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by

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Signaling with Performance and the Effect of Competition*

Toru Suzuki†

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Abstract

Candidates compete to persuade a decision maker. The decision maker wishes to select a candidate who possesses a certain ability. Then, as a signaling, each candidate decides whether to perform a task whose performance statistically reflects the ability. However, since the cost of the performance is the same across all candidates, the performance is a poor signaling device. This paper analyzes a "signaling game with performance" in which the standard single crossing condition is violated. It is shown that more competition makes the equilibrium signaling more informative when the level of competition is moderate. Moreover, the equilibrium signaling can perfectly reveal the ability under a certain level of competition. On the other hand, too much competition always makes the equilibrium signaling less informative.

Keywords. Signaling, Competition

JEL Codes. D82, D83

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1 Introduction

Suppose a decision maker (DM) wishes to select a qualified candidate. When the ability is not observable, one natural way to persuade the DM is performing a costly task which reflects the candidate's ability. This kind of signaling can be observed in many situations. For instance, a worker takes an exam to show his ability. A political candidate discloses information to demonstrate his budgetary management ability. A firm announces a new project to signal his technological advance. However, in many cases, these performances depend not only on the ability but also on the luck. Moreover, the cost of the performance often does not depend on the ability, e.g., the exam fee, the cost of hiring auditors etc. Then, unqualified candidates can take advantage of the noisiness of the performance and perform the task pretending qualified candidates, that is, the performance is a poor signaling device. This paper analyzes a "signaling game with performance" in which the standard single crossing condition is violated.

Even though the performance is not an ideal signaling device, since the performance statistically reflects the ability, it is not cheap talk neither. Especially, when candidates compete to outperform each other, it can be too costly for unqualified candidates to perform the task pretending qualified candidates. Then, this paper answers the following question: Can more competition make the equilibrium signaling more informative? It is shown that the answer is not simple: more competition makes the equilibrium signaling more informative *as long as competition is moderate*.

The following sender-receiver game is introduced in Section 2. There are candidates and a decision maker (DM). Each candidate may or may not possess a certain ability which is private information. On the other hand, the DM wishes to select one candidate who possesses the ability. Each candidate tries to persuade the DM by performing a certain task. The cost of the performance is independent of the ability but the performance statistically reflects the ability. The game consists of two periods: In period 1, each candidate decides whether to perform a task. Then, in period 2, the DM selects one candidate or rejects all based on their performances.

The equilibrium of the game is analyzed in Section 3. The signaling game always has a

trivial equilibrium in which no type performs. Since the trivial equilibrium can be counterintuitive, the focus of the analysis is on nontrivial equilibria where some type performs the task with positive probability. It is shown that there exists a unique nontrivial equilibrium outcome given the number of candidates. Then, the equilibrium is characterized given the number of candidates, i.e., the level of competition. It is shown that more competition makes the equilibrium signaling more informative as long as competition is not too strong. Especially, the separating equilibrium can exist for some moderate levels of competition. On the other hand, too much competition always makes the equilibrium signaling less informative. An intuition of these results is the following. As the number of candidates gets larger, it is more difficult to outperform other candidates. On the other hand, since the performance of the qualified type tends to be higher than that of the unqualified type, the expected payoff of the qualified type is higher than that from the unqualified type given the strategy profile. Then, when the level of competition is moderate, only the qualified type finds the signaling profitable. On the other hand, when competition is too strong, even the qualified type finds that the chance of being selected is too low to play the separating strategy.

Some implications of the model are provided in Section 4. First, we show that when the number of candidates is sufficiently large, a policy maker can make the equilibrium signaling more informative by subsidizing the performance. Moreover, there exists the optimal subsidy level in which the equilibrium perfectly reveals the ability. On the other hand, when the number of candidates is small, the subsidy always makes the equilibrium signaling noisier. Second, it is shown that the DM is better off with larger candidate pool whenever the size of the pool is not too large. On the other hand, it is not obvious that the DM is better off with larger candidate pool when the size is very large.

Finally, Section 5 introduces a simple refinement concept. We show that, when we restrict off-the-equilibrium beliefs so that the belief reflects the performance of candidates, a counterintuitive trivial equilibrium can be eliminated.

Related literature. Our model is a sender-receiver game in which the receiver observes not only message but also a noisy signal of private information. First, unlike standard signaling game, e.g., Spence (1973), Riley (1979), the cost of messages does not depend on the type

and the single crossing condition is violated. Thus, our signaling game has no separating equilibrium when the number of candidates is small. Second, as cheap talk games, e.g., Crawford and Sobel (1985), the set of feasible messages is independent of types and the type does not affect the cost of messages in our model. However, since the performance statistically reflects private information, the message is not cheap talk in this paper. Even though the receiver observes a noisy signal, our model is not same as noisy signaling models, e.g., Matthews and Mirman (1983), Carlsson and Dasgupta (1997). In our model, the action of the sender can be observed by the receiver without any noise.

There are some papers which analyze the effect of competition in a sender-receiver game. In cheap talk game, the effect of multiple senders can be dramatic: when there are more than one sender, private information can be perfectly revealed in equilibrium, e.g., Battaglini (2002). In persuasion game, Milgrom and Roberts (1986) shows that, if the decision maker is not strategically sophisticated, competition can improve the decision maker's payoff.

Since candidates compete to get a single opportunity, this model has a similarity to contest models. In most of contest models, a candidate gets the opportunity if his effort level or performance is the highest. However, in my model, the signaling of the candidate has to be informative enough to be selected. That is, getting the highest performance among candidates is not sufficient condition to be selected.

2 The model

Basics. There are a decision maker (DM) and candidates. Each candidate may or may not possess a certain ability which is private information. The DM wishes to select one candidate who possesses the ability. On the other hand, each candidate wishes to be selected by the DM. Then, let $\mathcal{I} = \{1, 2, .., I\}$ be a set of candidates. There are two types of candidates: type α possesses the ability and type β does not. Then, let $\Theta = \{\alpha, \beta\}$ be the set of types. The probability that candidate i has the ability is $p \in (0, 1)$ which is identical across candidates. Each candidate can perform a task with cost $c \in (0, 1)$. The performance is noisy and cannot perfectly reflect the ability. Then, let $x \in X = [\underline{x}, \bar{x}]$ be performance which is correlated with type θ. Concretely, candidate i's performance x_i is drawn from probability distribution

$F(.|\theta_i)$ with the continuous density $f(.|\theta_i)$ such that $\text{supp}(f(.|\theta)) = X$ for any θ. I assume that the distribution function has the following property.

