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## Abstract

Under bond-rate transmission of monetary policy, the authors show that a generalized Taylor Principle applies, in which the average anticipated path of policy responses to inflation is subject to a lower bound of unity. This result helps explain how bond rates may exhibit stable responses to inflation, even in periods of passive policy. Another possible explanation is time-varying term premiums with risk pricing that depends on inflation. The authors present a no-arbitrage model of the term structure with horizon-dependent policy perceptions and time-varying term premiums to illustrate the mechanics and provide empirical results that support these transmission channels.

*JEL classification: E3, E5, N1*

*Bank classification: Interest rates; Transmission of monetary policy*

## Résumé

Les auteurs montrent que, si les taux obligataires forment le canal de transmission de la politique monétaire, l'application du principe de Taylor est généralisable. Le nouveau principe exige que la moyenne des réactions du taux directeur à l'inflation soit perçue comme étant au moins égale à un. Ce résultat permet d'expliquer comment la réaction des taux obligataires à l'évolution de l'inflation peut se révéler stable même lorsque la politique monétaire est passive. Une autre explication possible est que les primes de terme varient dans le temps en fonction de l'inflation. Pour illustrer les mécanismes en jeu et obtenir des résultats empiriques à l'appui des deux canaux de transmission avancés, les auteurs présentent un modèle de structure des taux sans arbitrage où les perceptions de la politique monétaire dépendent d'un horizon temporel et où les primes de terme fluctuent avec le temps.

*Classification JEL : E3, E5, N1*

*Classification de la Banque : Taux d'intérêt; Transmission de la politique monétaire*

# 1. Introduction

“Monetary policy works largely through indirect channels—in particular, by influencing private-sector expectations and thus long-term interest rates.” Bernanke (2004)

“Financial markets are the channel through which our policy affects the economy, and asset prices contain valuable information about investors’ expectations for the course of policy, economic activity, and inflation, as well as the risks about those expectations.” Kohn (2005)

Most studies of monetary policy focus on the policy interest rate, typically a very short-term rate, such as an overnight rate. However, as suggested by the above quotations, longer-term bond rates are essential conduits for the transmission of monetary policy. As bond rates contain bond trader expectations of future policy rates, not recent policy rates, monetary policy effectiveness depends on the policy perceptions of the bond market. The connection of these perceptions to announced or recently observed policy is not fully understood.

A large literature has developed to explain the historical behavior of the term structure of interest rates. The importance of accounting for asymmetric information on the part of the private sector and the central bank has been shown to be critical for understanding the relationship between short- and long-term interest rates—particularly in the 1980s (Kozicki and Tinsley 2001 a,b, 2005; Dewachter and Lyrio (2006c)). Other studies, including Shiller, Campbell, and Schoenholtz (1983), Duffee (2002), and Dai and Singleton (2002) emphasize the the key role of time-varying term premiums for capturing time variation in yields. Both of these areas of research form a part of a large literature exploring the connections between yields of different maturities. Other research, such as Ang and Piazzesi (2003), and Dewachter, Lyrio, and Maes (2006b) relate yields to macro factors.

Rudebusch and Wu (2004), Hördahl, Tristani and Vestin (2005), and Dewachter and Lyrio (2006a) use no-arbitrage term-structure models and structural macroeconomic models to relate bond yields to macroeconomic variables through policy responses of short-term interest rates. However, even in these papers, the focus remains largely one of explaining yield-curve behaviour given macroeconomic data, and explaining macroeconomic behaviour given policy rate responses. Although they comment on the lack of a structural link, Rudebusch, Sack, and Swanson (2006) establish an empirical link between term premiums and economic activity. Ang, Dong, and Piazzesi (2005) is the closest to our paper in that both papers present no-arbitrage models of the term structure based on Taylor-type rules that relate policy responses to economic outcomes. New contributions to the literature in this paper include an examination of the conditions for policy stability

under bond-rate transmission, and the introduction of horizon-dependent policy perceptions in a no-arbitrage term-structure model.

The primary distinguishing characteristic of the current study is our interest in examining policy transmission and, in particular, the stability of policy under bond-rate transmission (i.e., the ability of policy to deliver determinate inflation when bond rates influence economic behaviour). Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), and Kozicki and Tinsley (2005) provide empirical evidence that in the period before Paul Volcker was appointed Chairman of the Federal Open Market Committee (FOMC) of the Federal Reserve, nominal policy rates exhibited an inelastic response (i.e., less than one-for-one) with respect to inflation.<sup>1</sup> However, to the best of our knowledge, the responsiveness of bond rates to inflation has not been examined. This missing feature of the literature implies important shortcomings in some interpretations of the Great Inflation. In particular, if the bond rate is the transmission channel for monetary policy, explanations that focus on the stability of a Taylor rule description of the policy rate or on central bank assumptions regarding natural rates are not sufficient to assess the stability of the economy and the determinacy of inflation.

In this paper we propose that examinations of the stability of policy in standard DSGE frameworks ignore some important real-world characteristics of bond rates that are important for policy transmission. A simple illustrative macroeconomic model suggests that if the bond rate is the principal transmission channel, then what matters for stabilizing policy is that the *bond-rate average* of the forward rates displays an elastic response to expected inflation. Moreover, as already recognized in the term-structure literature, accounting for asymmetric information and time-varying term premiums seems likely to be important.

Building on this intuition, the paper presents a no-arbitrage model of the term structure with horizon-dependent policy expectations and term premiums that reflect time-varying compensation for macroeconomic uncertainty. Within this structure, there are two possible explanations for different inflation sensitivities of bond rates compared to policy rates. As suggested by the simple macroeconomic model, one explanation is that perceived inelastic responses by the policy rate to inflation in the short run may be counterbalanced by elastic responses in the longer run. A second possible explanation is that forward-rate term premiums may also be responsive to macroeconomic variables, including inflation. If this is the case, term premiums demanded by traders may compensate for modestly unstable short-run policy.

The central roles of bond rates and the perceptions of bond traders in the transmission of pol-

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<sup>1</sup>The FOMC of the Federal Reserve is responsible for U.S. monetary policy.

icy are discussed in remaining sections of the paper. Section 2 investigates the responsiveness of historical bond rates to macro variables since the mid-1960s. Section 3 examines the recent use of indeterminacy analysis in interpretations of US monetary policy during the 1960s and 1970s and indicates that an alternative condition for determinacy may be required if the bond rate is the policy transmission channel. In addition, shortcomings of DSGE models with symmetric information are discussed. Section 4 briefly sketches a no-arbitrage model of the term structure with term premiums that reflect time-varying compensation for macroeconomic uncertainty and the possibility of horizon-dependent expectations by bond traders. Section 5 presents estimated responses of forward rates to forecasts of macro variables, and section 6 concludes.

## 2. The responsiveness of historical bond rates to macro variables

In this section, we present empirical evidence on the responsiveness of historical bond rates to macroeconomic variables, including inflation. Regression results are reported for two monthly samples: one includes the “passive policy” period prior to 1980 and the second covers a period of aggressive policy in the 1980s.

We estimate the long-run responses of nominal bond rates,  $R_{12h,t}$  for  $h = 1, 3, 5, 10$ , to inflation,  $\pi_t$ , the level of unemployment,  $u_t$ , and the difference of unemployment,  $\Delta u_t$ :

$$\begin{aligned} R_{12h,t} &= b_3 R_{12h,t-1} + b_{33}(L)\Delta R_{12h,t-1} + (1 - b_3)R_{12h,t}^* + a_{t,12h}, \\ R_{12h,t}^* &= b_0 + b_1\pi_{t-1} + b_{11}(L)\Delta\pi_{t-1} + b_2u_{t-1} + b_{22}(L)\Delta u_{t-1}, \end{aligned}$$

where  $b_{jj}(L)$  are 11th-order lag polynomials. Inflation is measured over the prior 12 months using the deflator for personal consumption expenditures (pce), and unemployment is measured by the civilian unemployment rate. The bond rates are the nominal rates on 1-year, 3-year, 5-year and 10-year zero-coupon bonds from McCulloch and Kwon (1993). Results are reported in Table 1.

Regressions results reported in the bottom panel of Table 1 span the period after the abandonment of nonborrowed-reserves targeting to the end of the FOMC chairmanship of Paul Volcker, 1982m1 - 1987m7. The results are consistent with bond-trader forecasts of aggressive policy responses to inflation. Long-run mean responses by bond rates to inflation are well above unity for all maturities. Significant long-run mean responses are also indicated for the change in unemployment by 1-year and 3-year bond rates. No mean responses to the level of unemployment are significant.

