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Working Paper No. 641 April 2009 ISSN 1473-0278

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Abstract

Despite a widespread interest in the warm glow model [Andreoni (1989,1990)], surprisingly most attention focused on the voluntary contribution equilibrium of the model, and only very little attention has been devoted to the competitive equilibrium. In this paper, we introduce the notion of competitive equilibrium for a warm glow economy [Henceforth, warm glow equilibrium]. Then, we establish (and prove), in the contest of our model, the three fundamental theorems of general equilibrium: (i) warm glow equilibrium exists; (ii) a warm glow equilibrium is Pareto efficient; and (iii) a Pareto efficient allocation can be decentralized as a warm glow equilibrium). The concept of a warm glow equilibrium may prove to be very useful to the normative and positive theory of public goods provision. First, it is a price based mechanism achieving efficient outcomes. Secondly, not only the warm glow equilibrium outcomes could serve as a point of reference to measure free-riding and welfare loss, but also due to warm glow effects, unlike Lindahl allocations, they are more likely to be achieved.

Keywords: Warm Glow, Altruism, Competitive Equilibrium, Free Riding, Public Goods Provision .

JEL Classification Numbers: H41, D64, C62.

*This paper was presented at PET 08 meetings, LAMES 08,Vanderbilt University, East Anglia University, Warwick University, Kent University. I am indebted to the various audiences for useful comments.

1 Motivation

Altruistic behavior in societies has puzzled economists ever since the "*homo economicus*" model advocated by Adam Smith and David Ricardo. The puzzle is to explain how a palpable altruistic behavior may emerge amongst supposedly selfish members of the society. Prosocial behaviors are a pervasive feature of economics and, not surprisingly, they are the subject of ongoing interest in many fields in economics ranging from development economics to experimental economics. In public economics, the standard neoclassical model of public goods provision assumes that consumers care only about the magnitude of their public goods provisions insofar as these provisions affect the aggregate level of provision. The lucidity of the standard model compounded with the straightforwardness of its policy suggestions, have made it of paramount significance to the study of all areas of public economics such as taxation, pensions, and charity giving.

The work of Sugden (1982), Warr (1983), Bergstrom, Blume, and Varian (1986), Bernheim (1986), Roberts (1987), Andreoni (1988) has revealed some surprising implications of the standard model of public goods provision. Amongst these implications, an almost complete *Dollar to Dollar* crowding out between internal and external fundings, the neutrality of the level of provision to a significant class of income redistributions and as the population grows large the per-capita level of provision goes to zero. These predictions are the consequence of the fact that public goods are merely perceived as an additive externality of the various contributions, where the identity of the contributor is immaterial. Recent empirical tests and laboratory experiments of the standard model of public goods (Kingma (1989), Payne (1998), and for surveys Camerer (2003) and Bernheim and Rangel(2005)) were in conflict with its theoretical implications. For instance, Kingma (1989) in studying the charitable contributions to public radio stations has shown that crowding out has much smaller effects than the Dollar to Dollar prediction. These observations have induced economists to question the ability of the standard model to explain some of the empirical and experimental evidence.

Andreoni (1989,1990), inspired by earlier works of Becker (1974), Cornes and Sandler (1984), has proposed a "*warm glow*" model of public goods provision. In the warm glow model, consumers do not solely benefit from the aggregate amount of public goods provision but they may also benefit from the warm glow effects of their own public goods provision.[1] Thus, one consumer's public goods provision could not constitute a perfect substitute for the other consumers' provision. For example, when someone paints his house on a residential street, (provided the color choice is in the taste his neighbors), his action not only benefits him but also his neighbors. At the same time, having one of his neighbors houses painted will not substitute for having his own house painted.[2] In the warm glow model, free riding is shown to be less severe, crowding out is not complete, and neutrality of income redistribution does not hold. The above theoretical predictions are broadly in line with both empirical and experimental evidence.

Despite the importance of the warm glow model in subsequent research, surprisingly no competitive equilibrium for this model has been developed yet. Not only a price based approach to implement efficient outcomes is a highly desirable mechanism for its normative prescriptions, but also the outcome could potentially serve as a point of reference to measure free-riding and welfare loss (see for example, Cornes and Sandler (1984,1996), Temimi (2001) and Gaube (2006)). The notion of competitive equilibrium in public goods economies originates in the early work of Lindahl (1919) who introduced the concept of personalized prices for public goods. With private goods, different people can consume different bundles, but in equilibrium they all must pay the same prices. With public goods, everyone is faced with an equal provision, but consumers may pay different prices according to their preferences. The Lindahl equilibrium is typically more of a normative price based mechanism for the allocation of public goods rather than a positive description of the market mechanism itself. Since, the individual aspect of personalized prices undermines the price taking behavior assumption of competitive markets. A consumer will quickly learn to conceal his true preferences in order to take advantage of the public goods provision. The work of Foley (1967,1970), Fabre-Sender (1969), Milleron (1972) Bergstrom(1976) and Roberts (1974) show the existence of a Lindahl equilibrium by expanding either the commodity spaces or the price system and appealing to a standard existence results for a private goods economy. Khan and Vohra (1987)

