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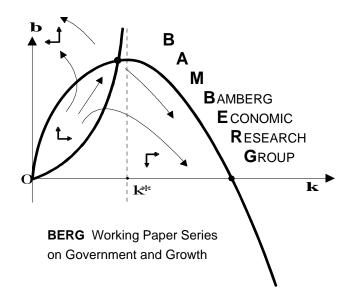
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# Removing systematic patterns in returns in a financial market model by artificially intelligent traders

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# **Redaktion:**

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Removing systematic patterns in returns in a financial

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**Abstract:** 

The unpredictability of returns counts as a stylized fact of financial markets. To reproduce this fact, modelers

usually implement noise terms - a method with several downsides. Above all, systematic patterns are not

eliminated but merely blurred. The present article introduces a model in which systematic patterns are removed

endogenously. This is achieved in a reality-oriented way: Intelligent traders are able to identify patterns and

exploit them. To identify and predict patterns, a very simple artificial neural network is used. As neural network

mimic the cognitive processes of the human brain, this method might be regarded as a quite accurate way of how

traders identify patterns and forecast prices in reality. The simulation experiments show that the artificial traders

exploit patterns effectively and thereby remove them, which ultimately leads to the unpredictability of prices.

Further results relate to the influence of pattern exploiters on market efficiency.

**Keywords:** 

financial markets; autocorrelations; artificial intelligence; agent-based modeling

**JEL Classification:** 

C45; G14; G17

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# 1. Problem Setting

As one of their most essential statistical properties, price returns on financial markets are free from significant autocorrelations. Being a stronger proposition, advocates of market efficiency believe that financial markets are virtually unpredictable (see Fama 1965 in this context). Whereas the absence of autocorrelations (AA) is easy to show econometrically, the Absence of Systematic Patterns in price dynamics (ASP) is harder if not impossible to verify, as the number of potential patterns is infinite and patterns can be highly complex. Technical trading takes the existence of such patterns as its central credo, and empirical studies provide some evidence that some systematic patterns do exist, e.g. the so-called January Effect (Thaler 1987). Nevertheless, AA and ASP remain accurate outlines of market behavior.

The general fulfillment of AA and ASP is a product of the profit-seeking behavior of traders. At the moment traders identify or believe that they have identified a systematic pattern in prices, they trade on it and thereby exploit it, which ultimately leads to the extinction of the particular pattern.<sup>1</sup>

Financial market models (surveys by Hommes 2006 and LeBaron 2006) seek to imitate the statistical properties of real markets. AA and ASP, therefore, constitute important criteria to evaluate the accuracy of the behavior of these models. To test econometrically if prices evolve more or less unpredictably, modelers usually limit themselves to the replication of AA. AA is a necessary condition for ASP but not a sufficient one as systematic patterns might be too complex to be mirrored in significant autocorrelations.

Table 1 provides an overview of selected financial market models with regard to the particular method used to reproduce AA. The table is based on the survey by Chen et al. (2009). The authors review 50 financial market models, classify them according to their origin and design, and report the particular stylized facts explained. The 27 models which, according to the authors, "explain" the absence of autocorrelations have been examined in more detail.

#### -- Table 1 about here --

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<sup>&</sup>lt;sup>1</sup> We admit that this logic is idealized. Regularities could be extremely complex such that traders do not recognize them, or some traders do recognize them but their trading capital could be too low to exploit the pattern entirely (analogue to the well-known "limits of arbitrage" by Shleifer 1997). Further, the argument is not valid for mid-term or long-term patterns. For example, financial dynamics appear to oscillate in the long term due to business cycles (Greenwald and Stiglitz 1993).

The table illustrates that to reproduce AA, modelers commonly use stochastic model components, and this applies for every model design and origin.<sup>2</sup> In some models (e.g. table entries 2, 4, 6 and 11), such noise terms are contained directly in the mechanism of price formation. Others chose more elegant, indirect solutions by implementing random terms into the behavior of agents (12, 21) or into the components agents react to, which can be news (10, 13, 20), trading signals (18, 19, 22) or dividends (13, 26, 27). A popular argument to implement stochastic components refers to the behavior of so-called "noise traders" (1, 4, 6, 25). According to Black (1986), noise traders "trade on noise as if it were information". As the particular behavior of noise traders does not follow a uniform logic, it can be nicely approximated stochastically.

The use of stochastic components is a viable and effective way to replicate AA. It is effective since by adding a sufficient amount of noise, the systematic behavioral patterns the deterministic model framework would generate can be blurred and prices evolve largely unpredictably. Nevertheless, the method is not free from considerable downsides. First, the amount of noise needed to eliminate detectable patterns can be very large, such that merely a small part of the movements of prices remains attributable to explicit model components. In general, the amount of noise needed declines with the structural complexity of the model as the interaction of deterministic mechanisms can lead to richer behavior. Greater model complexity, however, deteriorates the model's tractability. Second, if random demand terms are chosen, a significant proportion of the total demand originates model-exogenously. It would be more desirable to model the sources of this demand explicitly, and thereby to improve the subjective completeness of the model. Third, the greater the share of total demand which arises randomly, the worse prices react to shifts of fundamentals. As a realistic behavior, we would expect that fundamental news is reflected more or less in changes of prices, at least when the news is considerable. Fourth, through stochastic components, systematic patterns are never removed in a strict sense but merely blurred. A mechanism causing the elimination of patterns endogenously is absent. Therefore, a genuine explanation for the absence of systematic patterns is not given.

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<sup>&</sup>lt;sup>2</sup> Of course, the reproduction of AA is not the only purpose of using stochastic components. Producing volatility clustering or simply capturing effects which should not be modeled explicitly are examples for other functions. The modification of the grand-canonical minority game (e.g. Slanina et al. 1999) in table entry 17 constitutes an exception, as the authors attribute the AA-property to the fact that "speculators are exploring all available information". Unfortunately this feature is not presented in detail.

The present article presents a model in which systematic patterns are removed endogenously, and without exogenous noise (apart from fundamental news) being added. The method is inspired by reality as it assumes traders are able to detect patterns in price dynamics and to exploit them. To detect and predict patterns, a linear regression model is used. The linear regression model can be interpreted as the simplest form of an Artificial Neuronal Network (ANN). ANNs mimic the information processing of the human brain technically and thus represent a relatively accurate way to model the perception of financial traders. As a second contribution, the simulations provide insights into the effect of pattern exploitation on market efficiency. It is shown that exploiters enforce the tendency of prices to reflect changes of value. On the other hand, the discrepancy between prices and value can rise.

The remainder of this article is organized as follows. Treating methodical issues, section 2 illustrates theoretically how systematic patterns in prices can be removed endogenously. Section 3 deals with the application. At first, it introduces a simple model in which systematic patterns are recognized by traders and exploited. Then, the dynamic features of the model are illustrated while varying the impact of pattern exploiters. Section 4 summarizes the insights gained and highlights needs for future research.

# 2. Method: Endogenous Eliminating of Systematic Patterns

The endogenous elimination of systematic patterns embraces three components: (i) traders able to identify systematic patterns and to trade on them; (ii) a technique for the identification of these patterns; (iii) specific model features facilitating the effective implementation of (i) and (ii). In the following, the three components will be explained in the given order.