Assumption 1. $\frac{f(.|\alpha)}{f(.|\beta)}$ *is strictly increasing in* x.

That is, f satisfies the strict monotone likelihood ratio property.

We analyze the following sender-receiver game. First, each candidate i simultaneously chooses either $a_i = Y$ "performing the task" or $a_i = N$ "not performing the task." Then, after observing the performance of each candidates, the DM selects one candidate or rejects all. The payoff of candidate i is the following: Let $1_{i^*}(i)$ be the indicator function which assigns 1 if the DM selects i and $1_Y(a_i)$ be the indicator function which assigns 1 if $a_i = Y$. Then, the payoff of candidate i is $1_{i^*}(i) - 1_Y(a_i)c$.[1] On the other hand, when the DM selects candidate i, his payoff is $u(\theta_i)$. The DM wishes to select a qualified candidate but reject any unqualified candidates. Thus, we assume that $u(\beta) < 0 < u(\alpha)$. If the DM rejects all candidates, his payoff is 0.[2]

We focus on the situation where no candidate can be selected without performing the task. In other words, when the DM chooses one candidate randomly, the expected payoff of the DM is negative, that is,

Assumption 2. $p \leq \frac{-u(\beta)}{u(\alpha)-u(\beta)}$.

This assumption is for analytical simplicity.

Strategy and Equilibrium. Let $A_i = \{N, Y\}$ be candidate i's set of actions. Moreover, let $\Delta(A_i)$ be the set of all possible probability distributions over A_i. Then, candidate i's strategy is a mapping $s_i : \Theta \to \Delta(A_i)$. On the other hand, let z_i be the outcome of candidate i's action where $z_i = x_i$ if $a_i = Y$ and $z_i = \varnothing$ if $a_i = N$. Then, the DM's selection rule is a mapping $r : (X \cup \{\varnothing\})^I \to \mathcal{I} \cup \{R\}$ where R denotes "reject all." To analyze this game, we employ *Perfect Bayesian equilibrium* (PBE). Moreover, this paper focuses on symmetric strategy, i.e., $s_i(\theta_i) = s_j(\theta_j)$ whenever $\theta_i = \theta_j$. Finally, $q(\theta)$ denotes $s_i(\theta_i)(Y)$ and then $s_i(\theta_i)(N) = 1 - q(\theta)$.

[1] The benefit from being selected is normalized to 1.

[2] This can be interpreted as staying with a status quo.

3 The equilibrium

3.1 Trivial equilibrium

This signaling game has a trivial equilibrium which can be counterintuitive.

Observation 1 (Trivial equilibrium). *There always exists an equilibrium with* $q(\theta) = 0$ *for all* θ.

Notice that if both types do not perform the task, no information is revealed. That is, the DM selects one of candidates if prior probability p is such that $p \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}$. If $p < \frac{-u(\beta)}{u(\alpha)-u(\beta)}$, the DM rejects all candidates. The following off-the-equilibrium belief supports the pooling equilibrium: whenever a candidate performs the task, the DM believes that he is type β.

Note that this off-the-equilibrium belief can be counterintuitive. Our intuition suggests that whenever a candidate performs the task and the performance is high, the candidate should be selected with higher probability. Later, we introduce a refinement tool which reflects this intuition. Henceforth, this paper focuses on *nontrivial equilibria* in which $q(\theta) > 0$ for some θ.

3.2 Nontrivial equilibrium

After observing the performances z, the DM chooses the optimal action. Let $\psi(z_i|s)$ be the DM's posterior belief about $\theta_i = \alpha$ after observing z_i, which is consistent with strategy profile s. Then, when the DM chooses candidate i, the expected payoff of the DM given z_i is

$$\psi(z_i|s)u(\alpha) + (1 - \psi(z_i|s))u(\beta).$$

Then, if candidate i is selected, the following two conditions have to be satisfied. First, the expected payoff of the DM from selecting i has to be positive. That is,

$$\psi(z_i|s) \geq \frac{-u(\beta)}{u(\alpha) - u(\beta)} \ .$$

The second condition is that the expected payoff of the DM from selecting candidate i is higher than that from selecting other candidates. That is,

$$\psi(z_i|s) \geq \max_{j \neq i} \psi(z_j|s).$$

Note that the optimal reaction of the DM is not always unique. Concretely, given separating strategy $q(\alpha) = 1 - q(\beta) = 1$, the posterior belief is $\psi(z_i|s) = 1$ for all i who performs. Then, any choice from such set of candidates is the optimal reaction. However, the optimal reaction of the DM is unique when candidates' strategy profile is not separating. Moreover, when a candidate can make a mistake in his action, the DM's optimal reaction is always unique.

The next lemma provides some properties of nontrivial equilibria.

Lemma 1. *In any equilibrium,*
(i) *if* $q(\alpha) \in (0,1)$, *then* $q(\beta) = 0$.
(ii) *if* $q(\beta) \in (0,1)$, *then* $q(\alpha) = 1$.
(iii) $q(\beta) \leq q(\alpha)$.

Proof. To prove (i), note that the expected payoff from N is 0 for all types. On the other hand, since Assumption 1 implies that $F(.|\alpha)$ first-order-stochastically dominates $F(.|\beta)$, the expected payoff of type α from Y is always higher than that of type β. If $q(\alpha) \in (0,1)$ in equilibrium, then Y and N are indifferent for type α. However, it implies that the expected payoff of type β from Y is strictly negative.

To prove (ii), note that if $q(\beta) \in (0,1)$, Y and N are indifferent for type β. Then, by Assumption 1, the expected payoff from Y is always strictly larger than 0 for type α.

Finally, since we already proved (i) and (ii), we can prove (iii) by showing that if $q(\beta) = 1$, then $q(\alpha) = 1$. First, suppose $q(\beta) = 1$ and $q(\alpha) \in (0,1)$. Then, it contradicts (i). Second suppose $q(\beta) = 1$ and $q(\alpha) = 0$. Then type β strictly prefers to play N. Q.E.D.