[1] In the standard public good provision consumer i is assumed to have a utility function of the form $u_i(x_i, G)$ where x_i is consumer's i private goods consumption, and G is the aggregate supply of public goods. Andreoni's approach introduces a direct utility for each consumer from his own provision of the public good. In his formulation, if consumer's i provides g_i to the public good, his utility function is of the form $u_i(x_i, g_i, G)$.

[2] This example was kindly suggested to us by Ted Bergstrom.

and Bonnisseau (1991) prove the existence of Lindahl equilibrium with non-convex production technology including possibly increasing-returns to scale technologies. Conley (1994), Wooders(1997) and Florenzano and del Mercato (2006) show the existence of Lindahl equilibrium through subtle context specific core-equilibrium convergence. More recently, the work of Boyd and Conley (1997), Conley and Smith (2005) and Murty (2006) show the existence of competitive equilibrium in economies with externalities, in which the first welfare theorem could be interpreted as a Coase-like Theorem. It is worth noticing that, pure public goods could be accommodated in their model as a special case of additive externalities.

Our warm glow equilibrium resembles the Lindahl equilibrium by considering personalized prices. However, due to the warm glow effects, each consumer is endowed with two sets of personalized prices, one personalized price to finance his own public good provision and another one personalized price to finance other consumers provisions. Hence, unlike the Lindahl mechanism, each consumer values differently his own provision and other consumers' provisions. Having defined these personalized prices a warm glow equilibrium is then defined as a decentralized competitive equilibrium. It turns out that the warm glow equilibrium has interesting properties. Indeed, the warm glow equilibrium is Pareto efficient (first welfare theorem) and a Pareto efficient could be decentralized as a warm glow equilibrium (second welfare theorem). In addition, under general hypothesis, a warm glow equilibrium is shown to exist. It is worth noticing that, in the absence of warm glow effects, i.e. the standard model of public goods provision, our warm glow equilibrium coincide with the Lindahl equilibrium. It remains to be seen whether other solution concepts in the Lindahlian tradition such as ratio equilibrium Kaneko (1977) and cost-share equilibrium Mas-Colell and Silvestre (1989) could be defined for a warm glow economy.

The plan of the paper is as follows. Section 2 introduces a warm glow economy and defines the warm glow equilibrium. We show the welfare theorems in Section 3. In Section 4, We establish the existence of warm glow equilibrium. We discuss different formulations of warm glow utility functions in Section 5. Section 6 concludes the paper.

2 The Model

We consider a public goods economy E, with L private goods, K public goods. There are N consumers in the economy, each of whom is characterized by his consumption set $\mathbb{R}_+^{L+K}$. We will often write consumption bundles for consumer i in the form $(x_i, g_i) \in \mathbb{R}_+^{L+K}$, where $x_i \in \mathbb{R}^L$ is interpreted as consumer's i private goods consumption and $g_i \in \mathbb{R}^K$ refers to consumer's i public goods provision. The aggregate production technology is described by the production set $Y \subset \mathbb{R}^L \times \mathbb{R}_+^K$. For simplicity, we assume that Y is a closed convex cone with vertex the origin; satisfying the usual properties of irreversibility, no free production, and free disposal. Each consumer has a positive initial endowment of private goods $w_i \in \mathbb{R}_{++}^L$. We assume that there are no initial endowments of public goods. Each consumer i preferences can be represented by a utility function $u_i(x_i, g_i, G_{-i})$, where $G_{-i} = \sum_{j \neq i} g_j$ is the aggregate supply of public goods of other consumers. [3]

We assume that each utility function u_i satisfies the following properties:

[**A.1**] **Monotonicity**: The utility function $u_i(\cdot, \cdot, \cdot)$ is increasing and is strictly increasing on $\mathbb{R}_+^L \times \mathbb{R}_{++}^K \times \mathbb{R}_+^K$.

[**A.2**] **Continuity**: The utility function $u_i(\cdot, \cdot, \cdot)$ is continuous.

[**A.3**] **Convexity**: The utility function $u_i(\cdot, \cdot, \cdot)$ is quasi-concave.