#### 2.1 The Basic Idea

Consider an arbitrary asset market in which prices are formed in discrete steps of time. In such a framework, the price in time t,  $P_t$ , necessarily results from a set of information given at t. Let  $\Phi_t$  denote this information set. Fundamental trading (Greenwald et al. 2001; Damodaran 2002) rests on the belief that  $\Phi_t$  includes fundamental information – the price of some asset is not completely independent from its fundamental value,  $F_t$ . Consequently, fundamental traders seek to identify  $F_t$  and to exploit mispricing. In contrast, technical trading (Murphy 1999; Pring 2002) assumes that  $\Phi_t$  also includes past prices,  $(P_{t-1}, P_{t-2}, ..., P_{t-n})$ . If the price

in *t* is indeed influenced by the evolution of prices in the past, systematic patterns in prices exist. These basic insights can be formalized as follows.

$$P_t = g_t[\Phi_t], \quad \Phi_t = [F_t, P_{t-1}, P_{t-2}, \dots, P_{t-n}],$$
 (1)

where  $g_t$  is a deterministic or stochastic function of arbitrary complexity describing the behavior of prices at time t. Note that (1) does not state that prices are necessarily influenced by their history nor by value (the respective coefficients in  $g_t$  could equal zero) but merely that these are possible determinants for prices.

Financial trading implies the formation of expectations about prices, where  $E_t^i[P_{t+\Delta}]$  should denote the price of time  $t + \Delta$  ( $\Delta > 0$ ) expected by trader or trader group i at t. The expectation of a financial speculator is a central determinant for her<sup>3</sup> demand of assets. Taking this into account, in many financial market models, the formulation of the net demand is based on the following principle:

$$D_t^i = \alpha^i \left( \left( E_t^i [P_{t+1}] | \Phi_t^i \right) - P_t \right), \tag{2}$$

with

$$E_t^i[P_{t+1}]|\Phi_t^i = h_t^i(\Phi_t^i), \tag{3}$$

where  $\alpha^i$  is a constant parameter, and  $\Phi^i_t$  is a set of fundamental and/or technical information available in t and considered by i for the formation of expectations. The equation stipulates that traders buy (/sell) if the price they would pay (/receive) is below (/above) their expectation of the price in the next period, and their net demand rises with the difference between their expectation and the transaction price.  $\alpha^i$  can be interpreted as i's reaction intensity as it regulates the net demand for a given value of  $(E^i_t[P_{t+1}] - P_t)$ .  $h^i_t$  is an arbitrary deterministic or stochastic expectation function. Popular examples are  $h^F_t(P_t, F_t) := P_t + \delta^F(F_t - P_t)$  or  $h^C_t(P_t, P_{t-1}) := P_t + \delta^C(P_t - P_{t-1})$ , with  $\delta^F$  and  $\delta^C$  being positive parameters.  $h^F_t$  represents a stylized description of the philosophy of fundamentalists, who expect prices to return to value to some degree.  $h^C_t$  expresses the trend extrapolation by technical traders.

Deterministic features of the rules of trading, as the ones above, are the origin of systematic patterns in prices. For example, the behavior of technical traders with expectation function  $h_t^C$  induces a piece of positive feedback into the dynamics of prices. However, any mechanism which would work against these patterns is absent in the above framework.

<sup>&</sup>lt;sup>3</sup> For the sake of inclusive language, the author will use the feminine pronouns to represent individual actions.

Before an endogenous mechanism to remove systematic patterns is developed, it is helpful to recall the mechanism of how patterns may be removed in real markets. This mechanism involves three necessary steps:

- I. Pattern recognition: Traders recognize systematic pattern in price dynamics.
- II. Pattern exploitation: Traders seek to take profits by trading on the patterns identified.
- III. Pattern reduction: The exploitation of a pattern reduces its appearance and ultimately removes it.

A model in which all traders follow the rules  $h_t^F$  or  $h_t^C$ , or similar static rules, has problems fulfilling the very first necessary step (I): Traders do not recognize systematic patterns but stick to a constant trading behavior. The fulfillment of (I) requires that some traders seek to estimate the function  $g_t(\Phi_t)$ , which describes the actual behavior of prices. The index X should denote these "pattern exploiters". Their expectation formation can be written as:

$$E_t^X[P_{t+1}]|\Phi_t^X = \widehat{g_{t+1}}_t^X(\Phi_t^X), \tag{4}$$

where  $\widehat{g_{t+1}}_t^X$  stands for the estimation of  $g_{t+1}$  (the function giving the price  $P_{t+1}$ ) by exploiters in t. A trader forming his expectation according to (4) and formulating his demand according to (1), buys (/sells) for each price  $P_t$  for which she predicts a rise (/fall) of prices from t to t+1. This behavior is perfectly profit-oriented as the price change from t to t+1 (i.e. the return  $P_{t+1} - P_t$ ) determinates the immediate profit of the trader. The trading behavior specified by (1) and (2), hence, have satisfies step (II), as patterns are exploited.

Whether the exploitation of the identified pattern reduces the pattern identified or not, such that step (III) is fulfilled, depends on the mechanism of price adaption. Financial market models often use a stylized market maker as proposed by Farmer and Joshi 2002. The marker maker acts as an intermediary between supply and demand which reacts to the amount of excess demand in the market through the adjustments of prices. This behavior can be formalized as:  $P_{t+1} = P_t + \alpha^M D_t$ , where  $D_t$  represents the excess demand in the market at time t and  $\alpha^M$  is a positive reaction coefficient. If the marker maker approach is chosen, step (III) can be violated. The reason is that prices do not necessarily reflect the expectation of traders, but the market maker herself can create systematic patterns in prices. For example, if the value of  $\alpha^M$  is too high, the market maker overacts, and prices tend to fluctuate around the equilibrium price. The problem is prevented if equilibrium prices are computed directly. The respective formalization is:

$$P_t = P_t | (D_t(P_t) = 0). (5)$$

To prove that in the market described, regularities can be removed endogenously, we formulate the following proposition:

**Proposition 1**: If the following conditions are fulfilled, prices will evolve completely unpredictably: (i) every trader behaves according to equation (2) and (4); (ii) traders are omniscient – they know the function  $g_t$  and all relevant information contained in  $\Phi_t$  in all t; (iii) prices result according to (5).

**Proof**: Condition (ii) implies that  $E_t^i[P_{t+1}] = E_t^j[P_{t+1}] = E_t[P_{t+1}] \, \forall i,j$  (If traders are omniscient, they will arrive at the same price expectation,  $E_t[P_{t+1}]$ ). Using this equality, the total demand in the market,  $D_t$ , is given by a reformulation of eq. (2):

$$D_t = N * \alpha(E_t[P_{t+1}](\Phi_t') - P_t), \tag{6}$$

where N is the number of traders. Inserting (6) into (5) yields for the equilibrium price  $P_t = P_t | N * \alpha (E_t[P_{t+1}](\Phi_t^i) - P_t) = 0$ . Since  $N, \alpha > 0$ , the only solution of latter equivalence is the price for which  $(E_t[P_{t+1}](\Phi_t^i) - P_t) = 0$ . Hence,

$$P_t = E_t[P_{t+1}](\Phi_t^i) \,\forall t. \tag{7}$$

In other words, under conditions (i – iii), at each time t the price  $P_t$  will be such that traders neither expect a rise nor a fall of prices in the next period, as  $P_t$  already reflects all information that traders believe to be relevant. Therefore, any non-zero return  $r_{t+1} = P_{t+1} - P_t$  can only be due to information which traders did not consider in t. From the perspective of traders, the returns  $r_t$  are thus unpredictable for all t. Note that (7) alone does not imply prices are free from any systematic patterns but only from those patterns which traders have recognized. The absence of any pattern is dependent on the omniscience of traders as proposed by condition (ii). Omniscient traders know  $g_t$  and  $\Phi_t$  by definition. Hence, any divergence of  $P_t = E_t[P_{t+1}](\Phi_t) = g_t(\Phi_t)$  from  $P_{t+1} = g_{t+1}(\Phi_{t+1})$  can only be due to true news – exogenous events that could not be known nor be expected in t. It is easy to see that such news can either be equal in a change of  $g_t$  or  $\Phi_t$  from t to t+1. A change of  $g_t$  can be due to an alteration of the market structure (e.g. a new mechanism of price setting) or traders' behavior. A change of  $\Phi_t$ , on the other hand, can only be caused by a shift of the fundamental value  $F_t$  from t to t+1, as all other information included in  $\Phi_{t+1} = [F_{t+1}, P_t, P_{t-1}, \dots P_{t-n-1}]$  are already known to omniscient traders in t (including the price  $P_t$ , which, according to (1), is

