

Mayer, Eric; R uth, Sebastian; Scharler, Johann

**Working Paper**

## Government debt, inflation dynamics and the transmission of fiscal policy shocks

W.E.P. - W urzburg Economic Papers, No. 87

**Provided in Cooperation with:**

University of W urzburg, Department of Economics

*Suggested Citation:* Mayer, Eric; R uth, Sebastian; Scharler, Johann (2012) : Government debt, inflation dynamics and the transmission of fiscal policy shocks, W.E.P. - W urzburg Economic Papers, No. 87, University of W urzburg, Department of Economics, W urzburg

This Version is available at:

<https://hdl.handle.net/10419/55840>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor d urfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie d urfen die Dokumente nicht f ur  offentliche oder kommerzielle Zwecke vervielfaltigen,  offentlich ausstellen,  offentlich zuganglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verf ugung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewahrten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# W. E. P.

---

**Würzburg Economic Papers**

No. 87

## Government Debt, Inflation Dynamics and the Transmission of Fiscal Policy Shocks

Eric Mayer<sup>(a)</sup> Sebastian Rüth<sup>(a)</sup> Johann Scharler<sup>(b)</sup>

<sup>(a)</sup>University of Würzburg

<sup>(b)</sup>University of Innsbruck

February 2012

Universität Würzburg  
Department of Economics

Sanderring 2, D-97070 Würzburg

**Postal Address**

Eric Mayer  
University of Würzburg, Department of Economics, 97070 Würzburg, Germany  
*Email:* [Eric.Mayer@uni-wuerzburg.de](mailto:Eric.Mayer@uni-wuerzburg.de)

# Government Debt, Inflation Dynamics and the Transmission of Fiscal Policy Shocks

Eric Mayer\*      Sebastian R uth<sup>†</sup>      Johann Scharler<sup>‡</sup>

February 15, 2012

## Abstract

We analyze the influence of the fiscal position on the transmission of government spending shocks in a New Keynesian model. We find that once we allow for positive levels of government debt in the steady state, the sign and the size of the fiscal multiplier depend strongly on the horizon at which the multiplier is evaluated. While the long-run effect of a fiscal policy innovation is typically of a similar order of magnitude as in Gal  et al. (2007), short-run multipliers differ substantially. The reason for this non-monotonic behavior is the interaction between the dynamics of the inflation rate and the debt level in real terms, which is absent in standard models in which government debt is restricted to be equal to zero in the steady state.

Keywords: Fiscal Multiplier, New Keynesian Model, Government Debt, Inflation

JEL codes: E31, E62, H63

---

\*University of Wuerzburg, Department of Economics, Sanderring 2, 97070 Wuerzburg, Germany, Phone: ++49 931 3182948, Fax: ++49 931 888-7275, e-mail: eric.mayer@uni-wuerzburg.de.

<sup>†</sup>University of Wuerzburg, Department of Economics, Sanderring 2, 97070 Wuerzburg, Germany, Phone: ++49 931 3186398, Fax: ++49 931 888-7275, e-mail: sebastian.rueth1@uni-wuerzburg.de.

<sup>‡</sup>University of Innsbruck, Department of Economics, Universitaetsstrasse 15, A-6020 Innsbruck, Austria, Phone: ++43 (512) 507 7357, Fax: ++43 (512) 507 7357 2980, e-mail: johann.scharler@uibk.ac.at.

# 1 Introduction

How does fiscal policy influence the business cycle? This question has a long tradition and has received a tremendous amount of re-newed interest in academic discussions as well as policy debates in the aftermath of the global financial crisis (see e.g. Cwik and Wieland, 2011; Ramey, 2011, for recent surveys). Similarly, rising government debt levels have also attracted a lot of interest since debt-to-GDP ratios have been increasing strongly throughout the industrialized world. Nevertheless, the existing literature treats these two issues as being largely distinct and neglects potential interrelationships between the level of government debt and the effects of fiscal policy on the business cycle.<sup>1</sup> In fact, fiscal policy in the standard New Keynesian model is usually analyzed under the assumption that government debt is zero in the steady state. In this paper, we contribute to the literature by analyzing if and how positive levels of government debt in the steady state influence the responses of macroeconomic variables to a government spending shock in a New Keynesian model in which a fraction of the household sector is characterized by rule-of-thumb behavior as in Galí et al. (2007).

We find that once we allow for positive levels of government debt, the dynamics of the model become generally more persistent and less monotonic, albeit the long-run responses to a government spending shock are of a similar order of magnitude as in Galí et al. (2007). Put differently, the magnitude, and even the sign, of the fiscal multiplier depend strongly on the horizon at which the multiplier is evaluated for sufficiently large levels of steady state government debt. The intuition goes as follows: an expansionary government spending shock, for instance, puts upward pressure on inflation and reduces government debt in real terms. For an empirically plausible calibration of fiscal policy reaction functions, taxes decline in response to the reduction in the real level of debt and therefore fiscal policy exerts an additional, expansionary effect via the increase in the disposable income of rule-of-thumb agents which ultimately results in an increase in aggregate consumption. Over the medium run, however, the inflationary effects of the government spending shock lead to higher real interest rates via active monetary policy and therefore the real debt burden starts to increase. As a consequence, taxes increase and the initially expansionary effect is counteracted. Since these effects unfold slowly over time the dynamics of the model become more persistent and the adjustment back to the steady state is less monotonic.

