) \quad EC^{NR}_A = \begin{cases} \pi(x^\mu)[(D + K)\lambda + (1 - \lambda)(D - D)] = \pi(x^\mu)(D + K)\lambda & \text{if } x^\mu \geq x^* \\ 0 & \text{if } x^S < x^* \end{cases}$$

We can define the social cost for the rule of negligence that is settled at:

$$(15) \quad ECS^{NR} = \pi(x^\mu)(D + K)\lambda + x^\mu + ((1 - \lambda)D\pi(x^\mu)) = x^\mu + \pi(x^\mu)[\lambda K + D] \text{ if } x^\mu \geq x^*$$

And if the optimal level of care is fixed at  $x^\mu = x^*$  by the operator wishing avoiding the involvement in repairs, (i.e. if  $\frac{x^{\mu*} - x^{S*}}{\pi(x^{S*}) - ((1-\lambda)D\pi(x^{\mu*}))} > D$  is expected), the social cost is then:

$$ECS^{NR} = x^* + \pi(x^*)[\lambda K + D]$$

This value is higher than the social of a strict liability regime. Under negligence, victims should include the penalties associated to the possibility of having sued the injurer.

### ***Negligence and transaction costs***

We extend the analysis and we combine now the existence of transaction cost and uncertainty. These transaction costs ( $N$ ) are associated to the necessity to gather evidence and they can be borne either by the victims or the defendant because in each case they contribute to inflate the social cost. Here, we consider that the burden of the evidence is endured by the victims but, we could inverse the process and let them paid by the injurer without changing the final result. As before we express the expected cost of the polluter:

$$(16) \quad EC^{NR}_P = \begin{cases} \lambda x^\mu + (1-\lambda)(x^\mu + (D + p(d-D))\pi(x^\mu)) & \text{if } x^\mu \geq x^* \\ x^S + \pi(x^S)(D + p(d-D)) & \text{if } x^S < x^* \end{cases}$$

And, for the potential victims when submitted to bear the burden of the proof:

$$(17) \quad EC^{NR}_A = \begin{cases} \pi(x^\mu)[(D + N)\lambda + p(D(1-\lambda) - d)] & \text{if } x^\mu \geq x^* \\ \pi(x^\mu)[(D - d)] & \text{if } x^S < x^* \text{ if } x^S < x^* \end{cases}$$

Naturally, the expected social cost writes as:

$$(18) \quad ECS^{NR} = x^* + (D - dp\lambda + N((1-\lambda)p)\pi(x^*)) \text{ with}$$

$$(19) \quad ECS^{NR'} = 0 \Rightarrow \pi'(x^*) = -\frac{1}{D - dp\lambda + N((1-\lambda)p + \lambda)}$$

Using the same proof scheme than for establishing proposition 1, we get the expected result that the level of prevention will be weaker under the assumption of positive transaction costs. Naturally, the previous paragraph and this one could be combined, the distance between the social cost of a strict liability regime and the one of a negligence rule increases because none of them incorporate the same components.

As a conclusion, the introduction of uncertainty raises question about the conditions of comparing liability regimes in tort law. The more precise are becoming the features of uncertainty, the more difficult it is becoming defining the frontiers of their mutual assessment.

## 2. Ambiguity theory and uncertainty about damages

In most theoretical contributions, the maximum level of damage is known. It can be considered as given or depending on the level of prevention. In the latter case, the higher the prevention effort, the lesser the consequences of a major accident are<sup>6</sup>. However, one can imagine that a given operator can experience difficulty in identifying with certainty the maximum level of damage. This uncertainty applies even when is reached the higher level of safety. Hence, the operator can consider that the maximum value of damage may be comprised inside an interval between a high and a low value. For example, the explosion of a fuel tank could produce harm equivalent to either  $x$  or  $y$  thousand euro. Polluting leakage of groundwater could cost either 500,000 euro or three millions, and so on. Generally, faced with this kind of accident, it is only exceptionally that the actual amount of damage could be a priori known and/or knowable. However, the probabilities of potential maximum damage may be estimated (by a regulatory agency for instance) and its distribution known by the operator. Then, he may form estimates about them and he can either overestimate this level (or thus underestimate the minimum damage) or the reverse. Hence, in determining the prevention effort the potential polluter will fix its own estimate of the maximum amount of the loss. This is the foundation of an alternative theory of utility based on the observation of behavior facing true uncertainty (unspecified or ambiguous probabilities) and not only risky uncertainty (specified probabilities).

Literature on this theme began in the early fifties with the Allais' criticisms and, then the Ellsberg's Paradox in 1961. We give a quick overview of the question but we ask to the interested lector to refer to Teitelbaum (2007)'s contribution which is more complete than ours. Let us consider that an agent has to select two alternative actions. In the first one, he can choose one actions for which the probabilities for results are known (for instance drawing a blue ball in an urn that contains blue and red balls in a known proportion). In the second one, the choice is the same but the proportions are unknown (balls blue and red are in an unknown ratio). Experiences have shown that most of people will prefer to select the first alternative, i.e. the urn in which the proportion of red and blue balls is known. Agents feel aversion for the ambiguous choice present in the second alternative. This leads to implicitly allocate prior probabilities to the second choice with the result that the sum of probabilities for a given event are higher than 1. Schmeidler (1989) systematized ambiguity by applying Choquet's integral to expected theory utility. Ambiguity is understood as the lack of confidence of an

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<sup>6</sup> See for instance Dari-Mattiacci and Parisi, (2003).

agent in his faith about the distribution of probability of uncertain events. For ambiguity theory, non-additive probability or capacity represents agent's beliefs about the likelihood of these uncertain events. Agents maximize an expected utility function with respect to capacity. This utility is computed by Choquet integral. This expression allows taking into account ambiguity and behaviors facing uncertainty. More precisely, concave capacity involves optimism (super-additivity) while a convex one entails pessimism (subadditivity)<sup>7</sup>.

