



NOTA DI LAVORO

51.2009

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Foresight in the
Shapley-Scarf Housing
Market**

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SUSTAINABLE DEVELOPMENT Series

Editor: Carlo Carraro

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Summary

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Keywords: Indivisible Goods Market, Dynamics, Competitive Allocation, Strict Core, Foresight, Stable Set

JEL Classification: D78, C71

This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).

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Dynamics, stability, and foresight in the Shapley-Scarf housing market

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This Version: February 2009
First Version: October 2008

Abstract

While most of the literature starting with Shapley and Scarf (1974) have considered a static exchange economy with indivisibilities, this paper studies the dynamics of such an economy. We find that both the dynamics generated by competitive equilibrium and the one generated by weakly dominance relation, converge to a set of allocations we define as strictly stable, which we can show to exist. Moreover, we show that even when only pairwise exchanges between two traders are allowed, the strictly stable allocations are attained eventually if traders are sufficiently farsighted.

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1 Introduction

Since Shapley and Scarf (1974) have considered a market with indivisible goods, called a housing market, as a game without side payments, several authors have explored the properties of this market. Shapley and Scarf themselves define the core of this market in the usual way, as the set of allocation which are not strictly blocked by any other allocation, and show that it is always non-empty. Moreover, they also show that the set of competitive allocations is also non-empty and is included in the core of this market.

It is, however, also known that there exist some troublesome properties in the housing market. First, the core may contain an allocation that is weakly Pareto dominated

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by other allocation. Second, the allocations in the core can be “unstable” if we consider the dynamics of exchange of indivisible goods in the following way. Roth and Postlewaite (1977) show that an allocation a in the core of the market with initial endowment w may not be stable once a is realized; a is not guaranteed to be in the core of the market with initial endowment a . This instability inspired Roth and Postlewaite (1977) and subsequent works to focus on the strict core. It is known that the strict core satisfies nice properties (Wako, 1984, 1991). However, it is known that the strict core may be empty. Shapley and Scarf (1974) and Roth and Postlewaite (1977) also consider competitive allocations, but this competitive “mechanism” is not Pareto-satisfactory. That is, even if a is a competitive allocation of some market, there may exist another allocation b in the market such that b weakly Pareto-dominates a . Moreover, in some market, competitive mechanism may lead to only the Pareto-inefficient allocations.

In this paper, we attempt to resolve these problems by explicitly defining the dynamics of exchanging the indivisible goods among traders and provide an alternative solution, which we call strict stability (a stronger stability notion than the one proposed by Roth and Postlewaite, 1977), defined from our dynamics. An allocation is strictly stable if this allocation belongs to the strict core of the market with this allocation itself being an initial allocation. We show that, unlike the strict core, a strictly stable allocation always exists.

To illustrate what kinds of dynamic property the set of strictly stable allocations possesses, we introduce two different dynamics: competitive dynamics and the weak dominance dynamics. In the competitive dynamics, each of the competitive allocations of the market is chosen with positive probability, and once chosen, this allocation is realized and go to next step; in the next step, one of the competitive allocation of this new market is chosen, and so forth. The weakly dominance dynamics is also similarly defined. We show that from any initial allocation, in the competitive dynamics, the dynamic process converges with probability one to some strictly stable allocation. This implies that the result like the first theorem of welfare economics holds in the dynamics of Shapley-Scarf housing market, since it is shown that a strictly stable allocation is Pareto-efficient. The same result holds for the weakly dominance dynamics.

Our dynamic models are critically different from the dynamic recontracting model introduced by Serrano and Volij (2008). Serrano and Volij (2008) consider the dynamic recontracting process in the housing market where in each step, one coalition is randomly chosen and the members in this selected coalition can propose to recontract the allocation of their endowment. Serrano and Volij (2008) consider a static exchange economy, in which recontracting is based on an initial endowment that is fixed throughout the model. They support this assumption by citing the labor market as an example and claiming that entrepreneurs “do not sell the labour units that they bought in the previous period.”

In contrast, our dynamic models describe a transition of allocations, reflecting the situation in which agents do sell what they obtained in the previous period. Our models are similar to Roth and Vande Vate (1990) in this respect. Roth and Vande Vate (1990) consider a transition of matching pairs in marriage market where in each step, one of the blocking coalitions is randomly chosen and selected coalition changes the current matching to the others by the blocking deviation. They show that this dynamics converges with probability one to a stable matching.

In some situations, it may be natural to consider the case in which only pairwise trading is allowed. However, we note that the pairwise weakly dominance dynamics, which is the weakly dominance dynamics admitting only two players deviation, may stop at a Pareto-inefficient allocation. This limitation of the dynamics is due to the

myopia of the traders. If we incorporate the foresight of the traders in the dynamics, the dynamics eventually reaches a Pareto-efficient allocation. Here, we introduce a slightly different notion, a pairwise weakly farsighted stable set, from the dynamics considered before. The pairwise weakly farsighted stable set is a stable set of von Neumann and Morgenstern (1953) according to the dominance relations that describes both pairwise trading and foresight of players. We show that the set of the strictly stable allocations is a unique pairwise weakly farsighted stable set of the housing market.

