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## Non-bossy Social Classification

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By **Dinko Dimitrov**, Department of  
Economics, University of Munich  
**Clemens Puppe**, Department of  
Economics, Karlsruhe Institute of  
Technology

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By Dinko Dimitrov, Department of Economics, University of Munich  
Clemens Puppe, Department of Economics, Karlsruhe Institute of Technology

### Summary

We consider the problem of how societies should be partitioned into classes if individuals express their views about who should be put with whom in the same class. A non-bossiness condition makes the social aggregator dependent only on those cells of the individual partitions the society members classify themselves in. This information is used to construct for each profile of views an opinion graph. By means of natural sovereignty and liberalism requirements, we characterize the non-bossy aggregators generating partitions in which the social classes are refinements of the connected components in the opinion graph.

**Keywords:** Social Aggregation, Group Identity, Liberalism, Non-bossiness

**JEL Classification:** D71

*This paper was partly written while the first author was visiting the Delhi Center of the Indian Statistical Institute. The hospitality of the Institute and the helpful comments and suggestions of Anirban Kar, Debasis Mishra, and Arunava Sen are gratefully acknowledged.*

*Address for correspondence:*

Dinko Dimitrov  
Department of Economics  
University of Munich  
Germany  
E-mail: [dinko.dimitrov@lrz.uni-muenchen.de](mailto:dinko.dimitrov@lrz.uni-muenchen.de)

# Non-bossy social classification\*

Dinko Dimitrov<sup>†</sup>      Clemens Puppe<sup>‡</sup>

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## Abstract

We consider the problem of how societies should be partitioned into classes if individuals express their views about who should be put with whom in the same class. A non-bossiness condition makes the social aggregator dependent only on those cells of the individual partitions the society members classify themselves in. This information is used to construct for each profile of views an opinion graph. By means of natural sovereignty and liberalism requirements, we characterize the non-bossy aggregators generating partitions in which the social classes are refinements of the connected components in the opinion graph.

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## 1 Introduction

In this paper we study the formation of groups or classes from a social choice perspective. Adapting the framework of Fishburn and Rubinstein (1986), we consider an environment in which every individual has a view about how the

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<sup>†</sup>Department of Economics, University of Munich, Germany; e-mail: dinko.dimitrov@lrz.uni-muenchen.de

<sup>‡</sup>Department of Economics, Karlsruhe Institute of Technology, Germany; e-mail: clemens.puppe@kit.edu

society should be partitioned into classes. A *group identity function* assigns then to each profile of views a societal decomposition into classes. In the aggregation problem considered here the number of classes is thus *endogenously* determined. This is in contrast to environments in which the number of social groups is assumed to be fixed and their names matter (cf. Çengelci and Sanver 2008, Dimitrov et al. 2007, Houy 2007, Kasher and Rubinstein 1997, Miller 2008, Samet and Schmeidler 2003, among others).

The most studied rule in the context of aggregating partitions is the *conjunctive* aggregator which classifies two individuals in the same social group if and only if everyone in the society thinks so. This function belongs to the class of rules characterized by Fishburn and Rubinstein (1986) in the context of the aggregation of equivalence relations (see also Mirkin 1975, Barthélemy et al. 1986, Barthélemy 1988) and it was recently axiomatized by Houy (2007) in the context of group identification. The central axiom in these characterizations is a binary independence condition requiring the decision of whether or not two individuals belong to the same social class to depend only on the individual classifications with respect to *these* two individuals.

By contrast, the central condition considered here is a *non-bossiness condition* which requires the group identity function to depend only on one cell from the individual partition of each society member - namely on the cell the corresponding individual classifies himself in. Intuitively, non-bossiness thus states that the decision of whether or not two individuals belong to the same class should not depend on the view of unconcerned individuals. Non-bossiness neither implies nor is implied by binary independence.

The non-bossiness condition allows one to describe the social classification in terms of an *opinion graph* on the set of individuals. An edge  $ij$  in this graph corresponds to the situation in which *either* individual  $i$  classifies himself in the same group with  $j$  *or* individual  $j$  classifies himself in the same group with  $i$ . The group identity functions we introduce in this paper correspond to

particular ways of decomposing this graph. Specifically, any group identity function satisfying a positive liberalism condition and a simple sovereignty requirement decomposes the graph into special refinements of its *connected components*. Having described the set of admissible partitions from which such a group identity function selects the societal classification, we then provide characterizations of two non-bossy rules generating the coarsest and the finest partition in the admissible set, respectively.

