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Using VARs to identify models of fiscal policy: A comment on perotti

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Using VARs to Identify Models of Fiscal Policy:
A Comment on Perotti

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Using VARs to Identify Models of Fiscal Policy:

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Ricardo Reis

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Abstract

This note comments on Perotti's (2008) estimates of the impact of a government spending shock on the economy. In the process, it makes two points. First, it notes that with enough freedom to pick the dynamics of policy variables, the neoclassical model can generate any set of observations for the non-policy variables. Second, it proposes a method to identify the policy dynamics in theoretical models by using the estimated impulse responses of the policy variables from VARs, and in this way generate testable predictions of the model for the non-policy variables.

*This paper is forthcoming as a comment on Perotti (2008) in the NBER Macroeconomics Annual 2007.

How should economists compare the predictions of a dynamic model with the data? A currently popular answer to this perennial question is to plot the impulse response functions of some variables to shocks, and compare the responses predicted by the model to those estimated in the data. This approach is simple, intuitive, and even fairly comprehensive since impulse responses contain a great deal of information. For a linear (or linearized) model with constant variances, the impulse response functions summarize all of the model's dynamics, and for covariance-stationary data, they capture all of the second-order properties of the data. Impulse responses have allowed economists to move from focusing solely on variances and covariances into assessing finer features like persistence, sluggishness, hump shapes, and lead-lag relations.

In practice, one difficulty with this methodology is how to estimate the empirical impulse responses. In the study of monetary policy, research has used vector autoregressions (VARs) and this is the recent growing approach in the study of fiscal policy. Perotti gives a thorough and insightful survey of this work, focusing on the impulse responses of output, hours, consumption, and real wages to government-spending shocks. These have led to a debate and a challenge.

The debate is between Ramey and Shapiro on one side, and Blanchard and Perotti himself on the other. All agree that output and hours rise following an exogenous expansion in government spending, but while Ramey and Shapiro find that consumption and real wages fall, Blanchard and Perotti find that they rise. Because these opposite results come from different empirical strategies to identify shocks to government sending, one "narrative" that uses war build-ups as exogenous dates, and the other "structural" that assumes government spending responds with a lag to other shocks, this has led to a more general debate on the relative merits of these two methods. Furthermore, Ramey and Shapiro's results are used to support the neoclassical model, while Blanchard and Perotti's are used to support the Keynesian model, so they become involved in the more general debate of what is the best model of economic fluctuations.

The challenge is that, if Blanchard and Perotti are right, it is hard to understand the rise in consumption following the increase in government spending for two reasons. First, since more government consumption uses resources and lowers private wealth, any model with a significant role for the permanent-income hypothesis will predict a fall in consumption. Second, since with standard parameters, the marginal rate of substitution between leisure

and consumption rises significantly with the increase in hours but the wage only slightly changes, so for households to be on their labor supply and the two to be equal, consumption must fall. One answer to this challenge is, of course, the old-fashioned IS-LM model since it violates the permanent-income hypothesis and has hours determined by labor demand, not supply. But more modern models, whether neoclassical or new-Keynesian, whether real or monetary, fail at the challenge. One exception is Gali, López-Salido and Valles's (2007) "truly-Keynesian" model where there are not only pricing frictions but also a large group of Keynesian hand-to-mouth consumers (who consume more with the rise in income counteracting the wealth effect) and Keynesian labor markets where unions sets hours and wages (so these are determined by labor demand not supply).

In this comment, I discuss the use of VAR evidence to test models via impulse responses. There has been an intense debate on the merits and flaws of VARs at estimating impulse responses.¹ Here, I am actually going to assume that Perotti's estimates are exactly right. Instead, I will focus on the use of these estimates to distinguish between models.