The rest of the paper is organized as follows. In the next section, the market with indivisible goods is explained. In Section 3, two types of dynamics are considered. In Section 4, we consider the foresight of traders and incorporate this foresight into the dynamics. Section 5 is a conclusion.

2 The market with indivisible goods

Let $N = \{1, 2, \dots, n\}$ be the set of players (traders or owners). A non-empty subset S of N is called a coalition. There are n indivisible goods, w_1, \dots, w_n in the market, and initially each i owns w_i as his endowment. Denote i th player's preference relation by \succsim_i , where $w_j \succsim_i w_k$ means trader i prefers the goods w_j at least as well as w_k . The strict preference relation \succ_i and the indifference relation \sim_i are obtained from \succsim_i by the usual manner.

Let $a = (a_1, \dots, a_n)$ be an allocation of the market where $a_i = w_j$ means trader i is assigned w_j by this allocation. Since each player owns at most one item, $a_i = w_j$ implies that there does not exist $k \neq i$ such that $a_k = w_j$. By definition, $w = (w_1, \dots, w_n)$ is an allocation. The set of allocations is denoted by A . Because there is no externality, for $i \in N$ and two allocations $a, b \in A$, we write $a \succsim_i b$ if $a_i \succsim_i b_i$. The strict preference relation and the indifference relation are also extended over the set of allocations.

For two allocations $a, b \in A$, we say that a weakly Pareto-dominates b if $a \succsim_i b$ for all $i \in N$ and $a \succ_j b$ for some $j \in N$. An allocation a is said to be Pareto-efficient if there does not exist $b \in A$ such that b weakly Pareto-dominates a .

Fixing the player set and the preference relations, the market depends only on the initial endowment, and thus we denote the market with initial allocation a by $M(a)$.

Given a market $M(a)$, we say that an allocation c (strongly) blocks¹ an allocation b if there is some coalition S such that

$$(b.i) \{c_i : i \in S\} = \{a_i : i \in S\},$$

$$(b.ii) c \succ_i b \text{ for all } i \in S.$$

The first condition states that the coalition S is effective for the allocation c , and the second condition states that every member of S strictly prefers c to b .

We define weak blocking by relaxing condition (b.ii) to (b.ii') $c \succsim_i b$ for all $i \in S$ and $c \succ_i b$ for some $i \in S$.

The core of the market $M(a)$, denoted by $C(a)$, is the set of allocations that are not blocked by any other allocation in the market $M(a)$. The strict core of the market $M(a)$, denoted by $SC(a)$, is defined by the set of allocations that are not weakly blocked by any other allocation.

¹In this paper, we distinctly use two phrases, "an allocation b blocks an allocation c " and "an allocation b dominates an allocation c ." The former implies that given an initial allocation a , the realization of allocation b can be hampered by some players who alternatively propose allocation c . On the other hand, the latter means that once an allocation b is realized, this allocation can be replaced by another allocation c .

In the market $M(a)$, an allocation b is competitive if there exists a non-zero vector of non-negative numbers $p = (p_1, \dots, p_n)$ such that for each $i \in N$, (i) $a_j \succ_i b_i$ implies $p_j > p_i$ where p_j denotes the price of a_j and p_i denotes the price of a_i and (ii) for $j \in N$ with $b_i = a_j$, $p_i \geq p_j$ holds. The set of competitive allocations of the market $M(a)$ is denoted by $CA(a)$. It is easily confirmed that if $b \in CA(a)$, then $b \succsim_i a$ holds for all $i \in N$. Moreover, if b is a competitive allocation of market $M(a)$, it is also a competitive allocation of $M(b)$. This is checked as follows. Let p be a competitive price supporting allocation b in market $M(a)$. Consider a price vector q such that $q_i = p_j$ if and only if $b_i = a_j$. Then, allocation b is competitive for this q in market $M(b)$.

Roth and Postlewaite (1977) define an allocation a to be stable if $a \in C(a)$ holds. Likewise, we define an allocation a to be strictly stable if $a \in SC(a)$.

We note that the set of strictly stable allocations, which we will denote at the moment by K , coincides with the set of all the Pareto-efficient allocations, which is defined by

$$A^{PE} = \{a \in A : \text{there does not exist } b \text{ such that } b \text{ weakly Pareto-dominates } a\}.$$

Take $a \in K$ and assume $a \notin A^{PE}$. Then, there exists b such that b weakly Pareto-dominates a . This implies that in the market $M(a)$, b weakly dominates a via coalition N . This contradicts $a \in SC(a)$. Next, take $a \in A^{PE}$ and assume $a \notin K$. Then there exists coalition S and allocation b such that (i) $\{a_i : i \in S\} = \{b_i : i \in S\}$, (ii) $a_i = b_i$ for all $i \in N \setminus S$, and (iii) $b \succsim_i a$ for all $i \in S$ and $b \succ_j a$ for some $j \in S$. However, this implies b weakly Pareto-dominates a — a contradiction.