Teitelbaum (2007) applies the results of ambiguity theory to the determination of the level of prevention and show that neither strict liability nor negligence rule can reach the socially optimal level of prevention. More precisely, he refers to a specific application of Choquet expected utility theory developed by Chateauneuf, Eichenberger and Grant (2007), (CEG in the following). These authors develop the concept of neo-additive capacity. The difference between capacity and neo-additive capacity is that this last one is additive on non-extreme outcomes. Neo-additive capacity allows systematizing optimistic and pessimistic attitudes towards uncertainty. This gives foundation to the empirical evidence that in real world, investors do not behave according the patterns of the theory of expected utility. Camerer and Weber (1992) reviewed and gathered the whole set of significant criticisms brought to standard expected utility theory. Gonzales and Wu (1999) or Abdellaoui (2000) and many others by empirical studies showed that, in real world, when they are led to bet, agents tend to overweight probabilities close to zero and underweight probabilities close to 1. This is shown by an inverse S-shaped curve that represents the willingness to bet  $\tau(p)$  weighting the probability  $p$  of events. Individuals prefer to bet when the probability of winning is low (for national lottery tickets for instance) and are more reluctant to bet when the probability of winning is high.

The neo-additive weighting scheme defined by CFG (2007) makes possible the modeling of the certainty and the possibility effect represented in the famous inverse-S-shaped probability put into evidence by the above mentioned empirical studies. Before explaining this point, CFG (2007) underline that neo-additive weighting issue on neo-additive capacity. This concept corresponds to a probability weighting function. We present in appendix 1 the mathematical foundations of the model that takes into account neo-additive capacity. To give a more fluent presentation here, we consider only that the polluter and the society cannot assess with certainty the exact value of a maximum damage. Let be  $\mathcal{E}$  the finite set of states to which correspond the catastrophic events  $\mathcal{A}$  ( $\sigma$ -algebra of  $\mathcal{E}$ ). We consider a

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<sup>7</sup> For a more detailed presentation including examples see Teitelbaum(2007).

finite set of outcomes ( $A \subset \mathbb{R}$ ) and a set of simple functions  $\Phi = \{f: \mathcal{E} \rightarrow A\}$  from states to outcomes which correspond to simple acts and takes on values:  $a_1 \geq a_2 \dots \geq a_n$ . We define the maximum damage function as the expected costs of the maximum damages  $E_p(a)$ :

$$(20) \quad E_p(a) = \int_a^{\bar{D}} a p(a) da$$

The neo-additive capacity is then (see appendix 1 for details):

$$(21) \quad \mu(A/p, \delta, \alpha) = \begin{cases} 0 & \text{for } A = \emptyset \\ \delta\alpha v_0(A) + \delta(1 - \alpha)v_1(A) + (1 - \delta)p(A) & \text{for } \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1 & \text{for } A = \mathcal{E} \end{cases}$$

Let us consider that  $v_0(A) = \text{Inf}(f) = d$  (i.e. the minimum of the maximum damage and  $v_1(A) = \text{Sup}(f) = \bar{D}$ ), the highest one. Then, for  $\emptyset \subsetneq A \subsetneq \mathcal{E}$  we define the neo-additive capacity as:

$$(22) \quad \mu(\cdot) = \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)p(A)$$

Replacing still these factors we get the Choquet integral that expresses the maximum damage:

$$(23) \quad V_p(A/p, \delta, \alpha) = \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)E_p(a)$$

We materialize here what has been announced above. Hence, here, the Choquet integral of a neo-additive capacity is the weighted sum of, respectively, the minimum, the maximum and the expectation of the damage value of a major harm.

Here, optimism and pessimism refer to the scope of the major accident. Optimism involves high value of  $\alpha$  (that it to say the lowest damage) and a low value of  $\alpha$  tends to overweight the highest harm  $\bar{D}$ . Conversely to Teitelbaum's analysis that distinguishes only two values (the probabilities of a major accident), here the range of possible values for the harm is high because it spans the set of events  $A$ .

Let us notice that, when  $\alpha = 0$  (pessimistic feeling),  $V(A/p, \delta, 0) = \delta\bar{D} + (1 - \delta)E_p(a)$ , depends on the behavior of the injurer towards ambiguity, i.e. here, the value  $\delta$ . To see its meaning, consider (besides the fact that  $\alpha = 0$ ), that  $\delta = 0$ . Then, the capacity reduces to  $E_p(a)$ , that means that the injurer feels no ambiguity on the probability distribution,  $\mu(A/p, 0, \alpha) = p(A)$ . The higher  $\delta$  is the less confident is the operator about the prevalent probability distribution of major accident. For an absolute distrust in it ( $\delta = 1$ ), we have:

$$(24) \quad V_p(A/p, 1, \alpha) = \alpha d + (1 - \alpha)\bar{D}$$

This expression corresponds to the Hurwitz criteria weighted by the degree of optimism and optimism of the injurer. If, furthermore, the injurer is very pessimistic ( $\alpha \rightarrow 0$ ), he will tend to consider that the highest level of the harm will occur  $\bar{D}$ . However, before going further, we have to note that the condition for having the Choquet integral of the expected cost of the injurer higher than the major damage cost expectation, is that:

$$(25) \quad \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)E_p(a) > E_p(a) \text{ or, still,}$$

$$(26) \quad \alpha > \frac{E_p(a) - \bar{D}}{d - \bar{D}} > 0$$

Hence,  $V(\cdot) > E_p(a)$  if  $\alpha > \frac{E_p(a) - \bar{D}}{d - \bar{D}}$  and  $V(\cdot) \leq E_p(a)$  if

$$(27) \quad \alpha \leq \frac{E_p(a) - \bar{D}}{d - \bar{D}}$$

(We recall that because of its definition (see appendix 1), the only condition on  $\alpha$  is positivity).