## 2 Basic definitions and notation

The society is denoted by  $N$ ,  $N = \{1, \dots, n\}$ , and  $\Pi$  is the set of all partitions of  $N$ . Recall that a *partition* of  $N$  is a collection of non-empty, pairwise disjoint subsets of  $N$  whose union is  $N$ . We call these subsets *groups* or *classes*. A partition  $\pi$  is said to *refine* another partition  $\pi'$ , denoted  $\pi \leq \pi'$ , if every group from  $\pi$  is contained in some group from  $\pi'$ ; we also say in this case that  $\pi'$  is *coarser* than  $\pi$ . The refinement relation is a partial ordering on  $\Pi$ .

For each  $i \in N$ , individual  $i$ 's view is  $\pi^i \in \Pi$ . Moreover, for each  $j \in N$ , we denote by  $\pi_j^i$  the cell in the partition  $\pi^i$  that contains individual  $j$ . For instance, if  $\pi_2^1 = \pi_3^1 = \pi_5^1 = \{2, 3, 5\} \in \pi^1$  then, according to individual 1, individuals 2, 3, and 5 should belong to the same social group. A profile of individual views is denoted by  $\boldsymbol{\pi} := (\pi^1, \dots, \pi^n) \in \Pi^N$ . For  $i \in N$  and  $\pi^{i'} \in \Pi$ , we write  $(\pi^{-i}, \pi^{i'})$  to denote the profile at which  $i$ 's view  $\pi^i$  is replaced by  $\pi^{i'}$ .

A *group identity function* is a mapping  $f : \Pi^N \rightarrow \Pi$  which assigns to each profile  $\boldsymbol{\pi} \in \Pi^N$  of individual views a partition  $f(\boldsymbol{\pi}) \in \Pi$  of the society into social groups. For all  $i \in N$ ,  $f(\boldsymbol{\pi})_i$  is the social group to which individual  $i$  belongs according to  $f$ .

For any  $i \in N$  and  $\pi^i \in \Pi$ , denote by  $\bar{\pi}^i \in \Pi$  any partition such that

$\bar{\pi}_i^i = \pi_i^i$ . That is,  $\pi^i$  and  $\bar{\pi}^i$  may differ only with respect to the cells individual  $i$  does not belong to.

**Non-Bossiness (NB):** A group identity function  $f$  satisfies Non-Bossiness iff for all  $i \in N$ , all  $\boldsymbol{\pi} \in \Pi^N$ , and all  $\bar{\pi}^i \in \Pi$ ,  $f(\boldsymbol{\pi}) = f(\pi^{-i}, \bar{\pi}^i)$ .

Non-bossiness thus requires that an individual can influence the social classification only via his individual opinion about the social group he himself belongs to.<sup>1</sup>

Note that Fishburn and Rubinstein's conjunctive aggregator does not satisfy NB. To see this, take  $\boldsymbol{\pi} \in \Pi^N$  to be such that  $\{1, 2\} \in \pi^i$  for all  $i \in N$ . Further, for some  $j \in N \setminus \{1, 2\}$ , let  $\bar{\pi}^j$  be such that  $\{1\}, \{2\} \in \bar{\pi}^j$ . Then, the conjunctive aggregator classifies 1 and 2 together if the profile is  $\boldsymbol{\pi}$ , while 1 is classified as being single if the profile is  $(\pi^{-j}, \bar{\pi}^j)$ ; thus, NB is violated.

As is easily shown by repeated application of NB, the non-bossiness condition restricts a group identity function to depend only on the individual views with respect to the groups the corresponding individuals themselves belong to:

**Fact** *A group identity function  $f$  satisfies NB if and only if  $f(\boldsymbol{\pi}) = f(\boldsymbol{\pi}')$  for all  $\boldsymbol{\pi}, \boldsymbol{\pi}' \in \Pi^N$  with  $\pi_i^i = \pi_i'^i$  for all  $i \in N$ .*