[**A.4**] **Warm glow indispensability**: For every $(x_i, g_i, G_{-i}) \in \mathbb{R}_+^{L+K}$, if $g_i \notin \mathbb{R}_{++}^K$ then $u_i(x_i, g_i, G_{-i}) = \inf u_i(\cdot, \cdot, \cdot)$.

The production set is assumed to satisfy:

[**A.5**] **Possibility of producing public goods**: $Y \cap (\mathbb{R}^L \times \mathbb{R}_{++}^K) \neq \emptyset$.

[**A.6**] **Public goods are inessential in production**: if $(y, g^1, \ldots, g^K) \in Y$, then $(y, \sup\{g^1, 0\}, \ldots, \sup\{g^K, 0\}) \in Y$.

Assumption [**A.1**]-[**A.3**] and [**A.5**]-[**A.6**] are standard for public goods economy. Assumption [**A.4**] guarantees that own public goods provision is essential for each consumer.

[3] In Section 4, we show how the competitive equilibrium for this paper's formulation of utility functions $u_i(x_i, g_i, G_{-i})$ is equivalent to the competitive equilibrium for Andreoni's formulation of utility functions $u_i(x_i, g_i, G_{-i})$.

2.1 Feasible allocations

An allocation $((x_i, g_i), i \in N)$ is feasible if

$$(\sum_i (x_i - w_i), \sum_i g_i) \in Y.$$

Thus , we require the net inputs of private goods and outputs of public goods to be consistent with the production technology.

2.2 Pareto efficient allocations

An allocation $((x_i, g_i), i \in N)$ is (weakly) Pareto efficient if it is feasible and if there exists no feasible allocation $((x_i', g_i'), i \in N)$ such that for each consumer i we have

$$u_i(x_i', g_i', G_{-i}') > u_i(x_i, g_i, G_{-i}).$$

2.3 Warm glow equilibrium

A warm glow equilibrium is $((\overline{x}_i, \overline{g}_i, \pi_i, \pi_{-i})_{i\in N}, p, p^g)$, where $((\overline{x}_i, \overline{g}_i), i \in N)$ is a feasible allocation, $p \in \mathbb{R}_+^L$ is a price system for private goods, $p^g \in \mathbb{R}_+^K$ is a price system for public goods, $\pi_i \in \mathbb{R}_+^K$ is the personalized price of consumer's i own public goods provision, and $\pi_{-i} \in \mathbb{R}_+^K$ is the personalized price of consumer's i for others' public goods provision, such that

(i). For all $(y, g) \in Y$,

$$(p, p^g) \cdot (y, g) \leq (p, p^g) \cdot (\sum_i (\overline{x}_i - w_i), \sum_i \overline{g}_i) = 0$$

(Profit maximization);

(ii). For each consumer $i \in N$,

$$p \cdot \overline{x}_i + \pi_i \cdot \overline{g}_i + \pi_{-i} \cdot \overline{G}_{-i} = p \cdot w_i,$$

and if

$$u_i(x_i, g_i, G_{-i}) > u_i(\overline{x}_i, \overline{g}_i, \overline{G}_{-i})$$

then

$$p \cdot x_i + \pi_i \cdot g_i + \pi_{-i} \cdot G_{-i} > p \cdot w_i,$$

(Maximization of utility given personalized prices and public goods provision);

(iii). For each consumer $i \in N$,

$$\pi_i + \sum_{j \neq i} \pi_{-j} = p^g$$

(For each public good provision personalized prices sum up to the price of the public goods).

In the voluntary contribution equilibrium consumer i faces the price p^g to finance his own public goods provision g_i. In the warm glow equilibrium, due to the external effects that consumer's i public goods provision g_i has on other consumers welfare, each consumer $j \in N \setminus \{i\}$ will finance g_i at a personalized price of π_{-j}. Then, consumer i faces a personalized price $\pi_i = p^g - \sum_{j \neq i} \pi_{-j}$ to finance his own public good provision.

If we consider the standard public good model then one has $u_i(x_i, g_i, G_{-i}) = u_i(x_i, g_i + G_{-i})$. The perfect substitution between g_i and G_{-i} for consumer i implies that $\pi_i = \pi_{-i}$. Thus, in the case of the standard public good provision model our warm glow equilibrium coincides with the Lindahl equilibrium.

3 Welfare theorems for a warm glow economy

We now state the first welfare theorem for a warm glow economy: a warm glow equilibrium is Pareto efficient. Hence, amongst other things the warm glow equilibrium provides a price based mechanism to achieve efficient outcomes. The argument is similar to the textbook proof of the first welfare theorem.

Theorem 1: *(First Welfare Theorem for a warm glow economy)* A warm glow equilibrium is Pareto efficient.