These relationships show that the importance of the degrees of optimism and pessimism in the assessment of the expected cost of major damage. We can notice that the level of ambiguity aversion vanishes in the comparison. Having defined the background of the analysis we can see how to apply it to our analysis.

## 2.1 Strict liability with solvent and unworthy operator

In what follows the relationships between  $E_p(a)$  and the expected Choquet cost of major damage are fundamental. The injurer selects a level of prevention effort that solves:

$$(28) \quad \min_x EC_p(x) = x + V(\cdot)\pi(x) \text{ sc } x \geq 0$$

The regulator cannot take into account the optimism or the pessimism of the operator. Consequently, the regulator is assumed risk neutral and, following Shavell(1986) and Beard(1990), we consider that the socially optimal level of care is the solution of :

$$(29) \quad \min_x SC(x, E_p(a)) = x + E_p(a)\pi(x) \text{ sc } x \geq 0$$

We can immediately check that the solutions of both the injurer and the regulator do not coincide. Indeed, when  $V(\cdot) > E_p(a)$  the injurer will tend to over-invest and to under-invest in safety in the converse case. We join not only the results of Teitenbaum (2007) or Cooter (1983) but still those of Beard (1990) that shows that under the threat of being considered as judgement-proof, operators may over-invest in safety compared to the first best safety level. To illustrate this particular point, let us assume that the wealth  $w_p$  of the operator is limited:

$$(30) \quad w_p \leq a_k < \bar{D}$$

Where  $a_k$  is the starting point from which the harm that goes beyond the operator's wealth. Let us assume now that the operator distrusts fully the official distribution of accident probability and, as a consequence, he has an absolute preference for ambiguity  $\delta = 1$ . We assume also that he is absolutely pessimistic ( $\alpha = 0$ ). Then  $V(A / p, 1, 0) = \bar{D}$ .

Then, the program of the polluter is then:

$$(31) \quad \min_x EC_p(V(A / p, 1, 0)) = x + (w_p - x)\pi(x) \text{ s.c. } x \geq 0$$

That means that the amount of his wealth binds the compensation amount to  $(w_p - x)$ . Then,  $(x^0)$  is this safety level that minimizes the expected cost of the polluter, this expected cost will be  $x^0 + (w_p - x^0)\pi(x^0)$  where  $(w_p - x^0) = a_{k'}$ .

The expected social cost is:

$$(32) \quad ESC = x^* + E_p(a)\pi(x^*)$$

Where  $x^*$  is the optimum social level of safety.

Let us assume that  $a_{k'} > E_p(a)$ . We recall that because of his absolute pessimism and his absolute reluctance to accept the distribution of the accident probability, he thinks that the total amount of the harm will be  $\bar{D}$ . Hence, in case of an accident he will have to compensate it by the amount of his whole asset which is here  $(w_p - x^0) = a_{k'}$ .

Because  $a_{k'} > E_p(a)$ , by simple calculus, we can see that  $x^* < x^0$ . That means that the operator over invests in safety compared to the regulator's requirement.

As a consequence, the result is robust and can take into consideration both the case of an unbounded wealth and the one of judgment proof under a strict liability regime. Furthermore, it is possible to understand how uncertainty about the scale of damage can issue either on overinvestment or underinvestment in safety.

## 2.2 Negligence rule and ambiguity

Ambiguity on the level of the maximum damage prevents an easy matching of strict liability versus negligence as in Teitelbaum (2007). Linking them reveals difficult because under negligence the operator will tend to forecast the judge's ambiguity aversion. Indeed, under negligence, the operator will not seek to determine his own level of prevention but the one of the judge. This point is not that amazing. For instance, in some countries, environment harm may be weakly considered and judges are usually lenient with polluters. Obviously, this state of matters inflects the injurer's behavior.

The components of the polluter's uncertainty are at least twice. Indeed, first, naturally, uncertainty bears on the level of damages: the polluter ignores the true scale of the major

harm. Second, uncertainty stands on the determination by the court on who is liable of the damage. Consequently, the potential polluter will tend to determine the level of due care by assessing the behavior of the court. The injurer can assess the capacity of the court as the following

$$(33) \quad \mu^C(.) = \tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})p(A)$$

Where  $\tilde{\delta}$  and  $\tilde{\alpha}$  are the parameters of optimism and ambiguity aversion of the Court as assessed by the operator. We deduce the Choquet integral:

$$(34) \quad V_p^C = \tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})E_p(a)$$

We define then the Choquet expected cost function of the operator under the rule of negligence.

$$(35) \quad VC^{NR}_P = \begin{cases} x^\mu & \text{if } x^\mu \geq x^* \\ x^s + \pi(x^s)V_p^C & \text{if } x^s < x^* \end{cases}$$

Or more explicitly:

$$(36) \quad VC^{NR}_P = \begin{cases} x^\mu & \text{if } x^\mu \geq x^* \\ x^s + \pi(x^s)[\tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})E_p(a)] & \text{if } x^s < x^* \end{cases}$$

We have gathered all the elements to settle our main proposition which is the following:

**Proposition 2:** *Under uncertainty, when agents feel aversion for ambiguity, neither the negligence rule nor the strict liability rule can be considered as a superior rule comparing them mutually.*

The formal proof of the proposition is somewhat tedious and we relegate it to Appendix 2. This proposition settles that choosing among the mentioned liability rule cannot be accomplished on the ground of the comparison of their relative performance under true uncertainty. Indeed, the regulator cannot *a priori* forecast how reluctant to ambiguity will be the injurer. Consequently, from a methodological viewpoint, it is difficult to conceive an enforcement rule depending on the state of mind of the potential polluters or tortfeasors.