This fact allows us to describe the social classification in terms of an underlying ‘‘opinion graph’’. Recall that a graph  $H = (V, E)$  consists of a set of vertices  $V$  and a set  $E$  of subsets of  $V$  of size 2. Thus,  $kk' \in E$  represents the edge connecting the vertices  $k, k' \in V$ . A path  $(k_1, k_2, \dots, k_m)$  in  $H$  is a sequence of vertices  $k_1, k_2, \dots, k_m \in V$  for some positive integer  $m$  such that  $k_\ell k_{\ell+1} \in E$  for each  $1 \leq \ell \leq m - 1$ , and we say that  $(k_1, k_2, \dots, k_m)$  is

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<sup>1</sup> Our condition is closely related to, but not a literal adaptation of, the non-bossiness condition introduced by Satterthwaite and Sonnenschein (1981) in the context of social choice functions.

a path between  $k_1$  and  $k_m$ . Let  $X \subseteq V$ . We say that  $X$  is *connected* if, for every  $i, j \in X$ , there is a path between  $i$  and  $j$  which contains only vertices belonging to  $X$ . We say that  $X$  is a *connected component* if  $X$  is connected and, for all  $Y \subseteq V$  which properly contain  $X$ ,  $Y$  is not connected.

To describe the decompositions we study in the next section, we construct an opinion graph  $H_\pi^*$  for each  $\pi \in \Pi^N$ , where its set of vertices is  $N$  and its set of edges is  $\{ij : i \neq j, i \in \pi_j^i \text{ or } j \in \pi_i^i\}$ . The group identity functions presented in this paper assign to each profile of individual views special refinements of the *connected components* in  $H_\pi^*$ .

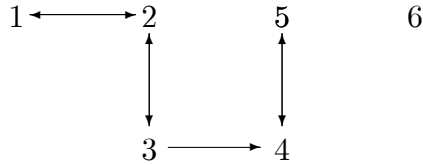


Figure 1: A six-member society

Figure 1 depicts a society consisting of six individuals where, for each  $i, j \in \{1, \dots, 6\}$ , there is an arrow from  $i$  to  $j$  if and only if  $j \in \pi_i^i$ . Thus, given the above definition of the opinion graph,  $H_\pi^*$  has set of edges  $\{12, 23, 34, 45\}$  and two connected components -  $\{1, 2, 3, 4, 5\}$  and  $\{6\}$ .

### 3 Connected components and their refinements

We consider now two appealing axioms and describe first the set of admissible partitions from which any group identity function that satisfies the two axioms selects the societal classification. We then provide characterizations

of two non-bossy rules generating the coarsest and the finest partition in the admissible set, respectively.

The first axiom has a liberal flavor and states that the aggregator puts two individuals in the same social group provided that *both* individuals think that they belong together (cf. Houy 2007).

**Weak Positive Liberalism** (WPL): A group identity function  $f$  satisfies Weak Positive Liberalism iff for every profile  $\boldsymbol{\pi} \in \Pi^N$  and all  $i, j \in N$ ,  $i \in \pi_j^j$  and  $j \in \pi_i^i$  imply  $f(\boldsymbol{\pi})_i = f(\boldsymbol{\pi})_j$ .

In order to explain our next axiom, Negative Group Sovereignty, imagine a situation in which the society  $N$  is partitioned into two non-empty subsets  $N_1$  and  $N_2$ . Consider the case in which every individual in  $N_1$  thinks that he belongs in the same group with individuals only from  $N_1$  and that every individual in  $N_2$  puts himself in the same group with individuals only from  $N_2$ . Then, it seems reasonable to require that an aggregator should not classify an individual from  $N_1$  and a second one from  $N_2$  as being members of the same social group.

**Negative Group Sovereignty** (NGS): A group identity function  $f$  satisfies Negative Group Sovereignty iff for every profile  $\boldsymbol{\pi} \in \Pi^N$  and for any two disjoint subsets  $N_1$  and  $N_2$  of  $N$  with  $N = N_1 \cup N_2$  we have that  $\pi_i^i \subseteq N_1$  and  $\pi_j^j \subseteq N_2$  for all  $i \in N_1$  and all  $j \in N_2$  imply  $f(\boldsymbol{\pi})_i \neq f(\boldsymbol{\pi})_j$  for all  $i \in N_1$  and all  $j \in N_2$ .

As we show next, a group identity function satisfying WPL and NGS necessarily selects, for each  $\boldsymbol{\pi} \in \Pi^N$ , a partition  $f(\boldsymbol{\pi})$  from the set  $\mathcal{R}(\boldsymbol{\pi})$  we introduce now.