Proof: Suppose the Theorem is false. Then there is a warm glow equilibrium $((\overline{x}_i, \overline{g}_i, \pi_i, \pi_{-i})_{i \in N}, p, p^g)$ with the property that the feasible allocation

$((\overline{x}_i, \overline{g}_i), i \in N)$ is not Pareto efficient. This means that there exists a feasible allocation $((x_i, g_i), i \in N)$ such that for each consumer i

$$u_i(x_i, g_i, G_{-i}) > u_i(\overline{x}_i, \overline{g}_i, \overline{G}_{-i})$$

From utility maximization, it holds that for each consumer i:

$$p \cdot x_i + \pi_i \cdot g_i + \pi_{-i} \cdot G_{-i} > p \cdot w_i.$$

Given the fact that for each $i \in N$, $G_{-i} = \sum_{j \neq i} g_j$, summing up over the above inequalities, one obtains

$$\sum_i (p \cdot x_i + \pi_i \cdot g_i + \pi_{-i} \cdot \sum_{j \neq i} g_j) > \sum_i p \cdot w_i$$

Given the fact that $\pi_i + \sum_{j \neq i} \pi_{-j} = p^g$, rearranging the terms with respect to each consumer public goods provision, will give us

$$(p, p^g) \cdot (\sum_i (x_i - w_i), \sum_i g_i) > 0.$$

Since the allocation $((x_i, g_i), i \in N)$ is feasible we have $(\sum_i (x_i - w_i), \sum g_i) \in Y$. This yields a contradiction to property (i) in the definition of warm glow equilibrium.$\square$

In the following we provide a decentralization of Pareto efficient allocations as warm glow equilibria. Our result shows that we can decentralize any interior Pareto efficient allocation with prices. This shows that an analogue to the second welfare theorem also holds for the warm glow economy. Similar results for standard public goods economy were first introduced by Foley (1970). In his approach, Foely (1970) considers the concept of public competitive equilibrium, which corresponds to a Lindahl equilibrium, after-tax redistribution.[4]

Theorem 2: *(Second Welfare Theorem for a warm glow economy)*
If the allocation $((\overline{x}_i, \overline{g}_i), i \in N)$ is Pareto efficient, then, there exists a price system $((\pi_i, \pi_{-i})_{i \in N}, p, p^g) \geq 0$, such that

[4] See also Milleron (1972), Boyd and Conley (1997), Conley and Smith (2005) and Murty (2006).

(i). For all $(y, g) \in Y$,

$$(p, p^g) \cdot (\sum_i (\overline{x}_i - w_i), \sum_i \overline{g}_i) \geq (p, p^g) \cdot (y, g).$$

(ii). For each consumer $i \in N$ if

$$u_i(x_i, g_i, G_{-i}) > u_i(\overline{x}_i, \overline{g}_i, \overline{G}_{-i}),$$

then

$$p \cdot x_i + \pi_i \cdot g_i + \pi_{-i} \cdot G_{-i} > p \cdot \overline{x}_i + \pi_i \cdot \overline{g}_i + \pi_{-i} \cdot \overline{G}_{-i}$$

(iii). For each consumer $i \in N$,

$$\pi_i + \sum_{j \neq i} \pi_{-j} = p^g.$$

Proof: The idea of the proof follows Foley (1970) in extending the commodity space for public goods. However, due to the warm glow effects, our extension is quite different from Foley's (1970). Indeed, for each consumer our extension distinguishes between own public goods provision and others' public goods provision. Thus, a larger number of commodities is considered.

We define the set:

$$F = \{(y, g_1, G_{-1}, \ldots, g_N, G_{-N}) \mid \text{ for each } i, \ G_{-i} = \sum_{j \neq i} g_j \text{ and } (y, \sum_{j \in N} g_j) \in Y\}.$$

Since Y is a convex cone F is also a convex cone. We next define the set

$$D = \{(\sum_{i \in N} y_i, g_1, G_{-1}, \ldots, g_N, G_{-N}) \mid \text{ for each } i, u_i(y_i + w_i, g_i, G_{-i}) > u_i(\overline{x}_i, \overline{g}_i, \overline{G}_{-i})\}.$$

The set D is convex and nonempty since the utility functions are increasing and quasi-concave.