To see an intuition, observe that the strict MLR property implies that the payoff of type α from Y is strictly higher than that of type β. On the other hand, since the expected payoff from N is 0 for all types, any equilibrium strategy is monotonic in type. Moreover, there is no equilibrium in which both types play totally mixed strategies. This is because when one of types finds Y and N indifferent, the other type never finds the two actions indifferent.

The next proposition characterizes the nontrivial equilibrium given I.

Proposition 1. *There exists* $I_1(c) \geq 0$, and $I_2(c) > I_1(c)$ such that

(i) *if* $I \leq I_1(c)$, *there exists a unique nontrivial equilibrium with* $q(\alpha) = q(\beta) = 1$

(ii) *if* $I_1(c) < I \leq I_2(c)$, *there exists a unique nontrivial equilibrium with* $q(\alpha) = 1$ *and* $q(\beta) \in (0, 1)$

(iii) *if* $I > I_2(c)$, *there exists a unique nontrivial equilibrium outcome with* $q(\alpha) \in (0, 1]$ *and* $q(\beta) = 0$.

Proof. See appendix.

(i) says that there exists a pooling equilibrium in which all types perform if the number of candidates is sufficiently small. Since this is a pooling equilibrium, the DM can learn their type only from the observed performance. The intuition of this result is the following. When the number of candidates is small, the chance to outperform all other candidates can be reasonably high for any candidate. Thus, when the DM believes that candidates who do not perform are type β, no candidate has incentive to deviate from this pooling strategy. On the other hand, as the number of candidates becomes larger, the probability to outperform other candidates becomes too low to justify the cost of the performance.

(ii) says that if the equilibrium cannot be pooling but the number of candidates is not too large, the nontrivial equilibrium is a semi-pooling equilibrium in which type α performs with probability one and type β performs with some probability. The idea of the proof is the following. First, given the separating strategy profile in which only type α performs, it is profitable for type β to imitate type α's action as long as I is not too large. On the other hand, if type β imitates type α and performs with probability one, then, the expected payoff is negative since $I > I_1(c)$. Observe that when type β imitates type α's action with higher probability, it makes (i) the signaling noisier and (ii) the performance more competitive. Hence, the expected payoff of type β from the performance is decreasing in the probability of type β's imitation. Then, by continuity, we can always find $q(\beta) < 1$ which makes Y and N indifferent for type β.

(iii) says that the number of candidates is sufficiently large, type β never performs the task in the nontrivial equilibrium. To obtain an intuition of (iii), observe that when I is large, the probability that type β outperforms type α is too low to justify the cost. Thus,

type β has no incentive to imitate type α. For larger I, even type α finds that the probability of being selected is too low to justify its cost and mixes his action between Y and N.

Note that the equilibrium outcome is unique but there can be multiple equilibria. This is because the optimal reaction of the DM is not unique when candidates play the separating strategy with $q(\alpha) = 1 - q(\beta) = 1$. Thus, when we can construct the separating equilibrium with more than one optimal reaction of the DM, we have outcome equivalent equilibria. However, when the strategy profile is non-separating, the nontrivial equilibrium is always unique. The uniqueness of the outcome relies on Assumption 2. For $p > \frac{-u(\beta)}{u(\alpha)-u(\beta)}$, there can be multiple nontrivial equilibria. This is because the candidate can be selected without performing the task if p is high.

The next proposition provides a condition in which a separating equilibrium exists. It claims that when the noisiness and the cost of the performance are sufficiently low, the separating equilibrium with $q(\alpha) = 1 - q(\beta) = 1$ exists. As a measure of the noisiness of the performance, let $\rho = \int F(x|\alpha)dF(x|\beta)$. Intuitively, when ρ is low, the probability that type β outperforms type α becomes lower. Let $\varphi(k|I) = \binom{I-1}{k} p^k (1-p)^{I-1-k}$ be the probability that the number of type α is k. Then, let $\hat{I}$ be

$$\max_I \left\{ I : \sum_k \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\alpha) > c \right\}.$$

This is the largest number of candidates in which the expected payoff of type α from Y is strictly positive given the separating strategy profile.

Proposition 2. *If ρ and c are sufficiently small, $\hat{I} > I_2(c)$. Then, the nontrivial equilibrium is such that*

(i) $q(\alpha) = 1 - q(\beta) = 1$ *if* $I_2(c) + 1 \leq I \leq \hat{I}$,

(ii) $q(\alpha) \in (0, 1)$ *and* $q(\beta) = 0$ *if* $I > \hat{I}$.

Proof. See appendix.

To provide an intuition of the result, note that if the noisiness of the performance is low, type α's probability of being selected becomes much higher than that of type β. Then, under a moderate competitive pressure, only type β finds that the chance of being selected is too low to justify the cost. Note that if the cost is too high, both types find the performance

too expensive under any level of competition. On the other hand, if the performance is very noisy, the chance of being selected is similar across types. Then, when the payoff of type α from the performance is positive, the payoff of type β is also positive. Thus, in order to have the separating equilibrium, the noisiness and the cost of the performance need to be sufficiently low.

4 Comparative statics

This section provides some comparative statics of the nontrivial equilibrium. Let $q(\theta|I,c)$ be the nontrivial equilibrium strategy of type θ given I and c.

Proposition 3.

1. $q(\theta|I,c)$ *is weakly decreasing in* I *for all* θ. *Especially,* (i) $q(\beta|I+1,c) < q(\beta|I,c)$ *whenever* $I \in \{I_1(c), I_2(c)-1\}$. (ii) $q(\alpha|I+1,c) < q(\alpha|I,c)$ *for* $I \geq I_2(c)$.

2. $q(\theta|I,c)$ *is weakly decreasing in* c. *Especially, if* $c' > c$ *and* $q(\theta|I,c) \in (0,1)$, *then* $q(\theta|I,c') < q(\theta|I,c)$.

.

Proof. See appendix.

