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## Abstract

This paper develops a multi-sector New Keynesian model of a small open economy that includes commodity, manufacturing, non-tradable, and import sectors. Price and wage rigidities are sector specific, modelled à la Calvo-Yun style contracts. Labour and capital are imperfectly mobile across sectors. Commodities, whose prices are exogenously set in world markets and denominated in a foreign currency, are divided between exports and home uses as direct inputs in production of manufactured and non-tradable goods. Structural parameters of monetary policy, wage and price rigidities, capital adjustment costs, and exogenous process shocks are econometrically estimated using Canadian and U.S. data for the period 1981–2005 and a maximum likelihood procedure. The estimates indicate significant heterogeneity across sectors. The model is then simulated to evaluate the effects of commodity price shocks on real exchange rate variability and to measure their welfare implications, by conducting welfare analysis employing a second-order solution method. The main results show that commodity price shocks, which are shocks to the terms of trade, significantly contribute to exchange rate fluctuations and business cycles in the small open economy. Moreover, because of different non-linearities in the model, fluctuating commodity prices lead to welfare gains when adopting a flexible exchange rate regime. This regime is also required to improve welfare gains and to offset negative effects of other domestic and foreign shocks.

*JEL classification: E4, E52, F3, F4*

*Bank classification: Economic models; Exchange rate regimes; International topics*

## Résumé

Le nouveau modèle keynésien élaboré par l'auteur décrit une petite économie ouverte composée de quatre secteurs : produits de base, biens manufacturés, biens non échangeables internationalement et biens importés. Le degré de rigidité des prix et des salaires varie d'un secteur à l'autre et est modélisé au moyen de contrats à la Calvo-Yun. Le travail et le capital sont imparfaitement mobiles d'un secteur à l'autre. Les produits de base, dont les prix sont établis de façon exogène sur les marchés mondiaux et exprimés en une monnaie étrangère, sont exportés ou servent d'intrants dans la production nationale de biens manufacturés et de biens non échangeables. Les paramètres structurels décrivant la règle de politique monétaire, le degré de rigidité des salaires et des prix, l'ampleur des coûts d'ajustement du capital et les processus exogènes sont estimés en appliquant la méthode du maximum de vraisemblance à des données canadiennes et américaines couvrant la période 1981-2005. Les résultats de l'estimation indiquent

une hétérogénéité considérable d'un secteur à l'autre. L'auteur procède ensuite à des simulations en vue d'évaluer l'incidence de variations des prix des produits de base sur la variabilité du taux de change réel et analyse leurs répercussions sur le bien-être en résolvant le modèle au moyen d'un schéma d'approximation d'ordre 2. Il montre que, en modifiant les termes de l'échange, les variations des prix des produits de base contribuent de façon significative aux fluctuations du taux de change et à la variabilité de la production au sein de la petite économie ouverte. En outre, du fait de la présence de plus d'un type de relation non linéaire dans le modèle, les variations des prix des produits de base entraînent des gains de bien-être si un régime de changes flottants est adopté. Un tel régime s'avère également nécessaire pour réaliser de nouveaux gains de bien-être et compenser les retombées négatives des autres chocs intérieurs et étrangers.

*Classification JEL : E4, E52, F3, F4*

*Classification de la Banque : Modèles économiques; Questions internationales; Régimes de taux de change*

# 1. Introduction

Exchange rates and commodity prices, which are shocks to the terms of trade, are among the most volatile variables in a small open economy.<sup>1</sup> A large empirical and theoretical research has been devoted to understanding the causes of exchange rate volatility and to explaining its macroeconomic effects. Only few studies have, however, examined the role of commodity price fluctuations in explaining exchange rate movements. For Canada, empirical studies by Amano and van Norden (1993 and 1995), Bailliu, Dib, Kano and Schembri (2007), and Issa, Lafrance and Murray (2008) find a long run-relationship between the real exchange rate and real commodity prices, split into energy and non-energy components. Using a theoretical small open economy model, Macklem, Osakwe, Pioro and Schembri (2000) examine the economic effects of alternative exchange rate regimes in Canada, focusing on the role of terms of trade shocks. They find that a flexible exchange rate regime helps insulate the Canadian economy from external shocks.

This paper quantitatively highlights the role of real commodity price shocks in determining exchange rate fluctuations and emphasizes their implications for welfare under alternative exchange rate regimes (flexible versus fixed exchange rate). This work is motivated by the recent experience in Canada, where since the beginning of 2002, mounting commodity prices have been accompanied by a significant appreciation of the Canadian dollar and an increasing share of commodities in total Canadian exports. Figure 1 shows that the appreciation of the bilateral Canada-U.S exchange rate has largely coincided with rising real commodity prices. This paper is related to previous studies that use new open economy models to determine welfare under alternative exchange rate regimes and derive optimized monetary policy rules. For example, Kollmann (2005) analyzes the effects of pegged and floating exchange rates in a two-country model. Obstfeld and Rogoff (2000) and Devereux and Engel (2003) compare the welfare effects of pegs and floats, using standard sticky price models. And Bergin, Shin and Tchakarov (2007) present

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<sup>1</sup>In HP-filtered Canadian data for the period 1981–2005, standard deviations of the real exchange rate, real commodity prices, and output are 3.8%, 7.35%, and 1.44%, respectively. The data also show a high correlation between the real commodity prices and the real exchange rate, with a correlation coefficient equals 0.61.

a quantitative investigation of the welfare effects of exchange rate variability in a two-country model.<sup>2</sup>

In this paper, we consider a multi-sector New Keynesian model of a small open economy that consists of monopolistically competitive households, three production sectors (commodity, manufacturing, and non-tradable sectors), an import sector, a government, and a central bank. The model incorporates nine different types of structural shocks: Commodity price, natural resources, manufacturing technology, non-tradable technology, government spending, monetary policy, the foreign interest rate, foreign inflation, and foreign output. Therefore, the model offers a more realistic economic environment for the Canadian economy.

It is assumed that labour and capital are imperfectly mobile across the sectors and it is costly to adjust capital. Sector-specific price and wage rigidities are modelled à la Calvo-Yun style contracts and solved using a non-linear recursive procedure, similar to that in Schmitt-Grohé and Uribe (2007). Commodities are produced using capital, labour, and a natural resource factor and divided between exports and domestic use as material inputs in the production of manufactured and non-tradable goods. Thus, movements of the real exchange rate and commodity prices directly affect marginal costs in manufacturing and non-tradable sectors. Natural resource supply evolves exogenously to ensure the coexistence of two tradable-goods producing sectors in this small open economy: commodity and manufacturing sectors. Commodity prices are exogenously set in world markets and denominated in the foreign currency (the U.S. dollar). The central bank conducts its monetary policy by following a standard Taylor-type rule.

The model's structural parameters are either calibrated or estimated. The estimated parameters are those associated with monetary policy, capital-adjustment costs, price and wage rigidities, and the exogenous shock processes. We estimate these structural parameters with Canadian and U.S. time series using maximum-likelihood procedure via the Kalman filter applied to the model's state-space solution. The estimates mainly indicate significant heterogeneity across sectors, as the estimates of price and wage

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<sup>2</sup>One strand in the literature uses highly stylized models that permit to analytically assess welfare effects. See, for example, Bacchetta and van Wincoop (2000), Obstfeld and Rogoff (2000), Clarida, Galí and Gertler (2001), Devereux and Engel (2003), Galí and Monacelli (2005), Sutherland (2005), and others. Another strand uses structural new open economy models to quantitatively evaluate welfare effects. See, for example Kolmann (2002, 2005), Ambler et al. (2004), Ortega and Rebei (2006), and Bergin et al. (2007).

rigidities, and capital adjustment costs are significantly different across the sectors. Moreover, real commodity price shocks are persistent and highly volatile. They significantly account for exchange rate movements: these shocks explain at least 32% of the short-run real exchange rate volatility and significantly contribute to the business cycles; whereas, foreign interest rate shocks account for at least 50% of real exchange rate volatility. On the other hand, the impulse responses indicate that, following a positive commodity price shock, the real exchange rate instantaneously appreciates; and commodity output, consumption and CPI inflation sharply rise beyond their steady-state levels. Commodity revenues increase and give a boost to imports and reduces exports of manufactured goods. Therefore, movements of commodity prices have impacts on the exchange rate through the wealth effect: Following a commodity price shock, the real exchange rate moves to balance the current account of the domestic economy. Interestingly, the model is successful in reproducing the unconditional correlation between the real exchange rate and real commodity prices.

For the welfare computations, we use a second-order approximation of the model's equilibrium conditions around the deterministic steady state. This procedure captures uncertainty effects, due to the presence of the shocks, on the means and variances of the endogenous variables of the economy. Then, welfare measures are calculated as an unconditional expectation of utility. The main results show that commodity price shocks entail nontrivial welfare gains when adopting a flexible exchange rate regime. They yield a stochastic welfare level that is higher than that of the deterministic steady state. The overall welfare gains are driven by the positive effects of commodity price shocks on the mean of consumption because of different non-linearities in the model.<sup>3</sup> Nevertheless, variance effects have negative welfare implications whatever the type of the shock.

When simulating the model with all the shocks, the overall welfare effect under a flexible exchange rate regime, measured by compensating variation, is about 0.042%, divided into the level effect of

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<sup>3</sup>For example, among others, marginal costs in manufacturing and non-tradable sectors are strictly concave in commodity prices; while real wages, real capital returns, and real returns of natural resource factor (the price of fixed factor in the production in the commodity sector) are strictly convex in commodity prices. Therefore, the average marginal costs with fluctuating commodity prices, as in the stochastic steady state, are lower than with a stabilized price, as in the deterministic steady state. Similarly, the average of real wages, capital and natural resource return rates are higher with fluctuating commodity prices than with stabilized prices.

0.067% and the variance effect of -0.025%. Nevertheless, under a fixed exchange rate regime, the overall welfare is about -0.164%, divided into the level effect of -0.111% and the variance effect of -0.053%. Thus, the negative effects of variances are much larger when the exchange rate is fixed. In all examined scenarios, all of the fixed exchange rate cases are inferior to those of the flexible exchange rate.

Real commodity price shocks have a positive effects on welfare because expected revenues in the commodity sector are higher when prices are variable; in this case, the commodity-producing firm increases its output when prices are higher and reduces it when prices are lower.<sup>4</sup> In addition, in response to uncertainty about real commodity prices, households increase their stocks of foreign bonds. Hence, they are wealthier on average and enjoy higher mean consumption and leisure.

This paper is organized as follows. Section 2 presents the salient features of the model. Section 3 describes the data and the calibration procedures. Section 4 reports and discusses the estimation and simulation results. Section 5 measures and discusses the welfare effects of commodity and exchange rate volatilities. Section 6 offers some conclusions.

## **2. The Model**

We consider a small open economy with a continuum of households, a perfectly competitive commodity-producing firm, a continuum of manufactured and non-tradable intermediate-goods producing firms, a continuum of intermediate-foreign-goods importers, a government, and a central bank. Households are monopolistically competitive in the labour market, and there is monopolistic competition in intermediate-goods markets. Domestic and imported intermediate goods are used by a perfectly competitive firm to produce a final good that is divided between consumption, investment, and government spending. Nominal wages, domestic and imported intermediate-goods prices are sticky à la Calvo-Yun style contracts. In the presence of nominal rigidities, exchange rate movements are partially passed through to domestic prices.

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<sup>4</sup>When the profit function is convex in prices, the expected profits are higher than those at a constant price.

## 2.1 Households

The economy is populated by a continuum of households indexed by  $h \in [0, 1]$ . Each household  $h$  has preferences defined over consumption,  $C_{ht}$ , and labour hours,  $H_{ht}$ . Preferences are described by the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{ht}, H_{ht}),$$

where  $E_0$  denotes the mathematical expectations operator conditional on information available at the period 0,  $\beta \in (0, 1)$  is a subjective discount factor, and  $U(\cdot)$  is a utility function, which is assumed to be strictly concave, strictly increasing in  $C_{ht}$  and strictly decreasing in  $H_{ht}$ .

The single-period utility function is specified as

$$U(\cdot) = \frac{C_{ht}^{1-\tau}}{1-\tau} - \frac{H_{ht}^{1+\chi}}{1+\chi}, \quad (1)$$

with  $H_{ht} = \left[ H_{M,ht}^{\frac{1+\varsigma}{\varsigma}} + H_{N,ht}^{\frac{1+\varsigma}{\varsigma}} + H_{X,ht}^{\frac{1+\varsigma}{\varsigma}} \right]^{\frac{\varsigma}{1+\varsigma}}$ , where  $H_{M,ht}$ ,  $H_{N,ht}$ , and  $H_{X,ht}$ , represent hours worked by the household  $h$  in manufacturing, non-tradable, and commodity sectors, which are indexed by  $M$ ,  $N$ , and  $X$ , respectively. The preference parameters,  $\tau$ ,  $\varsigma$ , and  $\chi$  are strictly positive. The parameter  $\tau$  is the inverse of the elasticity of intertemporal substitution of consumption;  $\varsigma$  denotes the labour elasticity of substitution across sectors; and  $\chi$  is the inverse of the Frisch wage elasticity of labour supply.

It is assumed that household  $h$  is a monopoly supplier of differentiated labour services to the three production sectors indexed by  $i (= M, N, X)$ . Household  $h$  sells these services to a representative competitive firm that transforms them into aggregate labour inputs supplied to each sector  $i$  using the following technology:

$$H_{i,t} = \left( \int_0^1 H_{i,ht}^{\frac{\vartheta-1}{\vartheta}} dh \right)^{\frac{\vartheta}{\vartheta-1}}, \quad i = M, N, X, \quad (2)$$

where  $H_{M,t}$ ,  $H_{N,t}$ , and  $H_{X,t}$  denote aggregate labour supplies to manufacturing, non-tradable, and commodity sectors, respectively; and  $\vartheta > 1$  is the constant elasticity of substitution among different types of labour.

The demand curve for each type of labour in the sector  $i$  is given by

$$H_{i,ht} = \left( \frac{W_{i,ht}}{W_{i,t}} \right)^{-\vartheta} H_{i,t}, \quad (3)$$

where  $W_{i,ht}$  is the nominal wage of household  $h$  in sector  $i$ , and  $W_{i,t}$  is the nominal wage index in the sector  $i$ , which satisfies:

$$W_{i,t} = \left( \int_0^1 (W_{i,ht})^{1-\vartheta} dh \right)^{\frac{1}{1-\vartheta}}. \quad (4)$$

Household  $h$  takes  $H_{i,t}$  and  $W_{i,t}$  as given.

Households have access to incomplete international financial markets, in which they can buy or sell bonds denominated in foreign currency. Household  $h$  enters period  $t$  with  $K_{i,ht}$  units of capital in the sector  $i$ ,  $B_{ht-1}$  units of domestic treasury bonds, and  $B_{ht-1}^*$  units of foreign bonds denominated in foreign currency. During period  $t$ , the household supplies labour and capital to firms in all production sectors and receives total factor payment  $\sum_{i=M,N,X} (Q_{i,t}K_{i,ht} + W_{i,ht}H_{i,ht})$ , where  $Q_{i,t}$  is the nominal rental rate of capital in the sector  $i$ , and receives factor payment of natural resources,  $\varpi_h P_{L,t}L_t$ , where  $P_{L,t}$  is the nominal price of the natural resource input  $L_t$  and  $\varpi_h$  is the share of the household  $h$  in natural resource payments.<sup>5</sup> Furthermore, household  $h$  pays a lump-sum tax  $\Upsilon_{ht}$  to the government and receives dividend payments from intermediate goods producing firms  $D_{ht}$ . The household uses some of its funds to purchase the final good at the nominal price  $P_t$ , which it then divides between consumption and investment in each production sector.