By this very fact, we can claim that there always exists an allocation that is strictly stable. Since every strictly stable allocation is in fact stable by definition, this fact provides a way, different from Roth and Postlewaite (1977) to show the existence of a stable allocation.

Next, we explain the top trading cycles algorithm introduced by David Gale in Shapley and Scarf (1974). Let $N' \subseteq N$ be a coalition and a be an allocation. A top trading cycle coalition for N' in the market $M(a)$ is a non-empty subset T of N' , whose members can be indexed in a cycle order,

$$T = \{i_{t+1} = i_1, i_2, \dots, i_t\},$$

in such a way that each trader i_k prefers the good of i_{k+1} , i.e., $a_{i_{k+1}}$, at least as good as any other goods possessed by any trader in N' . By the finiteness of N' , it is evident that every coalition N' has at least one top trading cycle coalition. Using this idea, we can obtain a top trading cycles partition of N ,

$$T_1, T_2, \dots, T_r, \text{ where } \bigcup_{k=1}^r T_k = N,$$

by taking T_k , $1 \leq k \leq m$, to be any top trading cycle coalition for $N \setminus \bigcup_{h=1}^{k-1} T_h$. We can also obtain the corresponding allocation b such that for each i_k in T_h , $b_{i_k} = a_{i_{k+1}}$, i.e., any $i_k \in T_h$ obtains the goods prescribed by the trading cycle in T_h .

Let $TTC(a)$ be the set of allocations that can be obtained as the result of top trading cycles algorithm for market $M(a)$.

In the literature, the following facts are known:

Fact 1 $SC(a) \subseteq CA(a) \subseteq C(a)$. Moreover, if no agent's preferences exhibit indifference between two objects, then $SC(a) = CA(a)$ and is a singleton but still

may be a strict subset of $C(a)$. (Shapley and Scarf, 1974; Wako, 1984; Roth and Postlewaite, 1977)

Fact 2 $CA(a)$ is not empty; thus $C(a)$ is also non-empty. (Shapley and Scarf, 1974)

Fact 3 $SC(a)$ may be empty. (Shapley and Scarf, 1974)

Fact 4 $b \in C(a)$ does not imply that b is stable. On the other hand, $b \in SC(a)$ implies that b is strictly stable. (The former statement is by Roth and Postlewaite, 1977; the latter is checked as follows. Assume $b \in SC(a)$ for some a is not strictly stable. This implies that in $M(b)$, there exists c satisfying conditions (b.i) and (b.ii'). However, this also implies that in $M(a)$, an allocation c can weakly block allocation b by coalition N — a contradiction.)

Fact 5 For competitive allocation $b \in CA(a)$, there may exist another competitive allocation $c \in CA(a)$ such that c weakly Pareto-dominates b . Moreover, for $b \in CA(a)$, there may exist allocation $c \in A \setminus CA(a)$ such that c weakly Pareto-dominates b . (Roth and Postlewaite, 1977)

Fact 6 Every competitive allocation in $CA(a)$ can be obtained from the “top trading cycles” algorithm and vice versa. (Shapley and Scarf, 1974) Thus, $TTC(a) = CA(a)$.

From these facts, we find that there is some difficulty in applying any of these solution concepts to both descriptive and prescriptive analyses of Shapley-Scarf housing market. If we adopt the core as our solution concept, then we immediately find that the allocation in the core is not stable in the sense of the dynamics by Fact 4. Fact 4 also suggests that the strict core is a robust concept in the sense of dynamics but Fact 3 says that in some case, the strict core is empty and it does not provide any prediction. The set of competitive allocations always exists and it is a refinement of the core and is Pareto efficient if all agents' preferences are strict (Fact 1 and Fact 2). However, in the presence of indifference, the competitive “mechanism” may not be Pareto-satisfactory, in contrast with the case of the exchange economy with divisible goods. That is, even if a is a competitive allocation of some market, there may exist another allocation b in the market such that b weakly Pareto-dominates a (Fact 5). This means that the first theorem of welfare economics does not generally hold in a Shapley-Scarf housing market. In fact, there exists an economy, which is given below, in which every competitive allocation is not Pareto-efficient.

Example 1. The following example is from Wako (1999). Consider three traders with the following preferences,

$$\begin{aligned} \text{Trader 1 : } & w_2 \succ_1 w_3 \succ_1 w_1 \\ \text{Trader 2 : } & w_1 \sim_2 w_3 \succ_2 w_2 \\ \text{Trader 3 : } & w_2 \succ_3 w_1 \sim_3 w_3 \end{aligned}$$

Then, it is easily confirmed that in market $M(w)$, there exist two competitive allocations $a = (w_2, w_1, w_3)$ and $b = (w_1, w_3, w_2)$. However, a is weakly Pareto-dominated by allocation (w_2, w_3, w_1) and b is weakly Pareto-dominated by allocation (w_3, w_1, w_2) .