The top panel in Table 1 reports on regressions for a 1966-79 sample, ending just prior to the

announcement of the well-known shift in operational policy in October 1979.<sup>2</sup> The second column in the top panel indicates that the mean long-run response to inflation is above unity for bond rates of all maturities. Perhaps consistent with bond-trader perceptions of the emphasis of 1970s central-bank policy on money-growth targeting, as discussed in Kozicki and Tinsley (2007), the mean long-run response to the change of the unemployment rate is also significant for all maturities.

Overall, the results of Table 1 are consistent with elastic bond-rate responses to inflation in both samples, raising the question of how to evaluate the determinacy of policy. The specific question we examine is whether it is possible for policy to be passive yet for inflation to be determinate. In the remaining sections of this paper, we show that such apparently contradictory observations are indeed possible.

### 3. Bond-rate transmission of monetary policy

In this section we use a simple model to show how determinacy conditions for policy may change when policy anticipations are allowed to be horizon dependent. The results highlight the importance of considering anticipations of the path of policy.

Specifications of interest rates in the output equations of empirical macroeconomic models vary widely. However, in structural macroeconomic models, most specifications use one-period rates, with few identifying possible *distinct* roles for long-term rates. That said, models with explicit one-period rates do not necessarily imply that bond rates are unimportant in policy transmission.

Consider, for instance, the purely forward-looking IS equation,

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - a(\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \quad (1)$$

relating equilibrium deviations in output  $\tilde{y}_t$  to equilibrium deviations in the ex ante one-period real rate. Here, the latter is represented as the difference between equilibrium deviations in the nominal rate,  $\tilde{r}_t$ , and equilibrium deviations in expected inflation  $E_t \tilde{\pi}_{t+1}$ . After recursive forward substitution, this expression implies a relationship between output and long-horizon averages of equilibrium real-rate deviations:

$$\tilde{y}_t \simeq -a E_t \left( \frac{1}{n} \sum_{i=1}^n \tilde{r}_{t+i} - \frac{1}{n} \sum_{i=1}^n \tilde{\pi}_{t+i+1} \right). \quad (2)$$

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<sup>2</sup>The federal funds market was not well-developed prior to 1966, vid. Tinsley et al. (1982) and Fuhrer (1996).

Assuming constant term premiums and equilibrium deviations that converge to zero in expectation,  $E_t \frac{1}{n} \sum_{i=1}^n \tilde{r}_{t+i}$ , is typically taken as the equilibrium deviation of an n-period nominal-bond rate,  $\tilde{R}_{n,t}$ . Thus, such structural models can be used to provide a link between macroeconomic activity and bond yields. Indeed, (1) motivates the simple macroeconomic model used in the next subsection to discuss conditions for determinate policy.

However, even more sophisticated model specifications that include, for example, habit formation in consumption or time-to-build in investment, may miss important aspects of bond-rate transmission of policy. Shortcomings are likely because DSGE models are generally constrained by restrictive assumptions such as time-invariant (or zero) risk premiums and symmetric information. The second subsection expands on limitations of typical DSGE models.

### 3.1 Determinacy conditions with horizon-dependent policy perceptions

The general presumption in the literature is that evidence of passive monetary policy implies that the central bank and the private sector may have been influenced by exogenous inflationary sunspots. However, if the principal transmission channel of monetary policy is through the responses embedded in private sector borrowing rates, then conditions for determinacy of inflation depend on passivity of the *anticipated path* of future policy rates. Thus, the sunspot interpretation is based on two untested assumptions: First, that bond traders can infer, in real time, that the central bank policy is passive.<sup>3</sup> And second, that the passivity of monetary policy is expected to persist over lengthy forecast horizons. With regard to these assumptions, Kozicki and Tinsley (2001a, 2001b) indicate that private sector perceptions of the implicit central-bank target for inflation were very slow to adjust—the mean lag of adjustment of the perceived inflation target consistent with Treasury bond rates exceeded 5 years in the 1970s and 1980s. In addition, FOMC announcements in the 1970s regarding explicit policy targets were limited to one-year horizons. Overall, it is not obvious that bond traders would extrapolate difficulties in reaching one-year objectives to policy failure in the long run.

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<sup>3</sup>In contrast to the regression analysis in Clarida, Gali and Gertler (2000), empirical analysis in Kozicki and Tinsley (2007) uses the retrospective advantage of access to the central bank real-time forecasts of explanatory variables. If external observers are not privy to central bank information, Beyer and Farmer (2004) illustrate that it is not always possible for the observers to discriminate between determinate and indeterminate policies.

We introduce horizon-dependent policy in a simple linear DSGE model:

$$\begin{aligned}
\tilde{y}_t &= -a(R_{2,t} - \frac{1}{2}E_t(\tilde{\pi}_{t+1} + \tilde{\pi}_{t+2})) + e_t \\
\tilde{\pi}_t &= E_t\tilde{\pi}_{t+1} + b\tilde{y}_t \\
R_{2,t} &= \frac{1}{2}(\tilde{r}_{t,1} + \tilde{r}_{t,2}) \\
\tilde{r}_{t,k} &= c_k E_t\tilde{\pi}_{t+k} \quad k = 1, 2
\end{aligned} \tag{3}$$

where  $\tilde{x}_t$  denotes the equilibrium deviation of variable  $x_t$ . The first equation in (3) indicates that deviations in output,  $\tilde{y}_t$  are determined by equilibrium deviations in the two-period ex ante real bond rate,  $E_t\{\tilde{R}_{2,t} - \frac{1}{2}E_t(\tilde{\pi}_{t+1} + \tilde{\pi}_{t+2})\}$ , and by a stochastic disturbance,  $e_t$ . To facilitate the derivation of closed-form results, a 2-period bond rate replaces the n-period rate in (2). In the second equation, equilibrium deviations in the inflation rate,  $\tilde{\pi}_t$ , are determined by a standard New Keynesian (NK) pricing equation under assumptions necessary for the output gap to be proportional to marginal cost. The notation  $\tilde{r}_{t,k}$  is used to represent the expectation in  $t$  of the policy rate in  $t + k$ . This notation will facilitate analysis of horizon-dependent policy. The third equation defines the two-period bond-rate deviation in  $t$  to be equal to the average of one-period nominal rate deviations for each of the next two periods as expected in  $t$ . The fourth equation describes the expected policy path and relates equilibrium deviations in the nominal policy rate to expected inflation deviations. Note that the perception of the future policy response,  $c_2$ , is not restricted to be identical to the response perceived in the current period,  $c_1$ .<sup>4</sup>

Substituting the first, third and fourth equations in (3) into the second equation gives a second-order equation for inflation

$$\tilde{\pi}_t = \left(1 - \frac{ab(c_1 - 1)}{2}\right)E_t\tilde{\pi}_{t+1} - \frac{ab(c_2 - 1)}{2}E_t\tilde{\pi}_{t+2} + be_t. \tag{4}$$

The solution for inflation has the form:

$$\tilde{\pi}_t = E_t\left\{\frac{be_t}{(1 - \lambda_1^{-1}F)(1 - \lambda_2^{-1}F)}\right\}, \tag{5}$$

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<sup>4</sup>Here, as elsewhere in this paper, conditions for determinacy can be extended to include policy responses to equilibrium deviations in real activity. Strictly speaking, conditions for determinacy are system properties and not just limited to the inflation responsiveness of current and anticipated real policy rates, such as models where nominal interest rates may play an important stabilizing role, vid. Beyer and Farmer (2004). To simplify exposition, discussion in this paper assumes real variables, such as output, are responsive only to real interest rates, consistent with responses by households and firms in conventional NK models.

where the roots of the characteristic equation are determined by<sup>5</sup>

$$\begin{aligned}\lambda_1\lambda_2 &= \frac{2}{ab(c_2 - 1)}, \\ \lambda_1 + \lambda_2 &= \frac{2 - ab(c_1 - 1)}{ab(c_2 - 1)}.\end{aligned}\tag{6}$$

The perceptions of the bond traders must satisfy three conditions for determinacy, vid. Woodford (2003)

$$\begin{aligned}\lambda_1\lambda_2 > 1 &\Rightarrow c_2 < 1 + \frac{2}{ab}, \\ (1 + \lambda_1)(1 + \lambda_2) > 0 &\Rightarrow c_1 < c_2 + \frac{4}{ab}, \\ (1 - \lambda_1)(1 - \lambda_2) > 0 &\Rightarrow \frac{c_1 + c_2}{2} > 1.\end{aligned}\tag{7}$$

In this formulation, upper bounds for the policy responses are summarized by the first two conditions in (7). These bounds will depend on the particular specifications of the model. In the case where  $c_1 = c_2$ , the second condition puts no constraints on the parameters of the perceived policy path.