equal to  $g_t(\Phi_t)$ ). As true news, such as new fundamental information, is by definition unsystematic and unpredictable, every price change from t to t+1 will be unpredictable, too. Proposition 1 describes an extreme, theoretical scenario, and aims at demonstrating that under specific conditions, any systematic pattern will be eliminated. The scenario is extreme and theoretical as it assumes the absence of any information deficit and perfectly rational behavior for all traders.4 Of course, in reality – as well as in the agent-based model presented later – these conditions are not fulfilled, such that some systematic patterns might occur at least temporarily – think of a sequence of positive returns during a speculative rally, for example. However, the consideration above can be used to derive the determinants for the degree to which systematic patterns are actually removed. The first determinant is the relative trading power of pattern exploiters. Only if their trading power is sufficiently great will prices indeed fully reflect their expectation such that eq. (7) is fulfilled. A violation of this condition can be read analogously to the "limits of arbitrage" (Shleifer 1997); Exploiters identify regularities in the evolution of prices but cannot exploit them entirely because their investment capital is too small. The second determinant is the knowledge of these traders. In principle, any pattern which is not identified correctly will not be exploited and, thus, can persist, although exploiters' trading power might be great. If traders miss existent patterns or misinterpret them, their trading activity moves prices towards some value  $P_t$  for which under c.p. assumptions  $P_{t+1} \neq P_t$ . Hence, the following price change is not completely unforeseeable, although eq. (7) might be true. In the ideal case, exploiters have perfect knowledge about  $g_t$  and  $\Phi_t$  at all t. In sum, the degree to which autocorrelations will be removed tends to be greater, the greater the trading power of pattern exploiters and the better their ability of pattern perception.

# 2.2 Techniques for Pattern Recognition and Price Prediction.

Eq. (1) has stated that the price in t is determined by  $P_t = g_t[\Phi_t]$ . The identification of patterns implies forming an estimation about  $g_t$ , denoted  $\widehat{g_t}$ . This estimation is then fundamental for price prediction. In this section, we briefly discuss three modeling

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<sup>&</sup>lt;sup>4</sup> Still, the assumptions made are not unusual in economic theory, particularly in studies dealing with market efficiency. Fama (1965), for example, writes: An "efficient" market is defined as a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants." Having made this definition, Fama argues that under similar assumption, the activity of "intelligent" traders cause prices to follow a random walk, which is free from any systematic patterns. Unfortunately, Fama does not derive this point from a formal proof but from a verbal argument.

alternatives to obtain  $\widehat{g_t}$ , including their pros and cons. Beside a) perfect knowledge, these alternatives are b) regression and c) artificial neuronal networks, where the latter two approximate  $g_t$  through price history.

# a) Perfect knowledge

Perfect knowledge of traders was occasionally assumed in section 2.1. It implies that  $\widehat{g_t} = g_t$ ,  $\forall t$ . As its greatest advantage, the method is the most effective one for the endogenous removal of systematic patterns, as any existent pattern will be exploited. Further, model complexity increases little, as no mechanism of pattern detection has to be implemented. On the other hand, perfect knowledge is not a realistic assumption for financial traders. Moreover,  $g_t$  can be very complex, because the behavior of pattern exploiters re-affects the law of motion, which in turn affects the behavior of exploiters. Solving such recursive problems can be intricate and identifying the true function  $g_t$  may be hardly possible.

#### b) Regression

If perfect knowledge is not given, traders have to identify regularities in prices from price history. The simplest method to do this is regression. As the first step, regression implies devising a reasonable regression model, which is a hypothetical relationship between the price  $P_t$ , representing the dependent variable, and the independent variables potentially determining  $P_t$ . For instance, agents could believe that prices  $P_t$  are possibly influenced by the present fundamental value  $F_t$  as well as by the prices in the two periods preceding,  $P_{t-1}$  and  $P_{t-2}$ , and that the relationship is linear. A corresponding generic form of  $\widehat{g_t}$  is:

$$\widehat{g_t}(F_t, P_{t-1}, P_{t-2}) := P_t = \beta_1 F_t + \beta_2 P_{t-1} + \beta_3 P_{t-2}$$
(8)

(In the model introduced in section 3, pattern exploiters will be realized by the very model above.) The second step consists of the estimation of the regression coefficients  $\beta_1$  to  $\beta_n$ . To this purpose, agents use a defined frame of historical data, e.g. the last N periods, and set the regression coefficients such that a certain error criterion, e.g. the mean square error (here:  $\sum_{k=t-N}^{-1} (E_k[P_k] - P_k)^2 / N$ ), is minimized for the time frame considered. For an advanced regression approach for stock market prediction, see Yang et al. (2002).

Choosing regression techniques for pattern detection is attractive as the method is relatively simple to implement and easy to understand. Nevertheless, by linear regression, autocorrelations in returns can be identified exhaustively. Hence, the method effectively

contributes to the fulfillment of the stylized fact of no correlations in raw returns. However, the complexity of the regularities learnable is limited by the regression model. For instance, if the regression model is linear, traders cannot identify non-linear regularities and so their predictions will imply considerable systematic errors. Thus, not all complex patterns will be exploited and removed. If pattern complexity is greater, a regression model of at least equal complexity is required. The design of such models implies many degrees of freedom. Simply choosing the correct model (if ascertainable) implies that traders have profound previous knowledge, which might be an unrealistic assumption.

## c) Artificial Neural Networks (ANNs).

ANNs (see Basheer and Hajmeer 2000 for an introduction) may be regarded as the most sophisticated method of establishing  $\widehat{g_t}$ . ANNs are inspired by the human brain, which consists of complex webs of densely interconnected neurons. When the aggregate input of a neuron exceeds a certain threshold, the cell "fires" and activates other linked neurons if their stimulation is sufficient. Invented by psychologist Frank Rosenblatt in 1958, ANNs replicate the biological process numerically. Here, neurons are represented by artificial units organized in layers. The first layer is the input layer whose units each represent a sensor for the value of one independent variable (here:  $\Phi_t = [F_t, P_{t-1}, P_{t-2}, ... P_{t-n}]$ ). The last layer is the output layer. The output unit yields the result, that is, the value of the dependent variable (here:  $P_t$ , respectively  $E_t[P_{t+1}]$ ). Between input and output layer, several hidden layers can be implemented. Units in the hidden and output layers each represent functions. (Often sigmoid functions are used.) The input of each of these functions is the sum of the outputs of the units in the upstream layer with each output multiplied by a weight factor. Through the respective setting of these weight factors, the ANN can represent a variety of relationships between dependent and independent variables. The step of training aims at "teaching" the ANN the relationships existent in the particular case of application. The so-called Backpropagation algorithm is common for this purpose. The algorithm takes the network output and compares it to the target value of a set of training examples. The discrepancy is used as an indicator for how to adapt the weights within the network. Backpropagation denotes the successive retracing of the estimation error from the output unit to earlier units to correct their weight factors.