An intuition of 1-(i) is the following. Given $q(\beta)$, when there are more candidates, the expected number of performers becomes larger and the expected payoff from the performance becomes lower. Then, since lower $q(\beta)$ makes the DM's posterior belief about $\theta_i = \alpha$ higher and improves the expected payoff from the performance, $q(\beta)$ which makes Y and N indifferent for type β is lower for larger number of candidates. To obtain the idea of 1-(ii), suppose the number of candidates is so large that type β cannot get positive payoff from the performance. Then, for larger I, the probability of being selected for type α also becomes too low to justify its cost. Since the probability of being selected is decreasing in $q(\alpha)$, $q(\alpha)$ which makes N and Y indifferent for type α is lower for larger number of candidates. Finally, to see why $q(\theta|I,c)$ is decreasing in c, note the probability that the DM selects type θ is decreasing in $q(\theta)$. Then, $q(\theta)$ which makes the probability equal to c is lower for larger c.

5 Implications

The model has two normative implications. One is about a subsidy policy and the other is about the size of the candidate pool.

5.1 Subsidy

Suppose there is a policy maker who can subsidize the performance to reduce the cost. Concretely, the policy maker can choose $\delta \in [0, \infty)$ so that each candidate pays $c - \delta$ to perform the task. Then, a subsidy $\delta \in [0, \infty)$ is *optimal* if the DM's expected payoff is maximized under δ. Let δ^* be the optimal subsidy.

Proposition 4. *If $I > \hat{I}$, then $\delta^* \in (0, c)$. If $I \leq I_2(c)$, then $\delta^* = 0$.*

To prove Proposition 4, we need to establish the next lemma first.

Lemma 2. *The DM's expected payoff from the optimal reaction is decreasing in $q(\beta)$ given $q(\alpha) = 1$ and I.*

Proof. See Appendix.

The intuition of this lemma is the following. Suppose the probability of the imitation by type β is decreasing from q' to q''. Then, the proportion of type α among performers becomes higher. Thus, when the DM reacts optimally under q', the probability of selecting type α becomes higher when type β imitates with probability q''. Then, the probability that the DM selects type α becomes even higher when he reacts optimally under q''.

Proof of Proposition 4. Suppose $I > \hat{I}$. Then, by Proposition 2,

$$\sum_k \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\alpha) < c.$$

On the other hand, by Assumption 1, there always exists $c' < c$ such that

$$\sum_k \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\alpha) > c' > \sum_k \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\beta).$$

Note that this is the condition guarantees the existence of the separating equilibrium. Thus, the DM's payoff is maximized when $\delta = c - c'$.

For the second part, note that, by Proposition 3, $q(\beta|I,c) < q(\beta|I,c-\delta)$ for any $\delta > 0$ if $I \leq I_2(c)$. Then, by Lemma 2, any $\delta > 0$ decreases the DM's expected payoff. Q.E.D.

5.2 The candidate pool size

The next implication is about the size of the candidate pool.

Proposition 5. *If $I < I_2(c)$, the DM is always better off with larger candidate pool $I' > I$ as long as $I' \leq I_2(c)$.*

The following lemma helps to prove Proposition 5.

Lemma 3. *Given $q(\alpha) = 1$ and $q(\beta) \in (0,1)$, the DM's expected payoff from the optimal reaction is increasing in I.*

Proof. See appendix.

To obtain an intuition of Lemma 3, suppose the DM selects candidate i. First, the probability that candidate i is being selected is decreasing in the number of candidates. On the other hand, since type α's performance is statistically higher than type β's, it becomes more difficult for type β to outperform all candidates when the number of candidates is increased. As a result, the DM has higher chance to select type α for larger number of candidates.

Proof of Proposition 5. Suppose $I \leq I_2(c)$. Let $U_0(I,q)$ be the DM's expected payoff given I and q. By Lemma 3, $U_0(I, q(\beta|I)) < U_0(I+1, q(\beta|I))$. Then, by Proposition 3 and Lemma 2, $U_0(I+1, q(\beta|I)) < U_0(I+1, q(\beta|I+1))$. Hence, $U_0(I, q(\beta|I)) < U_0(I+1, q(\beta|I+1))$. That is, the DM's equilibrium payoff is increasing in I as long as $I \leq I_2(c)$. Q.E.D.

Remark 1. It is not obvious that whether larger candidate pool increases the DM's payoff when $I > I_2$. To see the reason, note that when the number of candidates is too large, even qualified candidates do not perform the task with some probability and, thus, the probability that no one performs the task is always positive. If this probability is decreasing in I, the DM is better off with larger I. However, the effect of larger I on this probability is not obvious. Larger number of candidates increases the probability of having at least one

type α. On the other hand, by Proposition 3, the probability that type α performs the task is decreasing in I for $I > I_2(c)$. Then, the net effect depends on the parameters of the model.

If the DM knows that there is at least one type α in the candidate pool given $I > I_2$, the DM never prefers larger candidate pool. This is because the probability that no one performs the task is increasing in I in this case.

6 Discussion

6.1 Refinement

As we mentioned before, the trivial equilibrium can be counterintuitive. Our intuition suggests that when one candidate performs unexpectedly and the quality of the performance is high, the DM tends to believe that the candidate is type α. Unfortunately, most of well known refinement concepts for perfect Bayesian equilibrium such as "equilibrium dominance" based refinements[3], Perfect sequential equilibrium, proposed by Grossman and Perry (1986)[4] cannot eliminate the trivial equilibrium.

However, the trivial equilibrium can be eliminated when we restrict off-the-equilibrium belief based on our intuition. Suppose, for off-the-equilibrium paths, the DM evaluates the performance x_i as if he is a Bayesian statistician who ignores the signaling element. That is, suppose DM's off-the-equilibrium belief given the performance x_i is $\bar{\psi}(x_i) = \frac{f(x_i|\alpha)p}{f(x_i|\alpha)p+f(x_i|\beta)(1-p)}$. Then, the trivial equilibrium exists if and only if

$$\int_{\left\{x|\bar{\psi}(x_i)\geq \max\left\{p, \frac{-u(\beta)}{u(\alpha)-u(\beta)}\right\}\right\}} f(x_i|\alpha)dx_i - c \leq 0.$$

Thus, whenever the performance reflects the ability sufficiently well, the trivial equilibrium cannot exist with off-the-equilibrium belief $\bar{\psi}(x_i)$.

[3] To see the claim, consider the set of the DM's actions in which the expected payoff of a candidate from a deviation is as good as the equilibrium payoff. Obviously, for any candidates, such set is singleton, i.e., "being selected." Since the set is singleton for all candidates, the equilibrium dominance based refinement does not work.