In Sections 3 and 4, we consider these problems by explicitly outlining the dynamic process. In these sections, we show that a strictly stable allocation have nice properties in the dynamics of exchange of the commodities, thus solving the problems listed earlier. That is, a strictly stable allocation always exists, is Pareto efficient, and is an allocation that can be reached as a limit point of our dynamic processes. We find that all of the problems (instability of the core, emptiness of the strict core, and inefficiency of the competitive allocations) can be resolved when we consider the dynamics and adopt the strict stability as a solution concept because the dynamics converges with probability one to a strictly stable allocation, the strict core exists in the market with initial allocation being strictly stable, and the strictly stable allocation is Pareto-efficient.

3 Dynamics and stability

Let Φ be a correspondence that associates with any allocation a a non-empty subset $\Phi(a)$ of A . We define the Φ -dynamics of the indivisible goods market as follows:

Φ -Dynamics

Step 0. Start with any allocation a^0 .

Repeatedly apply the following step m from $m = 1$ ad infinitum.

Step m . Take any allocation a from $\Phi(a^{m-1})$ with positive probability. Set $a^m = a$. We assume that the probability that any particular allocation a is chosen from $\Phi(a^{m-1})$ depends only on allocation a^{m-1} .

Given Φ , an infinite sequence $\{a^m\}_{m=0}^\infty$ in A is called an outcome of the Φ -Dynamics if $a^0 = w$ and for $m = 1, 2, \dots$, $a^m \in \Phi(a^{m-1})$.

We say that an infinite sequence $\{a^m\}_{m=0}^\infty$ converges to an allocation a if there exists some integer M such that for some $m \geq M$, $a = a^m$, and for all $m \geq M$, $a \sim_i a^m$ holds for all $i \in N$. Thus, if the sequence converges to a , after some step M , only the allocations that are equivalent to a are realized.²

Now we introduce two candidates for Φ . First is the competitive allocations. Let $\Phi^{comp} = CA$. Then, Φ^{comp} -dynamics is called the competitive dynamics. Note that by Fact 2, $\Phi^{comp}(a) \neq \emptyset$ for all $a \in A$.

Next we define the weak dominance dynamics. Given coalition S and two allocations a and b , we write $a \xrightarrow{S} b$ if $\{a_i : i \in S\} = \{b_i : i \in S\}$ and $a_i = b_i$ for all $i \in N \setminus S$. We say that an allocation b weakly dominates an allocation a via coalition S , and we write $b \triangleright_S a$, if there exists coalition S such that

(d.i) $a \xrightarrow{S} b$, and

(d.ii) $b \succsim_i a$ for all $i \in S$ and $b \succ_j a$ for some $j \in S$.

If there exists some coalition S such that $b \triangleright_S a$, we say b weakly dominates a and we write $b \triangleright a$.

For any allocation $a \in A$, we define $D(a)$ by

$$D(a) = \{b \in A : b \triangleright a\}$$

²This does not imply that for any two integers $m, m' \geq M$, $\Phi(a^m) = \Phi(a^{m'})$.

Then, Φ^{wdom} is defined by $\Phi^{wdom}(a) = D(a)$ if $D(a) \neq \emptyset$ and $\Phi^{wdom}(a) = \{a\}$ otherwise. The Φ^{wdom} -dynamics is called the weakly dominance dynamics.

We say that coalition S and allocation b are a genuine deviant coalition and a genuine deviant allocation for a if (i) $b \triangleright_S a$ and (ii) S can be represented by $\{i_{s+1} = i_1, i_2, \dots, i_s\}$ such that $b_{i_k} = a_{i_{k+1}}$ for all $k = 1, \dots, s$. As the next lemma shows, whenever allocation a is weakly dominated by other allocation, there exist a genuine deviant coalition and a genuine deviant allocation for a .

Lemma 1. *If $b \triangleright_S a$ holds, there exist a genuine deviant coalition T and a genuine deviant allocation c for a .*

Proof. We first note without proof that if $b \triangleright_S a$, there exists a partition of S , $\{T_1, T_2, \dots, T_m\}$, such that each T_k can be represented by

$$T_k = \{i_{t_k+1}^k = i_1^k, i_2^k, \dots, i_{t_k}^k\}$$

and for each i_h^k , $b_{i_h^k} = a_{i_{h+1}^k}$. Because $b \triangleright_S a$, there must exist $i \in S$ such that $b_i \succ_i a_i$. Let T_k be the subcoalition of S such that $i \in T_k$. Define a new allocation c as follows: $c_j = b_j$ if $j \in T_k$ and $c_j = a_j$ otherwise. Then, T_k and c are a genuine deviant coalition and a genuine deviant allocation for a , respectively. \square

Two theorems below affirmatively answer the question of whether the competitive dynamics and the weakly dominance dynamics guarantee the paths that lead to convergent allocations. A consequence is that two dynamics converge to some allocations with probability one, beginning from an arbitrary initial allocation. Moreover, the convergent allocations must be strictly stable. These results are presented as corollaries to the two theorems.