The third requirement for determinacy in (7) is a generalization of the Taylor Principle. If policy is not horizon-dependent, then  $c_1 = c_2$  and the determinacy condition simplifies to the standard Taylor Principle that the nominal policy rate must be respond at least one-for-one to expected inflation.

With horizon-dependency in the perceived policy path, determinacy requires that the *average anticipated response* to expected inflation over the maturity of the bond should exceed unity. Thus, even if the current period response is passive,  $c_1 < 1$ , the average perceived response may satisfy the lower bound requirement for determinacy.<sup>6</sup>

### 3.2 Pitfalls of standard NK specifications

A feature of standard NK models is that the link between bond rates and policy rates is treated very simplistically—forward averages of policy rates are generally viewed as bond-rate proxies. This

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<sup>5</sup>The roots of the associated companion form system for (4) are derived in the appendix.

<sup>6</sup>Note that the exercise conducted here is different than that in McGough, Rudebusch, and Williams (2005), where the bond rate is the monetary-policy instrument. For comparison, in the case of the two-period bond rate, determinacy in their set-up requires that the bond rate in  $t$  respond at least one-for-one to inflation in  $t$ , i.e.,  $R_{2,t} = c_B\pi_t$  with  $c_B > 1$ .

mapping ignores two potentially important features of the real world—time-varying term premiums and asymmetric information on the part of the central bank and the private sector.

The treatment of time-varying term premiums in a no-arbitrage model of the term structure is explored in the next section. In NK models, term-premium effects are generally absent by assumption. A recent exception is Rudebusch, Sack, and Swanson (2006) who develop a structural model in which the sign of the correlation between the term premium and output depends on the nature of the shock hitting the economy. However, they note that their structural model is unable to reproduce the magnitude and variation of term premiums observed in the bond market. Moreover, while their model incorporates time-varying term premiums, it assumes symmetric information.

The problem of asymmetric information is an important drawback of most structural models that formulate output equations as functions of the one-period rate. Given that theory indicates forecasts of long-horizon returns in bond markets are important determinants of private-sector expenditures, the information set of bond traders is more pertinent than that of a macro modeler. The bond-trader information set is likely to differ, and may be larger than the macro modeler. Unless the modeler ensures that the averages of forward rates generated by the model are equivalent to observed bond rates, the model description of policy transmission will reflect the modeler's priors regarding long-horizon forecasts, and these may differ markedly from the long-horizon forecasts contained in bond-market observations.<sup>7</sup>

In contrast to the assumptions made in structural models, empirical results from reduced-form analyses are generally supportive of an independent role for long-term interest rates. Kozicki and Tinsley (2002) explore competing specifications of short-term and long-term interest rates in output equations. If frictions in adjusting real expenditures are important, we might expect long-term ex-ante real interest rates should dominate competing short-term real-rate regressors in reduced-form regressions. Indeed, they provide empirical evidence indicating that long-term real rates are relatively more important. Results are reported for a sequence of bivariate tests, where U.S. manufacturing utilization, a proxy for the output gap, is regressed on competing short-term and long-term ex-ante real interest rates over a 1967m1 - 1997m7 sample. The tests confirm that spreads between the long-term and short-term interest rates are statistically insignificant when regressions are conditioned on the long-term rates and, conversely, long-short spreads are significant when regressions are conditioned on the short-term interest rates. Rudebusch, Sack, and Swanson (2006) construct several estimates of time-varying term premiums and use these estimates to examine the

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<sup>7</sup>The sensitivity of long-horizon forecasts to alternative modeling assumptions regarding time-variation in conditional equilibria is illustrated in Kozicki and Tinsley (2001a, 2001b).

link between movements in the term premium and subsequent economic activity.<sup>8</sup> In general, they find that a decline in the term premium has typically been associated with higher future real GDP growth.

Overall, these results suggest that an important direction for future research will be to incorporate time-varying premiums and asymmetric information into structural models. In addition, while not examined in the current paper, the explanatory role of credit risk premiums in private borrowing rates is empirically supported in Kozicki and Tinsley (2002), arguing for consideration of other forms of private-sector risk.

#### 4. No-arbitrage bond pricing with time-varying risk premiums and horizon-dependent perceptions

As noted earlier, bond rates may exhibit elastic responses to inflation at the same time that policy-rate settings appear to indicate passive policy. One possibility, as suggested in the previous section, is that coefficients of the perceived policy response may vary over the forecast horizon. Another possibility—one not examined in section 3—is that term premiums may vary systematically with macroeconomic variables.<sup>9</sup>

The next subsection describes a no-arbitrage model of bond pricing that incorporates term-premium responses to macroeconomic determinants of policy rates, such as inflation, and also allows for horizon-dependent expectations.<sup>10</sup> To establish terminology, the nominal yield on an  $n$ -period zero-coupon bond is denoted,

$$\begin{aligned}
 R_{n,t} &= \frac{1}{n} \sum_{i=0}^{n-1} f_{t,i}, \\
 &= \frac{1}{n} \sum_{i=0}^{n-1} (r_{t,i} + \psi_{t,i}), \\
 &= \frac{1}{n} \sum_{i=0}^{n-1} r_{t,i} + \Psi_{n,t}.
 \end{aligned} \tag{8}$$

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<sup>8</sup>As Rudebusch, Sack, and Swanson note, estimates of term premiums depend on assumptions regarding long-horizon expectations of the equilibrium real rate and the inflation anchor.

<sup>9</sup>In the model of section 3, term premiums of bond rates were assumed to be constant over time and drop out of equilibrium deviations.

<sup>10</sup>Recent examples of empirical estimates of the term structure exploring macro variable determinants of term premiums include Ang and Piazzesi (2003), Rudebusch and Wu (forthcoming,2004), Ang, Dong, and Piazzesi (2005), Duffee (2006), and Dewachter and Lyrio (2006a) and Dewachter, Lyrio, and Maes (2006)).

The first line of (8) indicates that the nominal bond rate is the average of *forward rates*,  $f_{t,i}$ , over the lifetime of the bond. The second line shows that the forward rate in the  $i$ th period of the  $n$ -period forecast horizon is equal to the expected *policy rate* in the  $i$ th period,  $r_{t,i}$ , plus a possibly time-varying forward rate *term premium*,  $\psi_{t,i}$ . The last line shows that the term premium of the  $n$ -period bond rate,  $\Psi_{t,n}$ , is equal to the average of the forward-rate term premiums.<sup>11</sup>

This format clarifies possible roles of time-varying term premiums. Positive term-premium responses to inflation may reconcile elastic nominal bond-rate responses with passive policy. In addition, with systematic positive responses to inflation, time-varying term premiums may operate as automatic stabilizers, reducing the effective lower bound required for determinate policy.

## 4.1 The model

In the model we present, bond prices depend on the current state and future evolution of the macroeconomy. We represent the dynamics of macroeconomic variables with a first-order companion-form system, where, for notational simplicity, a constant is included in the  $X$  vector:<sup>12</sup>

$$X_t = \Phi X_{t-1} + \Sigma \epsilon_t, \quad \epsilon_t \sim N(0, I). \quad (9)$$

The policy rate in period  $t + h$  anticipated by bond traders in period  $t$ ,  $r_{t,h}$  is assumed to be a linear function of macroeconomic variables anticipated for  $t + h$ :

$$r_{t,h} = \delta'_{h+1} E_t X_{t+h}, \quad (10)$$

where the vector of response parameters,  $\delta_{h+1}$  may vary over the forecast horizon.

In the absence of arbitrage, the price of a multiperiod asset that does not pay dividends is determined by the expected product of stochastic discount factors,  $M_{t+i}$ , over the lifetime of the asset. In the case of a zero-coupon  $n$ -period nominal bond paying \$1 at maturity, the current price is:

$$\begin{aligned} P_{n,t} &= E_t \{ M_{t+1} M_{t+2} \dots M_{t+n} \}, \\ &= E_t \{ M_{t+1} P_{n-1,t+1} \}, \end{aligned} \quad (11)$$

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<sup>11</sup>In the terminology of Shiller (1990),  $\Psi_{n,t}$  is the rollover term premium.