[4] We can show that there is no set of types with which the "credible" belief updating is possible when a candidate deviates from an equilibrium.

6.2 More general setting

6.2.1 Set of actions

The set of feasible tasks to demonstrate the ability can be more than one. It is not difficult to extend the result for larger set of feasible tasks. Then, it can be shown that, for each task, we can construct a nontrivial equilibrium which is analogous to that in the basic setting. Thus, the number of nontrivial equilibria is at least as large as the number of feasible tasks. On the other hand, non of these nontrivial equilibria can be refined by standard refinement concepts, e.g., equilibrium dominance based refinements, perfect sequential equilibrium. Thus, in order to analyze the effect of competition, we need to specify one equilibrium for each I. When we choose the equilibrium in which the same task is performed for every I, the effect of competition is the same as that in the basic setting.

6.2.2 Set of types

For more general type space, the effect of competition on the equilibrium signaling is not obvious. To see the point, suppose $\Theta = \{\alpha, \beta, \gamma\}$ and $A = \{1, 2, N\}$ where 1(2) denotes "performing task 1(2)." Moreover, we assume that the DM's payoff from selecting each type is such that $u(\alpha) > u(\beta) > 0 > u(\gamma)$. Moreover, suppose the cost of the performance is the same for any task.

Suppose the following strategy constitutes an equilibrium given I : $s(\alpha)(1) = 1$, $s(\beta)(1) = q$, $s(\beta)(2) = 1 - q$ and $s(\gamma)(N) = 1$. Then, the question is whether larger I decreases q in equilibrium. To analyze the problem, let $U_a(q|\beta, I)$ be the expected payoff of type β from action a given q and I. Observe that both $U_1(q|\beta, I)$ and $U_2(q|\beta, I)$ are decreasing in q. Moreover, $U_a(q|\beta, I)$ is decreasing in I. Then, since the equilibrium q is chosen so that $U_1(q|\beta, I) = U_2(q|\beta, I)$, the effect of competition on equilibrium q is not obvious. Moreover, since both $U_1(q|\beta, I)$ and $U_2(q|\beta, I)$ are decreasing in q, the equilibrium q can be multiple given I. In this case, it is not obvious how to analyze the effect of competition.

Turning to the existence of a separating equilibrium, let $p(\alpha)$ be the prior probability of $\theta_i = \alpha$. Then, consider the following separating strategy: $s(\alpha)(1) = 1$, $s(\beta)(2) = 1$, and $s(\gamma)(N) = 1$. First, if $p(\alpha)$ is large, there is no separating equilibrium. This is because the expected payoff of type β from the separating strategy is negative in this case. Second, if

$p(\alpha)$ is low, the separating equilibrium cannot exist. To see the reason, observe that if $p(\alpha)$ is low, type β rarely faces type α in competition. Then, it is profitable for type β to perform task 1 pretending type α. Thus, the existence of the separating equilibrium is very sensitive to parameters of the model, e.g., the cost of each task, the distribution of the type.

7 Appendix: Proofs

7.1 Proof of Proposition 1-(i)

Suppose strategy profile s is such that $q(\theta) = 1$ for all θ. Then, given x_i, the posterior belief of the DM is

$$\psi(x_i|s) = \frac{f(x_i|\alpha)p}{f(x_i|\alpha)p + f(x_i|\beta)(1-p)}.$$

Note the DM selects candidate i only if $\psi(x_i|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}$. Moreover, by Assumption 1, $\psi(x_i|s)$ is strictly increasing in x_i. Hence, we can find $\hat{x}(s) \in X$ such that $\{x_i|x_i \geq \hat{x}(s)\} = \{x_i|\psi(x_i|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}\}$. Now, let $k = \#\{i' \neq i|\theta_{i'} = \alpha\}$. Then, the probability that performance x_i is the highest among all candidates given I and k is $w(k, I, x_i) = F(x_i|\alpha)^k F(x_i|\beta)^{I-1-k}$. Note that the probability of having k given $I-1$ candidates is

$$\varphi(k|I) = \binom{I-1}{k} p^k (1-p)^{I-1-k}.$$

Then, the expected payoff of candidate i from Y given θ is

$$\int_{x_i \geq \hat{x}(s)} \sum_{k=0}^{I-1} \varphi(k|I) w(k, I, x_i) dF(x_i|\theta) - c.$$

Given p, let $I_1(c)$ be the largest I such that

$$\int_{x_i \geq \hat{x}(s)} \sum_{k=0}^{I-1} \varphi(k|I) w(k, I, x_i) dF(x_i|\beta) - c \geq 0.$$

and if such I does not exist, $I_1(c) = 0$.

Note, it is easy to see that, given θ,

$$\int_{x_i \geq \hat{x}(s)} \sum_{k=0}^{I-1} \varphi(k|I) w(k, I, x_i) dF(x_i|\theta) > \int_{x_i \geq \hat{x}(s)} \sum_{k=0}^{I} \varphi(k|I+1) w(k, I+1, x_i) dF(x_i|\theta)$$

Thus, if $I_1(c) > 1$, the expected payoff of type β from Y is positive for any $I \leq I_1(c)$. Then, since Assumption 1 implies that the expected payoff of type α from Y is higher than that of type β, the expected payoff from Y is positive for all types if $I \leq I_1(c)$. For off-the-equilibrium belief, suppose that if candidate i chooses N, then $\psi(z_i|s)$ is such that $\psi(z_i|s) < \frac{-u(\beta)}{u(\alpha)-u(\beta)}$ for any x_i. Then, there is no incentive to choose N for all types.

To prove "only if" part, observe that, if $I > I_1(c)$, then by construction of $I_1(c)$, the expected payoff from the pooling strategy is strictly negative for type β. Hence, type β prefers N to Y.

Turning to the uniqueness of the nontrivial equilibrium, first, there is no equilibrium with $q(\alpha) = 1$ and $q(\beta) \in (0, 1)$ because the expected payoff of type β is strictly positive for $I \leq I_1(c)$ given the strategy profile. Second, there is no equilibrium with $q(\alpha) \in (0, 1]$ and $q(\beta) = 0$ because, again, the expected payoff from type β is strictly positive for $I \leq I_1(c)$ given the strategy profile. Then, by Lemma 1, the nontrivial equilibrium is unique. Q.E.D.