Since the allocation $((\overline{x}_i, \overline{g}_i), i \in N)$ is Pareto efficient we have $F \cap D = \emptyset$. From the Minkowski's separating hyperplane theorem, there is a hyperplane with normal $\alpha = (p, \pi_1, \pi_{-1}, \ldots, \pi_N, \pi_{-N}) \neq 0$, and a scalar C such that (i). for all $(y, g_1, G_{-1}, \ldots, g_N, G_{-N}) \in F$

$$(p, \pi_1, \pi_{-1}, \ldots, \pi_N, \pi_{-N}) \cdot (y, g_1, G_{-1}, \ldots, g_N, G_{-N}) \leq C,$$

(ii). for all $(y, g_1, G_{-1}, \ldots, g_N, G_{-N}) \in D$

$$(p, \pi_1, \pi_{-1}, \ldots, \pi_N, \pi_{-N}) \cdot (y, g_1, G_{-1}, \ldots, g_N, G_{-N}) \geq C.$$

Since F is a closed convex cone with vertex zero, it follows that we can choose $C = 0$. Moreover, since the allocation $((\overline{x}_i, \overline{g}_i), i \in N)$ is Pareto efficient we have

$$(\sum_i (\overline{x}_i - w_i), \overline{g}_1, \overline{G}_{-1}, \ldots, \overline{g}_N, \overline{G}_{-N}) \in F \cap \bar{D}.$$

Hence, it follows from (i) and (ii) in the separation theorem that

$$(p, \pi_1, \pi_{-1}, \ldots, \pi_N, \pi_{-N}) \cdot (\sum_i (\overline{x}_i - w_i), \overline{g}_1, \overline{G}_{-1}, \ldots, \overline{g}_N, \overline{G}_{-N}) = 0. \qquad (1)$$

Now, we claim that, for any two consumers j_1 and j_2, it holds that

$$\pi_{j_1} + \sum_{i \neq j_1} \pi_{-i} = \pi_{j_2} + \sum_{i \neq j_2} \pi_{-i}.$$

Suppose this was not the case, then, without loss of generality, one could assume that for some public goods, say the k^{th}, it holds that[5]

$$(\pi_{j_1})_k + \sum_{i \neq j_1} (\pi_{-i})_k > (\pi_{j_2})_k + \sum_{i \neq j_2} (\pi_{-i})_k.$$

Let δ_k be a vector in $\mathbb{R}^K_+$ consisting of one unit of the k^{th} public good and nothing else. Let us now consider the following public goods bundle, $\overline{G}' = (\overline{g}'_i, \ldots, \overline{g}'_N)$, defined as follows

$$\overline{G}' = \begin{cases} \overline{g}'_{j_1} = \overline{g}_{j_1} + (\overline{g}_{j_2})_k \delta_k, & \\ \overline{g}'_{j_2} = \overline{g}_{j_2} - (\overline{g}_{j_2})_k \delta_k, & \\ \overline{g}'_i = \overline{g}_i, & \text{if } i \in N \setminus \{j_1, j_2\}. \end{cases}$$

[5]For any $a \in R^K$, $(a)_k$ is the k^{th} component of a.

Since, for each consumer i, $\overline{g}_i \in \mathbb{R}^K_{++}$, it follows that

$$(\sum(\overline{x}_i - w_i), \overline{g}'_1, \overline{G}'_{-1}, \ldots, \overline{g}'_N, \overline{G}'_{-N}) \in F.$$

Moreover, from (1) it follows that

$$(p, \pi_1, \pi_{-1}, \ldots, \pi_N, \pi_{-N}) \cdot (\sum(\overline{x}_i - w_i), \overline{g}'_1, \overline{G}'_{-1}, \ldots, \overline{g}'_N, \overline{G}'_{-N}) > 0,$$

but, this contradicts property (i) of the separation theorem. Thus, we set up

$$p^g = \pi_i + \sum_{j \neq i} \pi_{-j}, \text{ for all } i \in N.$$

From monotonicity of preferences it follows that

$$(p, \pi_1, \ldots, \pi_N) \geq 0.$$

Otherwise, property (ii) of the separation theorem would be violated by increasing the consumption of any good with a negative price. Suppose that $p = 0$. Therefore, it holds that for some consumer i and public good k we have either $(\pi_i)_k > 0$ or $(\pi_{-i})_k > 0$. Then, there will be a point F with strictly positive profit, which contradicts property (i) of the separation theorem.