<sup>12</sup>This format nests the reduced form of linear structural macroeconomic models.

where the last line in (11) follows by the law of iterated expectations. The yield on this bond is:

$$R_{n,t} = -\frac{1}{n} \log(P_{n,t}). \quad (12)$$

The stochastic discount factor is assumed to satisfy:

$$M_{t+i+1} = \exp\{r_{t,i}\} \exp\{-\lambda'_{t+i} \epsilon_{t+i+1} - \frac{1}{2} \lambda'_{t+i} \lambda_{t+i}\} \quad (13)$$

where, following Duffee (2002), the price of risk is the essentially-affine formulation:<sup>13</sup>

$$\lambda_{t+i} = \lambda_0 + \lambda'_1 X_{t+i}. \quad (14)$$

The second term in (13) is the Radon-Nikodym derivative that translates the distribution of the discounted asset price to a martingale by removing predictable drift due to bond risk premiums.<sup>14</sup> If investors are risk neutral,  $\lambda_{t+i} = 0$ , and  $M_{t+i+1} = \exp\{r_{t+i}\}$ .

As in Campbell, Lo and MacKinlay (1997), discrete-time bond prices in this Gaussian affine model of the term structure can be represented by:

$$\begin{aligned} P_{n,t} &= E_t[\Pi_{i=1}^n M_{t+i}] \\ &= \exp\{-A_n - B'_n X_t\}, \end{aligned} \quad (15)$$

where expressions for  $A_n$  and  $B_n$  can be derived by substituting for  $M_{t+i}$  from (13). Under the

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<sup>13</sup>For identification, the elements of  $\lambda_1$  that multiply the constant in  $X_t$  are restricted to equal zero. Constants are included separately to facilitate the derivation. In the absence of an explicit specification of investor utility functions, no theoretical restrictions are imposed on the  $\lambda'_1$  matrix. As the dimensions of the pricing matrix can be large, empirical investigations of essentially affine formulations of asset pricing often impose zero restrictions on elements of the  $\lambda'_1$  matrix, such as Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2005), and Dewachter and Lyrio (forthcoming(a) and forthcoming(b)). Kim and Orphanides (2005) suggest fewer zero restrictions are required if measurements include both bond-rate data and surveys of interest-rate forecasts over short and long horizons. Depending on the structure of the  $\lambda_1$  matrix, term premium variation linked to a variable may not reflect uncertainty in that variable. For example, suppose  $\Sigma \lambda'_1 X_t$  is a  $2 \times 1$  vector,

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{21} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} s_{11}\lambda_{11} + s_{12}\lambda_{21} & s_{11}\lambda_{12} + s_{12}\lambda_{22} \\ s_{21}\lambda_{11} + s_{22}\lambda_{21} & s_{21}\lambda_{12} + s_{22}\lambda_{22} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}.$$

Note that responses of the term premium to movements in  $x_{2,t}$  may not be related to the scale of the  $x_2$  shock,  $s_{22}$ .

<sup>14</sup>Change of drift under the Girsanov theorem is discussed in Duffie(1996).

assumption that  $X_t$  is observable in  $t$ , the price of a one-period bond is

$$\begin{aligned}
P_{1,t} &= E_t M_{t+1}, \\
&= E_t \exp[-\delta'_1 X_t - \frac{1}{2} \lambda'_t \lambda_t - \lambda'_t \epsilon_{t+1}], \\
&= \exp[-\delta'_1 X_t - \frac{1}{2} \lambda'_t \lambda_t + \frac{1}{2} \text{Var}([\lambda_0 + \lambda'_1 X_t]' \epsilon_{t+1})], \\
&= \exp[-\delta'_1 X_t], \\
&= \exp[-A_1 - B'_1 X_t].
\end{aligned} \tag{16}$$

Since  $r_{t,1} \equiv R_{1,t} = -\log(P_{1,t})$  and  $r_{t,1} = \delta'_1 X_t$ , it follows that

$$\begin{aligned}
A_1 &= 0 \\
B_1 &= \delta_1.
\end{aligned}$$

More generally, Kozicki and Tinsley (2005c) show

$$\begin{aligned}
A_n &\equiv - \sum_{i=2}^n [(\sum_{j=i}^n \delta'_j (\Phi - \Sigma \lambda'_1)^{j-i}) \Sigma [\Sigma' (\sum_{j=i}^n \frac{1}{2} \delta'_j (\Phi - \Sigma \lambda'_1)^{j-i})' + \lambda_0]], \\
B'_n &\equiv \sum_{i=1}^n \delta'_i (\Phi - \Sigma \lambda'_1)^{i-1} \\
&= B'_{n-1} + \delta'_n (\Phi - \Sigma \lambda'_1)^{n-1}.
\end{aligned} \tag{17}$$

The notation convention is that negative entries of  $\lambda_1$  contribute towards positive risk premiums. Consequently, since  $R_{n,t} = (1/n)(A_n + B'_n X_t)$ , as can be seen in the final line of (17), the sensitivity of bond rates to variations in the macro variables,  $X_t$ , may be increased if elements of  $\lambda_1$  are negative.

## 4.2 Forward-rate expressions

The formulations in (17) are particularly convenient for analysis of forward rates. The forward rate,  $f_{n-1}$ , is

$$\begin{aligned}
f_{t,n-1} &= r_{t,n-1} + \psi_{t,n-1}, \\
&= p_{n-1,t} - p_{n,t} \\
&= A_n - A_{n-1} + (B_n - B_{n-1})' X_t \\
&= C_n + \delta'_n (\Phi - \Sigma \lambda'_1)^{n-1} X_t,
\end{aligned} \tag{18}$$

where the intercept in the last line of (18) is

$$C_n \equiv - \sum_{i=2}^n [\delta'_n (\Phi - \Sigma \lambda'_1)^{n-i} \Sigma [\Sigma' (\sum_{j=i}^n \frac{1}{2} \delta'_j (\Phi - \Sigma \lambda'_1)^{j-i})' + \lambda_0] \\ + (\sum_{j=i}^{n-1} \delta'_j (\Phi - \Sigma \lambda'_1)^{j-i}) \Sigma \Sigma' (\frac{1}{2} \delta'_n (\Phi - \Sigma \lambda'_1)^{n-i})'].$$

Thus, with horizon-dependent perceptions of policy responses, the forward rate,  $f_{t,h-1}$ , satisfies

$$\begin{aligned} f_{t,h-1} &= C_h + \delta'_h (\Phi - \Sigma \lambda'_1)^{h-1} X_t \\ &\equiv C_h + \delta'_h (I - \Sigma \lambda'_1 \Phi^{-1})^{h-1} \Phi^{h-1} X_t \\ &= C_h + \delta'_h (I - \Sigma \lambda'_1 \Phi^{-1})^{h-1} E_t X_{t+h-1}. \end{aligned}$$

Estimates of  $E_t X_{t+h-1}$  obtained from projections of the empirical model in (9) with estimated coefficients permit exploration of “Taylor rule” regressions for forward rates with possibly horizon-varying coefficients,  $\delta'_h (I - \Sigma \lambda'_1 \Phi^{-1})^{h-1}$ ,  $h = 1, 2, \dots$

To determine an expression for the term premiums, notice that in the absence of term premiums, the forward-rate regression reduces to

$$\begin{aligned} f_{t,h-1} &= C_h + \delta'_h \Phi^{h-1} X_t, \\ &= C_h + \delta'_h E_t X_{t+h-1}. \end{aligned}$$

Thus, forward rate term premiums are defined by

$$\psi_{t,h} = C_{h+1} + \delta'_{h+1} (\Phi - \Sigma \lambda'_1)^h X_t - \delta'_{h+1} \Phi^h X_t. \quad (19)$$

If the effect of the risk-pricing matrix,  $\lambda'_1$ , is such that  $(\Phi - \Sigma \lambda'_1)^h$  is slower to decay than  $\Phi^h$ , the slope contributions of forward rate term premiums will increase over the forecast horizon with  $h$ .<sup>15</sup>

## 5. Empirical responses of forward rates

This section provides a direct evaluation of the combined effects of term-premium and expected-policy-rate responses by estimating forward-rate response equations over different forward horizons. The possibility of time-varying forward-rate term premiums and horizon-dependent expecta-

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<sup>15</sup>The Jordan form of a matrix,  $A = P \Lambda P^{-1}$ , implies  $A^i = P \Lambda^i P^{-1}$ .

tions of future policy-rate responses suggests that forward-rate responses to expectations of future macro variables may differ from the current-period policy-rate responses estimated by Taylor rules.

The macro variables we use to summarize economic activity are inflation,  $\pi_t$ , and the unemployment rate,  $u_t$ . The unemployment rate appears to be an appropriate summary measure of real activity because, as was discussed in Kozicki and Tinsley (2007), it seems likely that the FOMC used aggregate unemployment as a measure of resource slack. Moreover, the change in the unemployment rate provides a proxy for economic growth.