We also find that higher levels of steady state debt increase the regions of the parameter

---

<sup>1</sup>A notable exception is Ilzetki et al. (2010), who explicitly analyze how the size of the fiscal multiplier varies with the debt to GDP ratio in a large panel data set including developed as well as developing countries.

space associated with indeterminate equilibria. The intuition behind this result is again closely related to the interaction between the inflation rate and the real debt level which increases the volatility of disposable income such that the standard Taylor principle does no longer guarantee determinacy. Specifically, we show that even relatively low degrees of price stickiness may give rise to multiple equilibria and sunspot fluctuations if the debt-to-GDP ratio is sufficiently high. In this sense our results extend the analysis in Galí et al. (2004) who show that rule-of-thumb behavior in conjunction with price stickiness increases indeterminacy regions.

Overall, our results suggest that when the government is permanently indebted, the effect of fiscal policy on macroeconomic variables becomes rather erratic over time. If, for instance, due to a fiscal stimulus package implemented during a recession the debt level increases permanently, then fiscal stabilization policy becomes harder to implement during future downturns. Albeit somewhat related, this point is different from the argument that high debt levels leave little flexibility to use fiscal policy in times of economic downturns (see e.g. Fatás and Mihov, 2009).

Since our results follow from the interaction between the inflation rate and the real debt level, the paper is related to Aizenman and Marion (2011) and Krause and Moyen (2011) who also emphasize the effect of inflation on real debt. However, in contrast to these two contributions, we highlight the endogenous reaction of inflation without unexpected policy interventions which are the focus of these papers. Corsetti et al. (2010) analyze the implications of government debt in times of deep recessions. They show that anticipated spending reversals can have expansionary effects in the short-run. Our analysis differs from this paper in the sense that we look at debt dynamics more generally and not in the context of severe downturns.

The remainder of the paper is structured as follows: Section 2 describes the model and the calibration. Section 3 discusses the implications of a positive debt to GDP ratio for equilibrium determinacy and Section 4 discusses how the debt-to-GDP ratio influences the fiscal multiplier. In Section 5 we allow for a more general maturity structure of government debt and Section 6 concludes the paper.

## 2 Model and Calibration

In this section we describe the structure of a standard New Keynesian model which forms the basis for our analysis. Firms operate under monopolistic competition and each firm  $j$  hires labor  $N_{j,t}$  and capital  $K_{j,t}$  to produce a differentiated good  $Y_{j,t}$  according to:  $Y_{j,t} = N_{j,t}^{(1-\alpha)} K_{j,t}^\alpha$ . The firm sells its output at a price  $P_{j,t}$  and faces the demand curve  $Y_{j,t}^d = (P_{j,t}/P_t)^{-\epsilon} Y_t$ , where  $Y_t$

and  $P_t$  denote aggregate output and the price level respectively. The elasticity of substitution between differentiated goods is denoted by  $\epsilon$ . As in Calvo (1983), each period, only a fraction  $(1 - \theta)$  of firms is able to adjust its price. The household sector consists of  $(1 - \lambda)$  optimizing households with access to capital markets and a fraction  $\lambda$  of rule-of-thumb consumers. Optimizing households maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t^o - \frac{N_t^{o1+\varphi}}{1+\varphi} \right), \quad (1)$$

where  $C_t^o$  and  $N_t^o$  are consumption and labor supply of optimizing households.  $\beta$  is the discount factor and  $\varphi$  is the inverse of the Frisch elasticity of labor supply. The sequence of budget constraints reads:  $P_t(C_t^o + I_t) + B_t = (W_t P_t N_t^o + Div_t + R_t^k P_t K_{t-1}) + B_{t-1} R_{t-1} - P_t T_t^o$ , where  $I_t$  denotes investment and  $B_t$  are government bond holdings. These bonds yield a gross interest rate of  $R_t$ . The capital stock,  $K_t$  is owned by optimizing households and  $R_t^k$  is the rental rate of capital. Optimizing households draw income from labor  $W_t P_t N_t^o$ , capital  $R_t^k P_t K_{t-1}$  and dividends  $Div_t$ .  $T_t^o$  denotes lump-sum taxes. The capital accumulation equation is:  $K_t = (1 - \delta)K_{t-1} + \phi \left( \frac{I_{t-1}}{K_{t-1}} \right) K_{t-1}$ . As in Galí et al. (2007), the capital adjustment cost function  $\phi(I_t/K_t)$  satisfies:  $\phi' > 0$ ,  $\phi'' \leq 0$ ,  $\phi'(\delta) = 1$ ,  $\phi(\delta) = \delta$ . The asset holding household solves the optimization problem by choosing  $B_t, K_t, C_t^o$  and  $I_t$ . With respect to the labor market we follow Galí et al. (2007) and assume that there exists a continuum of unions which represents workers of a certain type. Specifically, the labor input of firm  $j$  is the CES aggregate:  $\left( \int N_t(j, i)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ , where  $\epsilon_w$  denotes the elasticity of substitution across different types of labor ( $z$ ). Each period unions set wages such that the instantaneous surplus of its members is maximized subject to a downward sloping demand curve  $N_t(z) = (W_t(z)/W_t)^{-\epsilon_w} N_t$ .

In addition to optimizing households, the household sector of the model economy also consists of rule-of-thumb agents, who consume their entire current labor income and maximize  $u(C_t^r, N_t^r)$  subject to  $P_t C_t^r = (W_t P_t N_t^r) - P_t T_t^r$ , which yields optimal consumption:  $C_t^r = W_t N_t^r - T_t^r$ .

Galí et al. (2007) show that the aggregate labor supply scheme can be written as  $W_t = C_t (N_t)^\varphi$  under the assumption that there exists a governmental transfer scheme which equates steady state per capita income across groups of households:  $\bar{C} = \bar{C}^o = \bar{C}^r$ , where bars denote steady state values.