Suppose that for some consumer i, and some consumption bundle (x_i, g_i, G_{-i}) one has

$$u_i(x_i, g_i, G_{-i}) > u_i(\overline{x}_i, \overline{g}_i, \overline{G}_{-i}).$$

Then,

$$(x_i - w_i + \sum_{j \neq i}(\overline{x}_j - w_j), \overline{g}_1, \overline{G}_{-1} - \overline{g}_i + g_i, \ldots, g_i, \overline{G}_{-i}, \ldots, \overline{g}_N, \overline{G}_{-N} - \overline{g}_i + g_i) \in \bar{D}.$$

Therefore, from (ii) in the separation theorem it holds that

$$\alpha \cdot (x_i - w_i + \sum_{j \neq i}(\overline{x}_j - w_j), \overline{g}_1, \overline{G}_{-1} - \overline{g}_i + g_i, \ldots, g_i, \overline{G}_{-i}, \ldots, \overline{g}_N, \overline{G}_{-N} - \overline{g}_i + g_i) \geq 0$$

Since only the terms relating to i differ from (1) one obtains

$$p \cdot x_i + \pi_i \cdot g_i + \pi_{-i} \cdot G_{-i} \geq p \cdot \overline{x}_i + \pi_i \cdot \overline{g}_i + \pi_i \cdot \overline{G}_i$$

Assume that this was an equality. Since $x_i \in \mathbb{R}^L_{++}$ there exists $x'_i \ll x_i$. By quasi-concavity and continuity, along the line joining (x_i, g_i, G_{-i}) and (x'_i, g_i, G_{-i}) there is a point in the consumption set of i which, at the same time, is strictly preferred and costs less when compared to $(\overline{x}_i, \overline{g}_i, \overline{G}_{-i})$. This would contradict property (ii) of the separation theorem.$\square$

3.1 Existence of warm glow equilibrium

As with any proposed solution concept, it is of paramount importance to show the existence of an equilibrium in significantly meaningful class of economies. Fortunately, Likewise the existence of Lindahl equilibrium is derived from standard existence results of competitive equilibrium in private goods economy, the existence of warm glow equilibrium in this paper is derived from standard existence result for Lindahl equilibrium. Indeed, the idea of the proof is to consider each consumer's public goods provision as a bundle of public goods on its own merit. Thus, understandably, this extension will capture the warm glow effects of each consumer's provision.

Now, we state our main existence result for warm glow equilibrium.

Theorem 3:*(Existence of a warm glow equilibrium)* There exists a warm glow equilibrium.

Proof: We construct an auxiliary economy $\widehat{E}$, with N consumers, L private goods and KN public goods. A bundle of public goods for the auxiliary economy $\widehat{E}$ will be $\widehat{G} = (g_1, \ldots, g_N)$, and will list the individual public goods provisions of the N consumers.

The production set for the auxiliary economy $\widehat{E}$ is characterized by $\widehat{Y} \subset \mathbb{R}^L \times \mathbb{R}^{KN}_+$. A production plan is feasible for the economy $\widehat{E}$, i.e. $(y, \widehat{G}) \in \widehat{Y}$, if and only if $(y, \sum_i g_i) \in Y$. It is clear that $\widehat{Y}$ is a closed convex cone with vertex the origin; and satisfies the usual conditions of irreversibility, no free production, and free disposal.

We define the utility functions $\widehat{u}_i$ for the auxiliary economy $\widehat{E}$ such that $\widehat{u}_i(x_i, \widehat{G}) = u_i(x_i, g_i, G_{-i})$. Using standard existence theorems (see for example , Foley (1970), Milleron (1972), Robert (1974), and Florenzano and del Mercato (2006)) one could get the existence of a Lindahl equilibrium for the auxiliary economy $\widehat{E}$, $((x_i, \Pi_i)_{i\in N}, \widehat{G}, p)$, that is, $((x_i)_{i\in N}, \widehat{G})$ is a feasible allocation in $\widehat{E}$, p is the price for private goods, $(\Pi_i)_{i\in N}$ are the associated personalized prices, and

(i). For all $(y, \widehat{G}') \in \widehat{Y}$,

$$(p, \sum_i \Pi_i) \cdot (y, \widehat{G}') \leq (p, \sum_i \Pi_i) \cdot (\sum_i (x_i - w_i), \widehat{G}) = 0$$

(ii). For each consumer $i \in N$,

$$p \cdot x_i + \Pi_i \cdot \widehat{G} = p \cdot w_i,$$

and if

$$\widehat{u}_i(x_i', \widehat{G}') > \widehat{u}_i(x_i, \widehat{G})$$

then

$$p \cdot x_i' + \Pi_i \cdot \widehat{G}' > p \cdot w_i.$$

For each consumer i, let $\Pi_i = (\Pi_i^1, \dots, \Pi_i^N)$, where Π_i^j is the personalized price faced by consumer i to finance the public goods b g_j. We claim that

$$\sum_i \Pi_i^1 = \dots = \sum_i \Pi_i^N,$$

which means the prices of producing each consumer's public goods provision are equal. Suppose this was not the case, then, without loss of generality, one could assume that for two consumers j_1 and j_2 and for the k^{th} public good, it holds that

$$\sum_i (\Pi_i^{j_1})_k > \sum_i (\Pi_i^{j_2})_k.$$

Since

$$(\sum_i (x_i - w_i), \widehat{G}) = (\sum_i (x_i - w_i), g_1, \dots, g_N)$$

Let us consider again the following public goods bundle, $\widehat{G}' = (g_i', \dots, g_N')$, defined as follows

$$\widehat{G}' = \begin{cases} g_{j_1}' = g_{j_1} + (g_{j_2})_k \delta_k, & \\ g_{j_2}' = \overline{g}_{j_2} - (g_{j_2})_k \delta_k, & \\ g_i' = g_i, & \text{if } i \in N \setminus \{j_1, j_2\}. \end{cases}$$