Hence, here, conversely to Teitelbaum's conclusions that give preference for the rule of negligence, proposition 2 show that enforcing a liability regulation under uncertainty requires more information than the simple assessment of their economic mutual efficiency. If our results were to be combined with the ones of Teitelbaum's ones, then the indeterminacy about the choice of a liability rule would be absolute. Indeed, in such a situation the injurer would form beliefs on the distribution of probability of accident and on the one of the scale of accident.

We give now some illustration considering some particular cases.

### 2.3 Some particular cases

We can see that it is only if  $\tilde{\delta} = \delta$  and  $\tilde{\alpha} = \alpha$  that the correspondence with the usual analysis can be made. Hence, the injurer considers that the Court will behave like himself experiencing the same ambiguity aversion and the same degree of optimism/pessimism about the Court feeling. We can deduce then the following propositions:

**Proposition 3:** *If the operator thinks that he and the Court are sharing the same aversion for ambiguity, i.e.  $\tilde{\delta} = \delta$ , but not the same optimism level  $\tilde{\alpha} \neq \alpha$ , then  $V_p^C \leq V_p$*

**Proof:**

We consider the conditions for having:

$$\tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})E_p(a) > \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)E_p(a)?;$$

Then, this is false independently of the level of  $\tilde{\alpha}$  and  $\alpha$ , indeed, developing we get

$$(\tilde{\alpha} - \alpha)d + (\alpha - \tilde{\alpha})\bar{D} > 0 \text{ and, as a consequence } \frac{d}{\bar{D}} > 1, \text{ which is contradictory with}$$

$$\bar{D} > d$$

Consequently, when  $\tilde{\delta} = \delta$  and  $\tilde{\alpha} \neq \alpha$

$$\tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})E_p(a) \leq \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)E_p(a)$$

**Proposition 4:** *If the operator thinks that he shares with the Court the same optimism level  $\tilde{\alpha} = \alpha$ , but not the same aversion for ambiguity, i.e.  $\tilde{\delta} \neq \delta$  then*

$$V_p^C > V_p \text{ if } \alpha < \frac{\bar{D} - E_p(a)}{\bar{D} - d}$$

$$V_p^C \leq V_p \text{ if } \alpha \geq \frac{\bar{D} - E_p(a)}{\bar{D} - d}$$

**Proof:** obvious, similar to the proof of proposition 1 and sub-propositions 1,2 and 3 in appendix 2.

From proposition 3 we show that, if the operator thinks that the judge and he are sharing the same ambiguity aversion, the level of optimism of the judge does not “push” to overestimate the level of damage. Consequently, in this case, the negligence rule does not induce the injurer to overinvest in safety. From proposition 4, it appears that when the judge and the operator share the same level of optimism but diverge on the assessment of the distribution of probability of major accident, then either  $V_p^C > V_p$  or  $V_p^C \leq V_p$  according his level of optimism  $\alpha$  is higher or lesser than the ratio  $\frac{\bar{D} - E_p(a)}{\bar{D} - d}$ . We know that  $\frac{\bar{D} - E_p(a)}{\bar{D} - d} < 1$  (because  $\bar{D} > E_p(a) > d$ ). Then, if  $\alpha < \frac{\bar{D} - E_p(a)}{\bar{D} - d}$  that means that the injurer feels more pessimistic than in the opposite case. To see that, it sufficient to take a low value of  $\alpha$ , that

gives a high weight to the pessimistic situation. Then, this involves that the operator assesses that the judge will estimate the Choquet integral at a higher value than himself would have done under a strict liability regime. Consequently,  $V_p^C > V_p$  and then, this would induce him to produce a higher level of safety. However, in the opposite case he will decrease this level.

Even if the above cases appear as particular cases, these tend to confirm the previous results that the negligence rule is not more efficient than the strict liability rule because of their high dependence on the beliefs of the injurer.

#### **2.4 A step further : Negligence rule and the judge as a regulator**

What does the judge assess under a negligence rule? An obvious answer is that he checks whether the prevention effort made by the injurer suits well with the requirement of the social planner i.e. the first best level of safety. This answer conforms to standard theory which considers that the regulator determines this level. The direct consequence is that the judge is in the passive role to comply (under perfect information) or to mistake about this level. However, under uncertainty and a negligence rule, as shown above, the judge can find that the set of measures taken by the injurer is insufficient, then, he complements the regulator's assessment. This is particularly the case with the cases of administrative Courts. In most countries, administrative decisions are taken under the control of administrative courts. In several countries, administrative courts are separated from general courts with their own organization in local administrative court, appeals court and Supreme Administrative Court. This is the case in Western Europe (France, Italy and in most European Countries). In the United States, several federal agencies are gifted with administrative law judges. This is the case for environmental concerns with the Environmental Protection Agency (EPA). This state of matter involves that the citizens can question the administrative determinations. Hence, the administrative judge can modify the administrative decision by substituting his own rules. We can quote U. Desai (2002, p.187) "*Nevertheless, administrative courts play an important role in environmental policy and conflicts. They exercise comprehensive judicial control over administrative actions, (.), and they are often mobilized by third parties in the course of licensing or planning procedures, with the aim of achieving tighter environmental standards or stopping projects or operating plants*".

That means that, ex post, the court can define a legal standard different as the previous one determined by the regulator. Against the above consideration, it may be argued that administrative law is few concerned with negligence rule that belongs to the field civil law. Carroll, (2007), shows the difficulty involving authorities in negligence. But this is not the aim here. The relationship with administrative law has been induced for heuristic reasons only.

The aim was of showing that in an uncertain world, courts could complement and correct the regulator's assessment.