1. An aside into monetary policy and anticipated policy

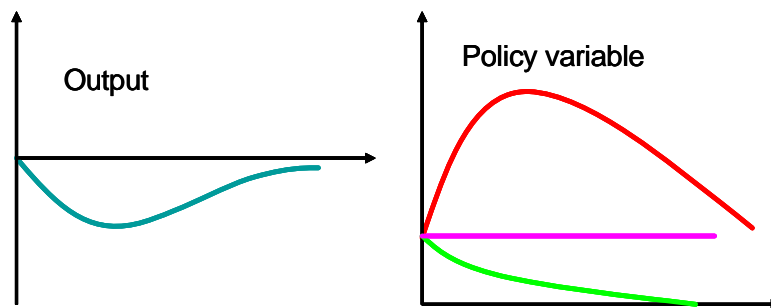
Before I start, it is worthwhile taking a short detour into the literature on monetary policy to make an observation inspired by Cochrane (1998). Imagine that 3 researchers estimated the response of output and a policy variable (say an interest rate) to an exogenous contraction in policy. All three found the same output response, in the left panel of figure 1, but each found a different response of the policy variable, in the right panel of the figure. Would they reach the same conclusion if they were interested in testing a theory of output fluctuations?

If that theory stated that only unanticipated policy matters, as in the classical models of Lucas and Barro, the answer is yes. All three estimated the same instantaneous impact on the policy variable, and that is all that matters for output. The path of policy afterwards is anticipated so it is neutral, whether it goes up, down, or stays the same.

If, however, they were examining a modern sticky-price model, the answer is no. In this model, the anticipated policy path after the shock affects by how much adjusting firms change their prices, which in turn affects by how much output falls. Each of the responses of the policy variable on the right side of the figure would lead to a different response of

¹In the Macro Annual conference alone, see Gali and Rabanal (2005) McGrattan (2005) Christiano, Eichenbaum and Vigfusson (2007) and Kehoe (2007).

Figure 1: Fictional impulse responses to policy shock



output, so only one (if any) could be consistent with the output response in the left side. In modern models of nominal rigidities, *policy rules matter*, and the response of policy variables to policy shocks provides information on these policy rules.

In the study of fiscal policy, anticipated policy also matters and, if anything, even more. Most fiscal policy changes are announced a few quarters in advance and they tend to persist, so fiscal policy is quite predictable. Moreover, changes in government spending typically come with future changes in fiscal policy to balance the budget (and intense debates on the best way to do it). And lastly, in models with intertemporal substitution, future fiscal policy affects relative trade-offs and therefore behavior in the present.

2. A neoclassical model of fiscal policy

Consider a simple neoclassical model of fiscal policy and the economy. Households maximize:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{C_t^{1-\theta} - 1}{1-\theta} - \frac{\varkappa N_t^{1+\psi}}{1+\psi} \right)$$

$$s.t. : (1 + \tau_t^C)C_t + K_{t+1} = R_{t+1}K_t + W_t N_t + T_t,$$

where C_t is consumption and τ_t^C a consumption tax, N_t hours worked and W_t their after-tax wage, K_t the capital stock and R_t the after-tax return on renting it, and T_t are lump-sum transfers from the government. Firms produce private output to maximize profits:

$$\max_{N,K} \left\{ K_t^\alpha N_t^{1-\alpha} - \frac{W_t N_t}{1 - \tau_t^N} - \frac{[R_t - (1 - \delta)(1 - \tau_t^K)] K_t}{1 - \tau_t^K} \right\}$$

where τ_t^N is the tax rate on labor and τ_t^K the tax rate on capital (with no depreciation exemption). Finally, the economy's resource constraint and total output Y_t are:

$$\begin{aligned} K_{t+1} &= (1 - \delta)K_t + Y_t - C_t - G_t \\ Y_t &= K_t^\alpha N_t^{1-\alpha} + \sigma G_t \end{aligned}$$

Total government spending is G_t and a fraction σ of it is used in the public sector to generate output, while the remaining $1 - \sigma$ is wasted or provides welfare through some additive extra term in the utility function.² The government chooses $\{G_t, \tau_t^C, \tau_t^N, \tau_t^K\}$ and T_t ensures a balanced budget every period.

This model has a steady-state where all variables are constant. It is described by 4 non-linear equations relating the 4 endogenous variables that Perotti wants to focus on, $\{Y, N, C, W\}$, to the 4 policy variables, $\{G, \tau^C, \tau^N, \tau^K\}$. My only assumption on the parameters is that the steady-state endogenous variables are positive. Evaluating the Jacobian determinant of this system at the point where all policy variables are zero:³

Proposition 1: *There is (locally) a one-to-one relation between (Y, N, C, W) and $(G, \tau^C, \tau^N, \tau^K)$.*

Therefore, given an appropriate choice of fiscal policy, the neoclassical model can generate *any* steady state that you want. This result is not surprising: observations of average output, hours, consumption and wages (properly scaled with growth) convey no information on the validity of the neoclassical model.