Since, for each consumer i, $g_i \in \mathbb{R}^K_{++}$,
It is obvious that $(\sum_i (x_i - w_i), \widehat{G}') \in \widehat{Y}$. Also, since

$$(p, \sum_i \Pi_i) \cdot (\sum_i (x_i - w_i), \widehat{G}) = 0,$$

it follows that

$$(p, \sum_i \Pi_i) \cdot (\sum_i (x_i - w_i), \widehat{G}') > 0.$$

This would contradict property (i) of the Lindahl equilibrium of the auxiliary economy $\widehat{E}$.

Now, we claim that for each consumer i and for any two consumers $j_1, j_2 \in N \setminus \{i\}$ one has $\Pi_i^{j_1} = \Pi_i^{j_2}$. Suppose this was not the case. Then, without loss of generality, one could assume that for some public goods, say the k^{th}, it holds that $(\Pi_i^{j_2})_k < (\Pi_i^{j_1})_k$. In this case, it holds that

$$p \cdot x_i + \Pi_i \cdot \widehat{G}' < p \cdot w_i.$$

Therefore, by monotonicity of preferences there exists $(x_i^*, \widehat{G}^*)$ such that

$$\widehat{u}_i(x_i^*, \widehat{G}^*) > \widehat{u}_i(x_i, \widehat{G}),$$

and

$$p \cdot x_i^* + \pi_i \cdot \widehat{G}^* \leq p \cdot w_i.$$

This yields a contradiction to property (ii) of the Lindahl equilibrium of the auxiliary economy $\widehat{E}$.

Let us set $\pi_i = \Pi_i^i$ and $\pi_{-i} = \Pi_j^i$. It is easy to check that $((x_i, g_i, \pi_i, \pi_{-i})_{i \in N}, p, p^g)$ is a warm glow equilibrium.$\square$

4 Andreoni's utility function formulation

In the warm glow model Andreoni (1989,1990), consumer's i public goods provision g_i enters into the arguments of his utility function twice, that is $u_i(x_i, g_i, G) = u_i(x_i, g_i, g_i + G_{-i})$. In the voluntary contribution equilibrium, the Andreoni's formulation of the utility function leads to an easier characterization of the theoretical results than this paper formulation of the utility function $u_i(x_i, g_i, G_{-i})$. Andreoni (2006) argues that, amongst other things, the straight forward expression of convexity in his formulation of utility function has proved beneficial to the characterization. Moreover, it has the methodological advantage of being flexible to accommodate the two opposite cases of standard public goods provision model and pure warm glow model. In our framework, since we are interested in the competitive equilibrium, the two formulations of the utility functions will lead us to same result. This is due to the fact that we are interested in the underlying preferences rather than their utility presentations. To illustrate the above observation, assume that instead of $u_i(x_i, g_i, G_{-i})$ representing the preferences of consumer i we have a utility function of the form $v_i(x_i, g_i, G)$. Then, the budget constraint of consumer i could be re-written as

$$p \cdot x_i + (\pi_i - \pi_{-i}) \cdot g_i + \pi_{-i} \cdot G = p \cdot w_i;$$

and the utility maximization of consumer i could be re-written as

$$v^i(x_i, g_i, G) > v^i(\overline{x}_i, \overline{g}_i, \overline{G})$$

implies

$$p \cdot x_i + (\pi_i - \pi_{-i}) \cdot g_i + \pi_{-i} \cdot G > p \cdot w_i.$$

5 Conclusion

Price based mechanisms achieving efficient outcomes are highly desirable in public economics for their insights to both normative and positive analysis. This paper introduces a competitive mechanism for a warm glow economy and shows the three fundamental principals of general equilibrium. In the public economics literature, the Lindahl competitive mechanism is often criticized for its lack of incentive compatibility since consumers have incentives to report false preferences. It will be interesting to investigate in a warm glow economy whether the in part private good aspect of consumers' public goods provision will lead to more truthful reports of consumers preferences and as a result the warm glow equilibrium allocations are more likely to be achieved.