Each macro variable is partitioned into its perceived equilibrium attractor or “natural rate,” such as  $(\bar{\pi}_t, \bar{u}_t)$ , and deviations from these natural rates.<sup>16</sup>

$$\begin{aligned}\pi_t &\equiv \bar{\pi}_t + \tilde{\pi}_t, \\ u_t &\equiv \bar{u}_t + \tilde{u}_t.\end{aligned}\tag{20}$$

The empirical macro model describes the dynamics of four variables: the perceived natural rate of unemployment,  $\bar{u}_t$ , deviations of the unemployment rate from its perceived natural rate,  $\tilde{u}_t = u_t - \bar{u}_t$ , the perceived central-bank target for inflation,  $\bar{\pi}_t$ , and deviations of inflation from this perception,  $\tilde{\pi}_t = \pi_t - \bar{\pi}_t$ . In the first subsection we provide more details on the data used and the structure of the empirical macroeconomic model.

For monthly observations, the instantaneous forward rates at twelve-month intervals in the forecast horizon are represented by

$$f_{t,12h} = (1 - \rho_1)f_{t,12h}^* + \rho_1 f_{t,12(h-1)} + \rho_2 \Delta^{(k)} f_{t,12(h-1)} + a_{t,12h},\tag{21}$$

in which the forward rate associated with the bond-trader expectation of the policy rate in the absence of policy lag adjustments is

$$f_{t,12h}^* = E_t\{c_0 + c_1 \bar{\pi}_t + c_2 \tilde{\pi}_{t,12h}^{(\bar{k})} + c_3 \tilde{u}_{t,12h} + c_4 \Delta^{(k)} u_{t,12h}\},$$

where  $\tilde{\pi}_{t,12h}$  is the projected deviation of inflation in the  $12h^{th}$  month of the forecast horizon;  $\tilde{u}_{t,12h}$  is the projected deviation of unemployment from bond trader perceptions of the unemployment natural rate,  $\bar{u}_t$ ; and the superscripts,  $(\bar{k})$  and  $(k)$ , denote  $k$ -period averages and  $k$ -period summations, respectively.<sup>17</sup> For monthly data,  $k = 12$ .

<sup>16</sup>In a slight abuse of conventional terminology, it is convenient to refer to the central bank target for inflation perceived by bond traders,  $\bar{\pi}_t$ , as the “natural rate” for inflation.

<sup>17</sup>The  $k$ -period summation of the first-difference operator is the  $k$ -period difference,  $\Delta^{(k)} = \Delta + \Delta L + \dots +$

As the specification in (21) is amenable to direct regression, it is straightforward to check if estimates of combined responses, such as  $c_2$ , are consistent with stable bond rate responses, and if responses vary over different partitions of the forecast horizon. The regressions reported here do not impose the cross-equation restrictions implied by no-arbitrage, as derived in section 4, on the forward-rate regressions of different horizons. Consequently, it is not possible to determine what proportions of combined responses are due to forward-rate term-premium responses,  $\Sigma\lambda'_1$ , or to expected policy-rate responses,  $\delta_h$ . There is one exception: under the physical probability measure, the expected response to the perceived inflation target is unity,  $c_1 = 1$ . Thus, significant deviations from  $c_1 = 1$  indicate time variation in forward-rate term premiums due to a time-varying inflation target,  $\bar{\pi}_t$ .

## 5.1 Forecast model

The structure of the empirical model, particularly the expressions describing the evolution of the natural rates, is very important when making long-horizon forecasts. Kozicki and Tinsley (2001a, 2001b) demonstrate that long-horizon predictions from VAR models are sensitive to specifications regarding the conditional equilibria of state variables. For instance if the perceived equilibrium or central bank target for inflation is fixed at the sample mean, then implied constructions of ex-ante 5-year and 10-year real bond rates appear to be trending up in the 1970s. By contrast, if  $\bar{\pi}_t$  closely tracks recent inflation, as would be the case if  $\bar{\pi}_t$  is estimated as the Beveridge-Nelson (1981) unit-root trend, ex-ante real bond rates are much more volatile and fall sharply below zero in the first half of the 1970s. Finally, more gradual movements in  $\bar{\pi}_t$  that are consistent with survey data imply ex-ante real rates on 5-year and 10-year bonds that move without much of a discernible trend in the 1970s.

For the forward-rate regressions reported below, the time-varying perceptions of bond traders for the central-bank target for inflation,  $\bar{\pi}_t$ , and the natural rate for unemployment,  $\bar{u}_t$ , are represented by the constant-gain learning equations:

$$\begin{aligned}\bar{\pi}_t &= \gamma_{\bar{\pi}}\pi_{t-1} + (1 - \gamma_{\bar{\pi}})\bar{\pi}_{t-1} + \varepsilon_{\bar{\pi},t}, \\ \bar{u}_t &= \gamma_{\bar{u}}u_{t-1} + (1 - \gamma_{\bar{u}})\bar{u}_{t-1} + \varepsilon_{\bar{u},t}.\end{aligned}\tag{22}$$

Kozicki and Tinsley (2001b) indicate that a monthly gain of  $\gamma_{\bar{\pi}} = .015$  provides an average approximation of private-sector long-horizon forecasts of inflation in the 1980s. This benchmark constant-gain proxy for bond-trader perceptions of the central-bank target for inflation,  $\bar{\pi}_t$ , is shown in Figure

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$\Delta L^{k-1} = 1 - L^k$ .

1, along with the 12-month moving average of pce inflation and the Hoey real-time survey of 5-10 year predictions of CPI inflation. Reductions in survey predictions of long-horizon inflation and in the perceived inflation target lag considerably the fall of inflation in the early 1980s.

The same benchmark learning rate is assumed for bond-trader perceptions of the natural rate of unemployment,  $\gamma_{\tilde{u}} = .015$ . The associated constant-gain proxy for the natural rate of unemployment is shown in Figure 2, along with the historical unemployment rate and the Congressional Budget Office (2004) retrospective estimate of the natural rate. As with many real-time estimates of the natural rate of unemployment, the constant-gain proxy tracks below the retrospective CBO estimate in the 1970s, with an average underestimation error of about 1.25 percentage points in the first half of the 1970s before the error sharply diminishes in the remainder of the 1970s.

Given the sensitivity of long-horizon forecasts to the specification of the perceived central-bank target for inflation, we examine the effects of three constant-gain learning rates. In the case of a fixed inflation target, the learning rate is set to zero,  $\gamma_{\tilde{\pi}} = 0.0$ , and the perceived inflation target is set to the sample mean. The benchmark perception of the central bank inflation target, shown in Figure 1, uses the constant-gain learning rate,  $\gamma_{\tilde{\pi}} = .015$ , which implies a mean learning lag of about 5.5 years. Finally, a faster learning rate is also examined for perceptions of the central bank target for inflation,  $\gamma_{\tilde{\pi}} = .03$ , with a mean learning lag of about 2.8 years.<sup>18</sup>

Time-variation in the natural-rate deviations of inflation and unemployment is captured by a  $p$ th-order, bivariate vector autoregression,

$$\begin{aligned}\tilde{\pi}_t &= a_{11,1}\tilde{\pi}_{t-1} + \sum_{i=1}^{p-1} a_{11,i+1}\Delta\tilde{\pi}_{t-i} + a_{12,1}\tilde{u}_{t-1} + \sum_{i=1}^{p-1} a_{12,i+1}\Delta\tilde{u}_{t-i} + \varepsilon_{\tilde{\pi},t}, \\ \tilde{u}_t &= a_{21,1}\tilde{\pi}_{t-1} + \sum_{i=1}^{p-1} a_{21,i+1}\Delta\tilde{\pi}_{t-i} + a_{22,1}\tilde{u}_{t-1} + \sum_{i=1}^{p-1} a_{22,i+1}\Delta\tilde{u}_{t-i} + \varepsilon_{\tilde{u},t},\end{aligned}\quad (23)$$

If the macro system is stable, each macro variable reverts to its natural rate in the long run. Monthly predictions of expected inflation,  $E_t\pi_{t,12h}$ , and unemployment,  $E_tu_{t,12h}$ , are generated by a 12th-order empirical model of inflation and unemployment, whose format is shown in (23).