Theorem 1. *Take any allocation $a \in A$. There exists a finite sequence of allocations a^0, \dots, a^k such that $a^0 = a$, a^k is strictly stable, and for each $m = 1, \dots, k$, $a^m \in \Phi^{comp}(a^{m-1})$.*

Proof. We first provide some notations. For any subset $A' \subset A$ of allocations, define $\Phi^{comp}(A') = \bigcup_{a \in A'} \Phi^{comp}(a)$. Also, for any allocation $a \in A$, let $\Psi^m(a)$, $m \geq 1$, be defined as follows:

$$\Psi^m(a) = \underbrace{\Phi^{comp}(\Phi^{comp}(\dots(\Phi^{comp}(a))\dots))}_{m \text{ times}}$$

Thus, $\Psi^m(a)$ is the set of allocations that are obtained from repeating the operation Φ^{comp} m times, starting with the initial allocation a . Since $c \in CA(b)$ for some allocation b implies $c \in CA(c)$, $\Psi^1(a) \subseteq \Psi^2(a) \subseteq \dots \subseteq \Psi^m(a) \subseteq \dots \subseteq A$ holds. Note that together with the fact that if $\Psi^m(a) = \Psi^{m+1}(a)$ implies $\Psi^m(a) = \Psi^{m'}(a)$ for all $m' \geq m$, this implies that there exists a convergent set $\Psi^\infty(a) \subseteq A$ such that there exists M such that for all $m \geq M$, $\Psi^m(a) = \Psi^\infty(a)$. This M must be less than or equal to $|A|$.

Now we show the following claim.

Claim 1: *There exists a finite sequence of allocations a^0, \dots, a^k such that $a^0 = a$, for each $m = 1, \dots, k$, $a^m \in \Phi^{comp}(a^{m-1})$, and for any $c \in \Psi^\infty(a^k)$, $c \sim_i a^k$ for all $i \in N$.*

Consider the set $\Psi^\infty(a) \subseteq A$. Then, since $\Psi^\infty(a)$ is a finite set, there must exist $b \in \Psi^\infty(a)$ such that b is not weakly Pareto-dominated by any other allocation in

$\Psi^\infty(a)$. Then, by the definition of $\Psi^\infty(a)$, there exists a finite sequence of allocations a^0, \dots, a^k starting from $a^0 = a$ and ending with $a^k = b$ such that for each $m = 1, \dots, k$, $a^m \in \Phi^{comp}(a^{m-1})$. Since $a^m \in CA(a^{m-1})$, allocation a^m weakly Pareto-dominates or is equivalent to allocation a^{m-1} , the construction of b implies that for any $c \in \Psi^\infty(b)$, $c \sim_i b$ for all $i \in N$.

By Claim 1, the proof of the theorem is complete if we show that b , which is defined in the above paragraph, is strictly stable. Assume, in negation, that b is not strictly stable. Then, for some $c \in A$ and some $S \subseteq N$, $c \succ_S b$ must hold. Because of Lemma 1, without loss of generality, let S and c be a genuine deviant coalition and a genuine deviant allocation for b .

We now show the following claim.

Claim 2: *In the market $M(b)$, there exists some competitive allocation d such that for some $i \in N$, $d \succ_i b$.*

This claim follows from the observation that because b is not strictly stable, it must be weakly dominated by another allocation. By proposition 2 of Wako (1991), allocation b must be weakly dominated by some competitive allocation. Label this allocation as d . Then, by definition of weak domination, there must exist some $i \in N$ such that $d \succ_i b$.

The above claim shows that there exists some competitive allocation d in the market $M(b)$ such that for some $i \in N$, $d \succ_i b$ holds. However, b is an allocation such that for any $c \in \Psi^\infty(b)$ — in particular, for any $c \in CA(b)$ — $c \sim_i b$ holds for any $i \in N$, which yields a contradiction. \square

Theorem 2. *Take any allocation $a \in A$. There exists a finite sequence of allocations a^0, \dots, a^k such that $a^0 = a$, a^k is strictly stable, and for each $m = 1, \dots, k$, $a^m \in \Phi^{wdom}(a^{m-1})$.*

Proof. Let $\{a^m\}_{m=0}^\infty$ be an outcome of the weakly dominance dynamics. If for some a^m , $D(a^m)$ is empty, this a^m must be strictly stable. On the other hand, whenever $D(a^m)$ is not empty, any allocation $a^{m+1} \in D(a^m)$ weakly Pareto-dominates a^m . Because A is finite, this implies that there must exist a finite integer M such that for all $m \geq M$, $D(a^m) = \emptyset$. Therefore, we have the desired result. \square

The preceding theorems show that there exists a sequence of allocations that result from either the competitive dynamics or the weak dominance dynamics which converges to a strictly stable allocation. One question to consider is whether all such sequences converge to some strictly stable allocation. The answer regarding the competitive dynamics is negative; there can exist a cycle consisting of two different allocations that may be Pareto-indifferent among agents. Instead, if we consider a random process as is assumed in Roth and Vande Vate (1990), we have the following.