Hence, under a rule of negligence, the courts are led to investigate and acquire more information as shown previously. This involves that even if he thinks that all prevention measures have been taken the operator is never sure of having fully complied with the socially required level of prevention.

We assume that Courts assess the optimum level of prevention effort by taking into account the set of information given by the investigation procedure. As a result, the injurer faces the situation in which either he made the right level of effort (first best) or, in the opposite, he supplied an insufficient one. From his view point the result is random and let  $\gamma$  be the probability that the court confirms his investment, (and, conversely,  $1 - \gamma$ , the probability that he did not invest enough in safety).

Reaching this stage, we cannot consider that the usual presentation of negligence rule can be maintained as such. This means that the injurer will dedicate only  $x^\mu$  in prevention investment if the court agrees with him, with a probability of  $\gamma$  and he will have to pay  $x^\mu + V_p^C \pi(x^\mu)$  in the opposite case with a probability of  $1 - \gamma$ . The consequence of the above consideration is that the injurer cannot be involved automatically if he does not perform the optimum level required by the regulator. In this case, the effective expected cost will be:

$$(37) \quad EC^{NR}_p = x + (1 - \gamma)V_p^C \pi(x)$$

We can show that this new factor of uncertainty can increase the ambiguity aversion or preference of the operator. Indeed after developing the above part of (37),

$$(38) \quad x + (1 - \gamma)V_p^C \pi(x) = x + (1 - \gamma)[\tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})\bar{D} + (1 - \tilde{\delta})E_p(a)] \pi(x^s)$$

Then the error factor can be introduced in the bracket.

$$(39) \quad x + [\tilde{\delta}\tilde{\alpha}d(1 - \gamma) + \tilde{\delta}(1 - \tilde{\alpha})(1 - \gamma)\bar{D} + (1 - \tilde{\delta})(1 - \gamma)E_p(a)] \pi(x)$$

That means that this supplementary factor of uncertainty can be "translated" in terms of optimism or pessimism. To do that, we are looking for the new expression of optimism, which is expressed by the variable  $g$  :

$$(40) \quad (1 - \gamma)\tilde{\delta}\tilde{\alpha}d + \tilde{\delta}(1 - \tilde{\alpha})(1 - \gamma)\bar{D} + (1 - \tilde{\delta})(1 - \gamma)E_p(a) \\ = \tilde{\delta}gd + \tilde{\delta}(1 - g)\bar{D} + (1 - \tilde{\delta})E_p(a)$$

And, then solving the above system gives the expression of  $g$  :

$$(41) \quad g = \alpha - \alpha\gamma + \frac{[E_p(a)(1 - \tilde{\delta}) + \bar{D}\tilde{\delta}]\gamma}{(D - d)\tilde{\delta}}$$

The condition to be respected is that  $g \leq 1$  which is true for:

$$(42) \quad E_p(a) < \frac{\bar{\delta}}{(1-\bar{\delta})} \frac{1}{\gamma} [D(1 - \alpha(1 - \gamma) - \gamma) - d]$$

In the present table we present the static analysis of the variation of  $g(\lambda, D, d, \alpha, \bar{\delta})$  according the variation each variable:

$g'(D)$	$g'(d)$	$g'(\alpha)$	$g'(\gamma)$	$g'(\bar{\delta})$
$-\frac{(E_p(a)(1 - \delta) + d\delta)\gamma}{(d - D)^2\delta}$	$\frac{(E_p(a)(1 - \delta) + D\delta)}{(d - D)^2\delta}$	$1 - \gamma$	$-\alpha + \frac{E_p(a)(1 - \delta) + D\delta}{(D - d)\delta}$	$\frac{E_p(a)\gamma}{(D - d)\delta^2}$
$g'(D) < 0$ (a)	$g'(d) > 0$ (b)	$g'(\alpha) > 0$ (c)	$g'(\gamma) > 0$ (d)	$g'(\bar{\delta}) > 0$ (e)

We just give a quick analysis of the above table. We can see that an increase in the maximum level of damage tends to decrease the level of optimism ((a) with  $g'(D) < 0$ ) while a raise in the minimum (of the maximum) damage  $d$  tends to increase it. As expected, a rise in the genuine level of optimism  $\alpha$  increase the new optimism level ((c)  $g'(\alpha) > 0$ ) and an augment in the degree of ambiguity involves, a raise in optimism.

We can show that when there is some uncertainty on the determination of the first rank efficiency of prevention the safety effort is lesser than when there is certainty.

**Proposition 5:** *Under negligence rule, if the probability to conform with the standard of the regulator is uncertain (probability  $\gamma > 0$ ), the first best level of prevention effort is below that level reached under no uncertainty about the standard.*

**Proof (appendix 3).**

The proof of this proposition is quite heavy and is developed in annex 3.

As a consequence, under uncertainty about the extent of damage, when it is known that the court investigates about the effective prevention effort, the injurer will tend to under invest in safety.

#### 4. Concluding remarks

Introducing uncertainty in the standard unilateral accident model involves ipso-facto the introduction of agency relationships. Indeed, if the regulator and the potential injurers disagree about the level of prevention effort to supply, in the interest of Society, regulator has to induce the tortfeasors to supply the first best level of safety. As a consequence, the comparison of performance between liability regimes is a direct consequence of the introduction of uncertainty. The first part of this paper has shown that defining each regime by its specificities leads to keep them away from each other. However, the task remained

incomplete and a more general treatment was needed. The application of ambiguity theory to this field allowed making the analysis more systematic.