The budget constraint of household  $h$  is given by:

$$\begin{aligned} P_t(C_{ht} + I_{ht}) + \frac{B_{ht}}{R_t} + \frac{e_t B_{ht}^*}{\kappa_t R_t^*} &\leq \sum_{i=M,N,X} (Q_{i,t}K_{i,ht} + W_{i,ht}H_{i,ht}) \\ &+ B_{ht-1} + e_t B_{ht-1}^* + \varpi_h P_{L,t}L_t + D_{ht} - \Upsilon_{ht}, \end{aligned} \quad (5)$$

where  $I_t = I_{M,t} + I_{N,t} + I_{X,t}$  is total investment in the manufacturing, non-tradable, and commodity sectors, respectively; and  $D_{ht} = D_{M,ht} + D_{N,ht} + D_{F,ht}$  is the total profit from the manufacturing, non-tradable and import sectors.

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<sup>5</sup>Note that,  $\int_0^1 \varpi_h dh = 1$ .

The stock of capital in the sector  $i$  evolves according to:

$$K_{i,ht+1} = (1 - \delta)K_{i,ht} + I_{i,ht} - \Psi(K_{i,ht+1}, K_{i,ht}), \quad (6)$$

where  $\delta \in (0, 1)$  is the capital depreciation rate common to all sectors and  $\Psi(\cdot) = \frac{\psi_i}{2} \left( \frac{K_{i,ht+1}}{K_{i,ht}} - 1 \right)^2 K_{i,ht}$  is the sector  $i$ 's capital-adjustment cost function that satisfies  $\Psi(0) = 0$ ,  $\Psi'(\cdot) > 0$  and  $\Psi''(\cdot) < 0$ .

The foreign bond return rate,  $\kappa_t R_t^*$ , depends on the foreign interest rate  $R_t^*$  and a country-specific risk premium  $\kappa_t$ . The foreign interest rate evolves exogenously according to the following AR(1) process:

$$\log(R_t^*) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^*,t}, \quad (7)$$

where  $R^* > 1$  is the steady-state value of  $R_t^*$ ,  $\rho_{R^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{R^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{R^*}$ .

The country-specific risk premium is increasing in the foreign-debt-to-GDP ratio. It is given by

$$\kappa_t = \exp \left( -\varkappa \frac{e_t \tilde{B}_t^* / P_t^*}{P_t Y_t} \right), \quad (8)$$

where  $\varkappa > 0$  is a parameter that determines the ratio of foreign debt to GDP,  $Y_t$  is total real GDP,  $\tilde{B}_t^*$  is the total level of indebtedness of the economy, and  $P_t^*$  is a foreign price index. The introduction of this risk premium ensures that the model has a unique steady state. It is assumed that the world inflation rate,  $\pi_t^* = P_t^* / P_{t-1}^*$ , evolves according to:

$$\log(\pi_t^*) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*,t}, \quad (9)$$

where  $\pi^* > 1$  is the steady-state value of the world inflation rate,  $\rho_{\pi^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{\pi^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{\pi^*}$ .

Household  $h$  chooses  $C_{ht}$ ,  $K_{i,ht+1}$ ,  $B_{ht}$ ,  $B_{ht}^*$  to maximize its lifetime utility, subject to Eqs. (5) and

(6). The first-order conditions, expressed in real terms, are:

$$C_{ht}^{-\tau} = \lambda_t; \quad (10)$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( q_{i,t+1} + 1 - \delta + \psi_i \left( \frac{K_{i,ht+2}}{K_{i,ht+1}} - 1 \right) \frac{K_{i,ht+2}}{K_{i,ht+1}} - \frac{\psi_i}{2} \left( \frac{K_{i,ht+2}}{K_{i,ht+1}} - 1 \right)^2 \right) \right] = \psi_i \left( \frac{K_{i,ht+1}}{K_{i,ht}} - 1 \right) + 1, \quad i = M, N, X; \quad (11)$$

$$\frac{\lambda_t}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right]; \quad (12)$$

$$\frac{S_t \lambda_t}{\kappa_t R_t^*} = \beta E_t \left[ \frac{\lambda_{t+1} S_{t+1}}{\pi_{t+1}^*} \right], \quad (13)$$

in addition to the budget constraint, Eq. (5), to which the Lagrangian multiplier,  $\lambda_t$ , is associated;  $q_{i,t} = Q_{i,t}/P_t$ ,  $\pi_t = P_t/P_{t-1}$ , and  $S_t = e_t P_t^*/P_t$  denote real capital return in the sector  $i$ , the CPI inflation rate, and the real exchange rate, respectively. Eqs. (12) and (13) together imply the uncovered interest rate parity (UIP) condition:

$$\frac{R_t}{\kappa_t R_t^*} = \frac{e_{t+1}}{e_t}. \quad (14)$$

Furthermore, there are three first-order conditions for setting nominal wages in each sector  $i$ ,  $\widetilde{W}_{i,ht}$ , when household  $h$  is allowed to revise its nominal wages. As in Calvo (1983), this happens with probability  $(1 - \varphi_i)$  in the sector  $i$ , at the beginning of each period  $t$ . If household  $h$  is not allowed to change its nominal wage, it fully indexes its wage to the steady-state inflation rate,  $\pi$ , as in Yun (1996). Therefore, household  $h$  sets his optimized nominal wage in the sector  $i$ ,  $\widetilde{W}_{i,ht}$ , to maximize the flow of its expected utility, so that

$$\max_{\widetilde{W}_{i,ht}} E_0 \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \left\{ U(C_{ht+l}, H_{i,ht+l}) + \lambda_{t+l} \pi^l \widetilde{W}_{i,ht} H_{i,ht+l} / P_{t+l} \right\} \right],$$

subject to  $H_{i,ht+l} = \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l}$ , where  $i = M, N, X$ . See Appendix A for wage setting details.

The first-order condition derived for  $\widetilde{W}_{i,ht}$  is

$$E_0 \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \lambda_{t+l} \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta - 1}{\vartheta} \frac{\widetilde{W}_{i,ht}}{P_t} \frac{\pi^l P_t}{P_{t+l}} \right\} \right] = 0, \quad (15)$$

where  $\zeta_{i,t} = -\frac{\partial U/\partial H_{i,ht}}{\partial U/\partial C_{ht}}$  is the marginal rate of substitution between consumption and labour type  $i$ . Dividing Eq. (15) by  $P_t$  and rearranging yields:

$$\tilde{w}_{i,ht} = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \zeta_{i,t+l} w_{i,t+l}^{\vartheta} H_{i,t+l} \prod_{k=1}^l \pi^{-\vartheta} \pi_{t+k}^{\vartheta}}{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} w_{i,t+l}^{\vartheta} H_{i,t+l} \prod_{k=1}^l \pi^{l(1-\vartheta)} \pi_{t+k}^{\vartheta-1}}, \quad (16)$$

where  $\tilde{w}_{i,ht} = \widetilde{W}_{i,ht}/P_t$  is household  $h$ 's real optimized wage in the sector  $i$ , while  $w_{i,t} = W_{i,t}/P_t$  is the real wage index in the sector  $i$ .

The nominal wage index in the sector  $i$  evolves over time according to the following recursive equation:

$$(W_{i,t})^{1-\vartheta} = \varphi_i (\pi W_{i,t-1})^{1-\vartheta} + (1 - \varphi_i) (\widetilde{W}_{i,t})^{1-\vartheta}, \quad (17)$$

where  $\widetilde{W}_{i,t}$  is the average wage of those workers who are allowed to revise their wage at period  $t$  in the sector  $i$ . Dividing (17) by  $P_t$  yields:

$$(w_{i,t})^{1-\vartheta} = \varphi \left( \frac{\pi w_{i,t-1}}{\pi_t} \right)^{1-\vartheta} + (1 - \varphi_i) (\tilde{w}_{i,t})^{1-\vartheta}, \quad (18)$$

In a symmetric equilibrium,  $\tilde{w}_{i,t} = \tilde{w}_{i,ht}$  and  $H_{i,t} = H_{i,ht}$  for all  $t$ . Therefore, we can rewrite the Eq. (16) in a non-linear recursive form which is similar to that in Schmitt-Grohé and Uribe (2007) as follows:

$$\tilde{w}_{i,t} = \frac{\vartheta}{\vartheta - 1} \frac{f_{i,t}^1}{f_{i,t}^2}; \quad (19)$$

where

$$\begin{aligned} f_{i,t}^1 &= E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \zeta_{i,t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{-\vartheta} \pi_{t+k}^{\vartheta} \right]; \\ &= \lambda_t H_{i,t} \zeta_{i,t} w_{i,t}^{\vartheta} + \beta\varphi_i E_t \left[ (\pi_{t+1}/\pi)^{\vartheta} f_{i,t+1}^1 \right]; \end{aligned} \quad (20)$$

and

$$\begin{aligned} f_{i,t}^2 &= E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{l(1-\vartheta)} \pi_{t+k}^{\vartheta-1} \right]; \\ &= \lambda_t H_{i,t} w_{i,t}^{\vartheta} + \beta\varphi_i E_t \left[ (\pi_{t+1}/\pi)^{\vartheta-1} f_{i,t+1}^2 \right]. \end{aligned} \quad (21)$$

In addition, Eqs. (16) and (18) permit us to derive the standard New Keynesian Phillips curve,  $\hat{\pi}_t^{w_i} = \beta \hat{\pi}_{t+1}^{w_i} + \frac{(1-\beta\varphi_i)(1-\varphi_i)}{\varphi_i} [\hat{\zeta}_{i,t} - \hat{w}_{i,t}]$ , where  $\pi_t^{w_i} = W_{i,t}/W_{i,t-1}$  is wage inflation in the sector  $i$  and hats over the variables denote deviations from steady-state values.

## 2.2 Commodity sector

The commodity sector is indexed by  $X$ . Production in this sector is modelled to capture the importance of natural resources in the Canadian economy. In this sector, there is a perfectly competitive firm that produces commodity output,  $Y_{X,t}$ , using capital,  $K_{X,t}(= \int_0^1 K_{X,ht} dh)$ , labour,  $H_{X,t}(= \int_0^1 H_{X,ht} dh)$ , and a natural-resource factor,  $L_t$ . The presence of the natural resource factor in the production of commodities limits the ability of the small open economy to specialize in the production of a single tradable good, either commodities or manufactured goods. Therefore, it allows the coexistence of both sectors in the equilibrium. The production function is the following Cobb-Douglas technology

$$Y_{X,t} \leq (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X}, \quad \alpha_X, \gamma_X, \eta_X \in (0, 1), \quad (22)$$

with  $\alpha_X + \gamma_X + \eta_X = 1$ , where  $\alpha_X$ ,  $\gamma_X$ , and  $\eta_X$  are shares of capital, labour, and natural resources in the production of commodities, respectively. It is assumed that the supply of  $L_t$  evolves exogenously according to the following AR(1) process:

$$\log(L_t) = (1 - \rho_L) \log(L) + \rho_L \log(L_{t-1}) + \varepsilon_{L,t}, \quad (23)$$

where  $L$  is a steady-state value of  $L_t$ ,  $\rho_L \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{L,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_L$ . A positive shock may be interpreted as an exogenous increase in the supply of the natural resource factor due to, for example, favorable weather or a new mining discovery.

We assume that commodity output is divided between exports and domestic uses as direct inputs in the manufacturing and non-tradable sectors, so that  $Y_{X,t} = Y_{X,t}^{ex} + Y_{X,t}^M + Y_{X,t}^N$ ; where  $Y_{X,t}^{ex}$  is the quantity of commodity goods exported abroad, while  $Y_{X,t}^M$  and  $Y_{X,t}^N$  denote the quantities of commodity goods used as material inputs in the manufacturing and non-tradable sectors, respectively.

Nominal commodity price,  $P_{X,t}^*$ , is determined exogenously in world markets and denominated in the foreign currency (i.e., the U.S. dollar in this case). Multiplying  $P_{X,t}^*$  by the nominal exchange rate,  $e_t$ , yields the commodity producer's revenues in terms of domestic currency. The commodity-producing firm takes commodity prices and the nominal exchange rate as given. Thus, given  $e_t$ ,  $P_{X,t}^*$ ,  $Q_{X,t}$ ,  $W_{X,t}$ , and  $P_{L,t}$ , the price of the natural-resource factor, the commodity-producing firm chooses  $K_{X,t}$ ,  $H_{X,t}$ , and  $L_t$  to maximize its real profit flows. Its maximization problem is<sup>6</sup>

$$\max_{\{K_{X,t}, H_{X,t}, L_t\}} [e_t P_{X,t}^* Y_{X,t} - Q_{X,t} K_{X,t} - W_{X,t} H_{X,t} - P_{L,t} L_t] / P_t,$$

subject to the production technology, Eq. (22).

The first-order conditions, with respect to  $K_{X,t}$ ,  $H_{X,t}$ , and  $L_t$ , in real terms are:

$$q_{X,t} = \alpha_X S_t p_{X,t}^* Y_{X,t} / K_{X,t}; \quad (24)$$

$$w_{X,t} = \gamma_X S_t p_{X,t}^* Y_{X,t} / H_{X,t}; \quad (25)$$

$$p_{L,t} = \eta_X S_t p_{X,t}^* Y_{X,t} / L_t, \quad (26)$$

where  $S_t = e_t P_t^* / P_t$  is the real exchange rate,  $p_{X,t}^* = P_{X,t}^* / P_t^*$  is the real commodity price with  $P_t^*$  denoting the foreign GDP deflator, while  $q_{X,t} = Q_{X,t} / P_t$ ,  $w_{X,t} = W_{X,t} / P_t$ , and  $p_{L,t} = P_{L,t} / P_t$  are real capital returns, real wages, and real natural resource prices in the commodity sector, respectively.

These first-order conditions give the optimal choices of inputs that maximize commodity producer's profits.<sup>7</sup> The demand for  $K_{X,t}$ ,  $H_{X,t}$ , and  $L_t$  are given by Eqs. (24)–(26), respectively. These equations stipulate that the marginal cost of each input must be equal to its marginal productivity.

Because the economy is small, the demand for commodity exports and their prices are completely determined in the world markets. It is assumed that real commodity prices,  $p_{X,t}^*$ , exogenously evolves according to the following AR(1) process:

$$\log(p_{X,t}^*) = (1 - \rho_{p_X}) \log(p_X^*) + \rho_{p_X} \log(p_{X,t-1}^*) + \varepsilon_{p_X,t}, \quad (27)$$

<sup>6</sup>This profit maximization problem is static because there is no real or nominal frictions in the commodity sector.

<sup>7</sup>The profits in the commodity sector are equal zero because of the perfect competition and the constant-return-to-scale production function.

where  $p_X^* > 0$  is the steady-state value of the real commodity price,  $\rho_{p_X} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{p_X,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{p_X}$ . This shock is interpreted as a shock to the terms of trade in this small open economy model.

### 2.3 Manufacturing sector

The manufacturing sector is indexed by  $M$ . There are a continuum of manufactured-intermediate goods producing firms indexed by  $j \in [0, 1]$ . Firm  $j$  produces its output using capital  $K_{M,jt} (= \int_0^1 K_{M,jht} dh)$ , labour,  $H_{M,jt} (= \int_0^1 H_{M,jht} dh)$ , and commodity input,  $Y_{X,jt}^M$ . Its production function is given by,

$$Y_{M,jt} \leq A_{M,t} (K_{M,jt})^{\alpha_M} (H_{M,jt})^{\gamma_M} (Y_{X,jt}^M)^{\eta_M}, \quad \alpha_M, \gamma_M, \eta_M \in (0, 1). \quad (28)$$

where  $\alpha_M + \gamma_M + \eta_M = 1$ ;  $\alpha_M$ ,  $\gamma_M$  and  $\eta_M$  are the shares of capital, labour, and commodity inputs in the production of manufactured goods, respectively.  $A_{M,t}$  is a technology shock specific to the manufacturing sector. It is assumed that this shock evolves exogenously according to the AR(1) process:

$$\log(A_{M,t}) = (1 - \rho_{AM}) \log(A_M) + \rho_{AM} \log(A_{M,t-1}) + \varepsilon_{A,t}^M, \quad (29)$$

where  $A_M > 0$  is the steady-state value of  $A_{M,t}$ ,  $\rho_{AM} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{A,t}^M$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{AM}$ .