To study the predicted response to government-spending shocks, one must specify the dynamics of the shocks and the fiscal policy rules. I assume the shock follows an AR(1), $a_t = \rho a_{t-1} + \varepsilon_t$. Letting small letters denote the log of the respective capital letter relative to its steady state, the fiscal policy rules are:

$$\begin{aligned} g_t &= \gamma^G c_t + (1 + \lambda^G) a_t, \\ \tau_t^C &= \gamma^C c_t + \lambda^C g_t, \\ \tau_t^N &= \gamma^N c_t + \lambda^N g_t, \\ \tau_t^K &= \gamma^K c_t + \lambda^K g_t. \end{aligned}$$

²For simplicity, this assumes that the public-sector's output is a perfect substitute for private-sector's output, so there is only one consumption good.

³All results are proven in an appendix available from my website: <http://www.princeton.edu/~rreis>.

Total government spending responds to a 1% fiscal shock by $(1 + \lambda^G)\%$, and it is cyclical, adjusting to the level of consumption. Tax rates are also cyclical and respond to movements in government spending. These fiscal policy rules may not be optimal or realistic for developed economies, but they are plausible and roughly capture the cyclicity of fiscal policy and the interaction between taxes and spending.⁴ There are 8 policy-rule parameters: $\pi = (\gamma^G, \gamma^C, \gamma^N, \gamma^K, \lambda^G, \lambda^C, \lambda^N, \lambda^K)$.

The log-linear approximate solution of the model implies an ARMA(2,1) structure for the impulse response to an ε_t shock:

$$(1 - \eta(\pi)L)(1 - \rho L)x_t = \mu_x(\pi)\varepsilon_t + \nu_x(\pi)\varepsilon_{t-1}$$

where x_t is either y_t, n_t, c_t or w_t . The autoregressive coefficients are common to all variables, so differences in dynamics depend on the 8 moving-average coefficients $\phi(\pi) = (\mu_y, \nu_y, \mu_n, \nu_n, \mu_c, \nu_c, \mu_w, \nu_w)$, which are functions of the policy parameters.

The neoclassical model's predictions for the variables of interest are fully described by $\phi(\pi)$. If Perotti's estimates fit this ARMA(2,1) structure, then he has effectively estimated $\hat{\phi}$. Asking if the neoclassical model fits the data then amounts to asking whether $\hat{\phi}$ is close to $\phi(\pi)$. Perotti finds that $\hat{\mu}_c > 0$ and $\hat{\mu}_w > 0$ and argues that the neoclassical model predicts the opposite signs, so he concludes against it. However, in the neighborhood of the point where all the elements of π are zero, and for conventional parameter values:⁵

Proposition 2: *There is (locally) a one-to-one relation between π and $\phi(\pi)$.*

That is, *whatever* were Perotti's estimated impulse responses of output, hours, consumption and wages, they are consistent with a neoclassical model with an appropriate choice of policy rules. Perotti's conclusion came from arbitrarily assuming that all the elements of π are zero. But, with freedom to pick the policy-rule parameters in π , the result on steady states applies also to the model's dynamics. No set of impulse responses could ever reject the model.

It is important to not over-state this result. This is not a claim that anything goes in the neoclassical model, nor is it necessarily specific to the neoclassical versus other dynamic models. The point is instead that looking only at a few impulse responses and having a lot

⁴For a careful empirical study of this interaction, see Romer and Romer (2007).

⁵The parameter values are $\beta = 0.99$, $\theta = 1$, $\psi = 4$, $\alpha = 0.34$, $\delta = 0.025$, $\sigma G/Y = 0.12$, $G/Y = 0.21$, $\tau^K = 0.54$, and $\rho = 0.8$. See the appendix for explanations.

of leeway to pick policy rules gives so much freedom that it leads to no predictions. This problem is familiar to empirical VARs, but here reversed on its head to apply to theoretical models: identification.