Corollary 1. *From any initial allocation a , the competitive dynamics converges to some strictly stable allocation a^* with probability one.*

Corollary 2. *From any initial allocation a , the weakly dominance dynamics converges to some strictly stable allocation a^* with probability one.*

For the weak dominance dynamics, however, we do have convergence for all such sequences. We state this fact without proof since it follows from the same argument as the preceding theorem.

Corollary 3. *Every sequence $\{a^m\}_{m=0}^{\infty}$ that is consistent with the weak dominance dynamics must converge to a strictly stable allocation.*

Remark 1. *We can show similarly that if we consider a (strong) dominance dynamic process, every resulting sequence of allocations must converge to a stable allocation. This argument sheds light on another interpretation of the stability concept defined by Roth and Postlewaite (1977). They argue that once a stable allocation is reached, there will be no further trading. Our interpretation is that a stable allocation can be reached in a finite number of exchanges using the dominance dynamics. Putting these two ideas together, we have an interpretation of the stability of stable allocations that is similar to that of the stable set of von Neumann and Morgenstern (1953).*

If an allocation b is a competitive allocation of some market $M(a)$, $b \in CA(b)$ holds by the definition of the competitive allocations, indicating that $b \in C(b)$. Thus, in the competitive dynamics, any allocation appeared in this dynamics is stable. However, the strict stability holds only for the convergent allocations.

The remaining question in the above discussion is the existence of strictly stable allocations. An affirmative answer is obtained in the next section. Moreover, it is also shown that the strictly stable allocation is Pareto-efficient (see the proof of Theorem 3 and the last paragraph of Section 4).

4 Pairwise exchange and foresight

Pairwise exchange of indivisible goods between two traders may be more convincing in some situation. In order to examine the difference between coalitional exchange environment and pairwise exchange environment, we consider the pairwise weakly dominance dynamics. For any allocation $a \in A$, we define $D^p(a)$ by

$$D^p(a) = \left\{ b \in A : \begin{array}{l} \exists S \text{ with } |S| = 2 \text{ s.t. } a \xrightarrow{S} b \\ b \succ_i a \text{ for all } i \in S, \text{ and } b \succ_j a \text{ for some } j \in S \end{array} \right\}.$$

Then, $\Phi^{p\text{wdom}}$ is defined by $\Phi^{p\text{wdom}}(a) = D^p(a)$ if $D^p(a) \neq \emptyset$ and $\Phi^{p\text{wdom}}(a) = \{a\}$ otherwise. The $\Phi^{p\text{wdom}}$ -dynamics is called the weakly pairwise dominance dynamics.

The question is whether convergent results like Theorems 1 and 2 hold for pairwise weakly dominance dynamics. The following example shows the negative answer to this question. This is quite in contrast with Roth and Vande Vate (1990) and Diamantoudi et al. (2004) that respectively consider the dynamics in the marriage problem and the roommate problem and show that the convergent results to the stable outcomes holds in these environments even though they do not consider deviation of more than two players coalition.

Example 2. Let there be three traders with the following preferences,

Trader 1 : $w_2 \succ_1 w_1 \succ_1 w_3$

Trader 2 : $w_3 \succ_2 w_2 \succ_2 w_1$

Trader 3 : $w_1 \succ_3 w_3 \succ_3 w_2$

The allocation $a = (w_2, w_3, w_1)$, in which every trader owns his most preferred items, clearly belongs to the strict core of the initial market $M(w)$ and is also unique competitive allocation of $M(w)$. However, the pairwise weakly dominance dynamics predicts that the initial Pareto inefficient allocation $w = (w_1, w_2, w_3)$ prevails because any pairwise trade between two players inevitably makes one of the two worse off.

Thus, in the weakly pairwise dominance dynamics, the dynamics may stop at Pareto-inefficient allocation. This is due to the “myopia” of the players presumed in this dynamics. In the example above, if traders are assumed to have some kinds of “foresight,” trader 2 may agree to trade with trader 1, despite the fact that this trade allocations to 2 his worst commodity w_1 . The reason for this is that 2 can now trade with 3, resulting in 3 receiving w_1 and 2 receiving w_3 .

To demonstrate this point, we consider the dominance relation that captures the farsightedness of traders.