The central bank follows a Taylor rule:

$$R_t = (R_{t-1})^{\rho_r} \left( \bar{R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_Y} \right)^{1-\rho_r}, \quad (2)$$

where  $\phi_\pi, \phi_Y$  characterize the responses of the interest rate to fluctuations in the inflation rate

and the output gap and  $\rho$  denotes the degree of interest rate smoothing.

The fiscal policy bloc closely follows Galí et al. (2007). Government debt evolves according to:

$$B_t = B_{t-1}R_{t-1} + P_t(G_t - T_t), \quad (3)$$

where new debt in period  $t$ ,  $B_t$ , is issued to finance the primary deficit  $P_t(G_t - T_t)$  and to revolve old debt at last periods gross nominal interest rate  $R_{t-1}$ . Government expenditures  $G_t$  are exogenous and follow an AR(1) process:

$$\left(\frac{G_t}{\bar{Y}}\right) = \left(\frac{G_{t-1}}{\bar{Y}}\right)^{\rho_g} \exp(\epsilon_t), \quad (4)$$

where  $\epsilon_t$  is a spending shock and  $\rho_g$  measures the persistence. Taxes are set according to the rule:

$$\left(\frac{T_t}{\bar{Y}}\right) = \left(\frac{B_t}{\bar{Y}}\right)^{\phi_b} \left(\frac{G_t}{\bar{Y}}\right)^{\phi_g}, \quad (5)$$

where  $\phi_b$  and  $\phi_g$  determine the responses to the ratio of debt to steady state output,  $B_t/\bar{Y}$ , and expenditures as a fraction of output in the steady state,  $G_t/\bar{Y}$ . The tax rule reflects the fact that government expenditure programs need to be financed either by taxes at an early stage, or by taxes at a later stage if expenditures are financed by debt in the first place (see also Corsetti et al., 2009; Bohn, 1998; Galí, 2003). A sufficient response to the level of debt assures that the debt-to-GDP ratio remains stationary.

**Aggregation and market clearance** We conjecture that market clearing requires:

$$N_t = \int N_t(j) dj \quad (6)$$

$$K_t = \int K_t(j) dj \quad (7)$$

$$Y_t(j) = X_t(j) \forall j. \quad (8)$$

$$Y_t = C_t + I_t + G_t. \quad (9)$$

**The Log-Linearized Model** The equilibrium dynamics of the model around the steady state are summarized by the following set of equations. Lower-case letters denote log-deviations from steady state values (*i.e.*,  $x_t \equiv \log(X_t/\bar{X})$ ). Based on the price setting behavior of firms we obtain the New Keynesian Phillips curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda_p \mu_t^p, \quad (10)$$



where  $\lambda_p = (1 - \beta\theta)(1 - \theta)\theta^{-1}$  and  $\mu_t^p = (y_t - n_t) - w_t$  denotes the log-deviation of the average mark-up from its steady state. The aggregate production technology is a standard Cobb-Douglas technology in the intermediate good sector which can be approximated up to first-order by  $y_t = (1 - \alpha)n_t + \alpha k_t$ . Consumption dynamics of asset holding households is described by the consumption Euler equation:

$$c_t^o = E_t\{c_{t+1}^o\} - (r_t - E_t\{\pi_{t+1}\}). \quad (11)$$

Labor supply of asset holding households evolves according to the labor supply schedule:  $w_t^o = c_t^o + \varphi n_t^o$ .

Investment dynamics are summarized as follows:

$$i_t = \eta q_t + k_t, \quad (12)$$

where the elasticity of investment with respect to the price of capital  $q_t$  is  $\eta = -1/(\phi''(\delta)\delta)$ . The price of capital evolves according to:

$$q_t = \beta E_t\{q_{t+1}\} + [1 - \beta(1 - \delta)]E_t\{r_{t+1}^k\} - (r_t - E_t\{\pi_{t+1}\}). \quad (13)$$

Rule-of-thumb agents consume their disposable income:

$$c_t^r = \left(\frac{WN^r}{C^r}\right)(w_t + n_t^r) - \left(\frac{Y}{C^r}\right)(t_t^r), \quad (14)$$

where  $\left(\frac{WN^r}{C^r}\right) = \frac{(1-\alpha)}{\mu^p \gamma_c}$  and  $\left(\frac{Y}{C^r}\right) = 1/\gamma_c$ . With respect to the steady state consumption share it holds that:  $\gamma_c = 1 - \gamma_g - \gamma_I$  and  $\gamma_I = \frac{\delta\alpha}{(\rho+\delta)\mu^p}$ . Optimal labor supply is:  $w_t^r = c_t^r + \varphi n_t^r$ . Since  $\bar{C}^o = \bar{C}^r = \bar{C}$  and  $\bar{N}^o = \bar{N}^r = \bar{N}$ , as in Galí et al. (2004), we can aggregate consumption and hours as  $c_t = (1 - \lambda)c_t^o + \lambda c_t^r$  and  $n_t = (1 - \lambda)n_t^o + \lambda n_t^r$ . Aggregate consumption  $c_t$  evolves according to:

$$c_t = E_t\{c_{t+1}\} - \sigma(r_t - E_t\{\pi_{t+1}\}) - \Theta_n E_t\{\Delta n_{t+1}\} + \Theta_\tau E_t\{\Delta t_{t+1}^r\}, \quad (15)$$

where  $\sigma = ((1 - \lambda)\Phi\gamma_c\mu^p)$ ,  $\Theta_n = (\lambda\Phi(1 - \alpha)(1 + \varphi))$ ,  $\Theta_\tau = (\lambda\Phi\mu^p)$ ,  $\Phi = (\gamma_c\mu^p - \lambda(1 - \alpha))^{-1}$ .  $\Delta n_{t+1}$  and  $\Delta t_{t+1}^r$  reflect the influence of disposable income on consumption.