The credit goes to Teitelbaum (2007) for having applied first ambiguity theory to the accident standard model. We extend his approach to understand the consequences when maximum damages are uncertain. Then, we show that comparing liability regimes on the basis of their relative performances (safety effort level) is no longer possible. In each case, the injurer is not only led either to overinvesting or under-investing in safety compared to the social first best care level as in Teitelbaum(2007), but also, it is particularly difficult to establish definitively which regime outstrips the other one. Indeed, we cannot define a clear decision rule about the best liability instrument that should be enforced. Putting it otherwise, sometimes strict liability fits better, sometimes it is negligence.

This result does not mean that a government should not implement a liability scheme as a regulatory instrument. Liability rules keep their deterrent effect on negligent behavior of potential injurers. It indicates that enforcing a liability regime cannot be based on safety performance only. The government should consider many other kinds of variables rather than strict efficiency in terms of level of safety effort. For instance, he could regard the transaction costs associated with the necessary investigation about evidence for fault under a rule of negligence, the fairness of letting the burden of the recovery to society when mistake cannot be proved from the polluter side. We find again the Calabresi (1970)'s considerations who takes every liability regime as a whole. Hence, if general result cannot be defined, this involves a close examination of the condition of application of liability rule according the specificity of the potential harm, the causal link, the number of potential victims, the time necessary to gather evidence and the associated transaction costs, etc. For instance, the European directive on environmental liability<sup>8</sup> distinguishes between activities which require allowances or permits and activities that need not. Facilities that can have far reaching consequences on the environment are submitted to a strict liability regime, while the other ones, because they work a lower scale will resort to a negligence rule only.

Other secondary results consist in specific consideration about both regimes. For instance, we show that the application of ambiguity theory to the maximum damage can be extended to the judgment-proof question by finding again the Beard (1990)'s conclusions. We show also that, under negligence and uncertainty, the Courts are led to investigate for determining who should bear the burden of repairs. This can induce court to define a different

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<sup>8</sup> Directive 2004/35/CE of the European parliament and of the Council of 21 April 2004 on environmental liability with regard to the prevention and remedying of environmental damage, L143/56, 30/4/04).

optimum level of safety effort compared to the one defined by the regulator. We materialized this change by a probability distribution. We show that this factor deters the injurer to provide the highest level of care. The direct consequence of such a possibility is to increase the evidence that strict liability and negligence cannot be directly compared as it is usually the case.

## Appendix 1

### Neo-additive capacity and Choquet utility function

We do not propose here a full formal mathematical presentation. The interested lector may refer to the clear exposition of Chateauneuf, Eichberger and Grant S., (2007).

A capacity is an extension of a probability. Formally, this is a function  $\tau(p)$  that assigns real numbers to events  $\mathcal{E}$ , where  $\mathcal{E}$  is the set built from the set  $\mathcal{S}$  of the states of nature. To be a capacity the following two conditions should be fulfilled. First, for all  $E, F \in \mathcal{E}$ , and  $E \subseteq F$ , then  $\tau(E) \leq \tau(F)$  as monotonicity condition and second, as normalization conditions,  $\tau(\emptyset) = 0$  and  $\tau(\mathcal{S}) = 1$ .

The best way to integrate capacities is the Choquet integral. To do that, it is assumed that exists a simple function of finite range  $f$  such that it takes values  $\mu_1 \geq \mu_2 \dots \geq \mu_n$ . A Choquet integral of a simple function  $f$  with respect to a capacity  $\tau(\cdot)$  is defined as:

$$V(f/\tau) = \sum_{\mu \in f(\mathcal{S})} \mu [\tau(\{s/f \geq \mu\}) - \tau(\{s/f > \mu\})]$$

Through the concept of neo-additive capacity the Choquet integral overweight high outcomes if the capacity is concave or overweigh low income if the capacity is convex. Convexity of a capacity is verified by the following relationships:

$$\tau(E \cup F) \geq \tau(E) + \tau(F) - \tau(E \cap F) \text{ (and concave in the opposite situation).}$$

Applying this to our model, we consider that the polluter and the society cannot assess with certainty the exact value of a maximum damage. Let be  $\mathcal{E}$  the finite set of states to which correspond the catastrophic events  $\mathcal{A}$  ( $\sigma$ -algebra of  $\mathcal{E}$ ). We consider a finite set of outcomes ( $A \subset \mathbb{R}$ ) and let  $\Phi = \{f: \mathcal{E} \rightarrow A\}$  be a set of simple functions from states to outcomes which correspond to simple acts and takes on values  $a_1 \geq a_2 \dots \geq a_n$ .

The polluter is gifted with a Choquet objective function which corresponds here to an expected cost function. His beliefs on the level of damage correspond to a neo-additive capacity ( $\mu$ ) based on ( $p$ ). Hence, the operator will form beliefs about the level of the damage. This is a supplementary uncertainty. We can define now the neo-additive capacity. To do that let us consider that the  $\sigma$ -algebra  $\mathcal{A}$  is partitioned in three subsets that we present and characterize (for a more complete information see CFG (2002, 3).

- The set of null events  $\mathcal{N}$ , where  $\emptyset \in \mathcal{N}$  and for  $G \subset H$ , and  $G \in \mathcal{N}$  if  $H \in \mathcal{N}$ .
- The set of "universal events"  $\mathcal{W}$ , in which an event is certain to occur, (complement of each member of the set  $\mathcal{N}$ ).
- The set of essential events,  $\mathcal{A}^*$ , in which events are neither impossible nor certain.

This set is composed of the following:

$$\mathcal{A}^* = \mathcal{A} - (\mathcal{N} \cup \mathcal{W})$$

Before going further, we define the following capacities  $\nu$  (see appendix):

$$\nu_0(A) = 1 \text{ if } A \in \mathcal{W} \text{ and } 0 \text{ otherwise and } \nu_1(A) = 0 \text{ for } A \in \mathcal{N} \text{ and } \nu_1(A) = 1 \text{ otherwise.}$$

Furthermore, we define a finite additive probability  $p(\cdot)$  such that  $p(A) = 0$ , if  $A \in \mathcal{N}$  and 1 otherwise.