The coarsest partition contained in  $\mathcal{R}(\boldsymbol{\pi})$  is  $\pi^*(\boldsymbol{\pi})$ , the partition of  $N$  into connected components in  $H_{\boldsymbol{\pi}}^*$ . The finest partition included in  $\mathcal{R}(\boldsymbol{\pi})$  replaces each  $D \in \pi^*(\boldsymbol{\pi})$  by its finest partition  $\pi^D = \{D_1, \dots, D_K\}$  for which the following condition is satisfied: for all  $k \in \{1, \dots, K\}$  and all  $i \in D_k$  and  $j \in D$ , we have that  $j \in \pi_i^i$  and  $i \in \pi_j^j$  imply  $j \in D_k$ .



The set  $\mathcal{R}(\boldsymbol{\pi})$  is then defined as follows:

$$\mathcal{R}(\boldsymbol{\pi}) = \left\{ \pi \in \Pi : \{ \pi^D \}_{D \in \pi^*(\boldsymbol{\pi})} \leq \pi \leq \pi^*(\boldsymbol{\pi}) \right\}.$$

Thus, each social class in every partition  $\pi$  in  $\mathcal{R}(\boldsymbol{\pi})$  belongs to a refinement of  $\{D\}$  and to a coarsening of  $\{\pi^D\}$  for some connected component  $D$  in  $H_{\boldsymbol{\pi}}^*$ .

Let us have a look again at Fig. 1 and describe the way the corresponding axioms restrict the decomposition of the depicted society. First, by NGS, the sixth individual cannot be grouped in the same class with any other individual. Second, by WPL, the following individuals have to be classified together: 1, 2, and 3; and 4 and 5. Hence, the coarsest partition compatible with these restrictions is  $\{\{1, 2, 3, 4, 5\}, \{6\}\}$ , while the finest one is  $\{\{1, 2, 3\}, \{4, 5\}, \{6\}\}$ . These are also the members of the set  $\mathcal{R}(\boldsymbol{\pi})$  for the problem considered in Fig. 1.

**Proposition 1** *A group identity function  $f$  satisfies WPL and NGS if and only if  $f(\boldsymbol{\pi}) \in \mathcal{R}(\boldsymbol{\pi})$  for all  $\boldsymbol{\pi} \in \Pi^N$ . Moreover, the two axioms are independent.*

**Proof.** Let  $f$  be such that  $f(\boldsymbol{\pi}) \in \mathcal{R}(\boldsymbol{\pi})$  for all  $\boldsymbol{\pi} \in \Pi^N$ . Notice then that  $f$  satisfies WPL as, for all  $\boldsymbol{\pi} \in \Pi^N$ , any partition in  $\mathcal{R}(\boldsymbol{\pi})$  classifies in the same group any two individuals  $i$  and  $j$  with  $i \in \pi_j^j$  and  $j \in \pi_i^i$ . Suppose now that  $f$  violates NGS. This implies that there is some profile  $\boldsymbol{\pi}' \in \Pi^N$  and a partition of  $N$  into two non-empty subsets  $N_1$  and  $N_2$  with  $\pi_i^i \subseteq N_1$  and  $\pi_j^j \subseteq N_2$  for all  $i \in N_1$  and all  $j \in N_2$  such that  $f(\boldsymbol{\pi}')_{k_1} = f(\boldsymbol{\pi}')_{k_2}$  for some  $k_1 \in N_1$  and  $k_2 \in N_2$ . Notice that we have in such a case a direct contradiction to  $f(\boldsymbol{\pi}') \in \mathcal{R}(\boldsymbol{\pi}')$ .

Consider now a group identity function  $f$  which satisfies WPL and NGS, and take  $\boldsymbol{\pi} \in \Pi^N$ . In what follows we show that  $f(\boldsymbol{\pi}) \notin \mathcal{R}(\boldsymbol{\pi})$  leads to a contradiction.

(1) Consider first the case in which there is  $D' \in f(\boldsymbol{\pi})$  that strictly contains some  $D \in \pi^*(\boldsymbol{\pi})$  (and thus,  $|D'| \geq 2$ ). Define then  $N_1 := D$  and

$N_2 := N \setminus D$  and note that both sets are non-empty. Let  $k \in D = N_1$  and  $k' \in D' \setminus D \subseteq N_2$ . Since  $D \in \pi^*(\boldsymbol{\pi})$ , we have  $\pi_i^i \subseteq N_1$  for all  $i \in N_1$  and  $\pi_j^j \subseteq N_2$  for all  $j \in N_2$ . Hence, by NGS, we should have  $f(\boldsymbol{\pi})_k \neq f(\boldsymbol{\pi})_{k'}$  in contradiction to  $f(\boldsymbol{\pi})_k = f(\boldsymbol{\pi})_{k'} = D'$ .