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<sup>18</sup>Results in Kozicki and Tinsley (2001a, 2005b) indicate that learning rates need not be constant over time. Faster learning rates are more likely if agents perceive larger forecast errors for observable variables and can reduce the real consequences of perception errors in episodes with a time-varying inflation target. But, faster constant-gain learning rates are inefficient in more tranquil periods, as larger responses to transient disturbances increase the dispersion of the ergodic distribution of perceived inflation targets about a fixed central-bank target.

## 5.2 Forward-rate regressions

The empirical analysis of forward rates uses data for forward rates at 1-year, 3-year, 5-year and 10-year horizons. As forward rates at neighbouring horizons tend to move closely, the forward-rate regressions are grouped into three horizon partitions: 1-3 years, 4-6 years, and 7-10 years. Assuming that perceived policy-rate responses do not vary significantly within a partition, this grouping should increase precisions of estimates. That said, because the term-premium responses within a partition are not likely to be identical, unless they are zero, the regression residuals will be heteroskedastic reflecting deviations from estimated average responses.

Forward-rate regressions for the three partitions are presented in Tables 2a and 2b for a pre-Volcker sample, 1966 m1 - 1979 m7.<sup>19</sup> Forecasts of inflation and unemployment regressors for results in Table 2a are generated by fitting the empirical macro model to the 1960-79 sample under the assumption of the benchmark learning rates,  $\gamma_{\pi} = \gamma_{\bar{u}} = .015$ . The mean long-run response to the equilibrium deviation in inflation,  $\tilde{\pi}_t$ , is statistically insignificant for forward rates in the 1-3 year partition; is not statistically different from unity for forward rates in the 4-6 year partition; and is greater than unity for forward rates in the 7-10 year partition. Thus, the pattern of increasing responses over the forecast horizon is consistent with elastic responses to expected inflation by intermediate-maturity bond rates in the 1960s and 1970s. Although mean responses to the forecast level or difference of the unemployment rate are negative, mean responses are not significant in the 1-3 and 4-6 year partitions. Although the signs of term-premium responses may be opposite to the signs of expected policy-rate responses, it is unlikely the combined response would overturn the direction of the expected policy-rate response.<sup>20</sup>

Forward-rate responses to expected inflation under alternative learning rates are examined in Table 2b for the pre-Volcker sample. The pattern of increasing inflation responses over the forecast horizon is relatively insensitive to variation in the assumed learning rate. However, determinacy of bond-rate responses to expected inflation is better supported for perceptions of a fixed inflation target,  $\gamma_{\pi} = 0.0$ , or the time-varying inflation target generated by the benchmark learning rate,

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<sup>19</sup>During this interval, the FOMC was chaired by William McChesney Martin, Jr, Arthur Burns, and G. William Miller.

<sup>20</sup>Buraschi and Jiltsov (2005) report positive term premiums for inflation, with larger premiums for longer maturities. Dewachter and Lyrio (forthcoming(b)) estimate positive term premiums for inflation that rise with maturity and negligible term premiums for GDP gaps. Positive term premiums are also estimated for a time-varying central tendency for inflation, similar to  $\tilde{\pi}_t$ ; these premiums also rise with maturity and are nearly triple the size of the term premiums for inflation. By contrast, Duffee (2006) presents evidence of negative term-premium responses to inflation in a pre-Volcker sample. Note that negative term-premium responses to inflation could conceivably reverse the historical roles of the inflation responses by the central bank and bond traders suggested in section 3. That is, system indeterminacy could occur if an elastic policy-rate response to inflation is accompanied by inelastic bond-rate responses to expected inflation.

$$\gamma_{\bar{\pi}} = .015.$$

Tables 3a and 3b present forward-rate regressions for the sample 1982 m1 - 1987 m7, a period that encompasses the last six and one-half years of the FOMC chairmanship of Paul Volcker but excludes the unusual interest-rate volatility in 1979-81, during the experiment with nonborrowed reserves as the operating policy instrument. In Table 3a, the forward-rate response to expected inflation,  $c_2$ , is not statistically different from unity in the 1-3 year partition, and greater than unity in the 4-6 year partition, although not significantly so. The mean inflation response in the 7-10 year partition is greater than unity but the associated p-value is marginally larger than .10. As with the earlier sample, forward rates do not appear to consistently respond to forecasts of unemployment.

The estimated forward-rate responses to the perceived inflation target,  $\bar{\pi}$ , in Table 3a are significantly greater than unity in the 1-3 year and 4-6 year partitions. Under the physical probability measure, the expected coefficient of the inflation target is one, so this suggests forward-rate term premiums responded positively to the perceived inflation target in the 1980s.

Forward-rate responses to expected inflation in the 1980s under alternative learning rates are examined in Table 3b. Here, the pattern of increasing responses to expected inflation over the forecast horizon is statistically supported when the inflation-target learning rate is equal to or exceeds the benchmark learning rate. The case with no learning is likely to be a particularly bad assumption for this sample, as it incorporates a period immediately after a large change in policy regime with sizable reductions in inflation. Positive responses of forward-rate term premiums to the perceived inflation target,  $c_1 > 1$ , are also indicated for the benchmark learning rate,  $\gamma_{\bar{\pi}} = .015$ , and the faster learning rate,  $\gamma_{\bar{\pi}} = .03$ .

Although the forward-rate regressions provide only rough approximations of combined forward-rate responses to macro variables, two results appear to be common to the pre-Volcker sample and the 1980s sample. First, forward-rate responses to equilibrium deviations in inflation are generally larger at more distant horizons and often greater than one, consistent with elastic bond-rate responses to inflation. Second, there is little evidence of systematic responses by forward rates to the level or difference of unemployment. A notable difference in sample results is that positive responses by forward-rate term premiums to a time-varying inflation target are supported in the 1980s sample although not in the pre-Volcker sample.

## 6. Concluding remarks

This paper re-examines the stability of monetary policy, taking into account the transmission role of bond yields. Some interpretations of the Great Inflation have focused on the stability of a Taylor-rule description of the policy rate or on central-bank assumptions regarding natural rates. However, these possible shortcomings in policy are not sufficient to assess the stability of the economy if the bond rate is the transmission channel for monetary policy. With bond-rate transmission, we show that conditions for determinate inflation require a lower bound on bond-rate responses to expected inflation. Consequently, mildly passive current policy may be compensated by bond-trader perceptions of aggressive policy later.

Such horizon-dependence in policy anticipations may explain elastic nominal bond-rate responses to inflation, even in the 1960s and 1970s—a period for which other studies have demonstrated that the policy rate did not keep pace with inflation. Another resolution of possible contradictory sensitivities to inflation by policy rates and bond rates is that risk prices in forward-rate term premiums may depend on expected inflation and operate as automatic stabilizers, reducing the lower-bound requirement for expected policy-rate responses.

In investigating historical behaviour of policy rates and bond rates, we also note the importance of allowing for asymmetric information on the part of the central bank and the private sector. Although structural dynamic expenditure equations are often formulated as functions only of the one-period interest rate, the elimination of market bond-rate observations substitutes the information set of the modeler for the more relevant information set of bond traders in long-horizon forecasts.

To accommodate these possibilities, we present a variant of the essentially affine model of no-arbitrage bond pricing that allows for horizon-dependent expectations of policy-rate responses and incorporates time-varying term premiums. This model provides a framework for interpreting forward-rate responses to equilibrium deviations in expected inflation. Forward-rate regressions provide empirical support for the conjecture that forward-rate responses at more distant horizons display larger long-run responses to equilibrium deviations in expected inflation. The regressions also suggest forward-rate term premiums responded positively to perceptions of a time-varying inflation target in the 1980s but not in the 1960s and 1970s.

In future work we look to isolate the separate contributions of time-varying term premiums and horizon-dependent expectations of future policy rates. This will require imposing no-arbitrage cross-equation restrictions on the forward-rate regressions. If horizon-dependent perceptions are confirmed, it would be useful to explore possible reasons for horizon dependency in expectations.

If long-horizon expectations are merely inertial, that inertia can partially insulate the economy from poor monetary policies, as may have occurred in the 1960s and 1970s, but may also attenuate responses to new monetary policies.