Domestic manufactured goods are divided between domestic use,  $Y_{M,jt}^d$ , and exports,  $Y_{M,jt}^{ex}$ , so that  $Y_{M,jt} = Y_{M,jt}^d + Y_{M,jt}^{ex}$ . Following Obstfeld and Rogoff (1995), we assume the producer currency pricing (PCP) behavior in the manufacturing sector. Under this assumption, the firm  $j$  sets the price  $\tilde{P}_{M,jt}$  for both home and foreign markets. Thus, the law of one price (LOP) holds and movements of the exchange rate are completely passed through into import prices. The foreign demand function for domestic manufactured-goods exports, under the assumption of PCP is

$$Y_{M,t}^{ex} = \omega_{ex} \left( \frac{e_t P_{M,t}}{P_t^*} \right)^{-\nu} Y_t^*, \quad (30)$$

where  $Y_t^*$  is foreign output. The elasticity of demand for domestic manufactured-goods by foreigners is  $-\nu$ , and  $\omega_{ex} > 0$  is a parameter determining the fraction of domestic manufactured-goods exports in

foreign spending. The economy is small, so domestic exports form an insignificant fraction of foreign expenditures and have a negligible weight in the foreign price index.

It is assumed that foreign output is exogenous and evolves according to

$$\log(Y_t^*) = (1 - \rho_{Y^*}) \log(Y^*) + \rho_{Y^*} \log(Y_{t-1}^*) + \varepsilon_{Y^*,t}, \quad (31)$$

where  $Y^*$  is the steady-state value,  $\rho_{Y^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{Y^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviation  $\sigma_{Y^*}$ .

The nominal profit of firm  $j$  in period  $t + l$ ,  $D_{T,jt+l}$ , is

$$D_{M,jt+l} = \pi^l \tilde{P}_{M,jt} Y_{M,jt+l} - Q_{M,t+l} K_{M,jt+l} - W_{M,t+l} H_{M,jt+l} - e_{t+l} P_{X,t+l}^* Y_{X,jt+l}^M. \quad (32)$$

Given  $Q_{M,t}$ ,  $W_{M,t}$ , and  $P_t$ , the intermediate-goods producer  $j$  chooses  $K_{M,jt}$ ,  $H_{M,jt}$ , and  $Y_{X,jt}^M$  that maximize its profits. As in Calvo (1983), the firm  $j$  is allowed to revise its prices with probability  $(1 - \phi_M)$  for  $l$  period. If the firm is not allowed to change its prices, it fully indexes them to the steady-state CPI inflation rate, as in Yun (1996). Therefore, it sets the prices  $\tilde{P}_{M,jt}$  that maximizes the expected discounted flows of its profits. The domestic manufactured-goods producer's maximization problem is:

$$\max_{\{K_{M,jt}, H_{M,jt}, Y_{X,jt}^M, \tilde{P}_{M,jt}\}} E_0 \left[ \sum_{l=0}^{\infty} (\beta \phi_M)^l \lambda_{t+l} D_{M,jt+l} / P_{t+l} \right],$$

subject to (28), (32), and the demand function:  $Y_{M,jt+l} = \left( \frac{\tilde{P}_{M,jt}}{P_{M,t+l}} \right)^{-\theta} Y_{M,t+l}$ , where the producer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ ;  $\lambda_{t+l}$  denotes the marginal utility of consumption in period  $t + l$ . The first-order conditions in real terms with respect to  $K_{M,jt}$ ,  $H_{M,jt}$ , and  $Y_{X,jt}^M$  are:

$$q_{M,t} = \alpha_M Y_{M,jt} \xi_{M,t} / K_{M,jt}; \quad (33)$$

$$w_{M,t} = \gamma_M Y_{M,jt} \xi_{M,t} / H_{M,jt}; \quad (34)$$

$$S_t p_{X,t}^* = \eta_M Y_{M,jt} \xi_{M,t} / Y_{X,jt}^M, \quad (35)$$

where  $\xi_{M,t}$  is the real marginal cost in the manufacturing sector, which is common to all intermediate-goods-producing firms, while  $q_{M,t} = Q_{M,t} / P_t$  and  $w_{M,t} = W_{M,t} / P_t$  are real capital return and real

wages in the manufacturing sector, respectively. The condition (35) indicates that productivity in the manufacturing sector (the real marginal cost) is directly affected by real exchange rate and commodity price movements, which is a result of using commodities as material input in Eq. (28).

When producer  $j$  is allowed to change its price, which happens with the probability  $(1 - \phi_M)$ , it chooses  $\tilde{P}_{M,jt}$  for both domestic and foreign markets, so that

$$\tilde{P}_{M,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_M)^l \lambda_{t+l} \xi_{M,t+l} p_{M,t+l}^\theta Y_{M,t+l} \prod_{k=1}^l \pi^{-\theta l} \pi_{t+k}^\theta}{E_t \sum_{l=0}^{\infty} (\beta \phi_M)^l \lambda_{t+l} p_{M,t+l}^\theta Y_{M,t+l} \prod_{k=1}^l \pi^{l(1-\theta)} \pi_{t+k}^{\theta-1}}; \quad (36)$$

where  $\tilde{p}_{M,jt} = \tilde{P}_{M,jt}/P_t$ , is the real optimized price for domestic manufactured goods, while  $p_{M,t} = P_{M,t}/P_t$  is the relative price of domestic manufactured goods sold on home and foreign markets.

The manufacturing price index evolves as follows:

$$(P_{M,t})^{1-\theta} = \phi_M (\pi P_{M,t-1})^{1-\theta} + (1 - \phi_M) \left( \tilde{P}_{M,t} \right)^{1-\theta}. \quad (37)$$

Dividing Eq. (37) by  $P_t$  yields

$$(p_{M,t})^{1-\theta} = \phi_M \left( \frac{\pi p_{M,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_M) (\tilde{p}_{M,t})^{1-\theta}, \quad (38)$$

In a symmetric equilibrium,  $\tilde{p}_{M,jt} = \tilde{p}_{M,t}$  and  $Y_{M,jt} = Y_{M,t}$ . Therefore, we can rewrite Eq. (36) in a non-linear recursive form as:

$$\tilde{p}_{M,t} = \frac{\theta}{\theta - 1} \frac{x_{M,t}^1}{x_{M,t}^2}, \quad (39)$$

where

$$x_{M,t}^1 = \lambda_t Y_{M,t} \xi_{M,t} p_{M,t}^\theta + \beta \phi_M E_t \left[ (\pi_{t+1}/\pi)^\theta x_{M,t+1}^1 \right], \quad (40)$$

$$x_{M,t}^2 = \lambda_t Y_{M,t} p_{M,t}^\theta + \beta \phi_M E_t \left[ (\pi_{t+1}/\pi)^{\theta-1} x_{M,t+1}^2 \right]. \quad (41)$$

When log-linearizing Eqs. (36) and (37) around the steady-state of the variables, we derive the standard New Keynesian Phillips curve,  $\hat{\pi}_{M,t} = \beta \hat{\pi}_{M,t+1} + \frac{(1-\beta\phi_M)(1-\phi_M)}{\phi_M} \hat{\xi}_{M,t}$ , where  $\pi_{M,t} = P_{M,t}/P_{M,t-1}$  is price inflation in the manufacturing sector and hats over the variables denote deviations from steady-state of variables.

## 2.4 Non-tradable sector

The non-tradable sector is indexed by  $N$ . There are a continuum of non-tradable-intermediate-goods producing firms indexed by  $j \in [0, 1]$ . Firm  $j$  produces its intermediate goods using capital,  $K_{N,jt} (= \int_0^1 K_{N,jht} dh)$ , labour,  $H_{N,jt} (= \int_0^1 H_{N,jht} dh)$ , and commodity input,  $Y_{X,jt}^N$ . Its production function is given by

$$Y_{N,jt} \leq A_{N,t} (K_{N,jt})^{\alpha_N} (H_{N,jt})^{\gamma_N} (Y_{X,jt}^N)^{\eta_N}, \quad \alpha_N, \gamma_N, \eta_N \in (0, 1), \quad (42)$$

where  $\alpha_N + \gamma_N + \eta_N = 1$ ;  $\alpha_N$ ,  $\gamma_N$ , and  $\eta_N$  are shares of capital, labour, and commodity inputs in the production of non-tradable goods, respectively.  $A_{N,t}$  is a technology shock specific to the non-tradable sector. It is assumed that this shock evolves exogenously according to:

$$\log(A_{N,t}) = (1 - \rho_{AN}) \log(A_N) + \rho_{AN} \log(A_{N,t-1}) + \varepsilon_{A,t}^N, \quad (43)$$

where  $A_N > 0$  is the steady-state value of  $A_{N,t}$ ,  $\rho_{AN} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{A,t}^N$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{AN}$ .

As in Calvo (1983), the firm  $j$  is allowed to revise its prices with probability  $(1 - \phi_N)$  for  $l$  period. If the firm is not allowed to change its prices, it fully indexes them to the steady-state CPI inflation rate, as in Yun (1996). Therefore, it sets the prices  $\tilde{P}_{N,jt}$  that maximizes the expected discounted flow of its profits. The firm  $j$  chooses  $K_{N,jt}$ ,  $H_{N,jt}$ ,  $Y_{X,jt}^N$ , to maximize the expected discounted flow of its profits. Its maximization problem is:

$$\max_{\{K_{N,jt}, H_{N,jt}, Y_{X,jt}^N, \tilde{P}_{N,jt}\}} E_0 \left[ \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} D_{N,jt+l} / P_{t+l} \right],$$

subject to (42) and the following demand function:

$$Y_{N,jt+l} = \left( \frac{\pi^l \tilde{P}_{N,jt}}{P_{N,t+l}} \right)^{-\theta} Y_{N,t+l},$$

where the profit function is

$$D_{N,jt+l} = \pi^l \tilde{P}_{N,jt} Y_{N,t+l} - Q_{N,t+l} K_{N,t+l} - W_{N,t+l} Y_{N,t+l} - e_{t+l} P_{X,t+l}^* Y_{X,jt+l}^N. \quad (44)$$

The producer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ , where  $\lambda_{t+l}$  denotes the marginal utility of consumption in period  $t + l$ .

The first-order conditions in real terms with respect to  $K_{N,jt}$ ,  $H_{N,jt}$ , and  $Y_{X,jt}^N$  are:

$$q_{N,t} = \alpha_N Y_{N,jt} \xi_{N,t} / K_{N,jt}; \quad (45)$$

$$w_{N,t} = \gamma_N Y_{N,jt} \xi_{N,t} / H_{N,jt}; \quad (46)$$

$$S_t p_{X,t}^* = \eta_N Y_{N,jt} \xi_{N,t} / Y_{X,jt}^N, \quad (47)$$

where  $\xi_{N,t}$  is the real marginal cost in the non-tradable sector that is common to all intermediate firms,  $q_{N,jt} = Q_{N,jt}/P_t$  and  $w_{N,jt} = W_{N,jt}/P_t$  are real capital return and real wages in the non-tradable sector. The condition (47) indicates that the productivity in the manufacturing sector (the real marginal cost) is directly affected by real exchange rate and commodity price movements.

The firm that is allowed to revise its price, which happens with probability  $(1 - \phi_N)$ , chooses  $\tilde{P}_{N,jt}$ , so that

$$\tilde{p}_{N,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} \xi_{N,t+l} p_{N,t+l}^\theta Y_{N,t+l} \prod_{k=1}^l \pi^{-\theta k} \pi_{t+k}^\theta}{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} p_{N,t+l}^\theta Y_{N,t+l} \prod_{k=1}^l \pi^{l(1-\theta)} \pi_{t+k}^{\theta-1}}, \quad (48)$$

where  $\tilde{p}_{N,jt} = \tilde{P}_{N,jt}/P_t$  is the real optimized price in the non-tradable sector, while  $p_{N,t} = P_{N,t}/P_t$  is the relative price of non-tradable goods.

The non-tradable price index evolves as follows:

$$(P_{N,t})^{1-\theta} = \phi_N (\pi P_{N,t-1})^{1-\theta} + (1 - \phi_N) (\tilde{P}_{N,t})^{1-\theta}. \quad (49)$$

Dividing Eq. (49) by  $P_t$  yields

$$(p_{N,t})^{1-\theta} = \phi_N \left( \frac{\pi p_{N,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_N) (\tilde{p}_{N,t})^{1-\theta}. \quad (50)$$

In a symmetric equilibrium,  $\tilde{p}_{N,jt} = \tilde{p}_{N,t}$  and  $Y_{N,jt} = Y_{N,t}$ . Therefore, we can rewrite Eq. (48) in a non-linear recursive form as:

$$\tilde{p}_{N,t} = \frac{\theta}{\theta - 1} \frac{x_{N,t}^1}{x_{N,t}^2}, \quad (51)$$

where

$$x_{N,t}^1 = \lambda_t Y_{N,t} \xi_{N,t} p_{N,t}^\theta + \beta \phi_N E_t \left[ (\pi_{t+1}/\pi)^\theta x_{N,t+1}^1 \right], \quad (52)$$

$$x_{N,t}^2 = \lambda_t Y_{N,t} p_{N,t}^\theta + \beta \phi_N E_t \left[ (\pi_{t+1}/\pi)^{\theta-1} x_{N,t+1}^2 \right]. \quad (53)$$

When log-linearizing Eqs. (48) and (49) around the steady-state of variables, we derive the standard New Keynesian Phillips curve,  $\hat{\pi}_{N,t} = \beta \hat{\pi}_{N,t+1} + \frac{(1-\beta\phi_N)(1-\phi_N)}{\phi_N} \hat{\xi}_{N,t}$ , where  $\pi_{N,t} = P_{N,t}/P_{N,t-1}$  is price inflation in the non-tradable sector and hats over the variables denote deviations from steady-state of variables.

## 2.5 Import sector

The import sector is indexed by  $F$ . There are a continuum of domestic importers, indexed by  $j \in [0, 1]$ , that import a homogeneous intermediate good produced abroad for the foreign price  $P_t^*$ . Each importer uses its imported good to produce a differentiated good,  $Y_{F,jt}$ , that it sells in a domestic monopolistically-competitive market to produce an imported-composite good,  $Y_{F,t}$ . Importers can only change their prices when they receive a random signal. The constant probability of receiving such a signal is also  $(1 - \phi_F)$ . If it is not allowed to revise its prices, the importer fully indexes them to steady-state CPI inflation.