In general, the complexity of the function learnable by the ANN rises with the number of hidden layers and the number of units in these layers. An ANN which does not contain any hidden layers is equivalent to a linear regression model as it can only represent linear functions. The weights of such an ANN correspond to the regression coefficients  $\beta$  and training the ANN will lead to the solution that minimizes a defined error criterion. The regression model specified by eq. (8) can thus be realized by an ANN. Interpreted that way, (8) can be regarded as quite an accurate reproduction of the cognition of relatively simpleminded but still intelligent traders.

To sum up, ANNs provide two considerable advantages: First, ANNs are capable of learning regularities of arbitrary complexity, provided that the number of hidden layers and units is great enough. Hence, by using ANNs, virtually any pattern can be eliminated, theoretically. The elimination of complex patterns is even possible by an ANN with a tractable structure. Cybenko (1988), for example, proves that any continuous and multivariate function can be approximated with an error approaching zero by a feed-forward network with only one hidden layer. Due to their ability to detect complex patterns, ANNs are a popular tool for financial forecasting (books on this topic were authored by Azoff 1994 and Gately 1996. For recent research see Majhi et al. 2009 or Nair et al. 2011). Second, ANNs are a very accurate way of modeling the perception of financial traders, because the principle derives from the workings of the human brain. Regarding this, financial market models using ANN-traders are still relatively scarce (examples include Beltratti and Margarita 1992, Belratti et al. 1996, and Hommes 2001).

The scarcity of financial market models based on ANNs may be due to three reasons. First, although the ANN per se can be created quickly, using it in a reasonable way is an intricate endeavor. In particular, the configuration of the ANN involves many degrees of freedom and identifying an appropriate design requires much trial and error. (We do not want to discuss the design choices in detail here, as this is its own topic, but point to the respective works dealing with such problems, e.g. Haykin 2009). Second, training ANNs can require considerable computational power. The computational demand rises more than proportionally with the number of units in network, because every new unit implies another weight factor to be learned for every link-neighbor. In the case of financial markets and the respective models, the problem becomes even worse because the network must be retrained often as the behavior of prices is not necessarily constant but new systematic patterns may emerge. To guarantee

that traders can identify some pattern at the moment it becomes established, retraining must even be done periodically. Third, the ANN(s) usually appears as a "black box" to the observer. The transparency of the model and its dynamics is reduced.

# 2.3 Necessary and Convenient Model Features

The techniques and building blocks presented up to now can be the cornerstones of a model in which systematic patterns are removed endogenously. Yet, in the model presented in the following, their implementation brings about the need for other specific model features. In the following, we provide an overview of the necessary and convenient features of that model for the endogenous removal of systematic patterns. The necessary features have already been explained:

#### (1) Pattern exploiters

At least some traders must be able to identify the patterns to be removed and trade on them.

#### (2) Appropriate price setting mechanism

Price setting can be modeled in different ways, but some of them, e.g. a stylized market maker, do not necessarily lead to the reduction of the patterns exploiters are trading on. Computing equilibrium prices has been shown to be an appropriate approach to this end.<sup>5</sup>

The two conditions described are necessary for the endogenous removal of regularities. However, their fulfillment may still lead to problems concerning the implementation of pattern exploiters and the resulting model dynamics. To prevent these problems, two model features turn out to be convenient.

# (3) Fundamental News

In the proof of proposition 1 it has been shown that, if systematic patterns in prices are removed, changes of prices can only be due to true news. This implies that if no news occurs,  $P_{t+1} = P_t$ ,  $\forall t$ . Put differently, pattern exploiters tend to drive the market towards its steady

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<sup>&</sup>lt;sup>5</sup> Of course, this is still a simplification of price formation in real markets. For example, in stock markets prices are usually formed by the matching of sell or buy requests listed in an order book. Evidently, this mechanism is also appropriate for the removal of patterns although the market is not necessarily in equilibrium for the last trading price.

state, countering endogenous market dynamics. To preserve dynamic complexity, true news is required. The implementation of a news arrival process can be achieved by a reality-oriented reproduction of the evolution of the fundamental value. The fundamental value of an asset is usually regarded to follow a random walk (Fama 1965 and followers). Adopting the random walk approach for the fundamental value  $F_t$  prevents the model from converging on its steady state. In contrast, setting  $F_t$  constant, as practiced by many modelers, is an unfavorable simplification in the context of pattern exploitation.

#### (4) Discrete Time

Formulating the model in discrete time solves a purely technical difficulty. Perfect removal of regularities requires their identification at the moment they are established. In a continuous-time model this would create immense computation costs, in particular if using ANNs. The discrete time approach reduces the computation time and thus enhances the tractability of the model.

# 3 Application

This section introduces a simple financial market model in which systematic patterns tend to arise mainly through the activity of trend followers. The simulations demonstrate that pattern exploiters effectively identify and trade on these patterns, leading to a model dynamics which again evolves unpredictably. Further results concern the effect of pattern exploiters on market efficiency.

#### 3.1 The Model

The model introduced next can be interpreted as an adaption of the deterministic framework of the model presented in Dieci and Westerhoff (2006), which is based on the fundamentalist-chartist approach. The framework mentioned is an appropriate basis to illustrate the effect of pattern exploiters for four reasons: (i) it is relatively simple, (ii) it is formulated in discrete time, (iii) it produces systematic patterns in prices, (iv) it includes a profit-based switching mechanism between strategies by which some interesting emergent phenomena can be uncovered. Major differences of the model introduced here compared to Dieci and Westerhoff (2006) concern the following aspect.

- *Pattern exploiters*. Pattern exploiters who behave in the way described in section 2.1 are introduced as an additional third trader group.
- 2-day moving average prediction of chartists. Compared to the simple extrapolation of the most recent trend, the 2-day moving average prediction adds an additional variable  $(P_{t-2})$ , causing a more complex systematic pattern.

The other differences are due to the realization of the requirements identified in section 2.3:

- Random walk of the fundamental value, instead of constant value.
- Equilibrium pricing, instead of market maker approach.

The resulting model consists of four major components: the expectation formation, the demand formulation, the switching mechanism, and the mechanism of price and value formation. The components interrelate according to the following logic: In each period, traders formulate their demand relative to their expectation about the price in the next period. Price expectations are formed according to different trading strategies: a fundamental rule, technical trend extrapolation, and sophisticated pattern exploitation. Furthermore, traders switch between these strategies. A strategy is the more popular, the more profits it has generated in the past. Finally, prices are formed such that demand and supply are equal. In a formal fashion, the model can be described as follows:

#### 3.1.1 Expectation Formation

At each time t, traders form an expectation about the price of the asset in the next period. For fundamentalists this expectation is

$$E_t^{F}[P_{t+1}]|P_t = P_t + \kappa(F_t - P_t)$$
(9)

which stipulates that fundamentalists expect prices to adapt to the fundamental value to some degree specified by the parameter  $\kappa$  ( $\kappa \in [0; 1]$ ).

Chartists believe trends will continue. To identify trends, moving averages are computed (Brock and Hommes 1998). In our model, chartists rely on the 2-day weighted moving average. Their expectation then results from the extrapolation of this trend, formally:

$$E_t^{\mathsf{C}}[P_{t+1}](P_t) = P_t + (2r_t + r_{t-1})/3, \tag{10}$$

where  $r_t$  represents the most recent return in t:  $r_t = P_t - P_{t-1}$ . Due to their extrapolative expectation and the resulting demand, chartists induce positive feedback into the dynamics of prices, and thus create a source of systematic patterns of prices.