The linearized Taylor rule is standard:

$$R_t = \rho_r R_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t). \quad (16)$$

Linearizing the government budget constraint yields:

$$b_t = (1 + \rho)b_{t-1} + (g_t - t_t) + \bar{b}(1 + \rho)(R_{t-1} - \pi_t), \quad (17)$$

where  $\rho = \beta^{-1} - 1$  and  $b_t = B_t/P_{t-1}/\bar{Y} - \bar{b}$  denotes the deviation of the debt-to-GDP ratio from its steady state value  $\bar{b} = \bar{B}/\bar{P}/\bar{Y}$  in percentage points and  $t_t = (T_t - \bar{T})/\bar{Y}$ . If  $\bar{b} > 0$ , then the lagged real interest rate exerts some influences on  $b_t$ . If  $R_{t-1} - \pi_t$  is below its steady state value  $\bar{R} - \bar{\pi} = \beta^{-1}$ , then fiscal authorities have additional room to cut taxes or increase expenditures. If, in contrast,  $\bar{R} - \bar{\pi} < R_{t-1} - \pi_t$  taxes have to be adjusted upward or expenditures need to be cut. The log-linearized tax rule reads:

$$t_t = \phi_b b_t + \phi_g g_t, \quad (18)$$

**Calibration** We set the fraction of rule-of-thumb consumers to  $\lambda = 0.5$  (see Mankiw 2000). The elasticity of wages with respect to hours is equal to  $\varphi = 0.20$ . The discount factor is set to  $\beta = 0.99$ . With respect to the supply side of the model we assume that prices are fixed on average for four quarters with  $\theta = 0.75$ . The steady state price mark-up  $\mu^p$  is 1.2, which reflects a 20 percent mark-up in the intermediate good sector. We set the capital share to  $\alpha = 0.33$  and the elasticity of investment with respect to Tobin's Q to  $\eta = 1$ . The quarterly depreciation rate of capital is  $\delta = 0.025$ . For the aggregate resource constraint we assume that government expenditure accounts for  $\gamma_g = 0.2$ , investment for  $\gamma_I = 0.198$ , while the remaining fraction of  $\gamma_c = 0.602$  accounts for the consumption share.

For the policy parameters we choose the following calibration. We set the parameter that determines the responsiveness to inflation in the Taylor rule to  $\phi_\pi = 2.50$ . Although this value is somewhat higher than what is usually reported in empirical studies, a higher value is needed to guarantee determinacy in the baseline calibration, since the model features rule-of-thumb consumers (see Galí et al., 2004).

For the calibration of the fiscal policy block, we use parameter values reported in Galí et al. (2007). Their calibration strategy consists of choosing values for  $\phi_g$  and  $\phi_b$  such that the dynamics implied by the model are consistent with the estimated impulse responses to a government expenditure shock. For the elasticity of taxes with respect to spending, they argue that  $\phi_g = 0.10$  is consistent with the data, and the elasticity of taxes with respect to debt is set to:  $\phi_b = 0.33$ . We set the first-order autocorrelation in the fiscal expenditure shock to  $\rho_g = 0.90$ , which matches the half-life of the empirical impulse responses to a government spending shock.

### 3 Equilibrium Determinacy

In this section we investigate how the steady state debt-to-GDP ratio influences the existence and uniqueness of the rational-expectations equilibrium. It is well known that in the basic New Keynesian model, without rule-of-thumb consumers or investment, a necessary condition for determinacy is (see e.g. Woodford, 2003, p.256):

$$\phi_\pi + \frac{1-\beta}{\theta}\phi_y > 1. \quad (19)$$

If  $\phi_y = 0$ , then this condition reduces to the well established, standard Taylor principle:  $\phi_\pi > 1$ . The existence and uniqueness of the rational expectations equilibrium is guaranteed if the nominal interest rate is adjusted at least one-for-one in response to fluctuations in the inflation rate, since this behavior ensures that the real interest rate and, ultimately, aggregate demand are stabilized.

Galí et al. (2004) show that the introduction of rule-of-thumb behavior changes the determinacy properties of the model since monetary policy may not be able to stabilize aggregate demand by adjusting interest rates as rule-of-thumb consumers respond only to changes in disposable income. Consequently, if economic activity increases, for instance, due to some non-fundamental reason, income and consumption of rule-of-thumb consumers increase as well and the Taylor principle may not be sufficient to guarantee determinacy if the share of rule-of-thumb consumers is sufficiently large. Note, however, that a countercyclical mark-up is necessary to obtain this outcome. Otherwise, the real wage, and therefore disposable income, fall due to lower labor productivity. This link between the stickiness parameter  $\theta$ , the share of rule-of-thumb consumers  $\lambda$ , and the determinacy of the equilibrium is illustrated in the top-left sub-figure in Figure 1, where we set the steady state level of the debt-to-GDP ratio to  $\bar{b} = 0$ . We see that for a sufficiently large share of rule-of-thumb consumers and a sufficiently high degree of stickiness, the equilibrium is indeterminate although the calibration of the interest rate rule satisfies the Taylor principle.

The remaining three subfigures show that the size of the determinacy region is influenced by the debt-to-GDP ratio in the steady state. For higher values of  $\bar{b}$ , even smaller shares of rule-of-thumb consumers are associated with indeterminate equilibria. Thus, as a novel finding we can report that for high debt-to-GDP ratios the non-uniqueness region also comprises low degrees of price stickiness, a feature that is not present in the basic framework.