If the importer  $j$  is allowed to change its price in the period  $t$ , it chooses the price  $\tilde{P}_{F,jt}$  that maximizes its weighted expected profits, given the nominal exchange rate  $e_t$ , and the foreign price level  $P_t^*$ . The maximization problem is

$$\max_{\{\tilde{P}_{F,jt}\}} E_0 \left[ \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} D_{F,jt+l} / P_{t+l} \right],$$

subject to

$$Y_{F,jt+l} = \left( \frac{\pi^l \tilde{P}_{F,jt}}{P_{F,t+l}} \right)^{-\theta} Y_{F,t+l},$$

where the nominal profit function is

$$D_{F,jt+l} = \left( \pi^l \tilde{P}_{F,jt} - e_{t+l} P_{t+l}^* \right) Y_{F,jt+l}. \quad (54)$$

In period  $t + l$ , the importer's nominal marginal cost is  $e_{t+l}P_{t+l}^*$ , so its real marginal cost is equal to the real exchange rate,  $S_{t+l} = e_{t+l}P_{t+l}^*/P_{t+l}$ . The importer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ . The first-order condition of this optimization problem is

$$\tilde{p}_{F,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} S_{t+l} p_{F,t+l}^\theta Y_{F,t+l} \prod_{k=1}^l \pi^{-\theta l} \pi_{t+k}^\theta}{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} p_{F,t+l}^\theta Y_{F,t+l} \prod_{k=1}^l \pi^{l(1-\theta)} \pi_{t+k}^{\theta-1}}, \quad (55)$$

where  $\tilde{p}_{F,jt} = \tilde{P}_{F,jt}/P_t$  is the real optimized price in import sector, while  $p_{F,t} = P_{F,t}/P_t$  is the relative price of imports. The import price index evolves as:

$$(P_{F,t})^{1-\theta} = \phi_F (\pi P_{F,t-1})^{1-\theta} + (1 - \phi_F) (\tilde{P}_{F,t})^{1-\theta}. \quad (56)$$

Dividing Eq. (56) by  $P_t$  yields

$$(p_{F,t})^{1-\theta} = \phi_F \left( \frac{\pi p_{F,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_F) (\tilde{p}_{F,t})^{1-\theta}. \quad (57)$$

In a symmetric equilibrium,  $\tilde{p}_{F,jt} = \tilde{p}_{F,t}$  and  $Y_{F,jt} = Y_{F,t}$ . Therefore, we can rewrite Eq. (55) in a non-linear recursive form as:

$$\tilde{p}_{F,t} = \frac{\theta}{\theta - 1} \frac{x_{F,t}^1}{x_{F,t}^2}, \quad (58)$$

where

$$x_{F,t}^1 = \lambda_t Y_{F,t} S_t p_{F,t}^\theta + \beta \phi_F E_t \left[ (\pi_{t+1}/\pi)^\theta x_{F,t+1}^1 \right], \quad (59)$$

$$x_{F,t}^2 = \lambda_t Y_{F,t} p_{F,t}^\theta + \beta \phi_F E_t \left[ (\pi_{t+1}/\pi)^{\theta-1} x_{F,t+1}^2 \right]. \quad (60)$$

When log-linearizing Eqs. (55) and (56) around the steady-state of variables, we derive the standard New Keynesian Phillips curve,  $\hat{\pi}_{F,t} = \beta \hat{\pi}_{F,t+1} + \frac{(1-\beta\phi_F)(1-\phi_F)}{\phi_F} \hat{S}_t$ , where  $\pi_{F,t} = P_{F,t}/P_{F,t-1}$  is price inflation in the import sector and hats over the variables denote deviations from steady-state of variables.

## 2.6 Final good

We assume that a representative firm that acts in a perfectly competitive market and uses a fraction of domestically-produced manufactured output,  $Y_{M,t}^d$ , non-tradable output,  $Y_{N,t}$ , and imports,  $Y_{F,t}$  to

produce a final good,  $Z_t$ , according to the following CES technology:

$$Z_t = \left[ \omega_M^\nu \left( Y_{M,t}^d \right)^\frac{\nu-1}{\nu} + \omega_N^\nu Y_{N,t}^\frac{\nu-1}{\nu} + \omega_F^\nu Y_{F,t}^\frac{\nu-1}{\nu} \right]^\frac{\nu}{\nu-1}, \quad (61)$$

where  $\omega_M + \omega_N + \omega_F = 1$ ;  $\omega_M$ ,  $\omega_N$ , and  $\omega_F$  denote the shares of manufactured, non-tradable, and imported goods in the final good, respectively; and  $\nu > 0$  is the elasticity of substitution between domestically-used manufactured, non-tradable, and imported goods in the final good. It also denotes the price elasticity of domestic and imported goods.

Inputs in (61) are produced with a continuum of differentiated goods using the following CES technology:

$$Y_{\iota,t} = \left( \int_0^1 (Y_{\iota,jt})^\frac{\theta-1}{\theta} dj \right)^\frac{\theta}{\theta-1}, \quad \text{for } \iota = F, M, N. \quad (62)$$

where  $\theta > 1$  is the constant elasticity of substitution between intermediate goods. The demand function for domestic manufactured, non-tradable, and imported-intermediate goods  $\iota (= F, M, N)$  is

$$Y_{\iota,jt} = \left( \frac{P_{\iota,jt}}{P_{\iota,t}} \right)^{-\theta} Y_{\iota,t}, \quad (63)$$

where domestic manufactured, non-tradable, and imported goods prices satisfy

$$P_{\iota,t} = \left( \int_0^1 (P_{\iota,jt})^{1-\theta} dj \right)^\frac{1}{1-\theta}. \quad (64)$$

Given  $P_t$ ,  $P_{F,t}$ ,  $P_{M,t}$ , and  $P_{N,t}$ , the final good producer chooses  $Y_{F,t}$ ,  $Y_{M,t}^d$ , and  $Y_{N,t}$  to maximize its profit. Its maximization problem is

$$\max_{\{Y_{F,t}, Y_{M,t}^d, Y_{N,t}\}} P_t Z_t - P_{F,t} Y_{F,t} - P_{M,t} Y_{M,t}^d - P_{N,t} Y_{N,t}, \quad (65)$$

subject to (61). Profit maximization implies the following demand functions for manufactured, non-tradable, and imported goods:

$$Y_{F,t} = \omega_F \left( \frac{P_{F,t}}{P_t} \right)^{-\nu} Z_t, \quad Y_{M,t}^d = \omega_M \left( \frac{P_{M,t}}{P_t} \right)^{-\nu} Z_t, \quad \text{and} \quad Y_{N,t} = \omega_N \left( \frac{P_{N,t}}{P_t} \right)^{-\nu} Z_t.$$

Thus, as the relative prices of domestic and imported goods rise, the demand for domestic and imported goods decreases.

The zero-profit condition implies that the final-good price level, which is the consumer-price index (CPI), is linked to manufactured, non-tradable, and imported goods prices through:

$$P_t = \left[ \omega_F P_{F,t}^{1-\nu} + \omega_M P_{M,t}^{1-\nu} + \omega_N P_{N,t}^{1-\nu} \right]^{1/(1-\nu)}. \quad (66)$$

The final good is divided between consumption,  $C_t$ , private investment in the three production sectors,  $I_t$ , and government spending,  $G_t$ , so that  $Z_t = C_t + I_t + G_t$ , where  $I_t = I_{M,t} + I_{N,t} + I_{X,t}$ .

## 2.7 Government

It is assumed that government's revenues include lump-sum taxes,  $\Upsilon_t$ , and newly issued debt,  $B_t/R_t$ . The government uses its revenues to finance its spending,  $P_t G_t$  and repay its debt,  $B_{t-1}$ . The government's budget is given by

$$P_t G_t + B_{t-1} = \Upsilon_t + B_t/R_t. \quad (67)$$

Government spending evolves exogenously according to the following process

$$\log(G_t) = (1 - \rho_G) \log(G) + \rho_G \log(G_{t-1}) + \varepsilon_{Gt}, \quad (68)$$

where  $G$  is the steady-state value  $G_t$ ,  $\rho_G \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{Gt}$  is an uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_G$ .

## 2.8 Monetary authority

We assume that the monetary authority manages the short-term nominal interest rate,  $R_t$ , according to the following Taylor-type monetary policy rule:

$$\log\left(\frac{R_t}{R}\right) = \varrho_R \log\left(\frac{R_{t-1}}{R}\right) + \varrho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \varrho_Z \log\left(\frac{Z_t}{Z}\right) + \varrho_e \log\left(\frac{\Delta e_t}{\Delta e}\right) + \varepsilon_{Rt}. \quad (69)$$

where  $R$ ,  $\pi$ ,  $Z$ , and  $\Delta e = \pi/\pi^*$  are the steady-state values of  $R_t$ ,  $\pi_t$ ,  $Z_t$ , and  $\Delta e_t$ ;  $\varrho_R$  is a smoothing-term parameter, while  $\varrho_\pi$ ,  $\varrho_Z$ , and  $\varrho_e$  are the policy coefficients measuring central bank's responses to deviations of CPI inflation,  $\pi_t$ , final good output,  $Z_t$ , and the nominal exchange changes,  $\Delta e_t$ , from their

steady-state values, respectively; and  $\varepsilon_{Rt}$  is uncorrelated and normally distributed monetary policy shock with zero mean and standard deviations  $\sigma_R$ .

The policy rule coefficients are chosen by the monetary authority. When adopting a flexible exchange rate regime,  $\varrho_e = 0$ , the central bank only responds to inflation and final good output movements. Alternatively, when  $\varrho_e \rightarrow \infty$  and  $\varrho_\pi = \varrho_Z = 0$ , the monetary authority strictly targets the nominal exchange rate, leading to a fixed exchange rate regime.<sup>8</sup>

## 2.9 Symmetric equilibrium

In a symmetric equilibrium, all households, intermediate goods-producing firms, and importers make identical decisions. Therefore,  $C_{ht} = C_t$ ,  $B_{ht} = B_t$ ,  $B_{ht}^* = B_t^*$ ,  $I_{i,ht} = I_{i,t}$ ,  $\tilde{w}_{i,ht} = \tilde{w}_{i,t}$ ,  $H_{i,ht} = H_{i,t}$ ,  $K_{i,ht} = K_{i,t}$ ,  $Y_{M,jt}^X = Y_{M,t}^X$ ,  $Y_{N,jt}^X = Y_{N,t}^X$ ,  $Y_{\ell,jt} = Y_{\ell,t}$ , and  $\tilde{p}_{\ell,jt} = \tilde{p}_{\ell,t}$ , for all  $h \in [0, 1]$ ,  $j \in [0, 1]$ ,  $i = M, N, X$ , and  $\ell = F, M, N$ . Furthermore, the market-clearing conditions  $P_t G_t = \Upsilon_t$  and  $B_t = 0$  must hold for all  $t \geq 0$ .

The manufacturing and non-tradable sectors use commodity goods as material inputs in production of  $Y_{M,t}$  and  $Y_{N,t}$ , which are defined as gross output. The value-added output in each sector,  $Y_{M,t}^{va}$  and  $Y_{N,t}^{va}$ , can be constructed by subtracting commodity inputs as follow:  $Y_{i,t}^{va} = Y_{i,t} - S_t p_{X,t}^* Y_{X,t}^i / p_{i,t}$  for  $i = M, N$ . Hence, aggregate GDP is defined as:

$$Y_t = p_{M,t} Y_{M,t}^{va} + p_{N,t} Y_{N,t}^{va} + S_t p_{X,t}^* Y_{X,t}. \quad (70)$$

Combining the household's budget constraint, government budget, and single-period profit functions of commodity producing firm, manufactured and non-tradable goods producing firms, and foreign goods importers yields a current account equation. The current account equation in real terms, under the PCP assumption, is given by

$$\frac{b_t^*}{\kappa_t R_t^*} = \frac{b_{t-1}^*}{\pi_t^*} + p_{X,t}^* (Y_{X,t} - Y_{X,t}^M - Y_{X,t}^N) + \frac{p_{M,t}}{S_t} Y_{M,t}^{ex} - Y_{F,t}, \quad (71)$$

where  $b_t^* = B_t^* / P_t^*$  is the stock of real foreign debt in the domestic economy.

<sup>8</sup>In the welfare analysis, when considering the fixed exchange rate regime, we set  $\varrho_e = 1$  and  $\varrho_\pi = \varrho_Z = 0$ .

To estimate the non-calibrated structural parameters, we solve the model by taking a log-linear approximation of the equilibrium system around deterministic steady-state values. Using Blanchard and Kahn's (1980) procedure yields a state-space solution of the form:

$$\widehat{\mathbf{s}}_{t+1} = \Phi_1 \widehat{\mathbf{s}}_t + \Phi_2 \widehat{\boldsymbol{\epsilon}}_{t+1}; \quad (72)$$

$$\widehat{\mathbf{d}}_t = \Phi_1 \widehat{\mathbf{s}}_t, \quad (73)$$

where  $\widehat{\mathbf{s}}_t$  is a vector of state variables that includes predetermined and exogenous variables;  $\widehat{\mathbf{d}}_t$  is a vector of control variables; and the vector  $\widehat{\boldsymbol{\epsilon}}_t$  contains the model's shocks. The state-space solution in (72)–(73) is used to estimate the underlying parameters of the model via a maximum-likelihood procedure with a Kalman filter.

### 3. Calibration, Data and Estimation

As in previous studies, some parameters of the model should be assigned values prior to the estimation because they are non-identified or the data used contain only limited information about them. We calibrate the non-estimated parameters to capture the salient features of the Canadian economy. Table 1 reports the calibration values.

The discount factor,  $\beta$ , is set at 0.9902, which implies an annual steady-state real interest rate of 4% that matches the average observed in the estimation sample: 1981Q1–2005Q4. The curvature parameter,  $\tau$ , is given a value of 2, implying an elasticity of intertemporal substitution of 0.5. Following Bouakez et al. (2005), we set both  $\varsigma$  and  $\chi$ , the labour elasticity of substitution across sectors and the inverse of the elasticity of intertemporal substitution of labour, at unity. The capital depreciation rate,  $\delta$ , is assigned a value of 0.025; this value is commonly used in the literature and assumed to be common to the three production sectors.

The shares of capital, labour, and natural resources in the production of commodities,  $\alpha_X$ ,  $\gamma_X$ , and  $\eta_X$ , are assigned values of 0.41, 0.39, and 0.2, respectively. The shares of capital, labour, and commodity inputs in production of manufactured (non-tradable) goods,  $\alpha_M$  ( $\alpha_N$ ),  $\gamma_M$  ( $\gamma_N$ ), and  $\eta_M$  ( $\eta_N$ ) are set

equal to 0.26 (0.28), 0.63 (0.66), and 0.11 (0.06), respectively.<sup>9</sup> All these shares are taken from Macklem et al. (2000) who have calculated them from Canadian 1996 input-output tables.<sup>10</sup>

The parameters  $\theta$ , which measures the degree of monopoly power in intermediate-goods markets, is set equal to 6, implying a steady-state price markup of 20%. The parameter,  $\vartheta$ , which measures the degree of monopoly power in labour markets, is set equal to 8, implying a steady-state wage markup of 14%. The parameter  $\nu$ , which captures the price-elasticity of demand for imports and domestic goods (and it is also the elasticity of substitution between imports, manufactured and non-tradable goods in the final good), is set equal to 0.8. We calibrate this parameter based on previous studies that have estimated different versions of structural small open economy models for Canada. In particular, the parameter  $\nu$  is estimated at around 0.8 in Dib (2003), 0.6 in Ambler et al. (2004), and 0.71 in Ortega and Rebei (2006). The parameter  $\omega_{ex}$  is a normalization that ensures the ratio of manufactured exports to GDP is equal to the one observed in the data. Therefore,  $\omega_{ex}$  is set equal to 0.21. The parameters  $\omega_F$ ,  $\omega_M$ , and  $\omega_N$ , which are associated with the shares of imports, domestic manufactured and non-tradable goods in the final good, are calibrated to match the average ratios observed in the data for the estimation period. We set  $\omega_F$ ,  $\omega_M$ , and  $\omega_N$  equal to 0.33, 0.10, and 0.57, respectively. See Table 3 for steady-state ratios of GDP in the model. The parameter  $\varkappa$  is calibrated to match the net-foreign-asset-to-GDP ratio that is about -20% in the data. This calibration gives an average annual risk premium of about 50 basis points.