Pattern exploiters compute their expectation analogue to eq. (4) in section 2.1. For simplicity, we assume that exploiters do not expect the law of motion  $g_t$  to change from t to t+1. Hence, instead of  $\widehat{g_{t+1}}_t^X$ , we can write  $\widehat{g_t}_t^X$ , or in short,  $\widehat{g}_t^X$ . This leads to:

$$E_t^{X}[P_{t+1}]|P_t = \hat{g}_t^{X}[\Phi_t^{X}](P_t)$$
 (11)

The information set  $\Phi_t^X$  will be specified in section 3.2. Note that the expectation about  $P_{t+1}$  is a function of the price  $P_t$ .

# 3.1.2 Demand Formulation

Each trader group derives their demand from the expected return  $E_t^i[r_{t+1}]|P_t = E_t^i[P_{t+1}]|P_t - P_t$ . For any transaction price  $P_t$  they buy, if for  $P_t$  they expect a following price rise  $(E_t^i[r_{t+1}]|P_t$  positive) and sell if they expect a price fall  $(E_t^i[r_{t+1}]|P_t$  negative). Assuming a linear function, this logic can be expressed as

$$D_t^{i}(P_t) = \alpha^{i} \left( E_t^{i}[r_{t+1}] | P_t \right), \quad i \in \{F, C, X\},$$
 (12)

where  $D_t^i(P_t)$  is the net demand of trader group i at price  $P_t$ , and  $\alpha^i$  represents a positive reaction parameter, as explained in section 2.1.

### 3.1.3 Switching Mechanism

The switching mechanism relates to the switching of traders between strategies. A popular assumption is that a strategy tends to gain (/lose) followers if it produces more (/less) profits than alternatives (Brock and Hommes 1997, 1998; Hommes 2001). In Dieci and Westerhoff (2006), the weight of some trader group, respectively strategy i and time t, denoted  $w_t^i$ , is determined by i's attractiveness in t, denoted  $A_t^i$ , in the following fashion:

$$w_t^i = \frac{Exp[\gamma A_t^i]}{\sum_i Exp[\gamma A_t^i]},\tag{13}$$

where  $\gamma$  is a constant rationality parameter that regulates the sensitivity of traders' reaction to a shift of the level of the attractiveness of a particular strategy. The attractiveness  $A_t^i$  represents a stylized moving average over past profits:

$$A_t^i = \eta A_{t-1}^i + r_{t-1} D_{t-2}^i. \tag{14}$$

 $r_{t-1}D_{t-2}^i$  represents the value change of the long/short position built up, based on strategy i at price  $P_{t-2}$  due to the price change from t-2 to t-1. The influence of this most recent profit for  $A_t^i$  relative to preceding profits stored in  $A_{t-1}^i$  is greater, the lower the positive parameter  $\eta$ . In this sense,  $\eta$  reflects agents' memory.

Technically, the switching mechanism defined above produces alterations in the model structure, because it implies changes of the state variables  $A_t^i$ . As in reality, these structural changes can lead to the emergence of new systematic patterns while others disappear. To discover new patterns, traders have to remain in a permanent state of alertness and learn continuously. Vice versa, simply relying on old patterns to persist is not optimal to maximize profits.

#### 3.1.4 Price and Value Formation

The mechanism of price adaption is equal to the equilibrium pricing principle, specified by equation 5. The total demand, denoted  $D_t(P_t)$ , results from the weighted sum of the net demand of all trader groups i:

$$D_t(P_t) = \sum_{i} w_t^i D_t^i(P_t), \quad i \in \{F, C, X\}$$
 (15)

For the fundamental value, we adopt the random walk assumption:

$$F_t = F_{t-1} + Y_t, \quad Y_t \in N(\mu_i, \sigma_i^2)$$
(16)

 $Y_t$  stands for changes of value caused by fundamental news emerging after t-l and not later than t. These value changes are IID normally distributed with mean  $\mu$  and variance  $\sigma^2$ .

#### 3.1.5 Law of motion

Note that the model market specified above complies with the requirements identified in section 2.3. Hence, the necessary and convenient model features for the endogenous removal of systematic patterns are given.

If we abstract from pattern exploiters, the law of motion of prices results from a combination of eqs. (5), (8), (9), (10), (11), and (14):

$$P_{t} = g_{t}(\Phi_{t}) = \frac{3w_{t}^{F}\kappa\alpha^{F}}{Z}F_{t} - \frac{w_{t}^{C}\alpha^{C}}{Z}P_{t-1} - \frac{w_{t}^{C}\alpha^{C}}{Z}P_{t-2}$$
 (17)

with  $Z = 3w_t^F \kappa \alpha^F + w_t^C \alpha^C$ . The law of motion is equivalent to the function  $g_t(\Phi_t)$ . The input for this function is given by the information set  $\Phi_t$ , which is here  $\Phi_t = [F_t, P_{t-1}, P_{t-2}]$ . Hence, any pattern exploiter could exploit the deterministic features perfectly if she knew  $g_t$  and  $\Phi_t$ .

Still, the active intervention of pattern exploiters will influence the function  $g_t$ , and possibly extend  $\Phi_t$  by other fundamental or technical information. For the latter to occur, it is sufficient that pattern exploiters believe this information to be relevant and react to it. The identification and exploitation of patterns is thus a recursive process, in which the actual law of motion of prices changes.

#### 3.2 Model Calibration

The calibration of the model refers to two aspects: the setting of the parameters included in the model equations and the specification of pattern exploiters.

#### 3.2.1 Parameter setting

Table 1 gives an overview of the model parameters and their settings. The parameter setting of the switching mechanism,  $\gamma$  and  $\eta$ , was adopted identically from Dieci and Westerhoff (2006).  $\sigma$  is set to 1%, which should be a reasonable assumption for the volatility of the fundamental value. The reaction coefficients of trader groups could be set rather freely as there is no empirical data for these values. Setting  $\kappa$   $\alpha^F = \alpha^C = 1$  might be the most salient assumption, which further generates a stable model dynamics.  $\alpha^F$  can be regarded as the independent variable which regulates the influence of pattern exploiters and, hence, the degree to which pattern will be removed.

#### -- Table 2 about here --

#### 3.2.2 Pattern Exploiters

Pattern exploiters should be configured such that they are able to identify the true behavior of the deterministic model as specified by the law of motion  $g_t(\Phi_t)$  (eq. 17). As  $g_t(\Phi_t)$  is linear here, it can be represented perfectly by a linear prediction model. According to the independent variables of  $g_t(\Phi_t)$ ,  $\Phi_t^X$  is set to  $\Phi_t^X = \Phi_t = [F_t, P_{t-1}, P_{t-2}]$ . This yields the prediction model specified by eq. (8). The model is realized technically by an ANN with no hidden layer. In each period t, the regression coefficients, respectively weight factors  $\beta_1$  to  $\beta_3$ ,

are learned using the last 50 pairs of independent and dependent variables. The root mean squares are computed as the error criterion. <sup>6</sup>

# 3.3 Measures of predictability

In general, a process complying with ASP (Absence of Systematic Patterns) is called a martingale. In discrete time, a martingale can be defined as a stochastic process with observations  $X_1, X_2, ...$  for which  $E[X_t] < \infty$  and  $E[X_{t+1}|X_1, X_2, ..., X_t] = X_t$ , that is, all preceding observations do not contain any evidence to believe that the next observation will be greater or smaller than the last one. In this sense, the process is unpredictable. The ASP property includes the AA property (Absence of significant Autocorrelations), meaning that autocorrelations between changes  $X_t - X_{t-1}$  tend towards zero for all lags  $\tau$  as the number of observations rises. The autocorrelation  $R(\tau)$  between the return in t and the return in  $t - \tau$  is computed as:

$$R(\tau) = \frac{E[(r_t - \mu)(r_{t-\tau} - \mu))]}{\sigma^2},$$
(18)

where  $\sigma^2$  is the variance of returns observed. The existence of significant autocorrelations indicates the existence of systematic patterns in price dynamics. The reverse conclusion, however, does not necessarily hold. Though AA might be fulfilled, complex patterns in prices may still be present, such that ASP is violated. Ignoring this aspect, financial market models usually limited themselves to the  $R(\tau)$  indicator. For the goal of the present study, however, advanced indicators able to identify more complex patterns are needed. These indicators are provided by Challet (2005), who introduces two conditional measures of predictability.