7.2 Proof of Proposition 1-(ii)

Consider a strategy profile such that $q(\alpha) = 1$ and $q(\beta) \in (0, 1)$. Then, when the DM observes z_i, the DM's posterior belief about $\theta_i = \alpha$ which is consistent with the strategy profile is

$$\psi(z_i|s) = \begin{cases} \frac{q(\alpha)F(x_i|\alpha)p}{q(\alpha)F(x_i|\alpha)p+q(\beta)F(x_i|\beta)(1-p)} \text{ if } z_i = x_i \\ 0 \text{ if } z_i = \varnothing \end{cases}.$$

Let $I_2(c)$ be

$$\max\left\{ I : \sum_k \int_{x_i} \varphi(k|I)F(x_i|\alpha)^k dF(x_i|\beta) > c \right\}.$$

Note that, if $I = 1$, the candidate is selected for sure given the separating strategy and the consistent belief. Thus, $I_2(c)$ always exits. Obviously, $I_2(c) \geq I_1(c)$.

First, I show that, for any I such that $I_1(c) < I \leq I_2(c)$, there exists a unique nontrivial equilibrium with $q(\alpha) = 1$ and $q(\beta) \in (0, 1)$. Suppose type β performs with probability q. Let $m = \#\{i' \neq i | \theta_{i'} = \beta\}$. Then, the probability that candidate i faces m given I, q and k is

$$\phi(m|k, I, q) = \binom{I-k-1}{m} q^m (1-q)^{I-k-1-m}.$$

Note that the probability that performance x_i is the highest among all candidates given m and k is $w(k,m,x_i) = F(x_i|\alpha)^k F(x_i|\beta)^m$. Then, the expected probability that the DM selects candidate i is

$$B(q|I,\theta) = \int_{x_i \geq \hat{x}(s)} \sum_{k=0}^{I-1} \sum_{m=0}^{I-1-k} \phi(m|k,I,q)\varphi(k|I)w(k,m,x_i)dF(x_i|\theta_i).$$

where $\hat{x}(s)$ is such that $\{x_i|x_i \geq \hat{x}(s)\} = \{x_i|\psi(x_i|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}\}$.

I claim that $B(q|I,\theta)$ is decreasing in q. To see the claim, notice that $w(k,m,x_i)$ is decreasing in k and m but increasing x_i. Since m follows a binomial distribution, for any $q < q'$, $\phi(m|k,I,q')$ first order stochastically dominates $\phi(m|k,I,q)$. Moreover, since $\psi(x_i|s)$ is strictly decreasing in q, $\hat{x}(s)$ is increasing in q. Thus, $B(q|I,\theta)$ is strictly decreasing in q.

Now, observe that

$$\lim_{q\to 0} B(q|I,\theta_i) = \sum_k \int_{x_i} \varphi(k|I)F(x_i|\alpha)^k dF(x_i|\theta_i).$$

Thus, as long as $I \leq I_2(c)$, $\lim_{q\to 0} B(q|I,\beta) > c$. On the other hand, since $I > I_1(c)$, $\lim_{q\to 1} B(q|I,\beta) < c$. Then, since $B(q|I,\theta)$ is decreasing in q, by continuity, there exists a unique q^* such that $B(q^*|I,\beta) = c$. On the other hand, by Assumption 1, $B(q^*|I,\alpha) > c$ whenever $B(q^*|I,\beta) = c$.

Turning to the uniqueness of the nontrivial equilibrium, there is no equilibrium with $q(\alpha) = q(\beta) = 1$ because the expected payoff of type β is strictly negative for $I > I_1(c)$ given the strategy profile. There is no equilibrium with $q(\alpha) \in (0,1]$ and $q(\beta) = 0$ because the expected payoff of type β from Y is positive for $I \leq I_2(c)$ given the strategy profile. Then, by Lemma 1, the nontrivial equilibrium is unique. Q.E.D.

7.3 Proof of Proposition 1-(iii)

Suppose $I > I_2(c)$. Consider a separating strategy $1-q(\beta) = q(\alpha) = 1$. Note that $\psi(x_i|s) = 1$ for all i who performs, the optimal reaction of the DM is not unique. Then, suppose the DM uses the following optimal reaction

$$r^*(z) = \begin{cases} i \text{ if } x_i > x_j \text{ for all } j \neq i \text{ and } \psi(x_i|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)} \\ R \text{ otherwise} \end{cases}$$

Then, suppose the following condition holds.

$$\sum_{k}^{I-1} \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\alpha) - c \geq 0.$$

Then, this is a separating equilibrium since the expected payoff of type β from the performance is negative by construction of $I_2(c)$.

Second, consider the case that the above condition cannot be satisfied. Then, let $q_\alpha = q(\alpha)$ and $\hat{k}$ be the number of type α who performs. Then, the probability of having $\hat{k}$ given I, q_α is

$$\widetilde{\varphi}(\hat{k}|I, q_\alpha) = \binom{I-1}{\hat{k}} (pq_\alpha)^{\hat{k}} (1 - pq_\alpha)^{I-1-\hat{k}}.$$

. When $q(\beta) = 0$, the expected payoff of type α given q_α is

$$\sum_{k}^{I-1} \int_{x_i} \widetilde{\varphi}(\hat{k}|I, q_\alpha) F(x_i|\alpha)^{\hat{k}} dF(x_i|\alpha) - c.$$

It is easy to see that the above value is decreasing in q_α. Moreover, note that, since there is no separating equilibrium,

$$\lim_{q_\alpha \to 1} \sum_{k}^{I-1} \int_{x_i} \widetilde{\varphi}(\hat{k}|I, q_\alpha) F(x_i|\alpha)^{\hat{k}} dF(x_i|\alpha) - c < 0.$$

Moreover, as q_α goes to 0, the probability that candidate i can be selected becomes close to 1. Hence,

$$\lim_{q_\alpha \to 0} \sum_{k}^{I-1} \int_{x_i} \widetilde{\varphi}(\hat{k}|I, q_\alpha) F(x_i|\alpha)^{\hat{k}} dF(x_i|\alpha) - c > 0.$$

Then, by continuity, there exists a unique q'_α such that

$$\sum_{k}^{I-1} \int_{x_i} \widetilde{\varphi}(\hat{k}|I, q'_\alpha) F(x_i|\alpha)^{\hat{k}} dF(x_i|\alpha) - c = 0.$$

On the other hand, by Assumption 1, the expected payoff of type β from Y is strictly lower than that of type α. Hence, type β has no incentive to deviate.