(2) We show next that it is impossible for  $f(\boldsymbol{\pi})$  to contain a social class which is a strict subset of some class from the finest partition  $\{\pi^D\}_{D \in \pi^*(\boldsymbol{\pi})}$  contained in  $\mathcal{R}(\boldsymbol{\pi})$ . If this were the case, there would exist groups  $D', D''$  and  $D'''$  s.t.  $D' \subset D'' \in \pi^{D''}$  and  $D' \in f(\boldsymbol{\pi})$ . However, this would imply that we can find individuals  $i \in D'$  and  $j \in D'' \setminus D'$  such that  $i \in \pi_j^j$  and  $j \in \pi_i^i$ . By WPL,  $f(\boldsymbol{\pi})_i = f(\boldsymbol{\pi})_j$  in contradiction to  $f(\boldsymbol{\pi})_i = D'$  and  $j \notin D'$ .

(3) Notice finally that, by the same argument as in (1) and by NGS, it is impossible to have a class in  $f(\boldsymbol{\pi})$  which contains members from two different groups from  $\pi^*(\boldsymbol{\pi})$ .

We conclude that  $f(\boldsymbol{\pi}) \in \mathcal{R}(\boldsymbol{\pi})$ .

In order to show the independence of the axioms, let us consider the following two rules. Each rule satisfies one of the axioms but not the other one. Moreover, for each of these rules, there is a profile  $\boldsymbol{\pi} \in \Pi^N$  s.t.  $f(\boldsymbol{\pi}) \notin \mathcal{R}(\boldsymbol{\pi})$ .

(not WPL) Take the aggregator  $f'$  defined as follows: for all  $\boldsymbol{\pi} \in \Pi^N$ ,  $f'(\boldsymbol{\pi}) = \{\lambda_N\}$ , where  $\lambda_N$  denotes the partition of  $N$  into singletons. This aggregator clearly violates WPL while satisfying NGS. We have for this rule that  $f''(\{N\}, \dots, \{N\}) \notin \mathcal{R}(\{N\}, \dots, \{N\}) = \{\{N\}\}$ .

(not NGS) Consider the aggregator  $f''$  defined as follows: for all  $\boldsymbol{\pi} \in \Pi^N$ ,  $f''(\boldsymbol{\pi}) = \{N\}$ . This rule satisfies WPL but not NGS, and we have in addition that  $f''(\lambda_N, \dots, \lambda_N) \notin \mathcal{R}(\lambda_N, \dots, \lambda_N) = \{\lambda_N\}$ .

This completes the proof. ■

Let us now consider the following stronger version of the WPL axiom (cf. Houy 2007).

**Positive Liberalism (PL):** A group identity function  $f$  satisfies Positive Liberalism iff for every profile  $\boldsymbol{\pi} \in \Pi^N$  and all  $i, j \in N$ ,  $i \in \pi_j^j$  implies  $f(\boldsymbol{\pi})_i = f(\boldsymbol{\pi})_j$ .

Clearly, WPL is implied by PL in which the wish of *one* of the individuals  $i$  and  $j$  suffices to put them in the same social group. Imposing both PL and NGS on a group identity function has two implications: first, any such function is non-bossy as well (Lemma 1); second, these two axioms characterize the aggregation rule  $f^*$  defined by  $f^*(\boldsymbol{\pi}) = \pi^*(\boldsymbol{\pi})$  for all  $\boldsymbol{\pi} \in \Pi^N$  (Proposition 1). In other words,  $f^*$  selects the coarsest partition in  $\mathcal{R}(\boldsymbol{\pi})$  for all  $\boldsymbol{\pi} \in \Pi^N$ . For the special case from Fig. 1, one has as the unique decomposition the partition  $\{\{1, 2, 3, 4, 5\}, \{6\}\}$ .