**Definition 1:** Let  $\lambda, \gamma$  that belong to a simplex  $\Delta$  in  $\mathbb{R}^2$ , ( $\Delta := \{(\alpha, \beta) / \alpha \geq 0, \beta \geq 0, \alpha + \beta \leq 1\}$ ), a neo-additive capacity  $\mu$  based on the distribution of probability  $p(\cdot)$  is defined as:

$$\mu(A / p, \lambda, \gamma) = \begin{cases} 0 & \text{for } A = \emptyset \\ \lambda v_0(A) + \gamma v_1(A) + (1 - \gamma - \lambda)p(A) & \text{for } \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1 & \text{for } A = \mathcal{E} \end{cases}$$

We can check here that a neo-additive capacity is additive on non-extreme outcomes. Here  $p$  corresponds to the probability of a major accident of a given scale. This is a common belief and  $(1 - \gamma - \lambda)$  represents the degree of confidence of the agent in this belief. We will give below, after the presentation of the Choquet integral of the neo-additive capacity, more complete explanation on the concept of optimism.

Then, we can define the Choquet integral which is a weighted sum of the minimum, the maximum and the expectation of a simple function  $f: \mathcal{E} \rightarrow \mathbb{R}$  as the following relationship:

$$V(f / p, \lambda, \gamma) = \lambda \cdot \inf(f) + \gamma \cdot \sup(f) + (1 - \gamma - \lambda)E_p(f)$$

Where  $E_p(f)$  is the expected value of the expected costs of a major accident, and from the linearity of the Choquet integral with respect to the capacity,  $V(f / v_0(\cdot)) = \inf(f)$  and  $V(f / v_1(\cdot)) = \sup(f)$ , (proof see CFG(2002, 3) and CFG(2006, 3)).

Then for  $e \in \mathcal{E}$ ,  $f(e) = a$ , we put,  $f(e_1) = \sup(f) = a_1 = \bar{D}$  and  $f(e_n) = \inf(f) = a_n = d$ . As,  $p(\cdot)$  is a finitely additive probability distribution on  $\mathcal{A}$ , we define  $E_p(f)$  as:

$$E_p(f) = E_p(a) = \int_d^{\bar{D}} a p(a) da$$

Taking into account these factors, the Choquet integral writes now:

$$V_p = \lambda \cdot d + \gamma \bar{D} + (1 - \gamma - \lambda)E_p(a)$$

We can immediately check that if  $\gamma = \lambda = 0$ , we find the usual expected utility. With  $1 \geq \gamma > 0$ ,  $\lambda = 0$ , the subject is waiving between the lowest value and the expected value of the function. That corresponds to pessimism because the operator cannot consider that  $\bar{D}$  occurs with sufficiently high probability. Then, optimism is induced by  $\gamma = 0$ ,  $1 \geq \lambda > 0$ .

However, to keep a correspondence with the analysis of Teitelbaum (2007) we will make the following change of variable:

$$\lambda = \delta\alpha, \gamma = \delta(1 - \alpha), \text{ then we can check that } 1 - \gamma - \lambda = 1 - \delta \text{ with } \delta, \alpha \in (0,1)$$

The neo-additive capacity is then:

$$\mu(A / p, \delta, \alpha) = \begin{cases} 0 & \text{for } A = \emptyset \\ \delta\alpha v_0(A) + \delta(1 - \alpha)v_1(A) + (1 - \delta)p(A) & \text{for } \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1 & \text{for } A = \mathcal{E} \end{cases}$$

Or, still, for  $\emptyset \subsetneq A \subsetneq \mathcal{E}$

$$\mu(\cdot) = \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)p(A)$$

Replacing still these factors in the above integral, we get:

$$V_p = \delta\alpha d + \delta(1 - \alpha)\bar{D} + (1 - \delta)E_p(a)$$

## Appendix 2

**Proposition 2:** *Under uncertainty, when agents feel aversion for ambiguity, neither the negligence rule nor the strict liability rule can be considered as a superior rule comparing them mutually.*

**Proof**

The proof requires several steps, conceived as a set of sub-propositions. We compare the consequences of assessing the prevention cost under strict liability and negligence when the potential injurer feels aversion to ambiguity. ( $V_p$  is his Choquet integral under strict liability and  $V_p^C$  under negligence).

**Sub-proposition 1:**

$$V_p^C > V_p \text{ if } \frac{\bar{D}-E_p(a)}{\bar{D}-d} > \frac{\delta\tilde{\alpha}-\delta\alpha}{\delta-\delta} \text{ and}$$

$$V_p^C \leq V_p \text{ if } \frac{\bar{D}-E_p(a)}{\bar{D}-d} \leq \frac{\delta\tilde{\alpha}-\delta\alpha}{\delta-\delta}$$

**Proof:**

The proof is very simple. We compare  $V_p^C = \delta\tilde{\alpha}d + \delta(1-\tilde{\alpha})\bar{D} + (1-\delta)E_p(a)$  and  $\delta\alpha d + \delta(1-\alpha)\bar{D} + (1-\delta)E_p(a)$  and by simple arithmetic with deduce the above results.

**Sub-proposition 2:**

If  $V_p^C > V_p$ , then the level of effort of prevention made under the Choquet integral  $V_p^C$  by the operator is higher than under  $V_p$ . In the opposite, when  $V_p^C \leq V_p$ , the reverse is true.

**Proof:**

This comes from the comparison of the first order conditions of  $x + \pi(x)V_p^C$  and  $x + \pi(x)V_p$ .

Adopting the same argument than used in proposition 1 we can show that when  $V_p^C > V_p$ , then the optimum associated to  $V_p^C$ ,  $x_C^0$ , is higher than the one associated to  $V_p$ ,  $x^0$ , ( $x_C^0 > x^0$ ).