Turning to the uniqueness of the nontrivial equilibrium, there is no equilibrium with $q(\alpha) = q(\beta) = 1$ because the expected payoff of type β is negative for $I > I_1(c)$ given the strategy profile. Next, there is no equilibrium with $q(\alpha) = 1$ and $q(\beta) \in (0, 1)$ since the expected payoff of type β is negative for $I > I_2(c)$ given the strategy profile. Then, by Lemma 1, the nontrivial equilibrium is unique. Q.E.D.

7.4 Proof of Proposition 2

Note that $\hat{I}$ is decreasing in c. Thus, by choosing small c, we can make $\hat{I}$ arbitrarily large. Then, if c and ρ are sufficiently small,

$$\sum_{k}^{I-1} \int_{x_i} \varphi(k|\hat{I}) F(x_i|\alpha)^k dF(x_i|\beta) < c.$$

Recall that $I_2(c) = \max\left\{I : \sum_k \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\beta) > c\right\}$. Thus, by Assumption 1, $I_2(c) \leq \hat{I}$. Then, whenever $I_2(c) + 1 \leq I \leq \hat{I}$,

$$\begin{aligned} \sum_{k} \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\alpha) &> c, \\ \sum_{k} \int_{x_i} \varphi(k|I) F(x_i|\alpha)^k dF(x_i|\beta) &< c. \end{aligned}$$

Note that, under the above condition, type α has no incentive to play N and type β has no incentive to imitate type α.

When $I > \hat{I}$, the expected payoff from the separating strategy is negative for type α. Then, by Proposition 1-(iii), the nontrivial equilibrium consists of $q(\alpha) \in (0,1)$ and $q(\beta) = 0$.Q.E.D.

7.5 Proof of Proposition 3

We use $B(q|I,\beta)$, $\phi(m|k,I,q)$, $\varphi(k|I)$, and $\widetilde{\varphi}(\hat{k}|I,q_\alpha)$ which are defined in the proof of Proposition 1.

For the first part of 1, from Proposition 1, $q(\beta|I) = 1$ when $I \leq I_1(c)$. Moreover, when $I > I_2(c)$ from Proposition 3, $q(\beta|I) = 0$. Thus, consider the case where $I_1(c) \leq I < I_2(c)$. Then, the expected payoff of type β from Y given I is $B(q|I,\beta) - c = 0$. Observe that $\phi(m|k,I+1,q)$ first order stochastically dominates $\phi(m|k,I,q)$ and $\varphi(k|I+1)$ first order stochastically dominates $\varphi(k|I)$. Then, since $w(k,m,x_i)$ is decreasing in k and m, we have, $B(q|I,\beta) > B(q|I+1,\beta)$ for any $q \in (0,1)$. Since $B(q|I+1,\beta)$ is strictly decreasing in q, q which solves $B(q|I,\beta) = c$ is strictly higher than q which solves $B(q|I+1,\beta) = c$.

For the second part of 1, first, $q(\alpha|I) = 1$ as long as

$$\sum_{k}^{I-1} \int_{x_i} \varphi(k|I) F(x_i|\alpha)^{\hat{k}} dF(x_i|\alpha) - c \geq 0.$$

If I is too large to satisfy the above inequality, $q(\alpha|I)$ is chosen so that

$$\sum_{k}^{I-1}\int_{x_i}\widetilde{\varphi}(\hat{k}|I,q(\alpha|I))F(x_i|\alpha)^{\hat{k}}dF(x_i|\alpha)=c.$$

Now, for any q_α, it is easy to see that

$$\sum_{k}^{I-1}\int_{x_i}\widetilde{\varphi}(\hat{k}|I,q_\alpha)F(x_i|\alpha)^{\hat{k}}dF(x_i|\alpha)>\sum_{k}^{I}\int_{x_i}\widetilde{\varphi}(\hat{k}|I+1,q_\alpha)F(x_i|\alpha)^{\hat{k}}dF(x_i|\alpha).$$

Then, since $\sum_k^{I-1}\int_{x_i}\widetilde{\varphi}(\hat{k}|I+1,q_\alpha)F(x_i|\alpha)^{\hat{k}}dF(x_i|\alpha)$ is decreasing in q_α, we must have $q(\alpha|I)>q(\alpha|I+1)$.

For Proposition 3-2, first, suppose, $q(\beta)$ is such that $B(q(\beta)|I,\beta)-c=0$ given $q(\alpha)=1$. Then, since $B(q(\beta)|I,\beta)$ is decreasing in $q(\beta)$, $q(\beta)$ which makes $B(q(\beta)|I,\beta)=c$ is lower for higher c. Second, given $q(\beta)=0$, if $q(\alpha)$ is such that

$$\sum_{k}^{I-1}\int_{x_i}\widetilde{\varphi}(\hat{k}|I,q(\alpha))F(x_i|\alpha)^{\hat{k}}dF(x_i|\alpha)=c.$$

Then, since the right hand side of the above equation is decreasing in $q(\alpha)$, $q(\alpha)$ which satisfies the above condition is smaller for larger c. Q.E.D.