Motivated by Harsanyi (1974) and Chwe (1994), we define the dominance relation that captures the players’ foresight as follows. We say that an allocation b pairwise weakly farsightedly dominates or pwf-dominates an allocation a , and we write $b \blacktriangleright a$, if there exist finite sequences of pairs, S^1, S^2, \dots, S^q and allocations $a = a^0, a^1, \dots, a^q = b$ such that for each $k, 1 \leq k \leq q$,

$$(f.i) |S^k| = 2,$$

$$(f.ii) a^{k-1} \xrightarrow{S^k} a^k, \text{ and}$$

$$(f.iii) b \succsim_i a^{k-1} \text{ for all } i \in S_k,$$

and

$$(f.iv) \text{ for some } i \in S_1, b \succ_i a^0.$$

The interpretation is that there is a “leading” deviator that initiates this sequence of pairwise trading, and this leading deviator is better off in the final allocation. A pairwise weakly farsightedly stable set (PWFSS) is a subset K of A that satisfies the following two conditions:

(s.i) For any $a, b \in K$, $a \blacktriangleright b$ does not hold,

(s.ii) For any $a \in A \setminus K$, there exists $b \in K$ such that $b \blacktriangleright a$.

Thus, a PWFSS is a stable set for an abstract system (A, \blacktriangleright) . Conditions (s.i) and (s.ii) are refereed to as the “internal stability” and the “external stability,” respectively.

A PWFSS presumes the following standard of behaviors of individuals. Suppose that allocations in set K are commonly considered to be “stable” and allocations outside K to be “unstable” by all the individuals. Then, once an allocation a in K is reached, any deviation from a never occurs because there exists no stable allocation that pwf-dominates a , and if in time an outcome b outside K is reached, there exists stable outcome $a \in K$ that pwf-dominates b . Therefore, in a dynamic sense, it is interpreted as follows. From any initial allocation b outside K , there exists a sequence of allocations starting from b and ending with some allocation a in K such that in each step, two farsighted traders exchange their goods comparing the their current items with their final items in allocation a . On the other hand, from any allocation in K , such a sequence of allocations does not exist.

We show in the following that the set of strictly stable allocations is the unique stable set with respect to \blacktriangleright . Before doing so, we note two properties of \blacktriangleright that are used in the proof of the main result.

Lemma 2. *If $b \blacktriangleright a$, then b weakly Pareto dominates a .*

Proof. Suppose $b \blacktriangleright a$ holds. Then, by definition, there exists a sequence of pairs S^1, S^2, \dots, S^q and $a = a^0, a^1, \dots, a^1 = b$ such that conditions (f.i) through (f.iii) and (f.iv) hold. By definition, for some $i \in S^1$, $b \succ_i a$. It now remains to show that for all other $i \in N$, $b \succsim_i a$. Take any $i \in N$ and consider the first step $l \leq q$ that i appears in the sequence. By how the sequence is defined we have $a_i^{l-1} = a_i$. Then, by (f.iii) $b_i \succsim_i a_i^{l-1} = a_i$, and thus, $b \succsim_i a$. Therefore, b weakly Pareto dominates a . \square

For the next preliminary result, we need the following notation. If there exists a sequence of allocations and pairs from allocation a to b satisfying condition (f.i), (f.ii) and (f.iii), we say that there exists a weak pairwise path from a to b and write $a \succrightarrow b$. Note that by the same logic as the previous lemma, we must have that if $a \succrightarrow b$, then $b \succsim_i a$ for all $i \in N$.

Lemma 3. *If $b \in CA(a)$, then $a \succrightarrow b$.*

Proof. Since an allocation b is obtained as the results of the top trading cycles algorithm applied for the market $M(a)$, there is the corresponding top trading cycles partition T_1, \dots, T_m of N . Delete all the singleton coalitions in this partition, and we obtain $R_1, \dots, R_{m'}$, where for each k , $1 \leq k \leq m'$, there exists h , $1 \leq h \leq m$, such that $R_k = T_h$. Let, for each k , $1 \leq k \leq m'$, R_k be represented by

$$R_k = \{i_{r_k+1}^k = i_1^k, i_2^k, \dots, i_{r_k}^k\},$$

where the indices of the traders represent trading cycle prescribed by the algorithm.

Define a sequence of two-traders coalitions S^1, S^2, \dots, S^q and a sequence of allocations $a = a^0, a^1, \dots, a^q = b$ as follows:

$$q = \sum_{k=1}^{m'} r_k,$$

and for $h = r_1 + \dots + r_{k-1} + \ell$ and $1 \leq \ell \leq r_k$,

$$S^h = \{i_\ell^k, i_{\ell+1}^k\},$$

$$a^h \text{ is such that } a_{i_\ell^k}^h = a_{i_{\ell+1}^k}^{h-1}, a_{i_{\ell+1}^k}^h = a_{i_\ell^k}^{h-1} \text{ and } a_i^h = a_i^{h-1} \text{ for all } i \in N \setminus S^h.$$

Then, it is easily shown that these sequences of coalitions and allocations satisfy (f.i) and (f.ii).