**Lemma 1** *If a group identity function  $f$  satisfies PL and NGS, then it also satisfies NB.*

**Proof.** Let  $f$  satisfy PL and NGS, and take  $\boldsymbol{\pi} \in \Pi^N$ . By PL, the members of a connected component in  $H_{\boldsymbol{\pi}}^*$  belong to the same group. By the repeated application of NGS, there are no members of different connected components in  $H_{\boldsymbol{\pi}}^*$  who are classified in the same social group by  $f$ . Hence,  $f(\boldsymbol{\pi}) = \pi^*(\boldsymbol{\pi})$ . Notice finally that  $H_{\boldsymbol{\pi}}^* = H_{(\pi^{-i}, \bar{\pi}^i)}^*$  for all  $i \in N$  and all  $\bar{\pi}^i \in \Pi$ . We conclude that  $f(\boldsymbol{\pi}) = \pi^*(\boldsymbol{\pi}) = f(\pi^{-i}, \bar{\pi}^i)$  for all  $i \in N$ , all  $\boldsymbol{\pi} \in \Pi^N$ , and all  $\bar{\pi}^i \in \Pi$  as required by NB. ■

**Proposition 2** *A group identity function  $f$  satisfies PL and NGS if and only if  $f = f^*$ . Moreover, the two axioms are independent.*

**Proof.** The characterization follows from Lemma 1 and the fact that  $f^*$  satisfies the two axioms. Moreover, the aggregators  $f'$  and  $f''$  constructed in the proof of Proposition 1 serve also as examples for the independence of PL and NGS, respectively. ■

Let us now keep the WPL axiom and strengthen the NGS axiom by transforming it into the following one.

**Negative Group Liberalism (NGL):** A group identity function  $f$  satisfies Negative Group Liberalism iff for every profile  $\boldsymbol{\pi} \in \Pi^N$  and for any two disjoint subsets  $N_1$  and  $N_2$  of  $N$  with  $N = N_1 \cup N_2$  we have that  $\pi_i^i \cap \{j : i \in \pi_j^j\} \subseteq N_1$  for all  $i, j \in N_1$  and  $\pi_i^i \cap \{j : i \in \pi_j^j\} \subseteq N_2$  for all  $i, j \in N_2$  imply  $f(\boldsymbol{\pi})_i \neq f(\boldsymbol{\pi})_j$  for all  $i \in N_1$  and all  $j \in N_2$ .

The axiom requires that if no individuals  $i \in N_1$  and  $j \in N_2$  ever classify *each other* to belong to the same group, then the aggregator never puts an individual from  $N_1$  together with one from  $N_2$  in a social group. Again, imposing both WPL and NGP on a group identity function has two implications: first, any such function is non-bossy as well (Lemma 2); second, these two axioms characterize the aggregation rule  $f_*$  defined by  $f_*(\boldsymbol{\pi}) = \{\pi^D\}_{D \in \pi^*(\boldsymbol{\pi})}$ . In other words, the aggregator  $f_*$  selects the finest partition in  $\mathcal{R}(\boldsymbol{\pi})$  for all  $\boldsymbol{\pi} \in \Pi^N$ . For instance, in Fig. 1,  $f_*$  selects the partition  $\{\{1, 2, 3\}, \{4, 5\}, \{6\}\}$ .

**Lemma 2** *If a group identity function satisfies WPL and NGL, then it also satisfies NB.*

**Proof.** The proof is similar to the one of Lemma 1 by noticing that by WPL any two individuals who classify each other in the same group are classified together by  $f$ , and that by the repeated application of NGL any two individuals who do not classify each other in same group are not classified in the same group by  $f$  either. ■

**Proposition 3** *A group identity function  $f$  satisfies WPL and NGL if and only if  $f = f_*$ . Moreover, the two axioms are independent.*

**Proof.** The characterization follows from Lemma 2 and the fact that  $f_*$  satisfies the two axioms. As for the independence of the axioms, notice that the rule partitioning the society into singletons satisfies NGL but not WPL, while the aggregator  $f^*$  characterized in Proposition 2 satisfies WPL but not NGL. ■

## 4 Discussion and conclusion

This paper is devoted to the study of group identification problems in which the decision of whether or not two individuals belong to the same class does not depend on the view of unconcerned individuals. In this setup, a graph representation of the individual views allowed us to introduce new group identity functions and to characterize them in terms of appropriate (positive) liberalism and (negative) sovereignty axioms. It is worth noting that the two group identity functions characterized above are *anonymous* in the sense that, for any permutation  $\sigma : N \rightarrow N$  of individuals and all  $\boldsymbol{\pi} \in \Pi^N$ ,

$$f\left(\sigma\left(\pi^{\sigma^{-1}(1)}\right), \dots, \sigma\left(\pi^{\sigma^{-1}(n)}\right)\right) = \sigma\left(f(\boldsymbol{\pi})\right),$$

where  $\sigma(\pi)$  is the partition that results from permuting the partition  $\pi$  according to  $\sigma$ .