7.6 Proof of Lemma 2

Let s' denote a strategy profile such that $q(\alpha)=1$ and $q(\beta)=q'<1$. Then, suppose the DM follows the following decision rule: he chooses candidate i only if (i) $x_i\geq x_j$ for all j such that $a_j=Y$ and (ii) $x_i\geq\hat{x}'$ where $\hat{x}'=\sup\left\{x\in X:\psi(x|s')<\frac{-u(\beta)}{u(\alpha)-u(\beta)}\right\}$. Note that this is the optimal reaction for the DM given DM's strategy s'. Then, let m be the number of type β who performs, i.e., $\#\{i|a_i=Y,\,\theta_i=\beta\}$. Obviously, given the number of type α candidates, k, the payoff of the DM from the decision rule is decreasing in m. Let $\phi(m|k,I,q')$ be the probability of having m given (k,I,q'). Then, let s'' be the strategy profile such that $q(\alpha)=1$ and $q(\beta)=q''<q'$. Then, since $\phi(m|k,I,q')$ first order stochastically dominates $\phi(m|k,I,q'')$, the expected payoff of the DM from the decision rule with $\hat{x}'$ given s' is lower than that given s''. Thus, if the DM chooses the optimal reaction given s'', that is, $\hat{x}''=\sup\left\{x\in X:\psi(x|s'')<\frac{-u(\beta)}{u(\alpha)-u(\beta)}\right\}$, then, by definition, the DM's payoff becomes even higher than that from the decision rule with $\hat{x}'$. Q.E.D.

7.7 Proof of Lemma 3

Suppose type β performs with probability q. Then, given the performances of other candidates z_{-i}, let $\widetilde{x}(z_{-i}|q,I)$ be the cutoff performance level where the DM selects i if candidate i's performance is $x_i > \widetilde{x}(z_{-i}|q,I)$. Then, the probability that the DM selects candidate i whose type is θ_i given z_{-i}, q, and $\mathcal{I} = \{1,2,..,I\}$ is

$$\begin{aligned}\Pr(i,\alpha|z_{-i},q,\mathcal{I}) &= \int_{x_i>\widetilde{x}(z_{-i}|q,I)} dF(x_i|\alpha)p(\alpha),\\ \Pr(i,\beta|z_{-i},q,\mathcal{I}) &= \int_{x_i>\widetilde{x}(z_{-i}|q,I)} dF(x_i|\beta)p(\beta)q.\end{aligned}$$

Claim 1. The probability that $\theta_i = \alpha$ conditional on $r(z) = i$ is increasing in the size of the candidate pool I.

Suppose we add a new candidate i' to $\mathcal{I}$ who plays the same strategy as other candidates. First, consider the case where the performance of candidate i' is $x_{i'} > \widetilde{x}(z_{-i}|q,I)$. Then, $\Pr(i,\theta_i|x_{i'},z_{-i},q,\mathcal{I}\cup\{i'\}) = \Pr(x_i > x_{i'}|\theta_i)p(\theta_i)$. On the other hand, when the performance of candidate i' is $x_{i'} \leq \widetilde{x}(z_{-i}|q,I)$, $\Pr(i,\theta_i|x_{i'},z_{-i},q,\mathcal{I}\cup\{i'\}) = \Pr(i,\theta_i|z_{-i},q,\mathcal{I})$.

Then,

$$\begin{aligned}\Pr(i,\theta_i|z_{-i},q,\mathcal{I}\cup\{i'\}) &= \sum_{\theta_{i'}}\int_{x_{i'}>\widetilde{x}(z_{-i}|q,I)}\int_{x_i>x_{i'}} dF(x_i|\theta_i)p(\theta_i)q(\theta_i)dF(x_{i'}|\theta_{i'})p(\theta_{i'})\\ &\quad+\sum_{\theta_{i'}}\int_{x_{i'}\leq\widetilde{x}(z_{-i}|q,I)} dF(x_{i'}|\theta_{i'})p(\theta_{i'})\int_{x_i>\widetilde{x}(z_{-i}|q,I)} dF(x_i|\theta_i)p(\theta_i)\\ &< \Pr(i,\theta_i|z_{-i},q,\mathcal{I})\end{aligned}$$

By Assumption 1, $F(.|\alpha)$ first order stochastically dominates $F(.|\beta)$. Hence,

$$\frac{\int_{x_i>\widetilde{x}(z_{-i}q,I)} dF(x_i|\alpha)p(\alpha)}{\int_{x_i>\widetilde{x}(z_{-i}|q,I)} dF(x_i|\beta)p(\beta)q} < \frac{\sum_{\theta_{i'}}\int_{x_{i'}>\widetilde{x}(z_{-i},I)}\int_{x_i>x_{i'}} dF(x_i|\alpha)p(\alpha)dF(x_{i'}|\theta_{i'})p(\theta_{i'})}{\sum_{\theta_{i'}}\int_{x_{i'}>\widetilde{x}(z_{-i},I)}\int_{x_i>x_{i'}} dF(x_i|\beta)p(\beta)qdF(x_{i'}|\theta_{i'})p(\theta_{i'})}.$$

Therefore,

$$\frac{\Pr(i,\alpha|z_{-i},q,\mathcal{I})}{\Pr(i,\beta|z_{-i},q,\mathcal{I})} < \frac{\Pr(i,\alpha|z_{-i},q,\mathcal{I}\cup\{i'\})}{\Pr(i,\beta|z_{-i},q,\mathcal{I}\cup\{i'\})}.$$

and thus

$$\frac{\Pr(i,\alpha|q,\mathcal{I})}{\Pr(i,\beta|q,\mathcal{I})} < \frac{\Pr(i,\alpha|q,\mathcal{I}\cup\{i'\})}{\Pr(i,\beta|q,\mathcal{I}\cup\{i'\})}.$$

Hence,

$$\frac{\Pr(i,\alpha|q,\mathcal{I})}{\Pr(i,\alpha|q,\mathcal{I})+\Pr(i,\beta|q,\mathcal{I})} < \frac{\Pr(i,\alpha|q,\mathcal{I}\cup\{i'\})}{\Pr(i,\alpha|q,\mathcal{I}\cup\{i'\})+\Pr(i,\beta|q,\mathcal{I}\cup\{i'\})}$$

or $\Pr(\alpha|i,I) < \Pr(\alpha|i,I+1)$.

Claim 2. The probability that the DM rejects all candidates is decreasing in I,

Suppose, given I, the DM rejects all candidates. That is, there is no i such that $\psi(x_i|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}$. Then, whenever we add a new candidate i', there is always strictly positive probability that $\psi(x_{i'}|s) \geq \frac{-u(\beta)}{u(\alpha)-u(\beta)}$. Hence, the probability that the DM rejects all candidates is decreasing in I.

Note that, from Claim 1, the expected payoff of the DM is increasing in I whenever one candidate is selected. Thus, from Claim 2, we know that the expected payoff of the DM is increasing in I. Q.E.D.

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