3. Identification in the neoclassical model

In principle, identification in a theoretical model can follow the same strategies used in empirical estimation. For instance, it is popular in the literature on VARs to impose timing restrictions. These have a direct counterpart in the model. To see how they work, note that the neoclassical model has two static optimality conditions, one from the household's intra-temporal allocation of labor and consumption, and the other from labor demand by firms:

$$\begin{aligned} \varkappa N_t^\psi C_t^\theta &= W_t / (1 + \tau_t^C), \\ W_t &= (1 - \tau_t^N) Y_t / N_t. \end{aligned}$$

Now, imagine imposing the restrictions that the tax rates on consumption and labor income adjust only with a one-quarter delay to changes in spending. Then, these two conditions will pin down the impact response of two of (Y_t, N_t, C_t, W_t) as a function of the other two, independently of the policy-rule parameters. Proposition 2 will no longer hold, and the model has testable predictions on the impact response to spending shocks.

Another approach is to use institutional restrictions, using the details of how taxes are set in a country to learn about some of the policy-rule parameters directly (Blanchard and Perotti, 2002). In principle, one could impose exactly the same identifying restrictions on both the VAR and the model, solving both the empirical and theoretical identification problems in a coherent way.

I would like to propose a third approach to identification that uses the impulse responses of policy variables to policy shocks. These responses trace out the policy dynamics. The researcher can use them to pin down the policy rule parameters, tying his or her hands before looking at the impulse responses of the non-policy variables. In the model above, this would amount to using the estimated impulse responses of $(g_t, \tau_t^C, \tau_t^N, \tau_t^K)$ to pin down the policy-rule parameters. The resulting $\hat{\pi}$ can then be fed into $\phi(\hat{\pi})$ and compared with the empirical estimates $\hat{\phi}$.

This strategy accomplishes the coherence in identification between estimates and model, because the estimated impulse responses of the policy variables respect the empirical identifying assumptions by construction. When it is hard to map the empirical identifying restrictions to their theoretical counterparts, this procedure accomplishes it directly. Moreover, when the empirical identifying restrictions are not sufficient to identify the model, the policy-variables impulse responses include new information from the data to achieve identification.

To see this approach in action, I pursued an example using Perotti's baseline SVAR estimates with U.S. data from 1947. Because there are only two policy variables in his baseline VAR, government spending and an income tax, I consider a simpler version of the neoclassical model above where there is only an income tax (so $\tau_t^C = 0$ and $\tau_t^N = \tau_t^K = \tau_t$) and consider only the impulse responses of output and consumption. I solve the model for the theoretical impulse responses of g_t and τ_t , which follow the ARMA(2,1) structure above with 4 moving-average parameters. I pin down the 4 policy-rule parameters to match as closely as possible the first 16 elements of the empirical impulse responses of g_t and τ_t . Figure 2 shows the reasonably good match.

Using these policy-rule parameters, I then solve for the theoretical impulse responses of y_t and c_t and compare them to their empirical counterparts in figure 3. There are three results to note. First, after an expansion in spending, consumption rises on impact. Contrary to Perotti's claim, rising consumption is consistent with the neoclassical model. The reason is that in Perotti's estimates in figure 2, when spending rises, taxes rise and are expected to fall in the future. Households therefore realize it is relatively less rewarding to work today rather than in the future and so cut hours. Since consumption and leisure are complements, this pushes consumption up.

The second thing to note is that output also falls on impact. This example illustrates the perils of not taking into account the identification of the model. Perotti contrasted his estimates with the predictions of falling consumption and rising output coming from a neoclassical model where all the policy-rule parameter are equal to zero. In fact, given the policy rules for government spending and income taxes that he estimated, the neoclassical model predicts the opposite, a fall in output and a rise in consumption on impact.

The third result is that the neoclassical model is at odds with the facts. While consumption rises on impact in both data and theory, it stays positive in the former but falls to