The first one, H, applies the mean return conditional to different patterns in prices. With S being the number of relevant patterns  $\theta$  ( $\theta = 1, ..., S$ ), H can be formulated as

$$H = \frac{1}{S} \sum_{\theta=1}^{S} \langle r | \theta \rangle^2 \tag{19}$$

The intuition of H is that in a market which is completely unpredictable, a certain pattern in prices, e.g. a price rise, should not give any information about the return in the next period, i.e. the mean return following this pattern tends to zero. H averages these mean returns over a set of patterns which are believed to be relevant. A higher H points to a greater predictability of price dynamics.

<sup>&</sup>lt;sup>6</sup> Choosing the right learning horizon implies dissolving a trade-off. Increasing the learning horizon provides more information to exploiters but enhances the probability that a law of motion is learned which is no longer valid.

Still, the indicator H does not capture all aspects of predictability as it misses predictability associated with oscillatory behavior. The latter can be identified by the third indicator, the conditional price return auto-correlation function  $K(\tau)$ . It can be written as:

$$K(\tau) = \left(\frac{1}{S} \sum_{\theta=1}^{S} (R(\tau)|\theta) \,\sigma^2 - H\right) / \sigma^2,\tag{20}$$

where  $R(\tau)|\theta$  is the correlation of price returns subsequent to occurrences of pattern  $\theta$ .

As traders do not include more than the last two prices into their trading calculus, any predictability of  $P_t$  can be derived from  $[F_t, P_{t-1}, P_{t-2},]$ . Therefore,  $\tau \in [1,2]$  will be investigated. (Challet 2005 limits himself to  $\tau = 1$ ). To compute H and  $K(\tau)$ , four patterns  $\theta_k$ , are considered as relevant:  $\theta_1: r_t > 0$ ,  $\theta_2: r_t < 0$ ,  $\theta_3: (2r_t + r_{t-1})/3 > 0$ ,  $\theta_4: (2r_t + r_{t-1})/3 < 0$ . By representing a price rise and a price fall, respectively,  $\theta_1$  and  $\theta_2$  might be regarded as the simplest patterns possible.  $\theta_3$  and  $\theta_4$  are derived from the trading philosophy of chartists, as they stand for a positive and negative two-day moving average, respectively. Hence, if the trading behavior of chartists penetrates the dynamics of prices such that predictability is caused, this predictability should be reliably captured by H and  $K(\tau)$ .

#### 3.4 Simulation Results

To understand the influence of pattern exploiters, it is fundamental to understand the model when these traders are absent. Therefore, we begin by simulating a basic framework in which only fundamentalist and chartists are active.

#### 3.4.1 The model without pattern exploiters

Figure 1 shows the dynamics of model. The upper panel illustrates the price  $P_t$  (black) and the fundamental value  $F_t$  (gray). In principal, we observe excess volatility – prices evolve in a more volatile manner than value. More specifically, prices appear to be driven by certain momentum leading to fluctuations around value. The reason for this behavior is the interplay of trading strategies: Reacting to fundamental news, fundamentalist cause a movement of prices towards value. Technical traders interpret this movement as the beginning of a trend, and trade on it, thereby inducing momentum into the dynamics of prices by which prices overshoot value. The trend reverts once the mispricing is sufficiently great such that fundamental orders outweigh chartists, upon which the loop repeats.

The fluctuation of prices around value is a simple example of a systematic pattern in the dynamics of prices. It is simple, as it is captured well by the autocorrelation of returns. The autocorrelations  $R(\tau)$  for different lags  $\tau$  are indicated by the bottom panel, with the gray horizontal lines indicating the 1% level of significance. The panel shows that the basic framework produces great autocorrelations on the first two lags. The reason is that in eq. (9), chartists extrapolate the trends of the last two periods. For the third and fourth lag, the autocorrelations turn positive, which reflects the trend reversal due to fundamental trading. For the other lags, the scheme repeats. Formally, this pattern has been described by eq. 18. Note that in a market with fundamentalists only,  $P_t = F_t$ ,  $\forall t$ . This can be verified by simply combining eq. 5, 8, 11, and 14, meaning that systematic patterns in prices would be absent. From this perspective, the existence of the patterns can be attributed to the activity of chartists.

#### 3.4.2 The model with pattern exploiters

Next, pattern exploiters enter the market. To begin with, we conduct a series of runs in which patterns exploiters merely observe the dynamics of the deterministic model before and merely form predictions about prices – exploiters do not influence the dynamics themselves. Then, we measure the average prediction error of trader groups i, defined as  $\varepsilon^i = \frac{1}{T}\sum_{t=1}^T |E_t^i[P_{t+1}] - P_{t+1}|$ . We find that, due to their artificial intelligence, exploiters are able to predict prices most accurately. (For the exemplar run, we get  $\varepsilon^X = 11.67 * 10^{-3}$ ). Fundamentalists, who believe prices to follow value, perform second best ( $\varepsilon^F = 11.75 * 10^{-3}$ ), because the fundamental value is indeed the main anchor for the evolution of prices. However, fundamentalists ignore the predictive power of past returns and, thus, miss part of the information determining the evolution of prices. Chartists, who believe in the persistence of trends, commit the greatest prediction errors ( $\varepsilon^F = 16.67 * 10^{-3}$ ), as with the fundamental

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Note that a predictor who knows  $g_t$  and  $\Phi_t$  would still commit prediction errors because  $P_{t+1}$  is determined by  $g_{t+1}$  and  $\Phi_{t+1} = [F_{t+1}, P_t, P_{t-1}]$  which are still unknown in t. Example: Under the assumption that market efficiency is perfect such that  $P_t = F_t$ , the best prediction for  $P_{t+1}$  would be  $P_t$ . The resulting prediction error  $\varepsilon^i$  would then correspond to average absolute change of value  $(1/T)\sum_{t=1}^T |Y_t|$ . The latter can be computed as  $\left(\frac{1}{T}\right)\sum_{t=1}^T |Y_t| = \sqrt{2/\pi} * \sigma \approx \text{(with } \sigma = 0.01\text{) } 7.98 * 10^{-3}\text{ (Goldstein and Taleb 2007)}. 7.98 * 10^{-3}\text{ can thus be interpreted as the minimum achievable for } \varepsilon^i$  as the number of observation tends towards infinity. Nevertheless, in the present model this value can hardly be reached. The reasons is changes of the state variables  $A_t^i$ , i.e. changes of the model structure, continuously alter the law of motion via the weights  $w_t^i$ . Exploiters estimate the law of motion from historical data. Changes of the law in the meantime will thus lead to systematical errors.

value they ignore the most important determinant of prices. These results confirm that the prediction models of exploiters actually work. Furthermore, their superior prediction power potentially enables exploiters to achieve superior profits which potentially lead to a greater weight  $w_t^X$ .