## Appendix A: Eigenvalues of the companion form for equation (4)

The second-order inflation equation

$$\tilde{\pi}_t = \left(1 - \frac{ab(c_1 - 1)}{2}\right) E_t \tilde{\pi}_{t+1} - \frac{ab(c_2 - 1)}{2} E_t \tilde{\pi}_{t+2} + be_t,$$

is restated in the first-order companion form

$$A_1 y_{t+1} = A_0 y_t + a_e e_t + a_\eta \eta_t,$$

where  $y'_{t+1} = [\tilde{\pi}_{t+1}, E_t \tilde{\pi}_{t+2}]$ ,

$$A_1 = \begin{bmatrix} 0 & -\frac{ab(c_2-1)}{2} \\ 1 & 0 \end{bmatrix}, A_0 = \begin{bmatrix} 1 & -\frac{(2-ab(c_1-1))}{2} \\ 0 & 1 \end{bmatrix}, a_e = \begin{bmatrix} b \\ 0 \end{bmatrix}, \text{ and } a_\eta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Using

$$A_1^{-1} = \begin{bmatrix} 0 & 1 \\ -\frac{2}{ab(c_2-1)} & 0 \end{bmatrix},$$

the reduced form is

$$y_{t+1} = B_0 y_t + b_e e_t + b_\eta \eta_t,$$

where

$$B_0 = \begin{bmatrix} 0 & 1 \\ -\frac{2}{ab(c_2-1)} & \frac{2-ab(c_1-1)}{ab(c_2-1)} \end{bmatrix}.$$

The text equations for the eigenvalues  $(\lambda_1, \lambda_2)$  in (6) are provided by the trace and determinant of  $B_0$ ,

$$\begin{aligned} \lambda_1 \lambda_2 &= \frac{2}{ab(c_2 - 1)}, \\ \lambda_1 + \lambda_2 &= \frac{2 - ab(c_1 - 1)}{ab(c_2 - 1)}. \end{aligned}$$

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Table 1: Bond-rate responsiveness to macro variables <sup>1</sup>.

$$R_{12h,t}^* = b_0 + b_1\pi_{t-1} + b_{11}(L)\Delta\pi_{t-1} + b_2u_{t-1} + b_{22}(L)\Delta u_{t-1}.$$

$$R_{12h,t} = b_3R_{12h,t-1} + b_{33}(L)\Delta R_{12h,t-1} + (1 - b_3)R_{12h,t}^* + a_{t,12h}.$$

bond rate	$\pi$	$u$	$\Delta u$	$b_3$	Prob( $b_1 > 1$ )
	$b_1$	$b_2$	$b_{22}(1)$		
sample: 1966 m1 - 1979 m9					
h = 1	1.37 (2.8)	-1.28 (-2.0)	-1.94 (-3.1)	.880 (12)	.77
h = 3	1.06 (3.8)	-.889 (-2.3)	-1.44 (-3.2)	.864 (13)	.58
h = 5	1.05 (3.6)	-.853 (-2.1)	-1.15 (-3.2)	.864 (15)	.58
h = 10	1.12 (3.4)	-.904 (-1.8)	-.812 (-3.0)	.898 (20)	.64
sample: 1982 m1 - 1987 m7					
h = 1	1.96 (4.9)	.068 (.19)	-3.92 (-2.8)	.704 (7.8)	.99
h = 3	1.95 (3.7)	.214 (.50)	-2.97 (-2.1)	.757 (10)	.96
h = 5	1.73 (2.7)	.338 (.71)	-2.42 (-1.5)	.774 (11)	.87
h = 10	1.20 (1.6)	.551 (1.0)	-2.42 (-1.5)	.795 (12)	.60

1.  $b_{ii}(L)$  are 11th-order polynomials in L. Parentheses contain ratios of coefficients to asymptotic standard errors.

Table 2a: Forward-rate regressions, pre-Volcker 1966 - 1979,  
with benchmark learning rates <sup>1</sup>.

$$f_{t,12h}^* = E_t \left\{ c_0 + c_1 \bar{\pi}_t + c_2 \tilde{\pi}_{t,12h}^{(\bar{k})} + c_3 \tilde{u}_{t,12h} + c_4 \Delta^{(k)} u_{t,12h} \right\} .$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2 \Delta^{(k)} f_{t,12(h-1)} + (1 - \rho_1) f_{t,12h}^* + a_{t,12h} .$$

forward rate	$\bar{\pi}$ $c_1$	$\tilde{\pi}^{(k)}$ $c_2$	$\tilde{u}$ $c_3$	$\Delta^{(k)} u$ $c_4$	$\rho_1$	$\rho_2$
time-varying perceived natural rates, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.015$						
h = 1-3	.588 (9.2)	<b>-.042</b> <b>(-0.4)</b>	–	–	.513 (16)	-.009 (-0.2)
	.581 (9.1)	-.042 (-0.4)	-.023 (-0.3)	–	.509 (15)	-.005 (-0.1)
	.581 (9.0)	.056 (0.3)	–	-.172 (-0.6)	.518 (16)	-.017 (-0.4)
h = 4-6	.904 (10)	<b>.661</b> <b>(2.5)</b>	–	–	.877 (38)	.219 (2.5)
	.861 (8.5)	.375 (1.5)	-.564 (-1.5)	–	.873 (39)	.211 (2.4)
	.860 (8.4)	2.92 (1.8)	–	-2.65 (-1.5)	.873 (39)	.211 (2.4)
h = 7-10	.796 (25)	<b>2.70</b> <b>(3.9)</b>	–	–	.967 (178)	.877 (20)
	.813 (25)	1.32 (1.8)	-2.86 (-3.7)	–	.967 (179)	.873 (20)
	.813 (25)	13.8 (4.3)	–	-13.1 (-3.7)	.967 (179)	.873 (20)

1. Superscript  $(\bar{k})$  denotes  $k$ -period averages. For monthly data,  $k = 12$ . Parentheses contain ratios of coefficients to HAC standard errors.

VAR forecast model sample: 1960 m1 - 1979 m7.

Table 2b: Forward-rate regressions, pre-Volcker 1966 - 1979, with alternative learning rates <sup>1</sup>.

$$f_{t,12h}^* = E_t \left\{ c_0 + c_1 \bar{\pi}_t + c_2 \tilde{\pi}_{t,12h}^{(\bar{k})} + c_3 \tilde{u}_{t,12h} + c_4 \Delta^{(k)} u_{t,12h} \right\}.$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2 \Delta^{(k)} f_{t,12(h-1)} + (1 - \rho_1) f_{t,12h}^* + a_{t,12h}.$$

forward rate	$\bar{\pi}$	$\tilde{\pi}^{(k)}$	$\tilde{u}$	$\Delta^{(k)} u$		
	$c_1$	$c_2$	$c_3$	$c_4$	$\rho_1$	$\rho_2$
fixed perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\tilde{u}} = 0.0$						
h = 1-3	1.0	.320 (5.1)	-	-	.540 (14)	-.037 (-0.7)
h = 4-6	1.0	.929 (1.8)	-	-	.970 (54)	.312 (3.1)
h = 7-10	1.0	1.59 (13)	-	-	.981 (346)	.870 (20)
time-varying perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\tilde{u}} = 0.015$						
h = 1-3	.588 (9.2)	-.042 (-0.4)	-	-	.513 (16)	-.009 (-0.2)
h = 4-6	.904 (10)	.661 (2.5)	-	-	.877 (38)	.219 (2.5)
h = 7-10	.796 (25)	2.70 (3.9)	-	-	.967 (178)	.877 (20)
time-varying perceived inflation target, $\gamma_{\bar{\pi}} = 0.03; \gamma_{\tilde{u}} = .015$						
h = 1-3	.499 (10)	-.012 (-0.2)	-	-	.503 (17)	-.001 (-0.0)
h = 4-6	.670 (9.9)	.422 (2.1)	-	-	.866 (36)	.216 (2.3)
h = 7-10	.694 (26)	1.24 (4.0)	-	-	.965 (164)	.879 (20)

1. Superscript  $(\bar{k})$  denotes  $k$ -period averages. For monthly data,  $k = 12$ . Parentheses contain ratios of coefficients to HAC standard errors. VAR forecast model sample: 1960 m1 - 1979 m7.