Figure 2: Impulse responses of taxes and government spending

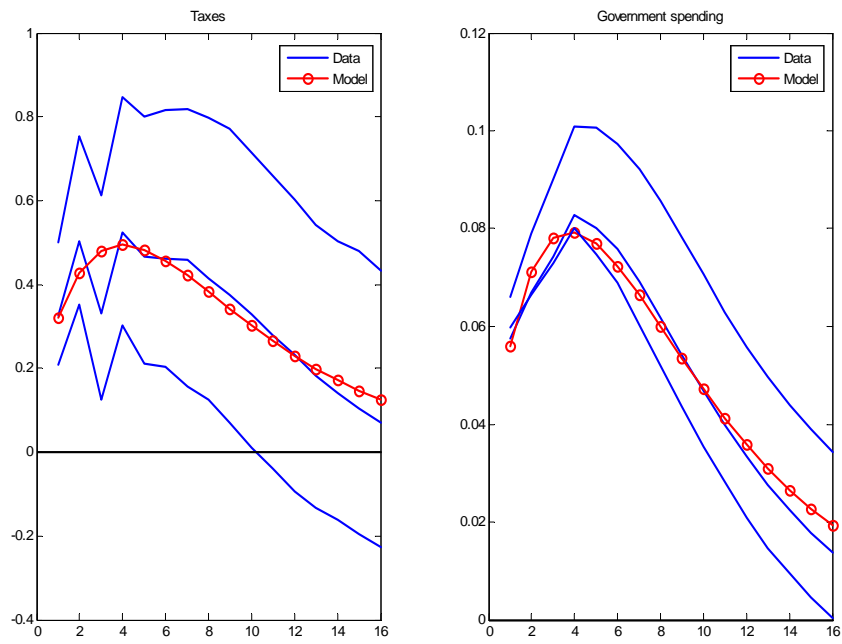
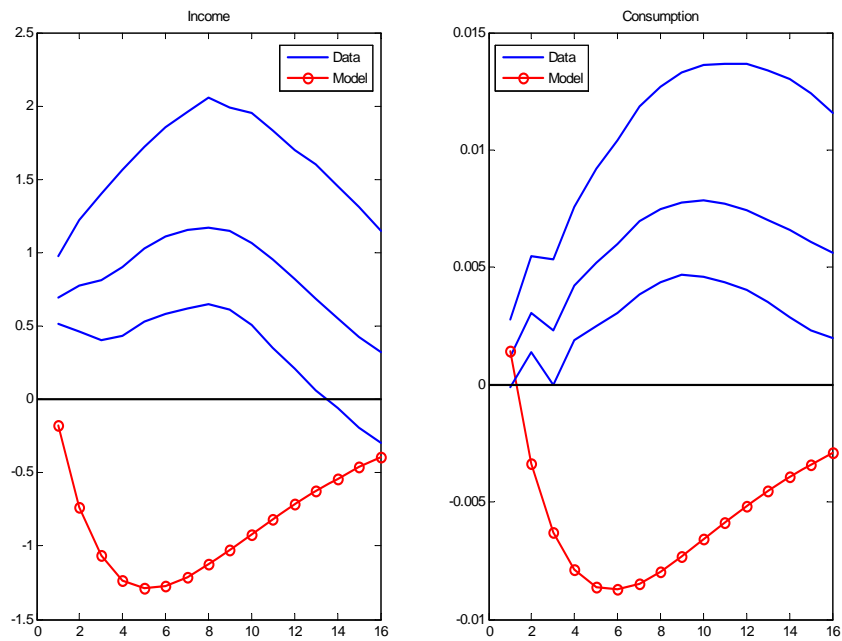


Figure 3: Impulse responses of income and consumption



negative in the latter. And the output response is positive in the data but negative in the theory at all horizons. In general, the theory predictions are quite far from the empirical confidence bands.

4. Conclusion

Perotti has done a tour de force on the difficult and important issue of estimating and identifying empirical impulse responses to government spending shocks. He used these estimates in part to test models and this comment focused on this application.

I have tried to make two points that apply more generally than to his paper. The first is well-known: policy rules and anticipated policy matter for the dynamics of intertemporal models. The second is perhaps less appreciated: theoretical models can suffer from identification problems that are as serious as those in empirical estimates. The theorist has many degrees of freedom in building his or her model, and some of the most important are the most difficult to pin down, the policy rules.

To be constructive, I proposed an approach to identify the theoretical model. It uses the empirical impulse responses of the policy variables to the policy shocks as a summary of both the data and the VAR's identification conditions to identify the policy rules in the model. Then, it compares the theoretical impulse responses for the non-policy variables with their empirical counterparts.⁶ When I applied this method to compare Perotti's empirical estimates with those of a neoclassical model, I agreed with him that they seem inconsistent, but for very different reasons.