Following Macklem et al. (2000), we assume that the shares of employment in manufacturing, non-tradable, and commodity sectors are 0.21, 0.64, and 0.15, respectively. We assume that, on the average, households allocate one third of their available time to market activities. Therefore, the steady-state hours worked,  $H_M$ ,  $H_N$ , and  $H_X$ , are set equal to 0.07, 0.21, and 0.05, respectively. The steady-state stock of natural resources,  $L$ , and the steady-state technology levels in manufacturing and non-tradable sectors,  $A_M$ ,  $A_N$ , are assigned values to match the ratios of commodity, manufactured, and non-tradable goods in Canadian GDP. The steady-state value of government spending,  $G$ , is calibrated so that the ratio

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<sup>9</sup>The model is also simulated with the shares of commodities in the production of manufactured and non-tradable goods set equal 0, i.e.,  $\eta_M = \eta_N = 0$ . The qualitative results are very similar to those reported in this paper.

<sup>10</sup>Macklem et al. (2000) have obtained these shares from 1996 current-dollar input-output tables at the medium level of aggregation, which disaggregates input-output tables into 50 industries and 50 goods.

$G/Y$  is equal to 0.23, matching the one observed in the data. The steady-state level of the exogenous variables,  $p_X^*$ , and  $Y^*$  are simply set equal to unity.

The remaining parameters are estimated using the maximum likelihood procedure: a Kalman filter is applied to the model's state-space form, given in Eqs. (72)–(73), to generate series of innovations, which are then used to evaluate the likelihood function for the sample.<sup>11</sup> Since the model is driven by nine shocks, the structural parameters embedded in the matrices  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$  are estimated using data for nine series: commodities, manufactured goods, non-tradables, real commodity prices, the domestic nominal interest rate, government spending, the real exchange rate, foreign inflation, and foreign output.

Commodities are measured by the total real production in primary industries (agriculture, fishing, forestry, and mining) and resource processing, which includes pulp and paper, wood products, primary metals, and petroleum and coal refining. The non-tradables are in real terms and include construction; transportation and storage; communications, insurance, finance, and real estate; community and personal services; and utilities. The manufactured goods are measured by the total real production in different manufacturing sectors in the Canadian economy. Real commodity prices are measured by deflating the nominal commodity prices (including energy and non-energy commodities) by the U.S. GDP deflator. The nominal interest rate is measured by the rate on Canadian three-month treasury bills. Government spending is measured by total real government purchases of goods and services. The real exchange rate is measured by multiplying the nominal U.S./CAN exchange rate by the ratio of U.S. to Canadian prices. Foreign inflation is measured by changes in the U.S. GDP implicit price deflator. Finally, foreign output is measured by U.S. real GDP per capita. The series of commodities, manufactured goods, non-tradables, and government spending are expressed in real terms and per capita using the Canadian population aged 15 and over. See Appendix B for further details about the used data.

The model implies that all variables are stationary and fluctuate around constant means; however, the series described above are non-stationary, with the exception of the foreign inflation rate. Thus, before estimating the model, we render them stationary by using the HP-filter. Using quarterly Canadian and

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<sup>11</sup>This estimation method is described in Hamilton (1994, chapter 13) and used by Ireland (2003), Bergin (2003), Dib(2003, 2006), Ambler et al. (2004), and others.

U.S. data from 1981Q1 to 2005Q4, we estimate the non-calibrated structural parameters of the above-described multi-sector model.<sup>12</sup>

## 4. Empirical Results

### 4.1 Estimation results

Table 2 reports the maximum-likelihood estimates of the structural parameters of the baseline described-above model. Because it is not easy to solve the model's steady-state equilibrium, we estimate only the parameters that do not affect the steady-state ratios. The estimated parameters are associated with monetary policy, price and wage rigidities, capital adjustment costs, and processes of the structural shocks. Almost all parameter estimates are highly significant at conventional confidence levels, consistent and economically meaningful. The estimates of price and wage rigidities, and capital adjustment cost parameters indicate significant heterogeneity across sectors.

The estimates of sticky price parameters  $\phi_F$ ,  $\phi_M$ , and  $\phi_N$ , which indicate the probability that prices remain unchanged for the next period, are around 0.72, 0.59, and 0.53, respectively. These values imply expected price durations in import, manufacturing, and non-tradable sectors of about 3.6, 2.4, and 2.1 quarters, respectively. The estimates of the price rigidity parameters are smaller in the manufacturing and non-tradable sectors than the one estimated in the import sector because of the presence of nominal wage rigidities in the former sectors. Therefore, the nominal stickiness in those sectors is the combination of price and wage rigidities.<sup>13</sup>

The estimates of nominal wage stickiness parameters,  $\varphi_M$  and  $\varphi_N$ , are around 0.49 and 0.91, respectively. These values imply that nominal wages remain unchanged, on average, for about two quarters in the manufacturing sector and about eleven quarters in the non-tradable sector. Surprisingly, the estimate of  $\varphi_X = 0.12$  indicates that nominal wages are almost fully flexible in the commodity sector. In this case, nominal wages change every 1.14 quarters.

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<sup>12</sup>The sample starts at 1981Q1 for the availability of the data (data on commodities, manufactured goods, and non-tradables are available only since 1981.)

<sup>13</sup>As in previous studies that estimate a DSGE models, when combining nominal price and wage rigidities, the estimates of price rigidity parameters are always smaller. See for example, Christiano, Eichenbaum and Evans (2005) and Dib (2006).

The estimates of capital-adjustment cost parameters in manufactured, non-tradable, and commodity sectors,  $\psi_M$ ,  $\psi_N$ , and  $\psi_X$ , are 2.03, 7.38, and 11.21, respectively. Thus, capital rigidity is different across sectors. It is less costly to adjust capital in the manufacturing sector than in the non-tradable and commodity sectors. With their standard errors, capital adjustment cost parameters are statistically different across the sectors.<sup>14</sup>

Next, the estimates of the monetary policy parameters are reported. They are all positive and statistically significant. The estimated value of the interest rate smoothing coefficient,  $\varrho_R$ , is 0.73. The estimates of  $\varrho_\pi$  and  $\varrho_Z$ , which measure the response of monetary policy to inflation and final good variations, are about 0.50 and 0.014, respectively. The estimated values of,  $\sigma_R$ , the standard deviation of monetary policy shocks, is about 0.0037, which is very close to the estimated value in Dib (2003).

Commodity price shocks,  $p_X^*$ , appear to be persistent and highly volatile, with an autoregressive coefficient,  $\rho_{P_X}$ , and standard deviation,  $\sigma_{P_X}$ , estimated at 0.86 and 0.041, respectively. Natural resource shocks,  $L_t$ , are moderately persistent but highly volatile, with estimated values of their autoregressive coefficient,  $\rho_L$ , and standard errors,  $\sigma_L$ , equal to 0.64 and 0.062, respectively. The autocorrelation coefficients of technology shocks in the manufacturing and non-tradable sectors,  $\rho_{AM}$  and  $\rho_{AN}$ , are estimated at 0.84 and 0.94, respectively, while the estimates of their standard deviations,  $\sigma_{AM}$  and  $\sigma_{AN}$ , are 0.028 and 0.004, respectively. Thus, technology shocks in the manufacturing sector are moderately persistent, but highly volatile, their standard deviations being seven times larger than those in the non-tradable sector. The remaining domestic and foreign shocks, save foreign inflation—government spending, the foreign interest rate, and foreign output—are persistent and volatile, with estimated values of their autoregressive coefficients and standard deviations similar to common findings in previous studies.

## 4.2 Impulse responses

Figure 2 plots the impulse responses of some key macroeconomic variables to a 1% positive commodity price shock. This shock is an exogenous increase in real commodity prices in the world markets.

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<sup>14</sup>Using a Bayesian estimation method and data for the period 1972Q1–2003Q3, Rebei and Ortega (2006), who assume that the adjustment cost parameter is common to tradable and non-tradable sectors, estimate it at around 10.

Therefore, it is interpreted as an exogenous shock to the terms of trade in this small open economy. Each response is expressed as a percentage deviation of a variable from its steady-state level. Since there is no significant wage rigidity in the commodity sector and the nominal exchange rate is perfectly flexible, several macroeconomic variables respond sharply to commodity price shocks.

Overall, following an increase in commodity prices, the real exchange rate immediately appreciates before gradually returning to its deterministic steady-state level. The exogenous increase in commodity prices induces the commodity-producing firm to instantaneously increase its production, which leads to an immediate jump in commodity exports. The appreciation of the real exchange rate should, however, negatively affect commodity-producing firm's revenues expressed in the home currency, leading to a reduction in the magnitude of the increase in the total commodity production.

On the other hand, as commodity prices increase, firms in manufacturing and non-tradable sectors, which use commodities as material inputs in their production technologies, are negatively affected. They immediately reduce their demand for commodity inputs. The demand for commodity inputs sharply decreases by about 0.8% in both sectors after the shock and persists for several quarters. Demand for labour and capital increase slightly, but not persistently, in manufacturing and non-tradable sectors. Since capital and labour are not perfect substitutes for commodity inputs in the production of manufactured and non-tradable goods, output in the manufacturing sector slightly, but persistently, decreases; whereas output in the non-tradable sector increases during the few quarters after the shock before settling marginally below its steady-state level in the long run. Nevertheless, the appreciation of the domestic currency helps in offsetting a fraction of negative effects of the increase in commodity prices in these two sectors. Even though real commodity price shocks lead to a significant decrease in manufacturing sector output, overall output (GDP in this economy) significantly and persistently increases (the total GDP increases by about 0.20% after the shock), due particularly to the rise in commodity production.

Conversely, a positive commodity price shock leads to an increase in consumption in both short and long terms. This increase in consumption is the result of the wealth effect associated with the jump in revenues in the commodity sector. Thus, the aggregate demand (consumption and investment) increases leading to some inflation pressures, as the CPI inflation rate jumps sharply, but temporarily, above its

steady-state level. The nominal interest rate increases slightly, but persistently, as a response of the monetary authority to the increases in inflation and aggregate demand. We also note a sharp increase in the real wage and in the hours worked in the commodity sector (real wages and hours worked increase by about 0.6% on impact). On the other hand, real wages marginally increase in the manufacturing sector, while they modestly decrease in the non-tradable sector. This latter response is explained by the presence of high wage rigidity in the non-tradable sector (as the Calvo coefficient is estimated at 0.91).

Figure 2 also shows that, following a positive commodity price shock, commodity exports sharply rise on impact and persist for a longer time. They immediately jump by about 1% above the steady state level. This happens because, after the shock, production of commodities increases, while home demand for commodities used as material inputs in manufacturing and non-tradable sectors decreases. Conversely, a positive commodity price shock leads to a gradual and persistent decrease in exports of manufactured goods. This is caused firstly by the appreciation of the domestic currency following this shock, which creates a Dutch disease effect that reduces foreign demand for domestic manufactured goods. Secondly, increases in commodity prices significantly reduce production in the manufacturing sector due to the reallocation of resources across sectors.

Finally, imports from abroad gradually and persistently rise because of the appreciation of domestic currency, the increase in the home aggregate demand, and the large increases in capital inflows from abroad. Therefore, the country is wealthier and enjoys more consumption and leisure. The positive response of commodity exports and the exogenous increase in commodity prices lead to large improvement in the home country's current account and trade balance, even though exports of manufactured goods decrease and imports increase. Thus, the foreign debt stock significantly falls, for a longer time, after a positive commodity price shock.

### **4.3 Volatility and correlations**

To assess the performance of our baseline model, we consider the model-implied volatilities (standard deviations), relative volatilities, autocorrelations, and correlations of some variables of interest. Table 4 reports these statistics for the baseline model and from the HP-filtered data for the estimation sample:

1981Q1–2005Q4. The standard deviations are expressed in percentage terms.

Columns 2 and 3, in Table 4, display standard deviations and relative volatilities of actual data and those simulated from the model. In the data, the real exchange rate and real commodity prices are about 2.6 and 5 times as volatile as output, respectively. The standard deviation of the real exchange rate is 3.80, 7.35 for real commodity prices, while the standard deviation of output is 1.44. We note that commodities and manufactured goods are also more volatile than total output; however, non-tradables, the nominal interest rate, and inflation are less volatile than output. The simulated results indicate that the model slightly overpredicts volatilities of all the considered variables, except inflation. This is a common feature in New-Keynesian models with price and/or wage rigidities. Conversely, the relative volatilities implied by the model for all the variables, save non-tradables, are smaller than those observed in actual data. The model is generally successful at matching volatilities and relative volatilities of the considered macroeconomic variables.

Column 4, in Table 4, shows unconditional autocorrelations of the data and those generated in the model. In general, the model does a better job at matching the unconditional autocorrelations shown in the data within a one-quarter horizon. Column 5 and 6, in Table 4, display the unconditional correlations of the data and those simulated in the model. They are calculated between a given variable,  $x_t$ , and either the real exchange rate,  $S_t$ , or total output,  $Y_t$ . The striking result is that the model is successful at replicating the unconditional correlation of the real exchange rate and commodity prices,  $p_{Xt}^*$ : The model-implied correlation between  $S_t$  and  $p_{Xt}^*$ , is -0.49, which is somewhat close to -0.61 observed in the data. Nevertheless, the model is relatively unsuccessful in generating other correlations as observed in the data; it produces either smaller correlations or correlations with opposite signs.

#### **4.4 Variance decomposition**

In this subsection, we examine the forecast-error variance decomposition of the real exchange rate in four variants of the above-described and estimated model: (1) the estimated baseline model, (2) a sticky-price model, (3) a sticky-wage model, and (4) a flexible-price-and-wage model. In the last three models, Calvo parameters of price and/or wage rigidities are set at zero (i.e.  $\phi_\iota = 0$  for  $\iota = F, M, N$ , and/or  $\varphi_i = 0$

for  $i = M, N, X$ ), but keeping all of the other parameters equal to their estimates from the baseline model. This decomposition enables us to calculate the proportion of real exchange rate variations owing to commodity prices, monetary policy, and foreign interest rate shocks.

Panel A in Table 5, shows that, in the baseline model when considering all the shocks, commodity price and foreign interest rate shocks account for about 32% and 51% of real exchange rate variations in a one-quarter-ahead horizon, respectively. Monetary policy shocks, however, account only for about 7% of these variations, while the other remaining shocks marginally contribute to exchange rate variations. Nevertheless, when commodity price shocks are turned off, by setting their standard deviation  $\sigma_{P_X} = 0$ , the contribution of foreign interest rate shocks becomes much larger; they account for about 75% of short-term real exchange rate variations. Monetary policy shocks, however, account for about 11% in one-quarter-ahead horizon. When excluding foreign interest shocks, by setting its standard deviation  $\sigma_{R^*} = 0$ , commodity price shocks become the dominant source of the short-term variations of the real exchange rate. In this case, these shocks account for about 65% of variations in a one-quarter-ahead horizon.

Similarly, Panels B, C, and D, in Table 4, display that commodity price shocks account for at least 37% of real exchange rate variations in the sticky-price, sticky-wage, and flexible price-and-wage models. Foreign interest rate shocks still explain at least 41% of this variation at a one-ahead-quarter horizon. When excluding either commodity price or foreign interest rate shocks, the contribution of either shock is at least 70% of real exchange rate fluctuations in the short term. Nevertheless, in these models, monetary policy shocks contribute very little to real exchange rate fluctuations, even when either commodity price or foreign interest rate shocks are excluded.

Thus, the variance decompositions show that commodity price shocks have a significant role in explaining the short-term fluctuations of the real exchange rate. These shocks are the second source of real exchange rate variations in all examined variants of the model. Note that natural resource shocks (land) contribute modestly to real exchange rate fluctuations. They account for only about 3% of total exchange rate variation in all the models.