The intuition goes as follows: Since interest payments on government debt are fixed in

nominal terms, any, perhaps non-fundamental, shock that influences the inflation rate, also influences the real interest burden. Thus, if inflation increases, interest payments in real terms decline and since taxes are set according to the tax rule (18), the resulting, lower level of debt in real terms translates into a tax reduction. Consequently, disposable income increases and so does the consumption of rule-of-thumb consumers. Although the interest rate rule ensures that the increase in inflation leads to a higher real interest rate, the central bank is not able to counteract this consumption boom. In essence, sufficiently high levels of debt-to-GDP amplify the volatility of disposable income such that self-sustaining booms undermine the Taylor principle.

## 4 Impulse Response Analysis

Having characterized the implications of government debt for the determinacy properties of the model, we now study how the steady state debt-to-GDP ratio influences the transmission of changes in government spending to aggregate economic activity. Figure 2 shows the impulse responses to an expansionary government spending shock for three different calibrations of the steady state debt-to-GDP ratio: the benchmark case with  $\bar{b} = 0$  (solid lines),  $\bar{b} = 1$  (broken lines) and  $\bar{b} = 2$  (dotted lines).

For the benchmark calibration with  $\bar{b} = 0$  we basically replicate the results reported in Galí et al. (2007): optimizing agents reduce consumption, due to the wealth effect associated with future tax increases and the intertemporal substitution effect induced by the higher real interest rate. Rule-of-thumb agents, in contrast, have higher disposable incomes due to the higher real wage in combination with an increase in hours worked. Note that along with the increase in demand, inflation increases since firms adjust prices to reflect higher production costs. Higher inflation in turn induces the central bank to tighten monetary policy. Following the increase in the rental rate of capital, investment declines. Since the consumption response of rule-of-thumb consumers is sufficiently strong to compensate both, the lower consumption spending of optimizing agents and the decline in investment, the increase in output is almost as large as the initial increase in government spending. Thus, with this calibration, we obtain a fiscal multiplier around unity.

Next, suppose that government debt amounts to 100 percent of GDP in the steady state:  $\bar{b} = 1$ . While the responses are qualitatively similar to those obtained with the benchmark calibration, the model economy responds initially more sluggishly to the government spending shock, and the responses are amplified over the medium run. Put differently, the impulse

responses become more persistent and the adjustment back to the steady state is less monotonic. This pattern is even more pronounced for  $\bar{b} = 2$ . Here, the peak responses are substantially larger and occur later than in the other two calibrations considered.

These differences in the dynamics are the consequence of the interrelationship between the inflation rate and the dynamics of government debt. The expansionary shock to government spending puts upward pressure on the inflation rate, since firms respond to higher demand, and consequently higher production costs, by increasing prices. Since interest payments on government debt are fixed in nominal terms for one period, the real interest burden declines. Figure 3 shows the dynamics of the government budget balance (the change in government debt), the primary deficit, the interest burden and tax revenues in real terms. We see that while the primary deficit increases by the same amount in the impact period for all three calibrations of  $\bar{b}$  considered, the interest burden declines when government debt is positive in the steady state. This effect initially counteracts the increase in debt associated with higher government spending. However, this dampening effect of the inflation rate is confined only to the impact period in which the interest payment is fixed in nominal terms. In later periods, the interest burden increases strongly along with the real interest rate. Since firms anticipate the higher demand associated with the temporary tax reduction, the increase in the inflation rate is more pronounced for higher levels of initial government debt, which - via the interest rate rule - leads to stronger increase in the real interest rate. Thus, although higher inflation initially reduces the tax burden, it ultimately leads to less favorable financing conditions.

In short, the government spending shock influences government debt dynamics not only directly, but also indirectly by altering the burden of existing debt. The second effect is absent when the debt-to-GDP ratio is zero in the steady state. Since taxes are closely related to debt dynamics, the lower burden of debt in real terms leads to a higher disposable income and consequently an increase in consumption spending of rule-of-thumb agents.

Note that in Figure 2 several impact responses change signs for sufficiently high levels of  $\bar{b}$ : aggregate consumption, consumption of rule-of-thumb agents, hours worked, the real wage, the rental rate and, to some extent, also output decline in the impact period. For higher levels of government debt, the increase in inflation in the impact period, partly due to the anticipation of higher demand in the future, is strong enough to reduce the real wage and therefore also income and consumption of rule-of-thumb agents, in the impact period. This effect counteracts, and for sufficiently high levels of government debt even overcompensates, the expansionary effect of government spending in the impact period.

Also note that the presence of rule-of-thumb agents, or limited asset market participation, is a crucial element of the transmission mechanism. If we set the share of rule-of-thumb agents to zero, then the debt level influences only the dynamics of the deficit and leaves the transmission of government spending shocks to macroeconomic variables unchanged. This result is somewhat reminiscent of Barro (1974), in the sense that the level of debt is neutral in a model with forward-looking agents and frictionless financial markets.

Figure 2 shows that the response of output to variations in government spending, i.e., the fiscal multiplier, now depends strongly on the level of debt-to-GDP in the steady state. To illustrate this point further, we report in Table 2 the cumulative fiscal multipliers for different values of  $\bar{b}$  and at different horizons. If  $\bar{b} = 0$ , then output increases by around 1 percent in response to a 1 percent shock to government spending in the impact period. In other words, the impact multiplier is roughly equal to unity. Over time, the multiplier declines to 0.91 after 4 quarters and to 0.85 after 8 quarters. When government debt amounts to 100 percent of GDP in the steady state,  $\bar{b} = 1$ , then the multiplier is reduced roughly by half to 0.53 percent in the impact period. However, after four quarters, the cumulative multiplier increases to 1.02 and falls again to 0.93 after 8 quarters. When  $\bar{b} = 2$ , the multiplier is negative in the impact period, increases strongly to 1.11 after four quarters and remains slightly above unity after eight quarters.