(The above proof is reproduced from the previous version of the paper)

To show that (f.iii) holds, take any h and let $S^h \equiv \{i, j\} \subset R^k$ for some k . By how the trading cycles are defined, it must be the case that for each $i \in S^h$, a_i^{h-1} is a good traded within R^k . Moreover, since b is a competitive allocation, we can take, without loss of generality, a price vector p such that

- the prices are the same for each good initially owned by members of the same (top) trading cycle
- for $k' \leq k''$, $p_{k'} \geq p_{k''}$ where $p_{k'}$ represents the price of a good traded within trading cycle $R^{k'}$

Thus, since b assigns i and j a good in R^k , by definition of a competitive allocation, we must have $b \succsim_i a_i^{h-1}$ and $b \succsim_j a_j^{h-1}$. \square

Further inspection of the proof shows that the order in which these coalitions deviate is not important for the relation \succrightarrow to hold. This observation is important when we use this lemma to construct a sequence such that the stronger relation \blacktriangleright holds.

Theorem 3. *The set of strictly stable allocations (denoted by K) is the unique stable set defined by \blacktriangleright .*

Proof. Internal stability is established by the first lemma. To show external stability, take any $a \in A \setminus K$. Because a is not weakly Pareto optimal, proposition 2 of Wako (1991) implies that there exists $b \in CA(a)$ such that b weakly Pareto dominates a . Let $i \in N$ be any agent such that $b \succ_i a$. Construct $a \rightsquigarrow b$ such that $S^1 = \{i, j\}$ where j is the agent that satisfies $b_i = a_j$. If b happens to be Pareto optimal, then $b \in K$ and $b \blacktriangleright a$, finishing the proof. If not, start from b and construct a sequence $\{b^k\}$ that converges to an allocation $c \in K$ such that for all integers $k \geq 1$, $b^{k+1} \in CA(b^k)$. Lemma 3 shows that if $a^m \in \Phi^{comp}(a^{m-1})$ and $a^m \neq a^{m-1}$, $a^{m-1} \rightsquigarrow a^m$. Furthermore, for any integers s, t with $t > s$, simply connecting weak pairwise paths $a^s \rightsquigarrow a^{s+1}, a^{s+1} \rightsquigarrow a^{s+2}, \dots, a^{t-1} \rightsquigarrow a^t$, we obtain $a^s \rightsquigarrow a^t$. In this manner, we can construct a sequence such that $b \rightsquigarrow c$. To check that external stability of K holds, recall that the competitive-dynamics converges to some allocation in K .

By lemma 3, $b^k \rightsquigarrow b^{k+1}$. Then, we can connect these sequences together to form a long sequence from a to c that satisfies (f.i), (f.ii), and (f.iv). (f.iii) follows from the observation that $a \rightsquigarrow b$ implies $b \succsim_i a$ for all $i \in N$ and from a similar logic used in the proof of lemma 2.

Uniqueness follows also from lemma 2, since no allocation in K cannot be pairwise weakly farsightedly dominated by another allocation. \square

5 Conclusion

In this paper, we consider several types of dynamics in Shapley-Scarf exchange economy with indivisible commodities and show that in all of these dynamics except for the pairwise weakly dominance dynamics, a Pareto-efficient allocation is reached. These results give useful insight on the market with indivisible commodities and how an efficient allocation can be reached.

First, if the market functions are in such a way that the competitive allocations are expected to be realized, the Pareto-efficient allocation will be attained after some steps of reallocation. This result hinges on the assumption that there is enough structure to the market that allows a competitive allocation to result.

Second, a Pareto-efficient allocation can be realized after reallocations using the weak dominance dynamics. In the weak dominance dynamics, a group of agents can convene together and redistribute their endowments at any time. Unlike the competitive dynamics, we do not require as much structure for this dynamic process to work. However, we still need some structure to allow a group of possibly large number of agents to cooperate together.

To get around this largeness, our third result shows that if agents are sufficiently farsighted, the set of strictly stable allocation can be attained by a sequence of bilateral trades, which are the simplest possible trades in the model. Our results then imply that even without the ability to be able to have a large group of agents to cooperate, farsighted bilateral trading can lead to an efficient allocation.

Acknowledgements

We would like to thank Yukihiro Funaki, Shigeo Muto, Ai Takeuchi, and Takumi Kongo for their helpful comments and discussion. We would also like to thank Jun

Wako for his helpful insights regarding this paper. We also thank Flip Klijn for very helpful comments and for pointing out an error in our previous version. Kawasaki would like to thank the Japan Society for the Promotion of Science as part of this research was funded through its Grants-in-Aid for Scientific Research (20310086, 20330036).

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(lxxxv) This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).