## 5. Welfare Analysis

To analyze welfare effects, we solve the model to a second-order approximation around its deterministic steady state using the Dynare program.<sup>15</sup> This procedure is similar to the one used in Schmitt-Grohé and Uribe (2004). Then, welfare measures are calculated as the unconditional expectation of utility in the deterministic steady state. This allows us to compare two alternative steady states: stochastic and deterministic.<sup>16</sup> Thus, we first calculate a second-order Taylor expansion of the single-period utility function that is given by:

$$U(\cdot) = \frac{C_t^{1-\tau}}{1-\tau} - \frac{H_t^{1+\chi}}{1+\chi}. \quad (74)$$

The second-order Taylor expansion of (74) around deterministic steady-state values of its arguments yields:

$$\begin{aligned} E(U_t) &= \frac{C^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi} + C^{1-\tau}E(\widehat{C}_t) - H^{1+\chi}E(\widehat{H}_t) \\ &\quad - \frac{\tau}{2}C^{1-\tau}E(\widehat{C}_t^2) - \frac{\chi}{2}H^{1+\chi}E(\widehat{H}_t^2), \end{aligned} \quad (75)$$

where  $\widehat{C}_t$  and  $\widehat{H}_t$  are the log deviations of  $C_t$  and  $H_t$  from their deterministic steady state values, while  $E(\widehat{C}_t^2)$  and  $E(\widehat{H}_t^2)$  are their variances, respectively.

We conduct welfare analysis assuming the historical (estimated) values of the Taylor rule coefficients from Table 2.<sup>17</sup> The welfare gain associated with a particular scenario is measured by the compensating variation. This measures the percentage change in consumption in the deterministic steady state that would give households the same unconditional expected utility in the stochastic economy. Because the model is solved using a second-order approximation of its equilibrium conditions, the variances of the shocks affects the means and the variances of the endogenous variables of the economy. Therefore,

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<sup>15</sup>Dynare, which uses Sims' (2002) programs, calculates a second-order approximation of the model around its deterministic steady state. We use the program to calculate the theoretical first and second moments of the model's endogenous variables, including period utility. See Juillard (2002).

<sup>16</sup>In stochastic steady state, there is a risk related to the shocks in the model, while in deterministic steady state the shock are set equal to zero, so there is no uncertainty.

<sup>17</sup>In this work, we do not calculate optimized monetary policy rules in which the monetary authority optimally chooses optimized Taylor rule coefficients to stabilize inflation, as in Kollmann (2002), Ambler et al. (2004), Ortega and Rebei (2006), and Bergin et al. (2007). We leave this to future work.

we decompose the total welfare effect that is measured by the compensating variation into a level effect, which influences the expected means of variables by implying a permanent shift in steady state of consumption, and into a variance effect which implies a permanent shift in steady state consumption associated with the effects of the shocks on the variables' variances.

Let  $\mu_m$  denote the level effect that is defined as:

$$\begin{aligned} EU((1 + \mu_m)C_t, H_t) &= \frac{((1 + \mu_m)C)^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi} \\ &= \frac{C^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi} + C^{1-\tau}E(\widehat{C}_t) - H^{1+\chi}E(\widehat{H}_t). \end{aligned} \quad (76)$$

Solving for  $\mu_m$  yields:

$$\mu_m = \left[ 1 + (1 - \tau)E(\widehat{C}_t) - (1 - \tau)\frac{H^{1+\chi}}{C^{1-\tau}}E(\widehat{H}_t) \right]^{\frac{1}{1-\tau}} - 1. \quad (77)$$

Similarly, let  $\mu_v$  denotes the variance effect. Therefore,

$$\begin{aligned} EU((1 + \mu_v)C_t, H_t) &= \frac{((1 + \mu_v)C)^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi} \\ &= \frac{C^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi} - \frac{\tau}{2}C^{1-\tau}E(\widehat{C}_t^2) - \frac{\chi}{2}H^{1+\chi}E(\widehat{H}_t^2) \end{aligned} \quad (78)$$

Solving for  $\mu_v$  yields:

$$\mu_v = \left[ 1 - \frac{\tau(1-\tau)}{2}E(\widehat{C}_t^2) - \frac{\chi(1-\tau)}{2}\frac{H^{1+\chi}}{C^{1-\tau}}E(\widehat{H}_t^2) \right]^{\frac{1}{1-\tau}} - 1. \quad (79)$$

We calculate welfare effects for the baseline model (the above-described model) and for other alternative models: Sticky-price, sticky-wage, and flexible-price-and-wage models, as well as the baseline model simulated under the local currency pricing assumption. Tables 6–8 report welfare analysis results: Standard deviations, stochastic steady state deviations, and welfare effects decomposed into level and variance effects of deviations of variables from their levels in the deterministic steady state. These results are for the real exchange rate,  $S_t$ , changes in the nominal exchange rate,  $\Delta e_t$ , consumption,  $C_t$ , aggregate labour,  $H_t$ , output,  $Y_t$ , total investment,  $I_t$ , the nominal interest rate,  $R_t$ , CPI inflation,  $\pi_t$ , and the trade balance,  $TB_t$ .

## 5.1 The baseline model

Table 6 reports the results from the baseline model. The results are for four experiments: (1) the baseline model simulated with all the shocks; (2) the baseline model simulated with only commodity price shocks; (3) the baseline model simulated with only foreign interest rate shocks; and (4) the baseline model simulated without commodity price and foreign interest rate shocks.<sup>18</sup> Of special interest in this study is the implication of exchange rate volatility on welfare. Hence, we focus on the shocks that account for large fractions of exchange rate fluctuations. The experiments are conducted under both alternative flexible and fixed exchange rate regimes.

Panel A shows firstly that, in all of the experiments, the volatility of the real exchange rate is much larger when adopting a flexible exchange rate than under a fixed exchange rate. Conversely, the volatilities of the other variables are much lower under a flexible exchange rate regime. Thus, fixing the exchange rate greatly raises the variability of consumption, labour, output, and investment. Secondly, whatever the adopted exchange rate regime, volatilities of the real exchange rate and the other variables are relatively lower when simulating the model with either commodity price or foreign interest rate shocks, or when excluding both shocks. Thirdly, under a flexible exchange rate regime, the presence of commodity price and/or foreign interest rate shocks increases exchange rate volatility. We also note that the volatility of the nominal interest rate is much larger when the model is simulated under a fixed exchange rate, except when excluding foreign interest rate shocks. This because, under a fixed exchange rate regime, the home nominal interest rate is mostly determined by foreign monetary policy, as shown in the UIP condition (14).

Panel B reports stochastic steady state deviations as a percentage of deterministic steady state levels. The presence of risk has negative effects on all examined variables, except consumption under a flexible exchange rate regime. The stochastic mean deviations of the real exchange rate are about -0.19% and -0.29% in the baseline model simulated with all the shocks with flexible or fixed exchange rate regimes, respectively. These exhibit a 0.19% and 0.29% appreciation relative to the deterministic steady state.

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<sup>18</sup>In experiments (2)–(4), we keep all other parameters at their calibrated or estimated values.

The negative effects on the other endogenous variables are much higher when pegging the exchange rate. Nevertheless, when simulating the model with all the shocks, stochastic consumption mean rises by about 0.03% when the nominal exchange rate is flexible; while it decreases by about -0.06% under the fixed exchange rate. Similarly, when excluding commodity price and foreign interest shocks, consumption marginally decrease, even under a flexible exchange rate regime. Since domestic output decreases, the increase in consumption is driven by the raise in imports, so the mean of the trade balance slightly decreases by about -0.043% in the baseline model simulated with all the shocks.

Panel C shows the total and decomposed welfare effects expressed as a percentage of the deterministic steady state of consumption. Welfare is higher in the stochastic steady state than in the deterministic steady state when adopting a flexible exchange rate regime, except when excluding commodity and foreign interest rate shocks. First, in the baseline model simulated with all the shocks and a flexible exchange rate, the overall welfare effect is 0.042% of consumption, indicating that consumption in the stochastic steady state is higher than deterministic steady state by about 0.042%. This overall effect is divided into the level effect of 0.067% and the variance effect of -0.025%. When simulating the model with only commodity price shock, the overall welfare effect slightly increases to 0.044%, divided into the level effect of 0.057% and the variance effect of -0.013%. Similarly, when simulating the model with only foreign interest rate shocks, the overall welfare effect is 0.044%, divided into the level effect of 0.047% and the variance effect of -0.003%.

Interestingly, the presence of uncertainty caused by either real commodity price or foreign interest rate shocks implies welfare gains in the stochastic economy. Real commodity price shocks have positive welfare effects because expected household's revenues would be higher with fluctuating commodity prices than with constant prices. This fact may be explained by the presence of different non-linearities in the model. For example, marginal costs in manufacturing and non-tradable sectors are strictly concave in commodity prices; whereas, real wages and real capital returns in the three production sectors, as well as the real price of the natural resource factor in the production of commodities, are strictly convex in real commodity prices. Thus, the average marginal costs with fluctuating commodity prices are lower than with stabilized prices. Similarly, the average of real wages, capital return rates, and prices of the natural

resource factor are higher with fluctuating commodity prices than with constant prices. In addition, both commodity price and foreign interest shocks lead the country to hold a larger stock of foreign bonds, as households engage in precautionary saving. Therefore, the country is wealthier on average and it enjoys higher means of consumption and leisure.<sup>19</sup> Furthermore, stochastic means of output and investment are lower because firms hedge against uncertainty by setting higher prices and lowering their production. Hence, labour demand decreases and the mean of hours worked is lower relative to deterministic steady state.

Under the fixed exchange rate regime, uncertainty negatively affects households' welfare whatever the simulated model. When the model is simulated with all the shocks, the overall welfare decreases by -0.164% of consumption, decomposed into the level effect of -0.111% and the variance effect of -0.053%. When simulating the model with only commodity price shocks, the overall welfare effect marginally rises by 0.009%, decomposed into the level effect of 0.031% and the variance effect of -0.022%. Nonetheless, compared to flexible exchange rate scenarios, the contributions of variance effects are more important under the fixed exchange rate regime. Therefore, all the fixed exchange rate cases are inferior to those under the flexible exchange rate regime. In the baseline model simulated with all the shocks, adopting a fixed exchange rate would lower the total welfare by about -0.2% of consumption if compared to the flexible exchange rate case. Importantly, exchange rate movements compensate for nominal rigidities by promoting adjustment in relative prices to domestic shocks.

## 5.2 Other variants of the model

Table 7 displays the welfare effects calculated from sticky-price, sticky-wage, and flexible-price-and-wage models. In this case, we report the results of two experiments: (1) the models simulated with all the shocks and (2) the model simulated with only commodity price shocks. In each experiment, we conduct welfare analysis under both alternative flexible and fixed exchange rate regimes.

The results in Table 7 confirm those in Table 6 for the baseline model. The real exchange rate is

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<sup>19</sup>Stochastic means of foreign bonds increase by 1.5% and 4.8% relative to the deterministic steady state when simulating the base model with either commodity price or foreign interest rate shocks, respectively.

more volatile under a flexible exchange rate; while all other variables are more volatile when adopting a fixed exchange rate. Moreover, the presence of uncertainty negatively affects the stochastic means of the main macroeconomic variables, except consumption in the sticky-price model, in the sticky-wage model simulated with a flexible exchange rate, and in the flexible-price-and-wage model.

Under the flexible exchange rate regime, the sticky-price model simulated with all the shocks implies an overall welfare effect of 0.103%, divided into level effect of 0.122%, and variance effect, -0.019%. However, in the case of commodity price shocks, the overall effect drops to 0.056%, decomposed to level effect of 0.066% and variance effect of -0.010%. Gains in welfare are smaller in the sticky-wage model; total effect is equal to 0.059% with all the shocks and 0.052% when simulating the model with only commodity price shocks. This is because the degree of nominal rigidities is much higher in the sticky-wage model than in the sticky-price model.

Under a fixed exchange rate regime, welfare gains are smaller. The total welfare effect is 0.035% in the sticky-price model simulated with all the shocks; this effect is divided into level effect of 0.06% and variance effect of -0.025%. However, when simulating the model with only commodity price shocks, overall effect slightly increases to 0.049%, driven mostly by the mean effect of 0.06%. Conversely, the negative impact of uncertainty is much larger in the sticky-wage model. Overall effect in the model simulated with all the shocks is -0.101%, decomposed into level effect of -0.053%, and variance effect of -0.048%. When simulating the model with only commodity price shocks, overall effect is slightly positive. It is about 0.018%, divided into level effect of 0.039% and variance effect of -0.021%.

On the other hand, the flexible-price-and-wage model generates the highest welfare gains, compared to the models with either nominal rigidities. With all the shocks, the overall effect rises by 0.145%, decomposed into level effect of 0.166% and variance effect of -0.021%; while with only commodity price shocks, the total welfare gain is 0.065%, divided into level effect of 0.075% and variance effect of -0.01%. We note that the choice of the exchange rate regime is irrelevant when prices and wages are perfectly flexible. The welfare gain is the same whatever the adopted exchange rate regime because relative prices freely adjusted, as argued by Friedman (1953). Therefore, the equilibrium with flexible prices and wages is efficient, as it gives the highest welfare gain. This result is in line with previous

findings in the literature on optimal monetary policies. See, for example, Aoki (2001), Kollmann (2002), Devereux and Engel (2003), and Bergin et al. (2007).

### 5.3 Model with local currency pricing

Motivated by the empirical failure of the LOP, several studies on optimal monetary policy in open economies have assumed local currency pricing (LCP) behavior by domestic firms. Examples of these studies are Betts and Devereux (1996), Kollmann (2002), Devereux and Engel (2003), and others. Under this assumption, prices of domestic manufactured goods exported abroad are set in foreign currency. Thus, firms may price-discriminate between home and foreign market and the LOP no longer holds.<sup>20</sup>

Table 8 reports the welfare results of the baseline model simulated under the LCP. The results indicate that the welfare effects of commodity prices and exchange rate volatilities are very similar to those reported in Table 6. The welfare gains when adopting LCP are very marginal under flexible exchange rate regime. The overall welfare effect under LCP is 0.055 compared to 0.042 in the baseline model with PCP assumption. Nevertheless, the loss in welfare is slightly higher under the fixed exchange rate regime. This result is not in contradiction with the finding in Devereux and Engel (2003): they find that an optimized monetary policy leads to higher welfare gains when adopting LCP and a fixed exchange rate. In our study, we do not consider optimized monetary policies under the alternative exchange rate regimes. We, however, calculate welfare effects using an estimated monetary policy rule. We also have a combination of PCP and LCP setting, as commodity exports that represent about 35% of total exports are invoiced in the foreign currency, while prices of manufactured goods exports are set in the domestic currency.

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<sup>20</sup>Under the assumption of PTM, the firm  $j$  sets the price  $\tilde{P}_{M,jt}$  for the domestic market and the price  $\tilde{P}_{M,jt}^*$  for the foreign market. The foreign demand for domestic manufactured-goods is  $Y_{M,t}^{ex} = \omega_{ex} \left( \frac{P_{M,t}^*}{P_t^*} \right)^{-\nu} Y_t^*$ . The nominal profit function is:  $D_{M,jt+l} = \pi^l \tilde{P}_{M,jt} Y_{M,jt+l}^d + \pi^{*l} e_{t+l} \tilde{P}_{M,jt}^* Y_{M,jt+l}^{ex} - Q_{M,t+l} K_{M,jt+l} - W_{M,t+l} H_{M,jt+l} - e_{t+l} P_{X,t+l}^* Y_{X,jt+l}^M$ . It is possible that  $\tilde{P}_{M,jt} \neq e_t \tilde{P}_{M,jt}^*$ , in the equilibrium. The current account equation is  $\frac{b_t^*}{\kappa_t R_t^*} = \frac{b_{t-1}^*}{\pi_t^*} + p_{X,t}^* (Y_{X,t} - Y_{X,t}^M - Y_{X,t}^N) + p_{M,t}^* Y_{M,t}^{ex} - Y_{F,t}$ .