Table 3a: Forward-rate regressions, Volcker 1982 - 1987  
with benchmark learning rates <sup>1</sup>.

$$f_{t,12h}^* = E_t \left\{ c_0 + c_1 \bar{\pi}_t + c_2 \tilde{\pi}_{t,12h}^{(\bar{k})} + c_3 \tilde{u}_{t,12h} + c_4 \Delta^{(k)} u_{t,12h} \right\} .$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2 \Delta^{(k)} f_{t,12(h-1)} + (1 - \rho_1) f_{t,12h}^* + a_{t,12h} .$$

forward rate	$\bar{\pi}$	$\tilde{\pi}^{(k)}$	$\tilde{u}$	$\Delta^{(k)} u$	$\rho_1$	$\rho_2$
	$c_1$	$c_2$	$c_3$	$c_4$		
time-varying perceived natural rates: $\gamma_{\bar{\pi}} = \gamma_{\tilde{u}} = 0.015$						
h = 1-3	3.63 (8.4)	.732 (2.6)	–	–	0.574 (9.9)	-.155 (-3.1)
	3.62 (9.5)	.731 (2.8)	.007 (0.0)	–	.574 (10)	-.155 (-3.1)
	3.85 (9.9)	.429 (1.3)	–	.759 (1.9)	.529 (9.1)	-.120 (-2.4)
h = 4-6	3.28 (3.6)	1.12 (1.8)	–	–	.918 (24)	-.232 (-2.3)
	.754 (0.2)	9.18 (1.1)	-20.2 (-1.0)	–	.955 (26)	-.265 (-2.7)
	.226 (0.1)	-1.50 (-0.6)	–	13.3 (1.0)	.958 (27)	-.268 (-2.8)
h = 7-10	-0.686 (-0.4)	6.64 (1.6)	–	–	.957 (60)	.450 (5.4)
	1.62 (2.0)	-76.1 (-2.6)	119 (2.7)	–	.942 (65)	.485 (6.4)
	2.15 (2.7)	-7.67 (-1.9)	–	-28.6 (-3.2)	.934 (60)	.467 (6.2)

1. Superscript  $(\bar{k})$  denotes  $k$ -period averages. For monthly data,  $k = 12$ . Parentheses contain ratios of coefficients to HAC standard errors. VAR forecast model sample: 1982 m1 - 1987 m7.

Table 3b: Forward-rate regressions, Volcker 1982 - 1987,  
with alternative learning rates <sup>1</sup>.

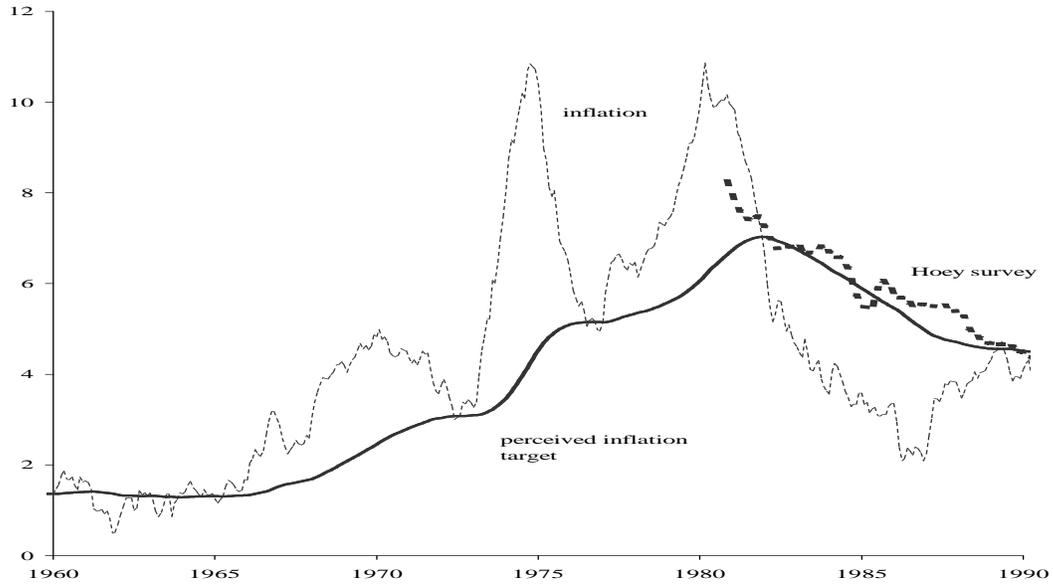
$$f_{t,12h}^* = E_t \left\{ c_0 + c_1 \bar{\pi}_t + c_2 \tilde{\pi}_{t,12h}^{(\bar{k})} + c_3 \tilde{u}_{t,12h} + c_4 \Delta^{(k)} u_{t,12h} \right\}.$$

$$\tilde{f}_{t,12h} = \rho_1 \tilde{f}_{t,12(h-1)} + \rho_2 \Delta^{(k)} \tilde{f}_{t,12(h-1)} + (1 - \rho_1) f_{t,12h}^* + a_{t,12h}.$$

forward rate	$\bar{\pi}$	$\tilde{\pi}^{(k)}$	$\tilde{u}$	$\Delta^{(k)} u$		
	$c_1$	$c_2$	$c_3$	$c_4$	$\rho_1$	$\rho_2$
fixed perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\tilde{u}} = 0.0$						
h = 1-3	1.0	4.37 (2.1)	-	-	.850 (24)	-.262 (-4.8)
h = 4-6	1.0	75.7 (1.1)	-	-	.978 (53)	-.285 (-2.9)
h = 7-10	1.0	36.2 (0.9)	-	-	.936 (80)	.412 (4.6)
time-varying perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\tilde{u}} = 0.015$						
h = 1-3	3.63 (8.4)	.732 (2.6)	-	-	.574 (9.9)	-.155 (-3.1)
h = 4-6	3.28 (3.6)	1.12 (1.8)	-	-	.918 (24)	-.232 (-2.3)
h = 7-10	-.686 (-0.4)	6.64 (1.6)	-	-	.957 (60)	.450 (5.4)
time-varying perceived inflation target, $\gamma_{\bar{\pi}} = 0.03; \gamma_{\tilde{u}} = .015$						
h = 1-3	2.22 (5.7)	.774 (2.3)	-	-	.617 (10)	-.171 (-3.3)
h = 4-6	2.04 (2.8)	1.38 (1.7)	-	-	.933 (26)	-.240 (-2.3)
h = 7-10	1.40 (3.1)	4.01 (3.2)	-	-	.934 (61)	.441 (5.6)

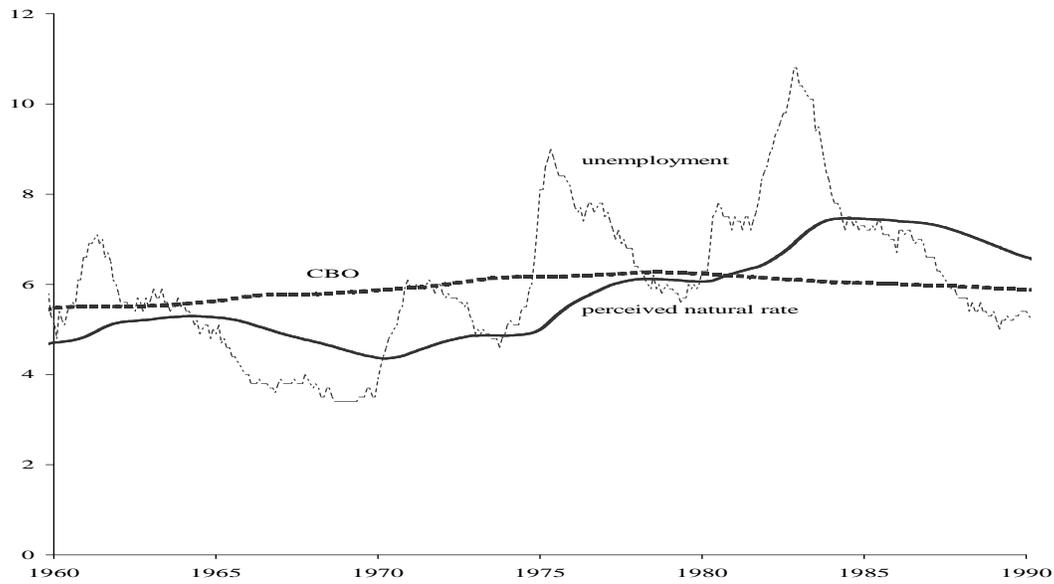
1. Superscript  $(\bar{k})$  denotes  $k$ -period averages. For monthly data,  $k = 12$ . Parentheses contain ratios of coefficients to HAC standard errors. VAR forecast model sample: 1982 m1 - 1987 m7.

Figure 1: Perceived central-bank target for inflation <sup>1</sup>



1. solid line: perceived inflation target with learning gain,  $\gamma_{\pi} = .015$  (see text).  
dashed: Hoey survey of expected 5-10 year inflation.  
dotted: inflation in personal consumption expenditure (pce) deflator, 12-month average.

Figure 2: Perceived natural rate of unemployment <sup>1</sup>



1. solid line: perceived natural rate of unemployment with learning gain  $\gamma_{\bar{u}} = .015$  (see text).  
dashed: Congressional Budget Office (2004) estimate of natural rate for unemployment.  
dotted: civilian unemployment rate.