Figure 2 depicts an exemplary simulation run in which exploiters actively participate in the market and their reaction intensity  $\alpha^X$  is set to 10. The upper panel shows that prices have stopped fluctuating around value; at first glance, the systematic pattern in price seems to have disappeared. The third panel, which shows the autocorrelations of returns, confirms that the predictability of prices has dropped. Autocorrelations between returns are considerably lower compared to the run without exploiters. Nevertheless, significant autocorrelations are still present. These observations indicate that exploiters successfully exploit systematic patterns. However, their trading volume is still too low to remove patterns entirely.

#### -- Fig. 2 about here --

The fourth panel depicts the weights of trader groups *i*. Two important insights are conveyed. First, the weight of exploiters (dark gray) on average exceeds the weight of fundamentalists (black) and chartists (light gray). Further, the weight of fundamentalists and the weight of chartists move in opposite directions to each other. The reason is that the beliefs of exploiters and chartist diverge greatly: chartists believe that trends will continue, whereas exploiters know that such patterns are almost always absent. Therefore, exploiters and chartists trade with each other, and, due to the great trading power of exploiters  $\alpha^X = 10$ , with great volume. However, as the prediction accuracy of exploiters ( $\varepsilon^X = 8.77 * 10^{-3}$ ) is significantly higher than the one of chartists ( $\varepsilon^C = 11.17 * 10^{-3}$ ), exploiters gain money from these transactions whereas chartists lose. As for every buy, there is one sale (eq. 14), the gains of exploiters mirror the losses of chartists. Because the weights of groups are dependent on profits, this leads to the mirror-inverted dynamics of both weights. In sum, pattern exploiters exploit those traders who are responsible for the emergence of systematic patterns.

The second insight is that the weight of exploiters does not remain on a certain level but goes up and down. To understand the cause, assume that a systematic pattern is present which is identified by exploiters. Then, exploiters will start to trade on the pattern, and thereby achieve superior profits compared to other traders. Due to the superior profits, the weight of exploiters tends to rise. However, the more exploiters are in the market exploiting the particular pattern, the more the pattern is reduced. Thereby, the opportunity for exploiters to achieve superior

profits diminishes and their weight tends to fall again. (In an extreme case, the dynamics of prices is completely unpredictable. Hence, profit opportunities are equal no matter if a trader buys or sells). The withdrawal of exploiters makes it possible that systematic patterns reappear, and the loop repeats itself. In conclusion, the activity of exploiters undermines their own superiority, and thereby opens the door for the survival of other, less informed and/or less intelligent traders. (Another reason for the variations in weights is that in some periods, prices indeed follow trends, simply due to a random trend of the fundamental value. Hence, the predictions of chartists are sometimes relatively good).

The exemplary run described above has confirmed the potential of our approach; systematic patterns are reduced successfully by the activity of intelligent traders. As a second step, we conduct a systematic analysis of the influence of these traders on market predictability. The results are displayed in figure 3. The setup of the large data experiment is as follows: We simulate the model with fundamentalists, chartists, and exploiters. The reaction parameter of exploiters  $\alpha^X$  represents the independent variable which is either set to 0, 1, 5, or 10. For each value of  $\alpha^X$ , 100 runs with 5,000 periods each have been simulated, including an initial transition period of 1,000 periods which has been rejected. The dependent variables are given by the indicators of predictability introduced in section 3.3. The results are summarized by Box-Whisker-Plots. The bottom (top) of the box represents the 25<sup>th</sup> (75<sup>th</sup>) percentile. The mid vertical line represents the median and the black dot the mean. The upper (lower) end of the whisker indicates the minimum (maximum) observation.

#### -- Fig. 3 about here --

The experiments show that all measures of predictability, including the measures of conditional predictability, H and  $K(\tau)$ , are continuously decreasing due to a greater influence of exploiters as specified by  $\alpha^X$ . For greater settings of  $\alpha^X$ , there are several runs in which predictability measures are not significant anymore. In conclusion, the predictability of prices can be reduced effectively, up to their disappearance, by traders who recognize the behavior of prices and trade on the patterns found.

Let us conclude by examining some remarkable incidental results on the influence of pattern exploiters on market efficiency. Figure 4 depicts three indicators of efficiency:

Market distortion, D:

$$D = \frac{1}{T} \sum_{t=1}^{T} D_t$$
, with  $D_t = |P_t - F_t|$ , (21)

where T stands for the number of observations. D captures the tendency of price to reflect value in terms of the absolute average mispricing.

Excess volatility,  $V^{ex}$ :

$$V^{ex} = \frac{1}{T} \sum_{t=1}^{T} r_t^{ex}$$
, with  $r_t^{ex} = |r_t| - |F_t - F_{t-1}|$ . (22)

 $V^{ex}$  compares shifts of prices and fundamentals, where  $r_t^{ex}$  can be interpreted as the excess return. If prices tend to overreact (/underreact) to fundamental news,  $V^{ex}$  is positive (/negative).

Ratio of correct price reactions, C:

$$C = \frac{N}{T}. (23)$$

where N is the number of returns  $r_t$  with  $sgn[r_t] == sgn[F_t - F_{t-1}]$ . C captures the degree to which prices react to fundamental news.

#### -- Figure 4 about here --

The results show that the effects on market efficiency are complex. On the one hand, market efficiency is improved as the tendency of prices to reproduce changes of fundamentals increases – a greater  $\alpha^X$  leads to an increase of C rises and a decline of  $V^{ex}$ . The reason is that, the more exploiters remove patterns, the more the fundamental value tends to be the only determinant of prices, and this is noted again by exploiters. As a result, exploiters tend to turn to fundamentalists, who only react to fundamental news. (With reference to the prediction model defined by eq. (17), the regression coefficients  $\beta_2$  and  $\beta_3$  tend to 0). This also leads to a decline in market distortion. On the other hand, if  $\alpha^X$  becomes very great, distortion tends to rise with relatively great variance between simulation runs. This result is due to the fact that pattern exploiters, in contrast to fundamentalists, do not trade on mispricing per se, but identify the fundamental value as the main determinant of prices. However, if  $\alpha^X$  becomes relatively great, the dynamics of prices is driven mainly by exploiters who, thus, learn from themselves. This complicates an adequate estimation about the influence of fundamentals. As a result, discrepancies between the levels of prices and value can emerge. (With reference to the prediction model,  $\beta_1$  deviates from 1).

Note that the positive effects of pattern exploitation on market efficiency are dependent on the condition that exploiters recognize the true behavior of prices. It would be also possible that

exploiters identify pseudo-patterns. In this case, exploiters adopt noise-trader behavior as they "trade on noise as if it were information" (Black 1986). The identification of pseudo-patterns can occur, if exploiters do not know all the variables actually determining the evolution of prices (for example, exploiters could be interpreted as purely technical traders who do not know the fundamental value) or if they misinterpret dynamics of prices (for example, if exploiters consider relatively narrow time ranges, patterns might appear to be systematic which are in fact random). Experiments with such agents have shown that the trading on pseudo-patterns destabilizes market dynamics and leads to the emergence of typical stylized facts such as volatility clustering or heavy tails in the distribution of returns. However, as these phenomena lay beyond the focus of the present study, we leave a closer examination to future research.

## 4. Conclusion

In financial market models, the absence of systematic patterns in prices – an important stylized fact of real markets – is usually replicated by means of stochastic components. The present article has introduced a model in which systematic patterns are eliminated endogenously and in a reality-oriented fashion. This is achieved by the activity of intelligent traders who are able to identify the patterns in prices and exploit them. With perfect knowledge, regression techniques and artificial neuronal networks, three methods for the pattern identification and forecasting of model agents have been discussed. In the present model, a linear prediction model is used, which can be interpreted as the simplest form of an Artificial Neural Network (ANN). As ANNs are technical reproductions of the human brain, they might be regarded as quite an accurate way of modeling the cognition of financial traders. The simulation experiments confirmed that if the trading power of pattern exploiters is sufficiently great, systematic patterns disappear completely and prices evolve unpredictably. In sum, we believe the model to reproduce the mechanisms of pattern detection, exploitation and reduction quite realistically. An explanation for the putative unpredictability of prices and, in particular, the absence of autocorrelations in returns seems to be given.