## 6. Conclusion

This paper develops a multi-sector New Keynesian model of a small open economy that includes commodity, manufacturing, non-tradable, and import sectors. Price and wage rigidities are sector specific, while labour and capital are imperfectly mobile across sectors. The model is used to emphasize the role of commodity price shocks in explaining exchange rate volatility and to assess their welfare implications. Some of the model's structural parameters are estimated using maximum likelihood procedure applied to Canadian and U.S. time series. The estimation results indicate significant heterogeneity among sectors.

Using a second-order procedure, the model is solved to calculate welfare measures. A general result indicates that commodity price shocks account for a large fraction of exchange rate fluctuations. Welfare benefits under commodity price and foreign interest shocks, the two dominant shocks that explain most of exchange rate variability, are larger when adopting a flexible exchange rate. In response to commodity price shocks, commodity-producing firms increase their output when prices are higher and reduces it when prices are lower. Therefore, their revenues are larger when commodity prices are variable. In addition, real commodity price and foreign interest rate shocks lead the country to hold a larger stock of foreign bonds, increase its imports, and enjoys higher means of consumption and leisure. Nonetheless, when excluding these shocks, the presence of uncertainty leads to large decreases in the welfare level; in particular, when adopting a fixed exchange rate regime. The main results indicate that a flexible exchange rate is necessary to increase the welfare gains of real commodity price shocks and to offset those of foreign interest rates.

While these results are interesting, future work requires some extensions, such as considering the case of optimized monetary policy rules to evaluate how a monetary authority would response to exchange rate fluctuations. Moreover, it would be interesting to conduct welfare analysis using conditional expectations of utility around the deterministic steady state.

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**Table 1: Calibration of the parameters**

Parameters	Definition	Values
$\beta$	discount factor	0.9902
$\tau$	curvature parameter	2
$\varsigma$	labour elasticity of substitution across sectors	1
$\chi$	intertemporal elasticity of labour	1
$\delta$	capital depreciation rate	0.025
$\alpha_X$	share of capital in commodity output	0.41
$\gamma_X$	share of labour in commodity output	0.39
$\eta_X$	share of natural resources in commodity output	0.20
$\alpha_M$	share of capital in manufactured goods	0.26
$\gamma_M$	share of labour in manufactured goods	0.63
$\eta_M$	share of commodity inputs in manufactured goods	0.11
$\alpha_N$	share of capital in non-tradable goods	0.28
$\gamma_N$	share of labour in non-tradable goods	0.66
$\eta_N$	share of commodity inputs in non-tradable goods	0.06
$\theta$	intermediate-goods elasticity of substitution	6
$\vartheta$	labour elasticity of substitution	8
$\nu$	elasticity of substitution between home and foreign goods	0.8
$\omega_F$	share of imports in the final goods	0.33
$\omega_M$	share of domestic manufactured goods in the final goods	0.10
$\omega_N$	share of non-tradables in the final goods	0.57
$\omega_{ex}$	constant associated with the share of exports in home GDP	0.21
$\varkappa$	constant associated with risk premium	0.0115
$R$	gross steady-state of domestic nominal interest rate	1.0185
$\pi$	gross steady-state of domestic inflation rate	1.0085
$R^*$	gross steady-state of the foreign nominal interest rate	1.0158
$\pi^*$	gross steady-state of the foreign inflation rate	1.0070

**Table 2: Maximum likelihood estimates: Sample 1981Q1–2005Q4**

Parameters	Definitions	Estimates	St. Errors
$\phi_M$	Calvo price parameter, manufacturing sector	0.589	0.004
$\phi_N$	Calvo price parameter, non-tradable sector	0.530	0.007
$\phi_F$	Calvo price parameter, import sector	0.718	0.011
$\varphi_M$	Calvo wage parameter, manufacturing sector	0.487	0.006
$\varphi_N$	Calvo wage parameter, non-tradable sector	0.914	0.019
$\varphi_X$	Calvo wage parameter, commodity sector	0.120	0.013
$\psi_M$	capital adjustment cost parameter, manufacturing sector	2.029	0.216
$\psi_N$	capital adjustment cost parameter, non-tradable sector	7.384	0.725
$\psi_X$	capital adjustment cost parameter, commodity sector	11.213	3.755
$\varrho_R$	monetary policy smoothing coefficient	0.733	0.083
$\varrho_\pi$	monetary policy inflation coefficient	0.496	0.025
$\varrho_Z$	monetary policy output coefficient	0.014	0.005
$\sigma_R$	standard deviations of monetary policy shocks	0.0037	0.0002
$\rho_{AM}$	technology autoregressive coefficient, manufacturing sector	0.842	0.092
$\sigma_{AM}$	technology standard deviations, manufacturing sector	0.0285	0.004
$\rho_{AN}$	technology autoregressive coefficient, non-tradable sector	0.942	0.051
$\sigma_{AN}$	technology standard deviations, non-tradable sector	0.0040	0.0005
$\rho_{PX}$	autoregressive coefficient of commodity price shocks	0.862	0.024
$\sigma_{PX}$	standard deviations of commodity price shocks	0.0413	0.003
$\rho_L$	autoregressive coefficient of natural resource shocks	0.639	0.020
$\sigma_L$	standard deviations of natural resource shocks	0.0622	0.004
$\rho_g$	autoregressive coefficient of government spending shocks	0.762	0.021
$\sigma_g$	standard deviations of government spending shocks	0.0123	0.002
$\rho_{R^*}$	autoregressive coefficient of foreign interest shocks	0.827	0.046
$\sigma_{R^*}$	standard deviations of foreign interest shocks	0.0057	0.002
$\rho_{\pi^*}$	autoregressive coefficient of foreign inflation shocks	0.429	0.010
$\sigma_{\pi^*}$	standard deviations of foreign inflation shocks	0.0066	0.001
$\rho_{Y^*}$	autoregressive coefficient of foreign output shocks	0.947	0.025
$\sigma_{Y^*}$	standard deviations of foreign output shocks	0.0073	0.001
<i>L.L.</i>	Log-Likelihood value	2880.9	

**Table 3: Steady-state ratios of GDP in the baseline model**

Manufacturing	0.261	Total exports	0.265
Non-tradables	0.589	Manufacturing exports	0.172
Commodities	0.150	Commodity exports	0.093
Consumption	0.628	Imports	0.316
Investment	0.192	Foreign assets	-0.20
Government spending	0.231		

**Table 4: Volatility and correlations: Data and the baseline model (HP-filtered series, sample 1981Q1–2005Q4)**

Variables	$\sigma_x$	$\sigma_x/\sigma_Y$	$E(x_t, x_{t-1})$	$E(S_t, x_t)$	$E(Y_t, x_t)$
<i>Data</i>					
$S_t$	3.80	2.64	0.96	1.00	0.11
$p_{X,t}^*$	7.35	5.10	0.82	-0.61	0.30
$Y_t$	1.44	1.00	0.91	0.11	1.00
$Y_{X,t}$	2.25	1.56	0.66	-0.13	0.46
$Y_{M,t}$	4.09	2.84	0.89	0.05	0.93
$Y_{N,t}$	1.04	0.72	0.88	0.18	0.92
$R_t$	0.36	0.25	0.78	-0.12	0.24
$\pi_t$	0.47	0.33	0.65	-0.06	0.55
<i>The Baseline model</i>					
$S_t$	4.69	2.11	0.82	1.00	-0.03
$p_{X,t}^*$	8.14	3.67	0.86	-0.49	0.61
$Y_t$	2.22	1.00	0.79	-0.03	1.00
$Y_{X,t}$	2.85	1.28	0.73	0.14	0.45
$Y_{M,t}$	4.46	2.01	0.90	0.48	0.21
$Y_{N,t}$	2.10	0.94	0.75	-0.01	0.69
$R_t$	0.46	0.21	0.79	0.27	-0.58
$\pi_t$	0.43	0.20	0.53	0.61	0.13

**Table 5: Forecast-error variance decomposition of the real exchange rate**

	Percentage owing to:			
	Comm. prices	Mon. Policy	Foreign Interest	Others
<i>A. Baseline model: Sticky prices and sticky wages</i>				
All of the shocks	32.21	7.15	50.66	9.98
Excluding com. price shock, $\sigma_{PX} = 0$	0.00	10.55	74.73	14.72
Excluding foreign int. shock, $\sigma_{R^*} = 0$	65.28	14.50	0.00	14.66
<i>B. Sticky prices</i>				
All of the shocks	37.17	1.85	51.77	9.21
Excluding com. price shock, $\sigma_{PX} = 0$	0.00	2.94	82.40	17.92
Excluding foreign int. shock, $\sigma_{R^*} = 0$	77.06	3.83	0.00	19.11
<i>C. Sticky wages</i>				
All of the shocks	41.05	3.08	41.20	14.67
Excluding com. price shock, $\sigma_{PX} = 0$	0.00	5.23	69.89	24.88
Excluding foreign int. shock, $\sigma_{R^*} = 0$	69.81	5.24	0.00	24.95
<i>D. Flexible prices and wages</i>				
All of the shocks	43.60	0.00	44.21	12.19
Excluding com. price shock, $\sigma_{PX} = 0$	0.00	0.00	78.39	21.61
Excluding foreign int. shock, $\sigma_{R^*} = 0$	78.15	0.00	0.00	21.85

**Table 6: Welfare effects: The baseline model**

	Flexible Exchange Rate				Fixed Exchange Rate			
	(A)	(B)	(C)	(D)	(A)	(B)	(C)	(D)
<i>A. Standard deviations (in %)</i>								
$S_t$	4.69	2.66	3.34	1.94	2.99	1.85	1.49	1.83
$\Delta e_t$	3.14	0.79	2.55	1.67	0.00	0.00	0.00	0.00
$C_t$	1.59	1.12	0.52	1.00	2.31	1.49	1.28	1.20
$H_t$	2.28	0.88	0.62	2.01	4.90	2.08	3.73	2.41
$Y_t$	2.22	1.38	0.35	1.70	3.50	2.26	1.94	1.82
$I_t$	9.02	2.35	5.14	7.03	17.6	5.08	15.1	7.31
$R_t$	0.46	0.15	0.28	0.34	1.04	0.17	1.02	0.07
$\pi_t$	0.43	0.12	0.27	0.32	0.66	0.27	0.34	0.49
$TB_t$	1.15	0.84	0.65	0.45	1.39	0.65	1.14	0.47
<i>B. Stochastic steady state deviations (in %)</i>								
$S_t$	-0.190	-0.057	-0.079	-0.054	-0.295	-0.079	-0.116	-0.100
$\Delta e_t$	-0.041	0.000	0.031	0.001	0.000	0.000	0.000	0.000
$C_t$	0.031	0.027	0.022	-0.018	-0.059	-0.014	-0.015	-0.058
$H_t$	-0.040	-0.012	-0.016	-0.012	-0.085	-0.021	-0.032	-0.033
$Y_t$	-0.043	0.025	-0.036	-0.032	-0.101	0.027	-0.043	-0.086
$I_t$	-0.005	0.007	-0.004	-0.008	0.022	0.011	0.030	-0.019
$R_t$	-0.014	-0.003	-0.001	-0.010	-0.038	-0.008	-0.020	-0.010
$\pi_t$	-0.007	-0.003	-0.001	-0.004	-0.009	-0.002	-0.006	-0.002
$TB_t$	-0.043	-0.003	-0.032	-0.008	-0.057	-0.018	-0.051	-0.008
<i>C. Welfare effects (as % of deterministic steady state of consumption)</i>								
$\mu_m$	0.067	0.057	0.047	-0.036	-0.111	0.031	-0.028	-0.114
$\mu_v$	-0.025	-0.013	-0.003	-0.010	-0.053	-0.022	-0.017	-0.014
Overall	0.042	0.044	0.044	-0.046	-0.164	0.009	-0.045	-0.128

In columns (A), the model is simulated with all of the shocks. In columns (B), the model is simulated with only commodity price shocks, i.e., only  $\sigma_{PX} \neq 0$ . In columns (C), the model is simulated with foreign interest rate shocks, i.e., only  $\sigma_{R^*} \neq 0$ . In columns (D), the model is simulated without commodity price and foreign interest shocks i.e.  $\sigma_{PX} = \sigma_{R^*} = 0$ .

**Table 7: Welfare effects: Sticky-price, sticky-wage, and flexible-price-and-wage models**

	Sticky prices				Sticky wages				Flexible P.&W.	
	Flexible E.R.		Fixed E.R.		Flexible E.R.		Fixed E.R.		Flexible E.R.	
	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
<i>A. Standard deviations (in %)</i>										
$S_t$	4.81	2.93	3.67	2.45	3.79	2.43	2.95	1.97	4.02	2.65
$\Delta e_t$	3.12	0.87	0.00	0.00	2.74	0.70	0.00	0.00	2.73	0.77
$C_t$	1.37	0.99	1.58	1.04	1.64	1.09	2.19	1.43	1.43	0.97
$H_t$	1.02	0.54	2.01	0.91	2.10	0.80	4.01	1.68	0.70	0.51
$Y_t$	1.66	1.07	2.09	1.51	2.18	1.55	3.14	2.10	1.68	1.28
$I_t$	6.44	1.95	11.2	2.63	10.5	2.36	15.5	4.13	8.01	1.73
$R_t$	0.33	0.13	1.04	0.17	0.54	0.14	1.04	0.18	0.41	0.12
$\pi_t$	0.54	0.11	1.23	0.53	0.92	0.11	1.07	0.45	0.90	0.09
$TB_t$	1.15	0.87	1.34	0.68	1.33	0.74	1.40	0.70	1.30	0.75
<i>B. Stochastic steady state deviations (in %)</i>										
$S_t$	-0.170	-0.050	-0.225	-0.064	-0.132	-0.052	-0.213	-0.069	-0.098	-0.047
$\Delta e_t$	0.041	0.001	0.000	0.000	0.027	-0.001	0.000	0.000	0.026	-0.001
$C_t$	0.058	0.032	0.027	0.028	0.040	0.031	-0.030	-0.018	0.080	0.036
$H_t$	-0.028	-0.009	-0.042	-0.012	-0.040	-0.012	-0.074	-0.019	-0.021	-0.009
$Y_t$	-0.023	0.027	-0.055	0.030	-0.027	0.039	-0.034	0.031	-0.065	0.042
$I_t$	-0.011	0.007	-0.023	0.003	0.029	0.011	0.043	0.013	0.028	0.010
$R_t$	-0.012	-0.003	-0.036	-0.007	-0.016	-0.004	-0.039	-0.008	-0.015	-0.004
$\pi_t$	-0.007	-0.003	-0.003	-0.001	-0.006	-0.003	-0.006	-0.001	-0.008	-0.003
$TB_t$	-0.043	-0.005	-0.047	-0.017	-0.044	-0.002	-0.054	0.001	-0.043	-0.003
<i>C. Welfare effects (as % of deterministic steady state of consumption)</i>										
$\mu_m$	0.122	0.066	0.060	0.060	0.086	0.064	-0.053	0.039	0.166	0.075
$\mu_v$	-0.019	-0.010	-0.025	-0.011	-0.027	-0.012	-0.048	-0.021	-0.021	-0.010
Overall	0.103	0.056	0.035	0.049	0.059	0.052	-0.101	0.018	0.145	0.065

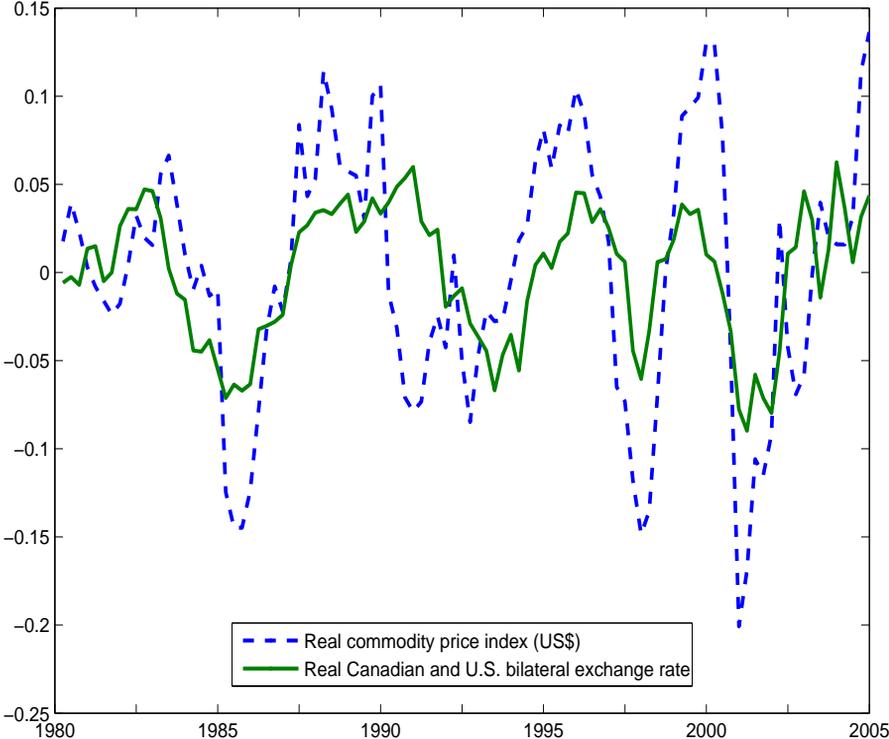
In columns (A), the model is simulated with all the shocks. In columns (B), the model is simulated with only commodity price shocks, i.e., only  $\sigma_{PX} \neq 0$ .