The typical debate on structural VARs focuses on how one can use information from models to help estimate and identify VARs. But, sometimes, the reverse can also be true: one can use information from VARs to help formulate and identify models.

⁶ Aside from the well-established practice of picking model parameters to fit estimated impulse responses (e.g., Christiano, Eichenbaum and Evans, 2005), there are two closer antecedents to this approach. Both also abide by the general principle that the policymaker's policy-rule parameters in the model are chosen to match the empirical impulse response function of the policy variables, but they impose stricter restrictions on the policy rules. Edelberg, Eichenbaum and Fischer (1999) and Burnside, Eichenbaum and Fischer (2004) assume the policy rules for government spending and taxes are a moving average of the exogenous fiscal shocks, without any feedback from endogenous variables, and pick the moving-average parameters to match their VAR empirical estimates. Rotemberg and Woodford (1996) make timing assumptions on monetary policy that ensure that the policy rule parameters can be identified from the VAR estimates without having to specify the rest of the model.

Appendix

This appendix defines the competitive equilibrium of the neoclassical economy, proves propositions 1 and 2 and outlines the calculations behind figures 2 and 3. The proofs of the propositions use an analytical-derivatives program (Mathematica) and the calculations for the figures use a numerical-computation software (Matlab). All programs are available from my website.

The neoclassical economy: In a competitive equilibrium, households and firms behave optimally and markets clear. Dropping the time index from variables and letting a subscript denote next period's variables, the household's optimality conditions are:

$$\begin{aligned} C^{-\theta} / (1 + \tau^C) &= \beta E \left[R' C'^{-\theta} / (1 + \tau^{C'}) \right] \\ W &= \varkappa N^\psi C^{-\theta} (1 + \tau^C). \end{aligned}$$

The firm's optimality conditions and a 0-profit condition imply:

$$\begin{aligned} W &= (1 - \tau^N)(1 - \alpha)(K/N)^\alpha \\ R &= (1 - \tau^K)[\alpha(N/K)^{1-\alpha} + 1 - \delta] \end{aligned}$$

and the production function and resource constraint serve as the relevant market-clearing conditions:

$$\begin{aligned} K' &= (1 - \delta)K + Y - C - G \\ Y &= K^\alpha N^{1-\alpha} + \sigma G \end{aligned}$$

These 6 equations, together with a transversality condition form consumer optimization, initial values for the capital stock and shocks, and the policy rules in the main text, define the competitive equilibrium of the model.

Proof of Proposition 1: At the steady-state, the Euler equation implies that $R = 1/\beta$ and the definition of output implies that $(K/N)^\alpha = (Y - \sigma G)/N$. Using these two results to replace out R and K from the remaining 4 equations for equilibrium, gives a system of 4 non-linear equations in the 4 endogenous variables $\{Y, N, C, W\}$ and the 4 policy variables

$\{G, \tau^C, \tau^N, \tau^K\}$:

$$\begin{aligned}
W &= \varkappa N^\psi C^{-\theta} (1 + \tau^C), \\
W &= (1 - \tau^N)(1 - \alpha) \left(\frac{Y - \sigma G}{N} \right) \\
1 &= \beta(1 - \tau^K) \left[\alpha \left(\frac{Y - \sigma G}{N} \right)^{1-1/\alpha} + 1 - \delta \right] \\
Y &= \delta N \left(\frac{Y - \sigma G}{N} \right)^{1/\alpha} + C + G
\end{aligned}$$

The determinant of the Jacobian of this system evaluated at the point where all elements of $\{G, \tau^C, \tau^N, \tau^K\}$ are zero is (computed by Mathematica):

$$\frac{[\alpha Y + (1 - \delta)K] [(\alpha - \sigma)Y + \sigma C]}{\alpha(\theta + \psi)}.$$

For the system to be one-to-one, this expression must not be zero. The first term in brackets in the denominator is non-zero as is the numerator. The second term in brackets in the denominator is also not zero as long as $C/Y \neq 1 - \alpha/\sigma$. But, the equations above imply that $C/Y = 1 - \alpha\delta/(\beta^{-1} - 1 + \delta)$, so this condition will hold as long as $(\sigma - 1)\delta \neq \beta^{-1} - 1$. Since the left-hand side is negative, but the right-hand side is positive, this must hold.