It is natural to ask whether similar results can be obtained if one replaces the positive liberalism axiom by its negative counterpart and the negative sovereignty condition by its positive counterpart. Somewhat surprisingly, the answer is, no! Specifically, consider the following negative liberalism axiom introduced in Houy (2007). It requires that two individuals belong to different social groups if at least one of them thinks so.<sup>2</sup>

**Negative Liberalism (NL):** A group identity function  $f$  satisfies Negative Liberalism iff for every profile  $\boldsymbol{\pi} \in \Pi^N$  and all  $i, j \in N$ ,  $i \notin \pi_j^j$  implies  $f(\boldsymbol{\pi})_i \neq f(\boldsymbol{\pi})_j$ .

As is easily seen, all social groups generated by any group identity function satisfying NL must form a *clique* as two individuals are put in the same social group only if they classify each other in the given profile as members of the same class. Formally, given a profile of individual partitions, a clique is a

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<sup>2</sup> It is also easily seen that NL and WPL cannot be jointly satisfied by a group identity function, cf. Houy (2007).

subset  $D \subseteq N$  such that for all  $i, j \in D$ ,  $j \in \pi_i^i$ . A trivial way to satisfy NL is to let the group identity function always select the partition into singletons, i.e., to never put two different individuals in the same group. The following condition represents a minimal requirement that prevents this; it can be seen as a positive counterpart of condition NGS.

**Positive Group Sovereignty (PGS):** A group identity function  $f$  satisfies Positive Group Sovereignty iff for no profile  $\pi \in \Pi^N$  there exist  $j$  and  $k$  such that  $j \notin f(\pi)_k$  and  $\{j\} \cup f(\pi)_k \subseteq \pi_i^i$  for all  $i \in \{j\} \cup f(\pi)_k$ .

Thus, PGS requires that the societal classification should not allow a situation in which an individual thinks he belongs to every single individual of a social group he is not a member of if all members of this group think they belong to this individual and moreover to any other member of the group. As is easily verified, PGS and NL jointly imply that the societal classification generated at any profile of individual views consists of *maximal cliques*. This implies that NL and PGS are incompatible with anonymity if  $n \geq 3$ . To see this, consider a three-person society and a profile  $\pi$  with  $\pi_1^1 = \{1, 2\}$ ,  $\pi_2^2 = \{1, 2, 3\}$ ,  $\pi_3^3 = \{2, 3\}$ . For any group identity function  $f$  satisfying NL and PGS one has  $f(\pi) \in \{\{\{1, 2\}, \{3\}\}, \{\{1\}, \{2, 3\}\}\}$ . Each selection, however, violates anonymity as can be seen by considering the permutation that exchanges individuals 1 and 3. The same example also shows that the negative conclusion of the non-existence of an anonymous group identity function (satisfying PGS) remains valid if NL is weakened to the negative counterpart of WPL by only requiring that two individuals should belong to different social groups if *both* of them think so.

Finally, we note that, unlike their counterparts in Section 3 above, conditions NL and PGS do not imply the non-bossiness condition NB. To verify this, consider a situation where there are different partitions into maximal cliques, for instance in the situation of Fig. 1 above these are the partitions  $\pi = \{\{1, 2\}, \{3\}, \{4, 5\}, \{6\}\}$  and  $\pi' = \{\{1\}, \{2, 3\}, \{4, 5\}, \{6\}\}$ . Define

a group identity function  $f$  as follows:  $f$  selects  $\pi$  if the individual views w.r.t. their own classification are as shown in Fig. 1 *and* individual 6 thinks that individuals 1 and 2 belong together,  $f$  selects  $\pi'$  if the individual views w.r.t. their own classification are as shown in Fig. 1 *and* individual 6 does not think that individuals 1 and 2 belong together, and  $f$  selects some partition into maximal cliques in any other case. Clearly,  $f$  then satisfies NL and PGS but violates non-bossiness.

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