**Sub-proposition 3:**

Sub-propositions 1 and 2 involves that :

- When  $V_p^C > V_p$ , then, the level of effort of prevention is higher under a negligence rule than under a strict liability regime.
- When  $V_p^C < V_p$ , then, the level of effort of prevention is higher under a strict liability regime than under a negligence rule.
- When  $V_p^C = V_p$ , both regime are equivalent.

**Proof :**

It is obvious from 2.

- **Sub-proposition 4:** From Sub-propositions 1 to 3, we can deduce that, whatever the liability regime, attitudes toward uncertainty is fundamental for determining the superiority of one regime on the other one.

**Proof:**

From sub-propositions 1 to 3, it is obvious that when enforcing a given liability regime, the regulator disposes of no means to know what will be (or what is) the state of mind of the operator. That means that the regulator cannot know the level of ambiguity aversion of the potential injurers. Hence, for instance, let us assume that a strict liability rule has been implemented. Then, the Choquet expected cost function  $V_p$  of a potential injurer yields a safety level equivalent to  $x^0$ . After calculation, the regulator can assess that a negligence rule would induce a Choquet integral equivalent to  $V_p^C$ , where  $V_p^C > V_p$  that from sub-proposition

2 yields  $x_c^0 > x^0$ . Then the negligence rule should have been enforced. The argument can be fully reverted.

### Appendix 3

**Proposition 5:** *Under negligence rule, if the probability to conform with the standard of the regulator is uncertain (probability  $\gamma > 0$ ), the first best level of prevention effort is below that level reached under no uncertainty about the standard.*

#### Proof

The proof needs two steps. First, we show that  $g > \alpha$ . To show this, we can notice that if we put  $g < \alpha$ , this involves that  $E_p(a) < \frac{\tilde{\delta}(\alpha-1)D - \alpha d \tilde{\delta}}{(1-\tilde{\delta})}$ . We can show that this relation is false, because the right hand side of this expression is negative and  $E_p(a) > 0$ . Indeed, as  $(1 - \tilde{\delta}) > 0$ ,  $\tilde{\delta}(\alpha - 1)D - \alpha d \tilde{\delta} < 0$ , indeed,  $\alpha - 1 < 0$ . Therefore,  $g > \alpha$ .

Second we compare  $V_p^C(\alpha)$  and  $V_p^C(g)$ . If we put:

$$V_p^C(\alpha) = \tilde{\delta} \tilde{\alpha} d + \tilde{\delta} (1 - \tilde{\alpha}) \bar{D} + (1 - \tilde{\delta}) E_p(a) > V_p^C(g) = \tilde{\delta} g d + \tilde{\delta} (1 - g) \bar{D} + (1 - \tilde{\delta}) E_p(a)$$

this involves, after developing that  $\bar{D} > d$  which is true.

Then if  $V_p^C(\alpha) > V_p^C(g)$  by the same argument used in proposition 1 we can show that the optimum effort corresponding to  $V_p^C(g)$ ,  $x_g^*$  is less than the one corresponding to if  $V_p^C(\alpha)$ ,  $x_\alpha^*$ , i.e.  $x_g^* < x_\alpha^*$  for  $g > \alpha$ .

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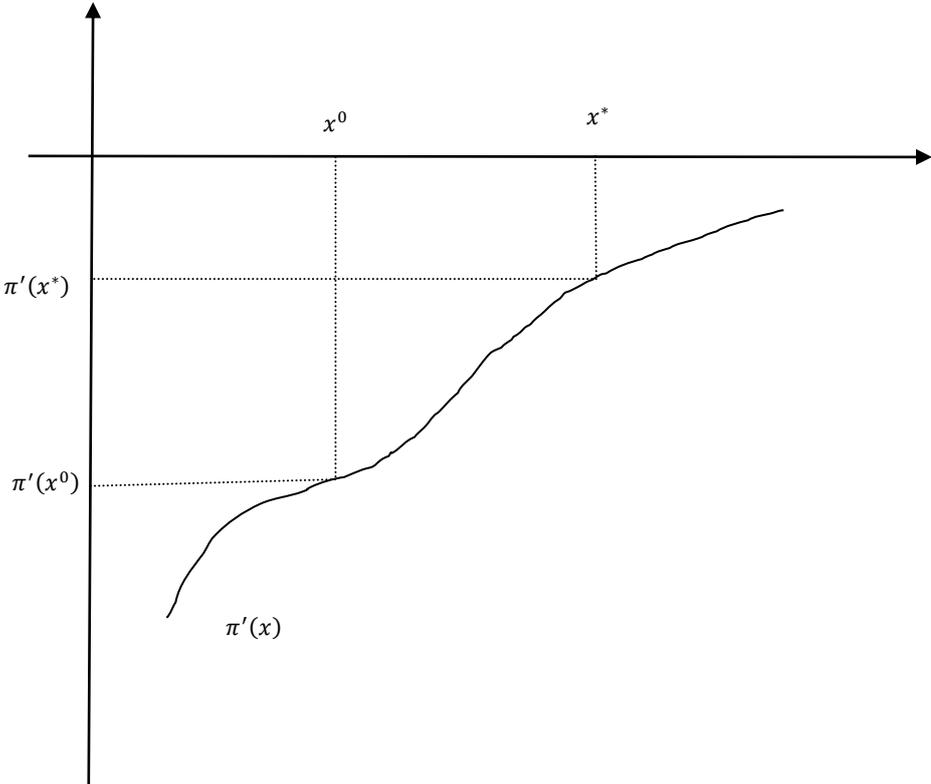
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Figure 1

The behavior of the first order derivative of the probability of major harm



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