Overall, the table shows that the size as well as the sign of the fiscal multiplier depend strongly on the horizon at which the multiplier is evaluated. Although the impact multiplier is lower for higher levels of government debt in the steady state, differences are less pronounced in the longer run and the multipliers are of similar order of magnitude as in Galí et al. (2007) after 8 quarters.<sup>2</sup>

## 5 Maturity Structure of Government Debt and the Fiscal Multipliers

So far, we have assumed that government debt has to be fully rolled over each quarter. In this section, we relax this assumption and allow for a more general maturity structure. To do so, we follow Calza et al. (2009) and assume that the government redeems only a fraction  $1/m$  of the total outstanding debt each quarter. Accordingly, the government budget constraint becomes:

$$B_{m,t} = P_t(G_t - T_t) + \frac{1}{m} \sum (B_{m,t-1} R_{t-m}). \quad (20)$$

---

<sup>2</sup>Since we assume a stronger response of the interest rate to inflation, we obtain multipliers which are somewhat smaller than in Galí et al. (2007).

Linearizing this expression around the steady state yields

$$b_{m,t} = (g_t - t_t) + \frac{1}{m}\beta^{-1} \sum b_{m,t-i} + \frac{1}{m}\bar{b}\beta^{-1} \left( \sum R_{m,t-j} - \sum \pi_{m,t-j+1} \right) \quad (21)$$

Since only a fraction of  $1/m$  of the outstanding debt matures in period  $t$ , the nominal interest rate on the remaining fraction,  $(m-1)/m$  is predetermined. Consequently, changes in the inflation rate will alter the real rate of interest  $(\sum R_{m,t-j} - \sum \pi_{m,t-j+1})$  and ultimately the real burden of government debt.

Since the asset holding households who take into account that their bonds will provide interest payments for  $t+m$  quarters in the future the Euler equation becomes

$$\frac{\lambda_t^o}{P_t} = R_{m,t} E_t \left( \frac{1}{m} \sum \beta^j \frac{\lambda_{t+j}^o}{P_{t+j}} \right). \quad (22)$$

Taking a log-linear approximation yields:

$$c_t^o = \left( \sum \beta^j \right)^{-1} E_t \left( \sum \beta^j c_{t+j}^o \right) - \left( R_{m,t} - \left( \sum \beta^j \right)^{-1} E_t(p_{t+j} - p_t) \right). \quad (23)$$

To see if and how the maturity structure matters, we will conduct our analysis along two dimensions: first, we keep the maturity fixed and report cumulative fiscal multipliers for different steady state debt-to-GDP ratios and second, we investigate the influence of longer maturities for a given level of government debt.

In Table 3 we report cumulative fiscal multipliers for different steady state debt-to-GDP ratios and  $m=2$ . We see that the impact multipliers still depend strongly on the steady-state debt-to-GDP ratio. Higher values for  $\bar{b}$  are generally associated with smaller multipliers in the impact period. However, for larger values of  $\bar{b}$  the multipliers increase strongly over the longer run. Thus, our main results do not appear to depend on the maturity structure.

In Table 4 we report cumulative fiscal multipliers for maturities ranging from  $m=1$  to  $m=4$  and a steady state debt-to-GDP ratio of  $\bar{b}=1$ . We see that increasing the maturity of government debt generally increases the multiplier. In particular, the impact multiplier increases from 0.53 to 1.01 when the maturity of debt increase from  $m=1$  to  $m=4$ . The reason for these amplified effects is that for longer maturities, a larger part of total debt is predetermined and therefore a surge in the inflation rate exerts a more pronounced effect on the real debt burden. That is, since the fiscal expansion is increasingly financed by inflation for higher  $m$ , the expansionary effects of the government spending shock are amplified to a greater extent. However, differences in the multipliers across different values of  $\bar{b}$  are more pronounced in the impact period than at longer horizons, suggesting that this effect works primarily in the impact period.

## 6 Summary

We show that higher levels of government debt result in less monotonic behavior of macroeconomic variables in response to spending shocks. Especially short-run and medium-run multipliers vary substantially depending on the level of government debt in the steady state. Moreover, our determinacy analysis suggests that the debt-to-GDP ratio does not only alter the responses to fundamental shocks, but also reduces the size of the determinacy region. That is, for high values of the debt-to-GDP ratio, sunspot fluctuations become more likely.

## References

- Aizenman, J., Marion, N., 2011. Using inflation to erode the US public debt. *Journal of Macroeconomics* 33 (4), 524 – 541.
- Barro, R. J., 1974. Are government bonds net wealth. *Journal of Political Economy* 82 (6), 1095–1117.
- Bohn, H., 1998. The behavior of u.s. public debt and deficits. *The Quarterly Journal of Economics* 113 (3), 949–963.
- Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12, 383–398.
- Calza, A., Monacelli, T., Stracca, L., Jul. 2009. Housing finance and monetary policy. Working Paper Series 1069, European Central Bank.
- Corsetti, G., Kuester, K., Meier, A., Müller, G. J., 2010. Debt consolidation and fiscal stabilization of deep recessions. *American Economic Review* 100 (2), 41–45.
- Corsetti, G., Meier, A., Müller, G. J., 2009. Fiscal stimulus with spending reversals. CEPR Discussion Papers 7302.
- Cwik, T., Wieland, V., 2011. Keynesian government spending multipliers and spillovers in the euro area. *Economic Policy* 26 (67), 493 – 549.
- Fatás, A., Mihov, I., 2009. The euro and fiscal policy. NBER Working Papers 14722, National Bureau of Economic Research, Inc.
- Galí, J., October 2003. Fiscal policy and monetary integration in europe. *Economic Policy* 18 (37), 533–572.