**Table 8: Welfare effects: Baseline model under PTM assumption**

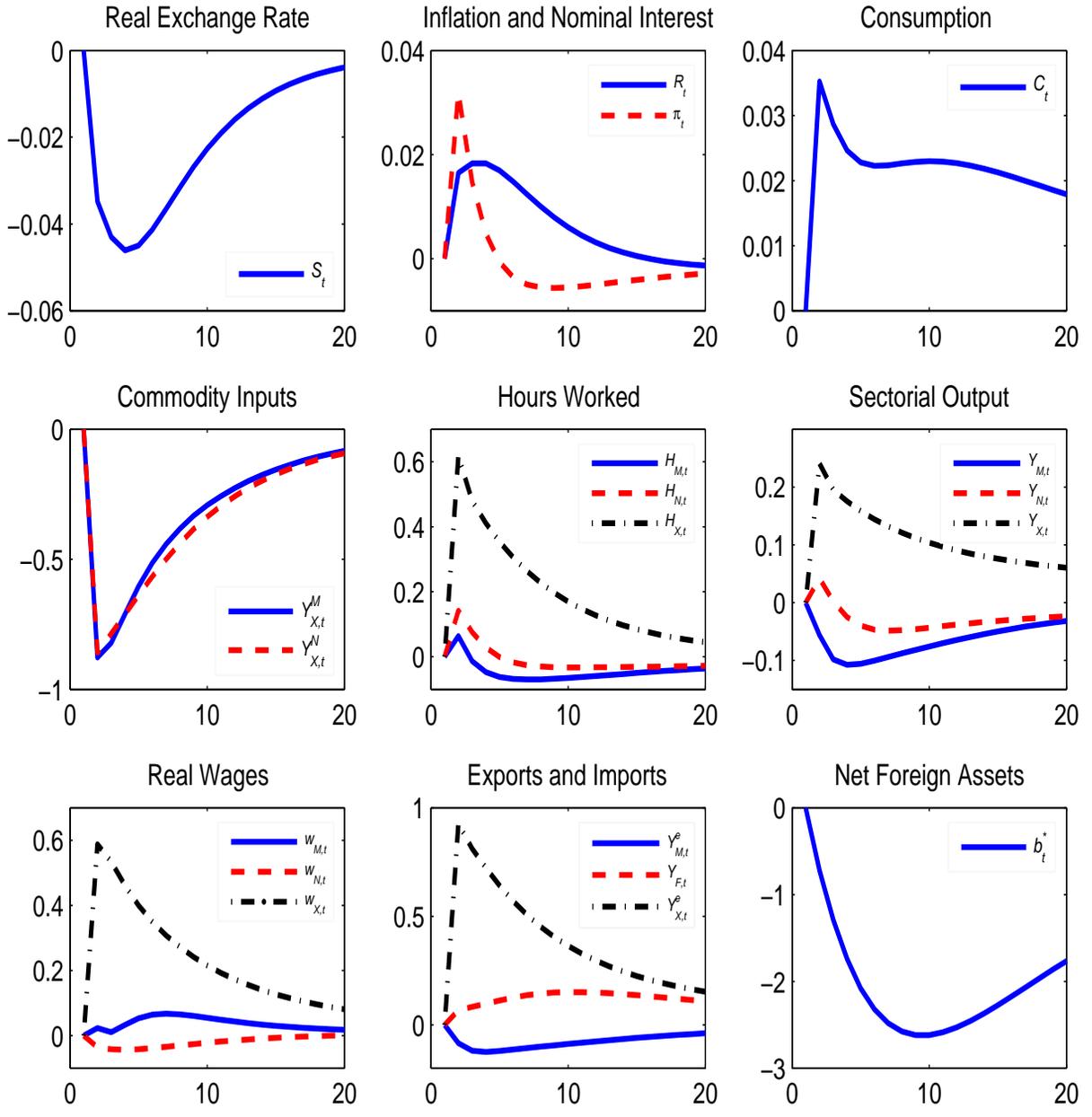
	Flexible Exchange Rate				Fixed Exchange Rate			
	(A)	(B)	(C)	(D)	(A)	(B)	(C)	(D)
<i>A. Standard deviations (in %)</i>								
$S_t$	4.66	2.64	3.33	1.93	3.00	1.84	1.50	1.83
$\Delta e_t$	3.13	0.78	2.53	1.665	0.00	0.00	0.00	0.00
$C_t$	1.58	1.11	0.50	1.01	2.32	1.49	1.27	1.22
$H_t$	2.26	0.88	0.63	1.98	4.91	2.08	3.74	2.39
$Y_t$	2.17	1.43	0.20	1.62	3.49	2.27	1.99	1.77
$I_t$	8.97	2.32	4.97	7.09	17.5	5.08	15.1	7.37
$R_t$	0.46	0.15	0.26	0.35	1.04	0.17	1.02	0.06
$\pi_t$	0.42	0.12	0.25	0.31	0.66	0.27	0.34	0.49
$TB_t$	1.16	0.89	0.53	0.52	1.38	0.64	1.10	0.53
<i>B. Stochastic steady state deviations (in %)</i>								
$S_t$	-0.191	-0.058	-0.076	-0.057	-0.300	-0.079	-0.117	-0.101
$\Delta e_t$	0.040	0.002	0.031	0.009	0.000	0.000	0.000	0.000
$C_t$	0.037	0.028	0.024	-0.015	-0.060	0.013	-0.017	-0.057
$H_t$	-0.043	-0.013	-0.016	-0.014	-0.086	-0.021	-0.032	-0.033
$Y_t$	-0.058	0.022	-0.037	-0.043	-0.101	-0.029	-0.041	-0.090
$I_t$	-0.004	0.007	-0.004	-0.009	0.021	0.011	0.029	-0.019
$R_t$	-0.007	-0.003	-0.012	-0.010	-0.038	-0.008	-0.020	-0.011
$\pi_t$	-0.008	-0.003	-0.001	-0.004	-0.009	-0.002	-0.006	-0.002
$TB_t$	-0.062	-0.057	-0.035	-0.021	-0.054	-0.004	-0.046	-0.012
<i>C. Welfare effects (as % of deterministic steady state of consumption)</i>								
$\mu_m$	0.080	0.058	0.050	-0.028	-0.114	0.030	-0.032	-0.112
$\mu_v$	-0.025	-0.012	-0.003	-0.010	-0.054	-0.022	-0.016	-0.015
Overall	0.055	0.046	0.047	-0.038	-0.168	0.008	-0.048	-0.127

In columns (A), the model is simulated with all the shocks. In columns (B), the model is simulated with only commodity price shocks, i.e., only  $\sigma_{PX} \neq 0$ . In columns (C), the model is simulated with foreign interest rate shocks, i.e., only  $\sigma_{R^*} \neq 0$ . In columns (D), the model is simulated without commodity price and foreign interest shocks i.e.  $\sigma_{PX} = \sigma_{R^*} = 0$ .

**Figure 1: Real commodity prices and the real CAN/US exchange rate (HP-filtered series)**



**Figure 2: The effects of a 1% positive commodity price shock**



## Appendix A. Wage Settings

For the production sector  $i = M, N, X$ , the household  $h$  chooses  $\widetilde{W}_{i,ht}$  to maximize:

$$\max_{\widetilde{W}_{i,ht}} E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi)^l \left\{ U(C_{ht+l}, H_{i,ht+l}) + \lambda_{t+l} \pi^l \widetilde{W}_{i,ht} H_{i,ht+l} / P_{t+l} \right\} \right], \quad (\text{A.1})$$

subject to

$$H_{i,ht+l} = \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l}. \quad (\text{A.2})$$

By substituting (A.2) into (A.1), the optimization problem may be written as:

$$\max_{\widetilde{W}_{i,ht}} E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \left\{ u(C_{ht+l}, H_{i,ht+l}) + \lambda_{t+l} \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{1-\vartheta} \frac{W_{i,t+l}}{P_{t+l}} H_{i,t+l} \right\} \right]. \quad (\text{A.3})$$

The first-order condition with respect to  $\widetilde{W}_{i,ht}$  is

$$E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \left\{ \frac{\partial U_{t+l}}{\partial H_{i,ht+l}} \frac{\partial H_{i,ht+l}}{\partial \widetilde{W}_{i,ht}} + (1-\vartheta) \lambda_{t+l} \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} \frac{H_{i,t+l} \pi^l}{P_{t+l}} \right\} \right] = 0, \quad (\text{A.4})$$

where  $\frac{\partial H_{i,ht+l}}{\partial \widetilde{W}_{i,ht}} = -\vartheta \left( \frac{\widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta-1} \frac{H_{i,t+l}}{W_{i,t+l}}$ .

Let  $\zeta_{i,t+l} = -\frac{\partial U_{t+l}}{\partial H_{i,t+l}} / \frac{\partial U_{ht+l}}{\partial C_{ht+l}}$  denote the marginal rate of substitution between  $C_{ht+l}$  and  $H_{i,ht+l}$ , where  $\frac{\partial U_{t+l}}{\partial C_{ht+l}} = \lambda_{t+l}$  is the marginal utility of consumption. Therefore, Eq. (A.4) may be rewritten as:

$$E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta-1}{\vartheta} \frac{\widetilde{W}_{i,ht} \pi^l P_t}{P_t P_{t+l}} \right\} \right] = 0. \quad (\text{A.5})$$

In symmetric equilibrium all households are identical, so  $\widetilde{w}_{i,ht} = \widetilde{w}_{i,t}$  and  $H_{i,jt} = H_{i,t}$ . Therefore, Eq. (A.5) can be rewritten in real terms as:

$$E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} H_{i,t+l} w_{t+l}^{\vartheta} \prod_{k=1}^l \pi^{-\vartheta k} \pi_{t+k}^{\vartheta} \left\{ \zeta_{i,t+l} - \frac{\vartheta-1}{\vartheta} \widetilde{w}_{i,t} \prod_{k=1}^l \pi^l \pi_{t+k}^{-1} \right\} \right] = 0, \quad (\text{A.6})$$

where  $\tilde{w}_{i,t} = \tilde{W}_{i,t}/P_t$  is the real contracted wage and  $w_{i,t} = W_{i,t}/P_t$  is the real wage index in the sector  $i$ . From Eq. (A.6),  $\tilde{w}_{i,t}$  is given by:

$$\tilde{w}_{i,t} = \frac{\vartheta - 1}{\vartheta} \frac{E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \zeta_{i,t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{-\vartheta l} \pi_{t+k}^{\vartheta} \right]}{E_t \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{l(1-\vartheta)} \pi_{t+k}^{\vartheta-1} \right]}. \quad (\text{A.7})$$

**Non-linear recursive procedure:**

Let  $f_{i,t}^1 = \sum_{l=0}^{\infty} (\beta\varphi)^l \lambda_{t+l} \zeta_{i,t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{-\vartheta l} \pi_{t+k}^{\vartheta}$ , which implies:

$$f_{i,t}^1 = \lambda_t \zeta_{i,t} H_{i,t} w_{i,t}^{\vartheta} + \beta\varphi (\pi_{t+1}/\pi)^{\vartheta} f_{i,t+1}^1; \quad (\text{A.8})$$

Similarly, let  $f_{i,t}^2 = \sum_{l=0}^{\infty} (\beta\varphi)^l \lambda_{t+l} H_{i,t+l} w_{i,t+l}^{\vartheta} \prod_{k=1}^l \pi^{l(1-\vartheta)} \pi_{t+k}^{\vartheta-1}$ , which implies:

$$f_{i,t}^2 = \lambda_t H_{i,t} w_{i,t}^{\vartheta} + \beta\varphi (\pi_{t+1}/\pi)^{\vartheta-1} f_{i,t+1}^2. \quad (\text{A.9})$$

Therefore, we can rewrite Eq. (A.7) as

$$\tilde{w}_{i,t} = \frac{\vartheta - 1}{\vartheta} \frac{f_{i,t}^1}{f_{i,t}^2}. \quad (\text{A.10})$$

Note that price settings in the intermediate-goods sectors are similar to this wage setting.

## Appendix B. Definitions of the Used Data

- *Commodity prices*: Bank of Canada's nominal total commodity price index (1997=1), US\$; Bank of Canada internal database.
- *Nominal exchange rate*: Calculated as the quarterly average of daily CDN\$ per U.S.\$ noon spot rate as reported by the Canadian Interbank Money Market.
- *Real exchange rate*: Calculated as the nominal CDN\$-US\$ exchange rate deflated by Canadian and U.S. CPI data.
- *Canadian nominal interest rate*: Canadian 3-month T-Bill interest rate.

- *CPI inflation rates*: The percentage change in the consumer price index as measured by Statistics Canada and the U.S. Dept. of Labour.
- *Government spending*: Government current expenditure on goods and services 1997 constant dollars, seasonally adjusted; Statistics Canada.
- *Commodities*: Commodity (Agriculture, fishing, forestry, Mining, and resources processing) 1997 constant dollars, seasonally adjusted; Statistics Canada.
- *Manufactured goods*: Manufacturing 1997 constant dollars, seasonally adjusted; Statistics Canada.
- *Non-tradable goods*: Services 1997 constant dollars, seasonally adjusted “Utilities, Construction, Wholesale trade, Retail trade, Transportation and warehousing, Information and cultural industries, Finance and insurance, real estate and renting and leasing and management of companies and enterprises, Professional scientific and technical services, Administrative and support, waste management and remediation services, Educational services, Health care and social assistance, Arts entertainment and recreation, Accommodation and food services, Other services, Public administration;” Statistics Canada.
- *U.S. real GDP*: calculated as Gross Domestic Product divided by total civilian population (16 and over) and the U.S. Implicit Price Deflator (1997=1.0); NIPA and Bureau of Labor Statistics.
- *U.S. inflation*: The percentage change in the U.S. Implicit Price Deflator (1997=1.0); NIPA.