As incidental results, pattern exploiters, who trade on the true behavior of prices, were found to improve market efficiency as the tendency of prices to replicate fundamental news increases. Yet price distortion may rise if the trading power of exploiters relative to fundamentalists becomes overwhelming.

Needs and potential for future research are various. When developing the model, the focus was on the endogenous removal of systematic patterns. Other typical statistical properties of financial market dynamics (surveys by Guillaume et al., 1997; Cont 2001), such as volatility clustering or heavy tails in the return distribution, are left beyond consideration. Experiments have indicated that these stylized facts can emerge if pattern exploiters trade on pseudopatterns as their information level or intelligence is not sufficient to interpret the behavior of prices correctly. These insights and the methods described here could lead to a model which replicates the stylized facts of financial markets, including the unpredictability of prices, without exogenous noise being added.

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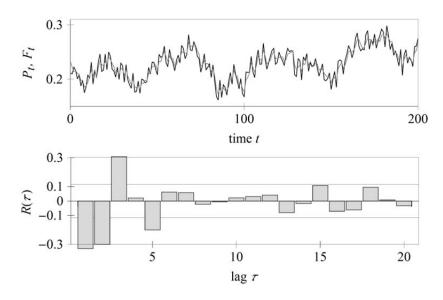
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Nr.	Article	Origin	Method for AA replication	Comment		
			2 Type Design			
1	Alfarano, Lux, and Wagner (2005)	IAH	Noise trader	AA not perfect		
2	Chiarella, He and Hommes (2006)	ABS	Random demand term in price adaption			
3	De Grauwe and Grimaldi (2005)	ABS	Exogenous noise: "Forecast errors of chartists and fundamentalists"	AA not shown		
4	Gaunersdorfer and Hommes (2007)	ABS	Random term in price adaption. "Noise Trader"			
5	Gilli and Winker (2003)	ANT	Random price shocks, noise in majority assessment			
6	He and Li (2007)	ABS	Random term in price adaption. "Noise Trader, unexpected market news"			
7	Hommes (2002)	ABS	Dividend noise, model approximation noise			
8	Kirman and Teyssiere (2002)	ANT	Random exchange rate, random interest rate			
9	Li and Rosser (2004)	ABS	None	AA not perfect		
10	Manzan and Westerhoff (2005)	ABS	Demand entirely determined by randomly arriving news			
11	Winker and Gilli (2001)	ANT	Random demand term			
	3 Type Design					
12	Lux and Marchesi (2000)	IAH	Transition probabilities	AA not perfect		
13	Parke and Waters (2007)	ABS	Martingale news, martingale dividends			
Many-Type Design						
14	Challet and Galla (2005)	MG	Probabilistic trading decision	AA for some setups only		
15	Cross, et al. (2007)	MG	Random term in price adaption representing "exogenous information stream"			
16	Dicks and van der Weide (2005)	ABS	Random news affecting traders expectation	Random Walk of Prices		
17	Ferreira, et al. (2005)	MG	Pattern exploitation?			
18	Ghoulmie, Cont and Nadal (2005)	TM	Random trading signal			
19	lori (2002)	IM	Random trading signal (individual)			
20	Pollard (2006)	TM	Gaussian trading signal ("morning news")	AA not shown		
21	Sallans, et al. (2003)	ABS	Random actions	AA not perfect		
22	Shimokawa, Suzuki and Misawa (2007)	PT	Noise trader, Gaussian private signals for price prediction	AA not shown		
	Autonomous Agent					
23	Arifovic and Gencay (2000)	SFI		No AA		
24	LeBaron, Arthur and Palmer (1999)	SFI	Stochastic dividend process	AA not shown		
25	Martinez-Jaramillo and Tsang (2007)	SFI	Pure noise traders	AA not perfect		
26	Reimann and Tupak (2007)	SFI	random dividend process			
27	Tay and Lin (2001)	SFI	Stochastic dividend process, Random expectations of prices and dividends			

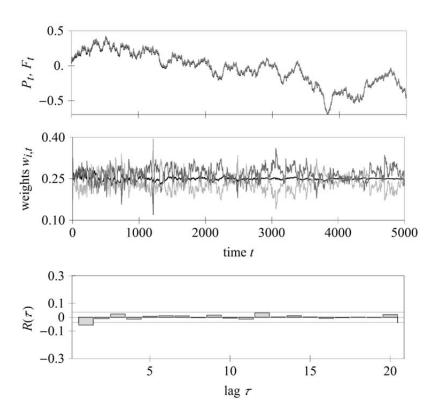
**Table 1**: Financial Market models listed in Chen (2009) method used to reproduce the absence of autocorrelations in returns (AA). Acronyms: ANT: Ant; ABS: Agent-based modeling; IAH: Interactive agent hypothesis; IM: Ising model; MG: Minority games; PT: Prospect-theory-based; SFI: Santa Fe artificial stock market; TM: Threshold model.

Parameter	Description	Value
$\kappa \alpha^{F}$	reaction intensity of fundamentalists	1
$\alpha^{C}$	reaction intensity of chartists	1
$\alpha^{X}$	reaction intensity of pattern exploiters	10
γ	rationality of strategy choice	350
$\eta$	weight of agents' memory	0.975
$\sigma$	Standard deviation of changes of	0.01
	fundamental value	

**Table 1**: The model parameters and their setting.



**Fig. 1**: Price and value (top), and autocorrelation of returns (bottom) without pattern exploiters. Autocorrelations based on 10,000 simulation periods. Top: Price and fundamental Value. Bottom: Autocorrelations of returns.



Fig, 2: Typical simulation run with pattern exploiters. Top: Price and fundamental Value. Center: Share of strategies. Bottom: Autocorrelations of returns.

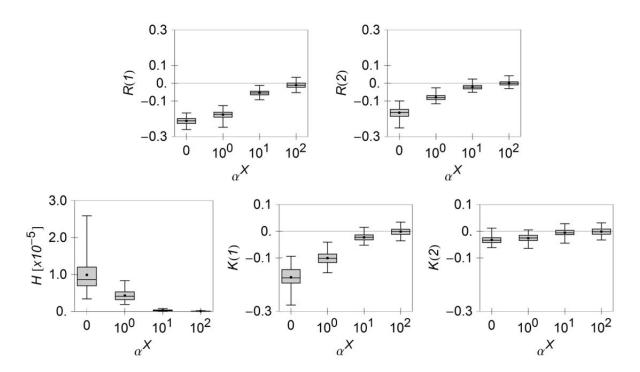
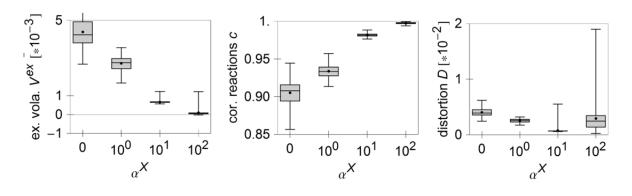


Fig. 3: Summary statistics – measures of predictability. 1% level of significance indicated by gray horizontal lines. Top: Autocorrelation with lag = 1 and lag = 2; Bottom: Conditional mean return, conditional price return auto-correlation with lag = 1 and lag = 2.



**Fig. 4**: Summary statistics – market efficiency.

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