- Galí, J., Lopez-Salido, J. D., Valles, J., 2004. Rule-of-thumb consumers and the design of interest rate rules. *Journal of Money, Credit, and Banking* 36 (4), 739–764.
- Galí, J., López-Salido, J. D., Vallés, J., 2007. Understanding the effects of government spending on consumption. *Journal of the European Economic Association* 5 (1), 227–270.
- Ilzetzki, E., Mendoza, E. G., Végh, C. A., 2010. How big (small?) are fiscal multipliers? NBER Working Papers 16479.
- Krause, M., Moyen, S., 2011. Public debt and changing inflation targets. manuscript.
- Perotti, R., 1999. Fiscal policy in good times and bad. *The Quarterly Journal of Economics* 114 (4), 1399–1436.
- Ramey, V. A., 2011. Can government purchases stimulate the economy? *Journal of Economic Literature* 49 (3), 673–85.
- Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review* 97 (3), 586–606.
- Woodford, M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Table 1: Baseline calibration

Parameter	Value	Description
Household Behavior		
$\lambda$	0.50	share of ROT
$\varphi$	0.20	real wage elasticity
$\beta$	0.99	discount factor
Price Setting		
$\theta$	0.75	Calvo parameter
$\eta$	1.00	elasticity to Tobin's Q
$\alpha$	0.33	capital share
$\delta$	0.025	depreciation rate
Fiscal Policy		
$\phi_b$	0.33	tax elasticity to debt
$\phi_t$	0.10	tax elasticity to spending
Monetary Policy		
$\phi_\pi$	2.50	response to inflation
$\phi_y$	0.00	response to output
$\rho_r$	0.00	interest rate smoothing
AR(1) Coefficient of the Government Spending Shock Processes		
$\rho_g$	0.90	government expenditures

Notes: The table displays the baseline calibration and largely relies on Galí et al. (2007)

Table 2: Cumulative Fiscal Multipliers

	$t = 1$	$t = 4$	$t = 8$
$\bar{b} = 0$	0.99	0.91	0.85
$\bar{b} = 1$	0.53	1.02	0.93
$\bar{b} = 2$	-0.24	1.11	1.02

Notes: The table reports the fiscal multiplier for steady state debt-to-GDP ratios of  $\bar{b} = 0$ ,  $\bar{b} = 1$ , and  $\bar{b} = 2$ .

Table 3: Cumulative Fiscal Multipliers for  $m=2$

	$t = 1$	$t = 4$	$t = 8$
$\bar{b} = 0$	1.41	1.17	0.88
$\bar{b} = 1$	0.81	0.99	0.97
$\bar{b} = 2$	0.19	1.44	1.21

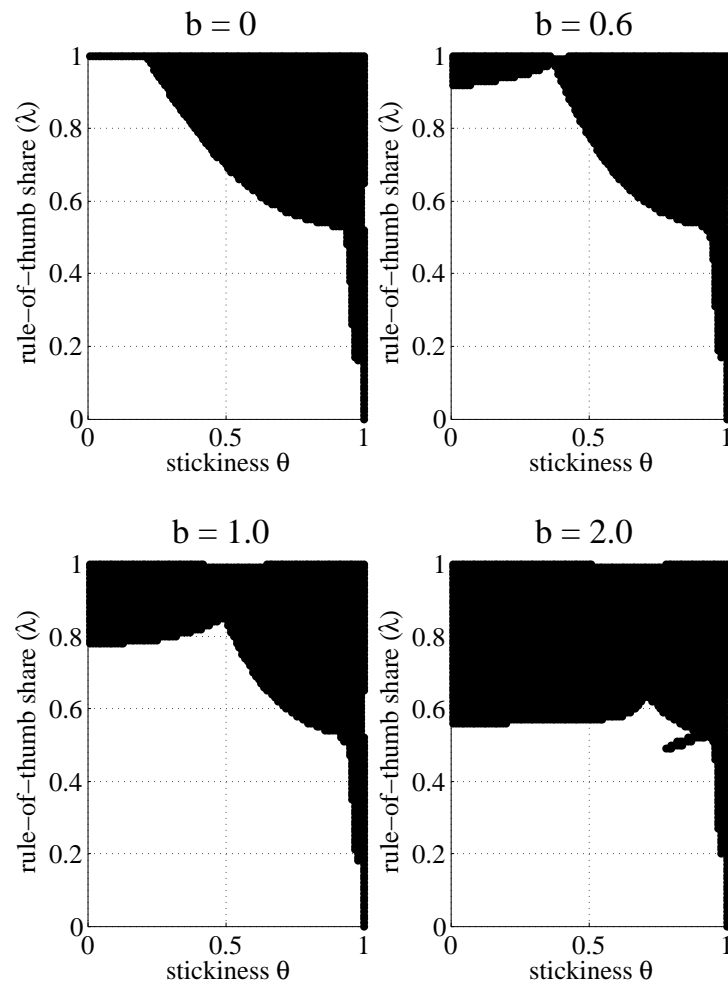
Notes: The table reports the fiscal multiplier for  $m = 2$  and steady state debt-to-GDP ratios of  $\bar{b} = 0$ ,  $\bar{b} = 1$ , and  $\bar{b} = 2$ .