References

[1] Andreoni, J. (1989) "Giving With Impure Altruism: Applications to Charity and Ricardian Equivalence," *Journal of Political Economy* 97, 1447-1458.

[2] Andreoni, J. (1990) "Impure Altruism and Donations to Public Goods: A Theory of warm glow Giving,"*Economic Journal* 100, 464-477.

[3] Andreoni, J. (2006) "Philanthropy,"in the L.-A. Gerard-Verat et. al. (eds) *Handbook of Giving, Reciprocity and Altruism* (Elsevier/ North-Holland).

[4] Becker, G.S. (1974) "A Theory of Social Interactions," *Journal of Political Economy* 82, 10631093.

[5] Bergstrom, T. (1976) "Collective choice and the lindahl allocation method,"In Steven A. Y. Lin, editor, *Theory and Measurement of Economic Externalities,* chapter IV, pages 107131. Academic Press, New York.

[6] Bernheim, D and A. Rangel (2005) "Behavioral public economics: welfare and policy analysis with non-standard decision makers," *NBER Working Paper* 11518, *forthcoming in Economic Institutions and Behavioral Economics, Peter Diamond and Hannu Vartiainen* XX

[7] Bonnisseau, J.M. (1991) "Existence of Lindahl equilibria in economies with nonconvex production sets," *Journal of Economic Theory* 54, 409-416.

[8] Boyd III, J. H., and J. P. Conley (1997), "Fundamental Nonconvexities in Arrovian Markets and the Coasian Solution to the Problem of Externalities," *Journal of Economic Theory* 72, 388407.

[9] Camerer, Colin (2003) "Behavioral Game Theory: Experiments in Strategic Interaction,"Princeton: Russell Sage Foundation.

[10] Conley, J. (1994) "Convergence Theorems on the Core of a Public Goods Economy: Sufficient Conditions," *Journal of Economic Theory* 62, 161-185.

[11] Conley, J. and J. Smith (2005) "Coalitions and clubs: Tiebout equilibrium in large economies," *in Group Formation in Economics: Networks, Clubs, and Coalitions, edited by Gabrielle Demange and Myrna Wooders,* Cambridge University Press.

[12] Cornes, R. and T. Sandler (1984) "Easy Riders, Joint Production and Public Goods," *Economic Journal* 94, 580-598.

[13] Fabre-Sender, F. (1969) "Biens Collectifs et Biens Qualit Variable," Paris: CEPREMAP.

[14] Florenzano, M and E. Del Mercato (2006) "Edgeworth and Lindahl-Foley equilibria of a General Equilibrium Model with Private Provision of Pure Public Goods," *Journal of Public Economic Theory* 8, 713-740.

[15] Foley, D. (1967) “Resource allocation and the public sector,” *Yale Economic Essays* 7, 43-98.

[16] Foley, D. (1970) “Lindahl’s Solution and the Core of an Economy with Public Goods,” *Econometrica* 38, 66-72.

[17] Khan, M, A. and R. Vohra (1987) “An Extension of the Second Welfare Theorem to Economies with Nonconvexities and Public Goods,” *The Quarterly Journal of Economics* 102, 223-241.

[18] Kingma, R. (1989) “An Accurate Measurement of the Crowd-out Effect, Income Effect, and Price Effect for Charitable Contributions,” Journal of Political Economy 97, 1197-1207.

[19] Ledyard, J. (1995) “Public Goods: A Survey of Experimental Research, ”in Handbook of Experimental Economics, John Kagel and Alvin Roth eds, Princeton University Press, 111-94

[20] Milleron, J.C. (1972) “Theory of Value with Public Goods: A Survey Article,” *Journal of Economic Theory* 5, 419-477.

[21] Murty, S. (2006) “Externalities and fundamental nonconvexities: a reconciliation of views and implications for decentralization,” *Warwick Economic Research Papers*, No 756.

[22] Myles, G. (1995) “Public Economics,” *Cambridge University Press.*

[23] Payne, A. A., (1998). “Does the Government Crowd Out Private Donations?: New Evidence from a Sample of Nonprofit Firms,” *Journal of Public Economics* 69, 323-345.

[24] Roberts, D.J. (1974) “The Lindahl solution for economies with public goods ” *J. Public Econ.* 3, 2342.

[25] Sugden, R. (1982): “On the Economics of Philanthropy,” *Economic Journal* 94, 341-350.

[26] Temimi, A. (2001) “Does Altruism Mitigate Free-riding and Welfare Loss?” *Economics Bulletin* 8, 1-8

[27] Wooders, M. (1997) “Equivalence of Lindahl equilibria with participation prices and the core,” *Econ. Theory* 9, 113-127.

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