Proof of Proposition 2: Log-linearizing the system of equations describing the competitive equilibrium around the steady state gives the system:

$$\begin{aligned}
\theta c &= \theta E(c') + E(\tau^{C'}) - \tau^C - E(r') \\
w &= \tau^C + \psi n + \theta c \\
w &= -\tau^N + \alpha(k - n) \\
r &= -\tau^K + \Xi(1 - \alpha)(n - k) \\
k' &= (1 - \delta)k + (Y/K)y - (C/K)c - (G/K)g, \\
y &= (1 - \sigma G/Y)(\alpha k + (1 - \alpha)n) + (\sigma G/Y)g
\end{aligned}$$

where $\Xi^{-1} = 1 + (1 - \delta)/[(Y/K)(1 - \sigma G/Y)]$. These 6 equations, together with the 4 policy rules in the main text provide the linear laws of motion for $\{c, y, n, k, r, w, g, \tau^C, \tau^K, \tau^N\}$. At any date, the two state variables are the current values of capital k and the

shock a . A solution to this system of 10 linear stochastic difference equation therefore has the form:

$$\begin{aligned} k' &= \eta k + H a \\ x &= A_x k + B_x a, \end{aligned}$$

where x_t is either y_t, n_t, c_t or w_t . The coefficients $\eta, H, A_x,$ and B_x are messy functions of all the parameters that are easily computed by Mathematica. Recalling that $a' = \rho a + \varepsilon$, the solution above implies that x follows the ARMA(2,1) processes in the text with $\mu_x = B_x$ and $\nu_x = A_x H - B_x \eta$.

This sequence of steps gives a linear map $\phi(\pi)$. This relation is one-to-one if there is an inverse map, which can be assessed by seeing whether the smallest eigenvalue in absolute value is above zero. I evaluate this for the following set of parameter values: $\beta = 0.99$, so that in the steady-state the annual real interest rate is approximately 4%, $\theta = 1$, so the utility function is logarithmic in consumption, $\psi = 4$, the high Frisch elasticity of labor supply that is commonly chosen in the literature on business cycles, $\alpha = 0.34$ so the capital share is 34% of private income, $\delta = 0.025$, so the annual depreciation rate is 10%, $\sigma G/Y = 0.12$ to match the U.S. annual average in 1929-2006 according to NIPA Table 1.3.5, $G/Y = 0.21$ to match the U.S. annual average in 1929-2006 according to NIPA Table 1.1.5, $\tau^K = 0.54$, the average tax rate on capital estimated by Poterba (1998) for the U.S. in the period 1959-1996, $\rho = 0.8$, so the shock to spending has a serial correlation of 0.8, and all the elements of π are zero. The smallest eigenvalue in absolute value is 0.002, so the map is one-to-one.

Calculations behind figures 2 and 3: The log-linearized model is the same as in the proof of proposition 2, but now $\tau_t^C = 0$, $\tau_t^N = \tau_t^K = \tau_t$ and $\tau_t = \gamma^T c_t + \lambda^T g_t$. Therefore, the same argument used in the proof of proposition 2 implies that τ and g follow ARMA(2,1) processes with MA coefficients $(\mu_\tau, \nu_\tau, \mu_g, \nu_g)$ that depend on the model's parameters. In particular, they depend on the four policy parameters $\pi = (\gamma^G, \gamma^T, \lambda^G, \lambda^T)$. Setting the structural parameters at the same values as in the proof of Proposition 2, then each choice of values for these 4 policy parameters implies values for the ARMA coefficients, which in turn imply values for the impulse response functions of taxes $\tau(t)$ and government spending $g(t)$. Using the estimates in the first column of figure 3 for $\hat{g}(t)$ and $\hat{\tau}(t)$, I pick π to numerically

minimize:

$$\sum_{t=1}^{16} \left[(g(t) - \hat{g}(t))^2 + (\tau(t) - \hat{\tau}(t))^2 \right].$$

This provides the estimated $\hat{\pi}$. Using these plus the structural parameters in the proof of proposition 2, I evaluate $(\mu_y, \nu_y, \mu_c, \nu_c)$ using the functions defined in proposition 2, and calculate the implied impulse responses. I scale the theoretical impulse responses so that their sum is the same as the one for the empirical impulse responses.

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