Table 4: Fiscal Multipliers for Different Maturity Structures of Debt

	$t = 1$	$t = 4$	$t = 8$
$m = 1$	0.53	1.02	0.93
$m = 2$	0.80	1.00	0.97
$m = 3$	0.94	1.04	1.02
$m = 4$	1.01	1.09	1.11

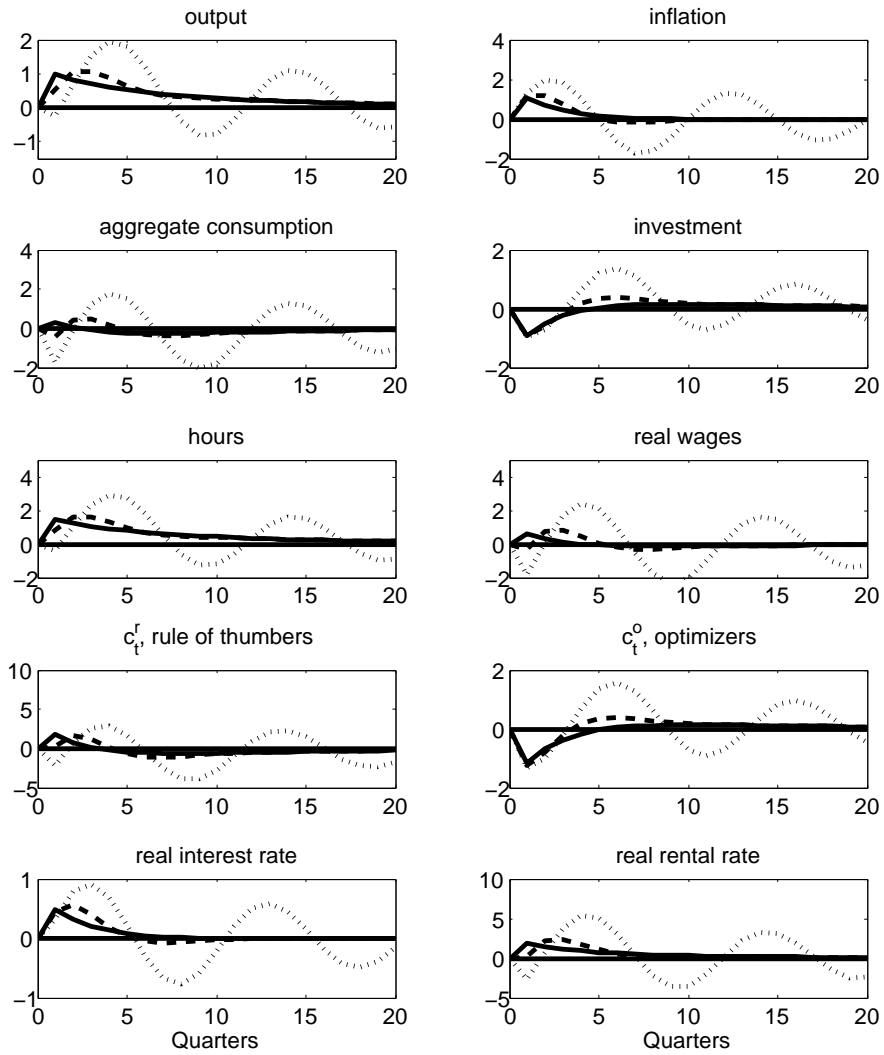
Notes: The table reports fiscal multipliers for different maturities with  $m = 1$  to  $m = 4$  and a steady state level of the debt-to-GDP ratio of  $\bar{b} = 1$ .

Figure 1: Determinacy



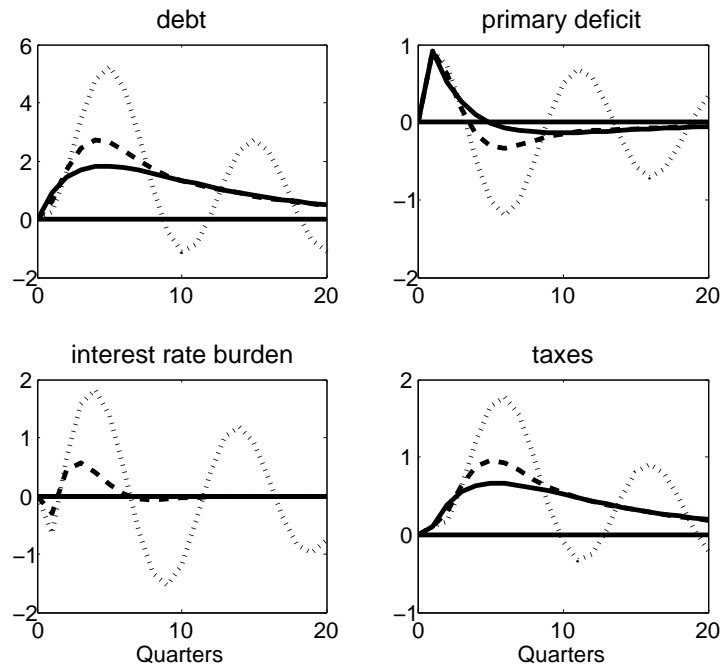
Notes: The Figure shows regions of the parameter space associated with determinate and indeterminate equilibria. Shaded areas indicate parameter combinations that result in indeterminacy.

Figure 2: Government Spending Shock



Notes: The Figure shows the responses to a positive government spending shock when the debt-to-GDP ratio in the steady state is zero,  $\bar{b} = 0$  (solid line), 100 percent,  $\bar{b} = 1$  (broken line), or 200 percent,  $\bar{b} = 2$  (dotted line).

Figure 3: Government Spending Shock - Details



Notes: The Figure shows the responses to a positive government spending shock when the debt-to-GDP ratio in the steady state is zero,  $\bar{b} = 0$  (solid line), 100 percent,  $\bar{b} = 1$  (broken line), or 200 percent,  $\bar{b} = 2$  (dotted line).